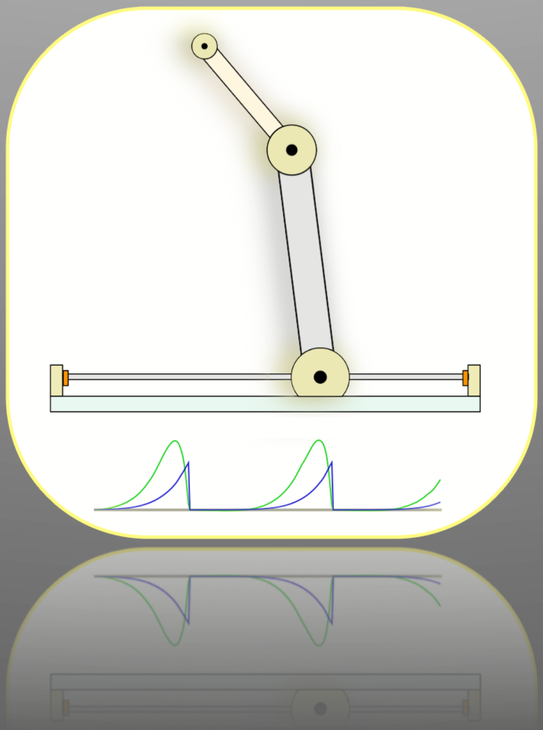
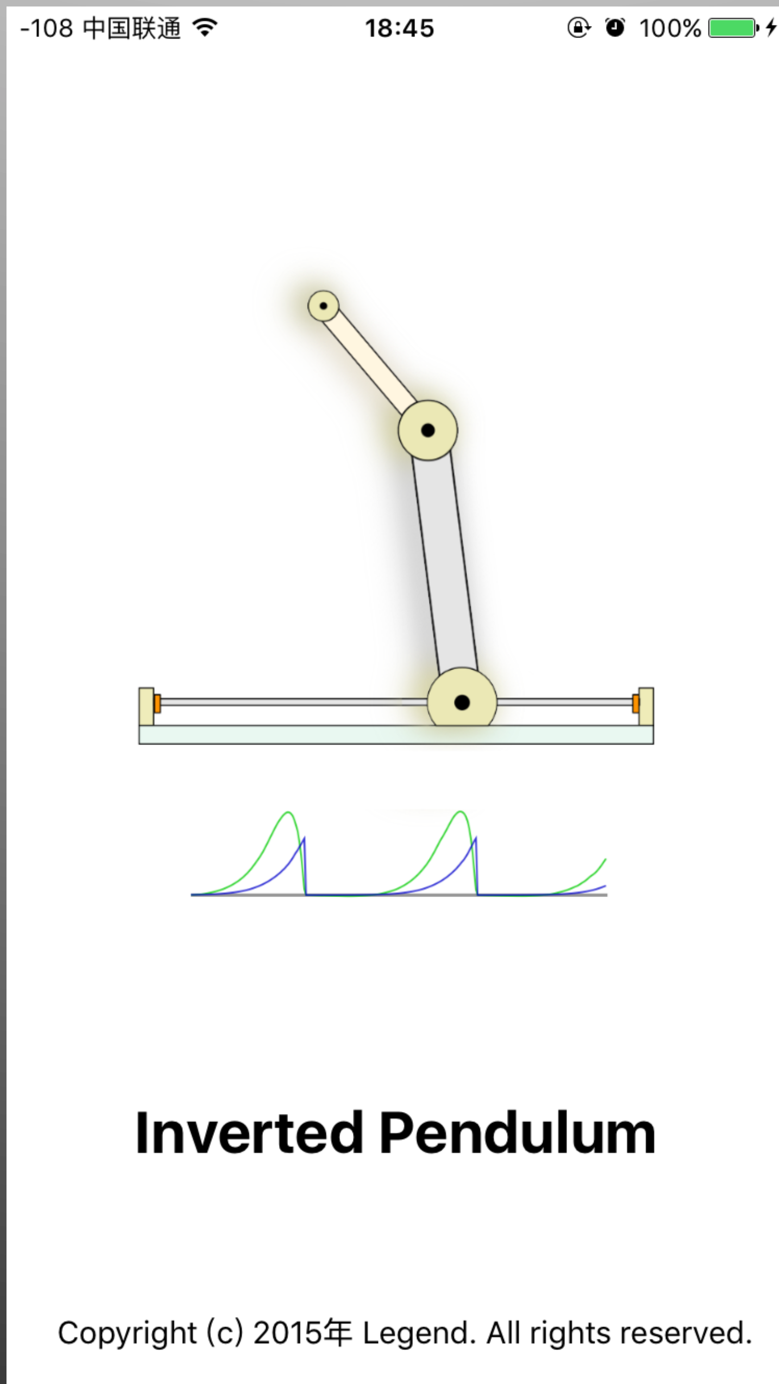


Inverted Pendulum

Jiahao Fu

7th, Dec, 2015


-108 中国联通
18:45
100%
Inverted Pendulum
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An App on iOS

- System Structure
- Mathematical Model
- iOS Implement
- Simulation
- Summary

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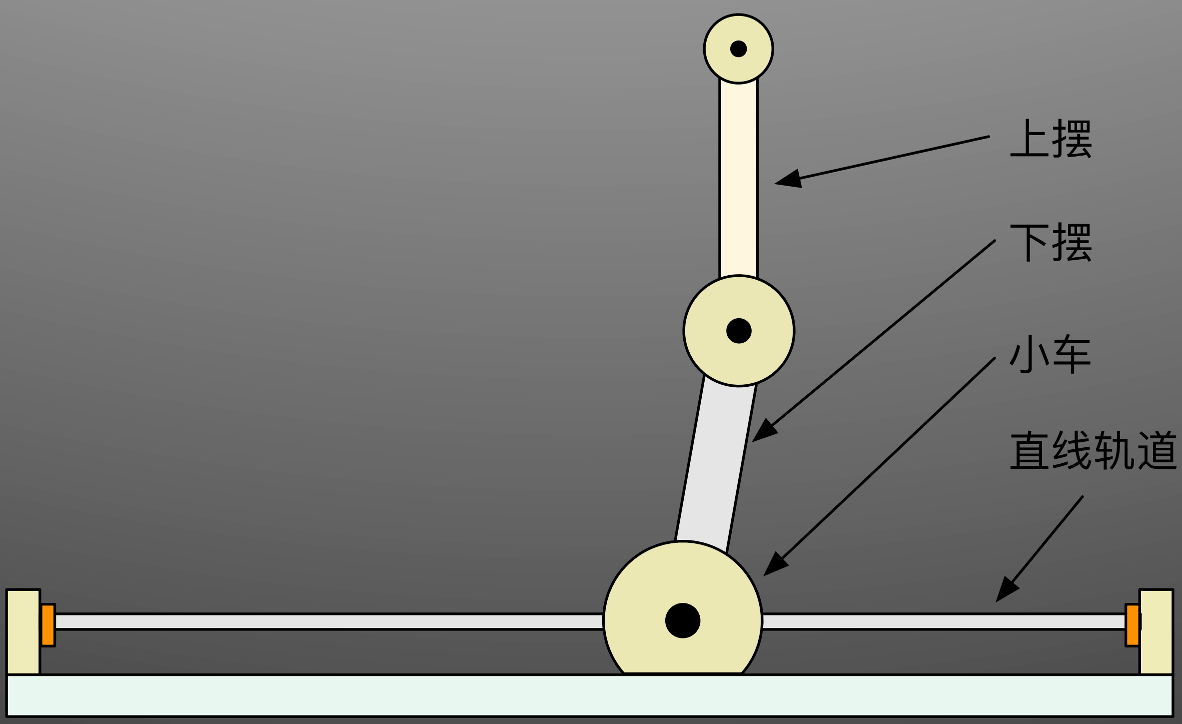
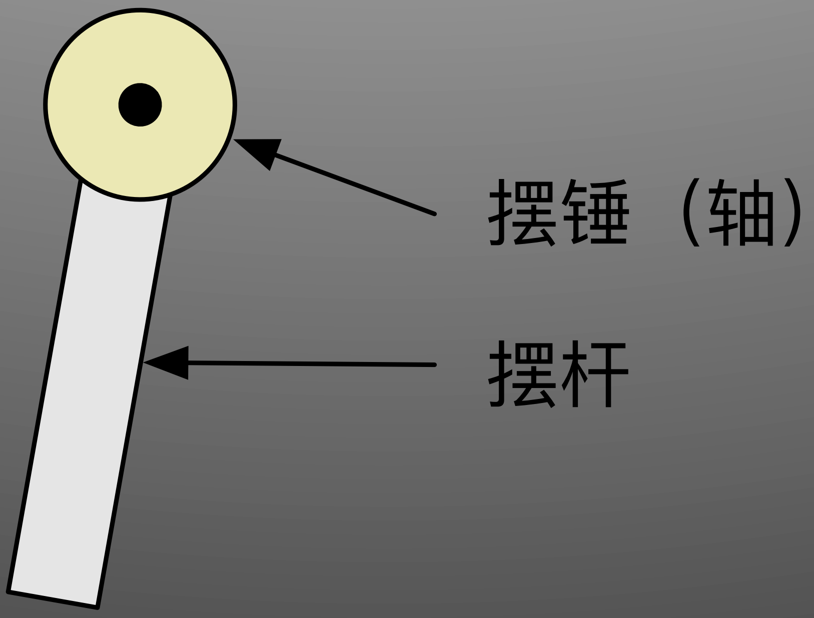
System Structure



System Structure



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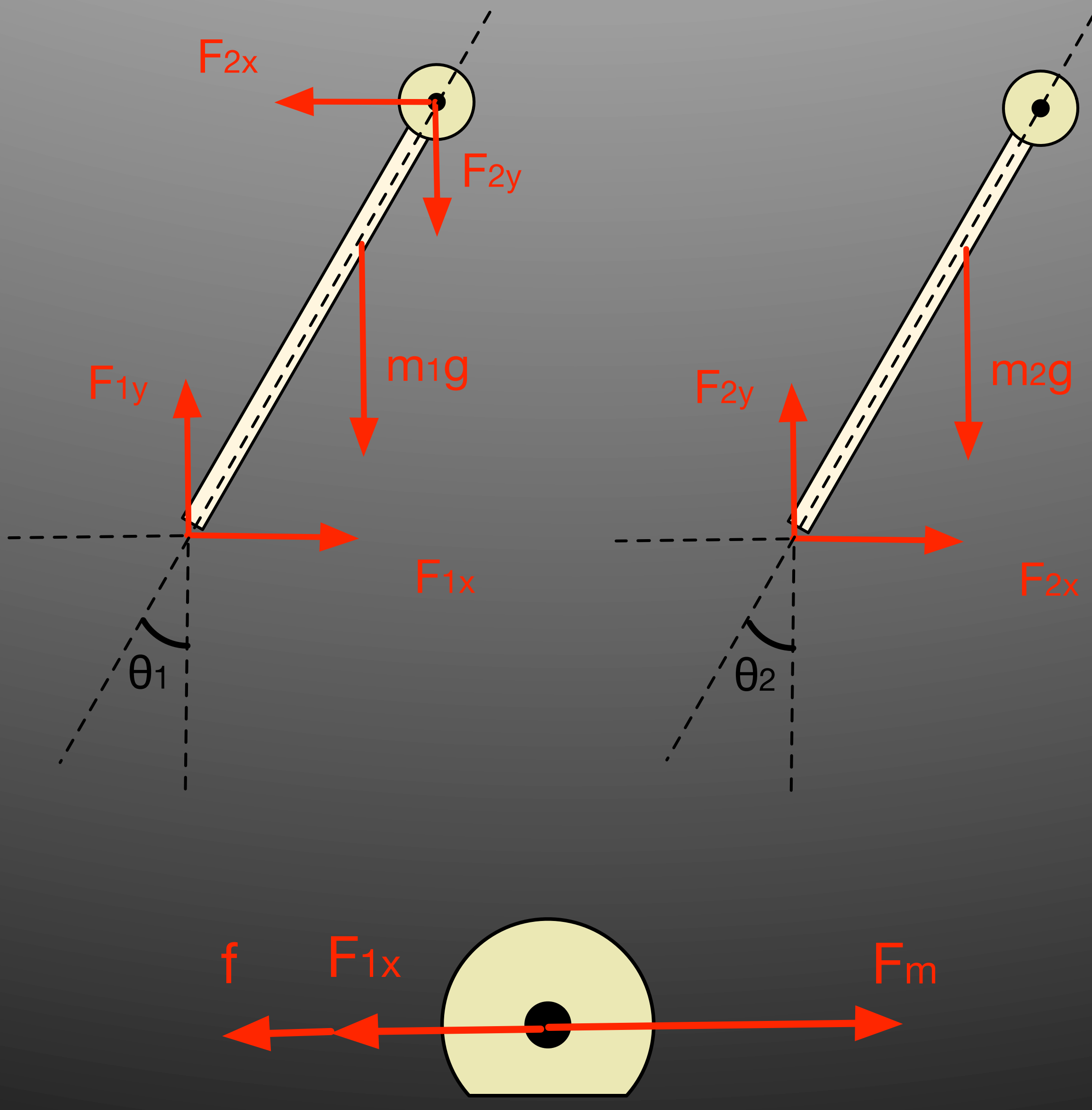
Model Simplification

- 小车、上下摆都是刚体;
- 上下摆的摆锤（轴）不相对于摆杆运动，作为质点处理;
- 上下摆的摆杆为均质细杆，只考虑长度和质量;
- 摆在绕轴运动过程中受到摩擦力矩的作用，大小正比于其转动角速度;
- 小车沿直线轨道运动过程中受到摩擦力的作用，大小正比于小车速度;
- 小车电机输出与控制电压成正比，无死区，无滞后，仅有饱和特性;

Force Analysis



Lagrange's Method

$$\mathbf{\Theta} = \begin{bmatrix} x \\ \theta_1 \\ \theta_2 \end{bmatrix}$$

$$\boldsymbol{M}\ddot{\mathbf{\Theta}} + \boldsymbol{C}\dot{\mathbf{\Theta}} + \boldsymbol{G} = \boldsymbol{F}$$

$$\begin{cases} \boldsymbol{M} = \begin{bmatrix} m_0 + m_1 + m_2 & (m_1 l_1 + m_2 L_1)\cos\theta_1 & m_2 l_2 \cos\theta_2 \\ (m_1 l_1 + m_2 L_1)\cos\theta_1 & J_1 + m_1 l_1^2 + m_2 L_1^2 & m_2 L_1 l_2 \cos(\theta_1 - \theta_2) \\ m_2 l_2 \cos\theta_2 & m_2 L_1 l_2 \cos(\theta_1 - \theta_2) & J_2 + m_2 l_2^2 \end{bmatrix} \\ \boldsymbol{C} = \begin{bmatrix} c_0 & -(m_1 l_1 + m_2 L_1)\sin\theta_1 \dot{\theta}_1 & -m_2 l_2 \sin\theta_2 \dot{\theta}_2 \\ 0 & c_1 + c_2 & m_2 L_1 l_2 \sin(\theta_1 - \theta_2)\dot{\theta}_2 - c_2 \\ 0 & -m_2 L_1 l_2 \sin(\theta_1 - \theta_2)\dot{\theta}_1 - c_2 & c_2 \end{bmatrix} \\ \boldsymbol{G} = \begin{bmatrix} 0 \\ -(m_1 l_1 + m_2 L_1) g \sin\theta_1 \\ -m_2 g l_2 \sin\theta_2 \end{bmatrix} \\ \boldsymbol{F} = \begin{bmatrix} \tau \\ 0 \\ 0 \end{bmatrix} \end{cases}$$

State Space Model

$$\ddot{\boldsymbol{\Theta}} = -\boldsymbol{M}^{-1}\boldsymbol{C}\dot{\boldsymbol{\Theta}} + \boldsymbol{M}^{-1}(\boldsymbol{F} - \boldsymbol{G})$$

$$\begin{bmatrix} \dot{\boldsymbol{\Theta}} \\ \ddot{\boldsymbol{\Theta}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{0} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Theta} \\ \dot{\boldsymbol{\Theta}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{M}^{-1}(\boldsymbol{F} - \boldsymbol{G}) \end{bmatrix}$$

Model Parameters

Table 2.1: 系统中所用到的符号说明

符号	物理量	单位	模型中取值
x	小车位移	m	/
θ_1	下摆较竖直方向偏角	rad	/
θ_2	上摆较竖直方向偏角	rad	/
L_0	直轨长度	m	1.400
m_0	小车质量	kg	1.000
m_1	下摆质量	kg	0.293
J_1	下摆转动惯量	$\mathrm{kg \cdot m^2}$	0.002569
L_1	下摆杆长度	m	0.3293
l_1	下摆杆质心到转轴距离	m	0.2769
m_2	上摆质量	kg	0.220
J_2	上摆转动惯量	$\mathrm{kg \cdot m^2}$	0.001227
L_2	上摆杆长度	m	0.2587
l_2	上摆杆质心到转轴距离	m	0.2156
c_0	小车与直轨摩擦系数	$\mathrm{N \cdot s/m}$	10.00
c_1	下摆杆与转轴摩擦系数	$\mathrm{N \cdot m \cdot s}$	0.00071
c_2	上摆杆与转轴摩擦系数	$\mathrm{N \cdot m \cdot s}$	0.00071
g	重力加速度	$\mathrm{m/s^2}$	9.80

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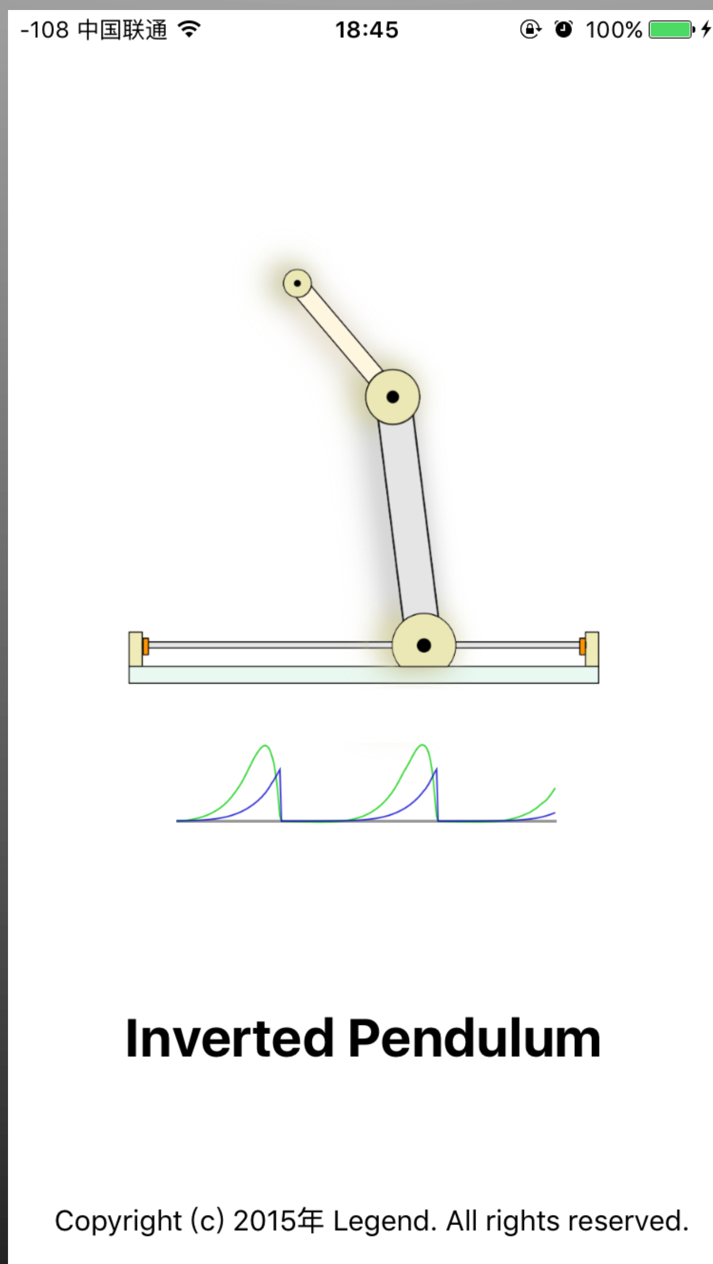
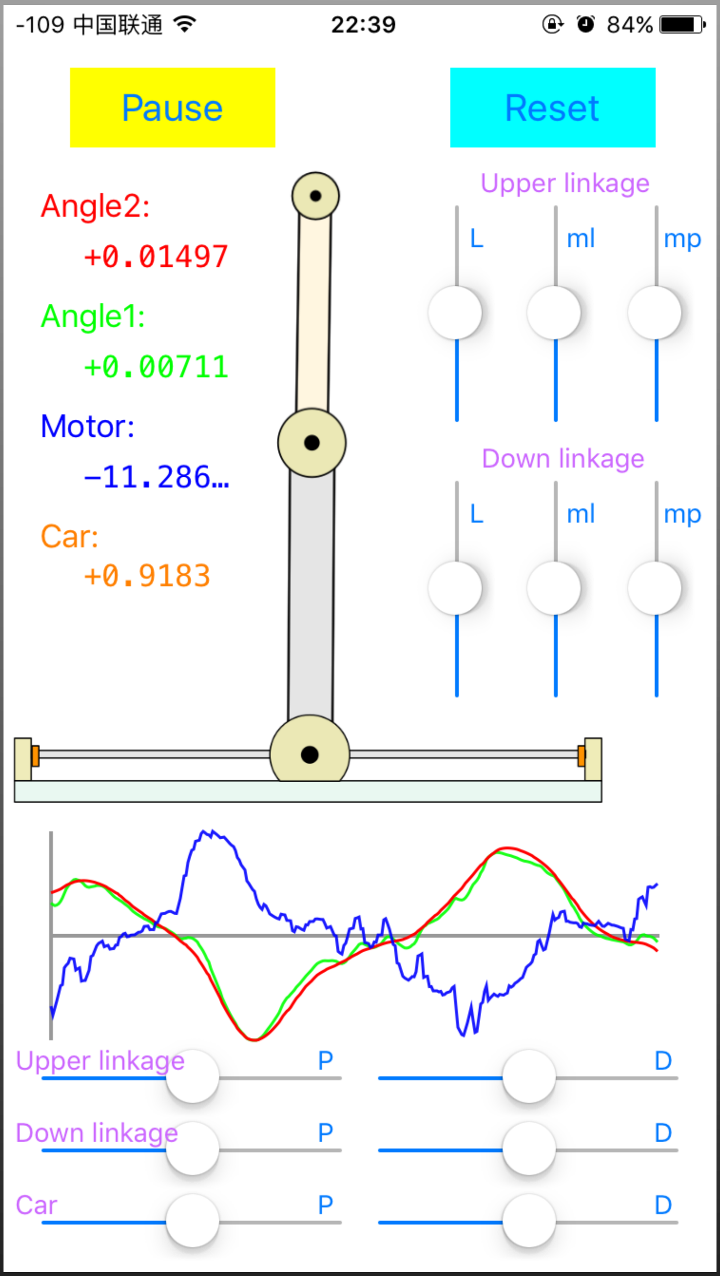
Launch Screen



Interface


-109 中国联通
22:39
84%
Pause
Reset
Angle2:
+0.01497
Angle1:
+0.00711
Motor:
-11.286…
Car:
+0.9183
Upper linkage
L
ml
mp
Down linkage
L
ml
mp
Upper linkage
P
D
Down linkage
P
D
Car
P
D

Function

- Run & Pause
- Labels
- Oscilloscope with Auto Scale
- Adjust Model Parameters
- Adjust Controller Parameters
- Exerting Disturbance

Code Implement

- class linkage
- class twoOrderInvertedPendulum
- class OscilloscopeCurve
- class ViewController
- mathSupport.swift

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Error Analysis

- 仿真过程存在舍入误差、截断误差等由于数字化、离散化造成的误差;
- 没有考虑角度、位置测量准确性与精确度;
- 电机过于理想，不存在电感等滞后环节;
- CoreMotion 框架采集到的加速度信号也存在一定误差;
- 未考虑各个环节的静摩擦力;

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Summary

- From Freescale to Course Design(by Prof. ZQF), inverted pendulum is a classic controlled object ;
- Less innovation on model and controller, but more on Platform ;
- Coding is disappointing and interesting ;
- Gain much & have a farctate life ;

Acknowledgements

- Prof. Xiuhua Du
- My roommate : Xiangze Liu
- Other classmates

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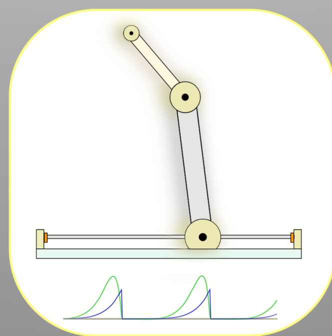

Thanks

Q & A

Jiahao Fu

7th, Dec, 2015