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Precision Electrical Measurement Experiment Using a Lock-in Amplifier that is Suitable for Science and Engineering Undergraduates

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Despite the lock-in amplifier's being one of the most frequently used precision measurement instruments in science and engineering, most undergraduates students in physics know very little about the lock-in detection technique and never get a chance to experience or learn about the beauty of precision measurements. In this article, an introductory-level electrical experiment using a lock-in amplifier and designed for upper-level undergraduate science and engineering students is described. In the presented experiment, students measure the resistance of a small piece of copper wire, a piece that is often too small to be accurately measured with a multimeter, and calculate the resistivity of copper. As additional topics for discussion, the effects of $1/f$ noise on measurement uncertainties can be demonstrated with a lock-in amplifier, and AC impedance measurements for capacitors or inductors are equally possible when using the same setup.

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I. INTRODUCTION

In today's fields of science and engineering, the ability to precisely measure quantities of interest is often critical for success. However, most of introductory physics laboratory classes targeted for undergraduate science and engineering students in many Korean universities do not have appropriate exercises for precision measurement techniques, and only a limited number of proposed experiments exists while it is unclear how many academic programs instituted such laboratory exercises [1]. There are many different precision measurement techniques to be learned, and each academic program, to a certain degree, is obligated to teach and introduce at least one of those precision measurement techniques to students going through its academic ors degree program. There are many physical quantities of interest depending on the specializations, but electrical measurements are often commonly required in many fields of science and engineering. So, it is our understanding that any science and engineering students should be trained to be well versed in typical electrical measurements techniques.

Typical introductory electrical measurement experiments for undergraduate science and engineering students are designed to use common laboratory equipments such as multimeters and oscilloscopes. Although multimeters and oscilloscopes are adequate for most cases, something as simple as measuring the resistance of a short piece of metallic wire is out of reach with ordinary multimeters due to the smallness of resistance. When asked to do such a task, surprisingly a great percentage of upper level undergraduate science and engineering students have no clear direction to take. Most experiments in introductory physics laboratory courses are rigid, out-of-the-box types that fail to draw much attention from students. Instead of traditional experiments, more well-thought and designed experiments that challenge and

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stimulate students curiosity are much needed in today's physics and engineering education. Unlike other scientific fields, physics is probably the most fundamental, and due to this reason, there have been a number of early publication articles about precision electrical measurement experiments suitable for upper level undergraduate students [2–4]. However, those proposed exercises rarely find their way to be included in other institutions' laboratory courses.

II. THEORETICAL BACKGROUND AND EXPERIMENTAL DESCRIPTION

First, a basic introduction about lock-in detection is in order [5,6]. Despite lock-in amplifiers are widely used in science and engineering these days, most undergraduate students graduate without even hearing about them, although they are routinely used in research laboratories and there are inexpensive, home-built lock-in amplifiers designs suitable for undergraduate laboratory exercises already exist [7–9]. A lock-in amplifier is essentially an AC voltmeter that can measure a small signal buried in much larger noise. Let us take a hypothetical situation where one wants to measure a 1 nV sinusoidal signal at 1 kHz. Obviously, this signal is hopelessly too small to be measured without some amplification by an ordinary voltmeter. Let us take the amplification of 10000 to get the signal to 10 μV, which is within a reasonable reach of a good voltmeter. An excellent, low noise voltage amplifier (for example, Model DLPVA-100-BUN-S, FEMTO Messtechnik GmbH) can have a noise spectral density of 420 pV/$\sqrt{\text{Hz}}$. When the measurement bandwidth of the amplifier is set to 10 kHz, the broadband white noise present in the background will be 0.42 mV (=420 pV/$\sqrt{\text{Hz}} \times \sqrt{10000\text{Hz}} \times 10000$). It is still impossible to measure a 10 μV signal in the background noise of 420 μV. As can be seen in this hypothetical thought experiment, even one of the best commercial amplifiers fails for the task at hand. Clearly, there must be some way to isolate or decrease the detection bandwidth to decrease the background noise level below the amplified signal level. It turns out typical electronic band-pass filters centered at 1 kHz also do not have a narrow enough bandwidth for this type of task. However, a lock-in amplifier can narrow the detection bandwidth even to a fraction of 1 Hz. If we take the lock-in amplifier's detection bandwidth of 0.1 Hz, the same hypothetical experiment will give roughly 1.3 μV of broadband white background noise. Now the signal to noise ratio is about 8, large enough to allow an accurate measurement of the desired signal.

A lock-in amplifier achieves such a narrow detection bandwidth by the technique called phase-sensitive detection (PSD). PSD utilizes the following well-known trigonometric identity that students should have learned from high school:

$$(\mathrm{A}\sin\theta)(\mathrm{B}\sin\phi) = \frac{\mathrm{AB}}{2}[\cos(\theta-\phi)-\cos(\theta+\phi)] \quad (1)$$

If the signal is in the form of $V_{\text{signal}} = V_{\text{sig}}\sin(\omega_{\text{sig}}\,t+\varphi_{\text{sig}})$ and the so called reference signal needed for PSD is in the form of $V_{\text{reference}} = V_{\text{ref}}\sin(\omega_{\text{ref}}\,t+\varphi_{\text{ref}})$, V_{PSD} can be written as

$$\begin{aligned} V_{\text{PSD}} &= V_{\text{signal}}V_{\text{reference}} \\ &= \frac{V_{\text{sig}}V_{\text{ref}}}{2}\{\cos([\omega_{\text{sig}}-\omega_{\text{ref}}]t+\varphi_{\text{sig}}-\varphi_{\text{ref}}) \\ &\quad -\cos([\omega_{\text{sig}}+\omega_{\text{ref}}]t+\varphi_{\text{sig}}+\varphi_{\text{ref}})\} \end{aligned} \quad (2)$$

A lock-in amplifier exploits the fact that when ω_{sig} matches ω_{ref}, the first term in Eq. (2) becomes time independent while the second term's frequency is twice ω_{sig}. When a low pass filter is used to filter out the second term, the filtered output of V_{PSD} is

$$V_{\text{filtered}} = \frac{V_{\text{sig}}V_{\text{ref}}}{2}\cos(\varphi_{\text{sig}}-\varphi_{\text{ref}}). \quad (3)$$

The bandwidth of this low-pass filter can be chosen by the appropriate selection of the so called "time constant" of a lock-in amplifier and it can be as narrow as 0.01 Hz for a digital lock-in amplifier. A dual phase lock-in amplifier has a second phase-sensitive detector that multiples the input signal by $V_{\text{reference},2} = V_{\text{ref}}\cos(\omega_{\text{ref}}\,t+\varphi_{\text{ref}})$, which is 90° out of phase compared with $V_{\text{reference}}$. By the virtue of knowing V_{ref}, one can process the outputs of two phase sensitive detectors to obtain

$$X = V_{\text{sig}}\cos\varphi \quad (4)$$

$$Y = V_{\text{sig}}\sin\varphi \quad (5)$$

where $\varphi = \varphi_{\text{sig}} - \varphi_{\text{ref}}$ is used. By definition, φ signifies the amount of phase difference between the signal and

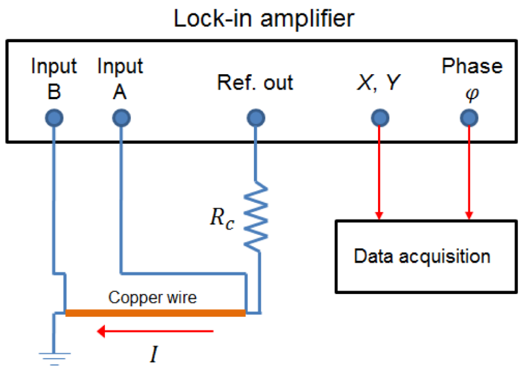

Fig. 1. (Color online) Connection diagram for measuring a metal wire using a lock-in amplifier.

the reference. X is called the in-phase output and Y is called the quadrature output. It is straightforward to calculate V_{sig} and φ using the following expressions:

$$\mathrm{MAG} = \sqrt{X^2+Y^2} = V_{\mathrm{sig}} \quad (6)$$

$$\varphi = \tan^{-1}\left(\frac{Y}{X}\right) \quad (7)$$

where MAG is the magnitude of the signal being measured. It is clear that by computing MAG from X and Y, the phase dependence can be removed.

Most undergraduate students have learned that the expression for resistance of a wire with a regular cross-section is given by

$$R = \rho\frac{L}{A} \quad (8)$$

where R is the resistance, ρ is the resistivity, L is the length of the wire, and A is the cross-sectional area of the wire. While resistivity is material specific and does not depend on the shape of the sample, the value of resistance is dependent on the shape of the material. For metals, resistivity is typically in the range of 10^{-8} Ωm to 10^{-7} Ωm at room temperature. For example, the resistivity of copper is $\rho_c = 1.69 \times 10^{-8}$ Ωm at 20 °C [10]. Thus, a 10 cm -long, 0.250 mm-diameter copper wire has the resistance of approximately 34.4 mΩ. It was the authors' experience that most undergraduate students, when asked to measure the resistivity of such a wire, did not have a clear way to tackle the problem because ordinary multimeters would not be able to measure such small resistance. One way to measure the resistance is to use a lock-in amplifier and this is schematically shown in Fig. 1.

Table 1. Measured resistance of 0.250 mm-diameter, 10 cm-long copper wire as a function of reference frequency.

Frequency [Hz]	Resistance [mΩ]	phase difference [deg]
10	34.18	0.000
20	34.19	0.017
50	34.21	0.033
100	34.22	0.067
200	34.23	0.218
500	34.27	0.518
1000	34.30	0.969
2000	34.35	1.902
5000	34.49	4.590
10000	34.93	8.961
20000	36.30	17.0423
50000	44.35	37.506
100000	61.70	54.365

The reference output from a lock-in amplifier supplies a sinusoidally varying voltage to both R_c and the copper wire. Since R_c is chosen to be much greater than the resistance of the copper wire, the current, I, flowing through the circuit will be nearly equal to $V_{\mathrm{ref}}/(R_c + R_{\mathrm{copper}}) \approx V_{\mathrm{ref}}/(R_c)$. For 1 mA current, 34.4 mΩ resistance would cause about 34.4 μV of voltage drop to be measured by a lock-in amplifier. The lock-in amplifier should be set for differential measurement, $V_{\mathrm{InputA}} - V_{\mathrm{InputB}}$. This means that the lock-in amplifier will only measure the voltage drop between the 10 cm-long junctions made to the copper wire.

III. RESULTS AND DISCUSSION

Since a lock-in amplifier is, in a sense, a sensitive AC voltmeter, purely DC measurements are not possible. However, the copper wire is theoretically treated to be purely resistive and the corresponding impedance of the copper wire should be frequency independent. Hence, the voltage drop measured across the 10 cm-long junction should be independent of the lock-in amplifier's reference frequency. We have set the frequency to several values from 10 Hz to 100 kHz. Table 1 summarizes the measurement results when $V_{\mathrm{ref}} = 1$ V and $R_c = 1$ kΩ. The lock-in amplifier's time constant and the filter slope were set to 3 s and 24 dB/oct, respectively.

Figure 2 clearly shows that the calculated resistivity of copper is very closed to the nominally accepted value

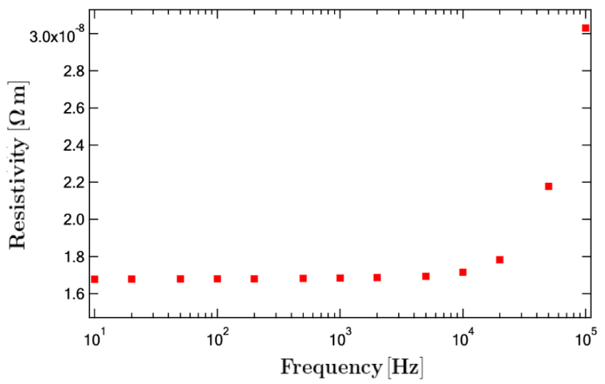

Fig. 2. (Color online) Calculated resistivity as a function of reference frequency.

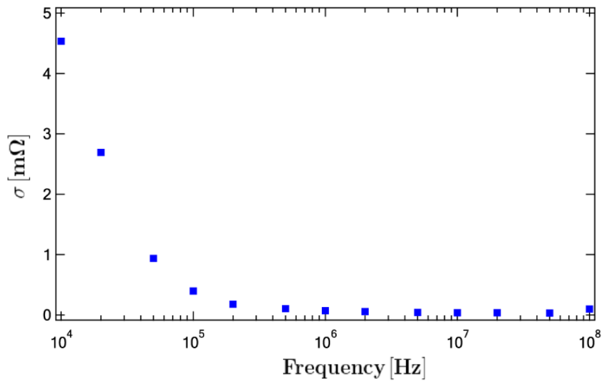

Fig. 3. (Color online) Standard deviations in measurements as a function of reference frequency.

of $1.69 \times 10^{-/8}$ Ωm and also frequency independent up to the frequency of about 10 kHz. However, a deviation can be seen for frequencies above 50 kHz. The measured resistance calculated from the magnitude of current flowing through the copper wire increases with frequency, and this is due to the lead inductance of the copper wire in addition to the stray capacitance of the circuit coupled with the finite output impedance of the lock-in amplifier's reference output resulting in an additional complex impedance term for the measurement circuit. This trend appears more clearly in the phase difference between the measured current through the copper wire and the lock-in amplifier's reference output as shown in Table 1. Even before the resistance calculated from the magnitude of current begins to deviate from a constant value, the phase difference starts to deviate from near 0. This phase difference is an indication that the copper wire and our measurement circuit are beginning to deviate from the ideal resistive behavior as the frequency increases. This phenomenon is, in a sense, a bonus topic for discussion for interested students. Students can be invited to further investigate this effect due to the complex impedance by varying connection cable lengths, using different reference sources instead of the lock-in amplifier's built-in reference output, *etc.*

Another topic for an extra discussion can be about the $1/f$ noise in the measurement involving a lock-in amplifier. The $1/f$ noise is a well-known electronic noise with the power spectrum inversely proportional to the signal frequency. Despite the average values of resistance measured at different frequencies were close as shown in Table 1, students should be taught that the measurement uncertainties at different frequencies are different due to the $1/f$ noise. Due to the power spectrum of the $1/f$ noise being inversely proportional to the frequency, the measurements at lower frequencies carry bigger uncertainties for the same given detection bandwidth. To demonstrate this point, a relatively short time constant ($\leq$0.3 s) for the lock-in amplifier should be used in order to acquire time-lapsed measurements for each frequency. A shorter time constant is necessary to introduce bigger effects of noise due to the increased detection bandwidth. Figure 3 shows the calculated standard deviations of time-lapsed 20 data points for each frequency. A clear $1/f$ dependence of the statistical uncertainty can be observed, and students can be advised to avoid a higher level of noise near DC by utilizing a higher reference frequency, but care must be taken not to use too high a reference frequency due to the unwanted complex impedance effects discussed earlier.

IV. CONCLUSION

We have introduced an introductory experiment that demonstrates the use of a lock-in amplifier for precision electrical measurements. In this laboratory exercise, students are expected to become acquainted with the concept of lock-in detection as well as precision measurement techniques involving a lock-in amplifier in general. The presented experimental can be readily applied to any undergraduate physics laboratory program at the junior or senior level due to its simplicity. Despite not having discussed all details of a lock-in amplifier's features and inner workings, the essence of lock-in amplifier usage was clearly demonstrated at an appropriate level for upper level undergraduate students in science and engineering.

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