Electromagnetism

JSW

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CONTENTS

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- Vector Analysis
  - ????
  - ????
  - ????
- Electrostatics
- Electrostatics in Matter
- Methods for Boundary Value Problems
- Magnetostatics
- Electrodynamics

Vectors do matter ?????!

??

Life is accumulation. Your height h is determined by not only your talent (velocity **v**) but also the path you walk along.

$$h = \int_{\text{now}}^{\text{future}} \mathbf{v} \cdot (\nabla h) dt = \int_{Path} \nabla h \cdot d\mathbf{l}$$

????????????????

???????? $d\mathbf{l}$??????? ∇h ??

??????????

CHAPTER

ONE

VECTOR ANALYSIS

- □□□□
  - inner product
  - cross product
- □□□□
  - gradient
  - divergence
  - curl
- □□□□
  - line integral
  - surface integral
  - volume integral
  - fumdamental theorom of calculus
  - Helmholtz theorem

1.1 Vector Algebras

1.1.1 Notations for Vectors

A vector is an object with an **amplitude** and a **direction**. □□□□□□□□□□□□□□□□□□ $\vec{A}$ □□□□□□□□ **A**□

- □□□□□: $\mathbf{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$
- $\hat{i}$ □□ x □□□□□□□□ $\hat{j}$ □□ y □□□□□□□□ $\hat{k}$ □□ z □□□□□□□□ □□□□□ $\hat{x}$, $\hat{y}$, $\hat{z}$ □ $\hat{e}_x$, $\hat{e}_y$, $\hat{e}_z$ □□□□□□
- The amplitude of **A** is

$$|\mathbf{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

1.1.2 Tensors

arrays index scalar index A_μ, where $\mu = x,\ y,\ z$ array indices $B_{\mu\nu}$, where $\mu = x,\ y,\ z$, $\nu = x,\ y,\ z$, index C_{ijk} (x, y, z) **index**

index μ, ν, i, j, k, l, m, n, α, β

Table 1.1: Tensor

	number of indice	example	name
	0	a	scalar
	1	A_μ	vector
	2	$B_{\mu\nu}$	
	3	C_{ijk}	

A_μ $\mu = x,\ y,\ z$

Einstein conventions

summations

$$\sum_j A_{ij}B_{jk}$$

$$\sum_j \sum_k A_{ij}B_{jk}C_{km}$$

$\sum$ Einstein convention

Definition 1.1 (Einstein convention)

index summation

$$A_{ij}B_{jk} = \sum_j A_{ij}B_{jk}$$

$$A_{ij}B_{jk}C_{km} = \sum_j \sum_k A_{ij}B_{jk}C_{km}$$

Exercise 1.1

1. $\mathbf{A} = (1, 2, 3)$ $\mathbf{B} = (6, 2, 5)$

$$A_\mu B_\mu = ?$$

2.

$$C_{ij} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$B_i C_{ij} A_j = ?$$

Solution to Exercise 1.1

1.

$$A_\mu B_\mu = \sum_{\mu=x,y,z} A_\mu B_\mu = A_x B_x + A_y B_y + A_z B_z = 25$$

2.

$$B_i C_{ij} A_j = \sum_i \sum_j B_i C_{ij} A_j = 11$$

1.1.3 Free index, contraction

□□□□□□index□□□□□□□□□□□□□□□□□A_μ□□□□□□□□□□□□ μ □□□□□□□□□□□ □□ A_μ □□□ A_x, A_y, A_z □□□□□□□□□□□□ μ □ free index□ □□□□$A_\mu B_\mu$ □□□□□□□□□□□□□ Einstein convention□□□□□□□ μ□□□□□□□□□□□□□□□□□□□□ $A_\mu B_\mu$ □□□□□□□□□□□□(□□□□)□□□□□□□ scalar□□□□□□□□ μ □ dummy index□ □□□□□□□□□□□□□□index $\sum_{i=0,1,2,...}$ □□□□□ i □dummy index□□□□□□□□□□□□□□□□□□□□□□□□□□□□**dummy index** □□□□□□□□□□(□□ $\sum_i = \sum_j$□□□□□□□□i□□□□□□□□□)□□□□□□□□□□dummy□□□□

CHAPTER

TWO

GALLERY OF ELECTROMAGNETISM

v-dipole

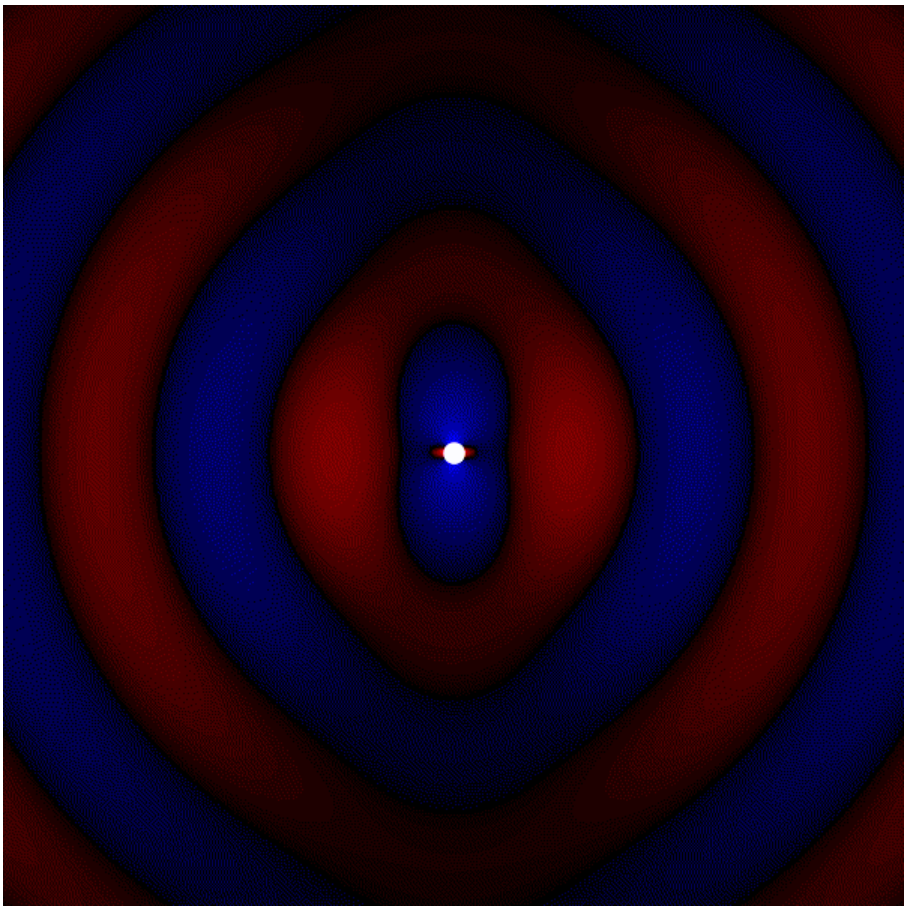

Fig. 2.1: Vertical dipole radition.

v-dipole SPP

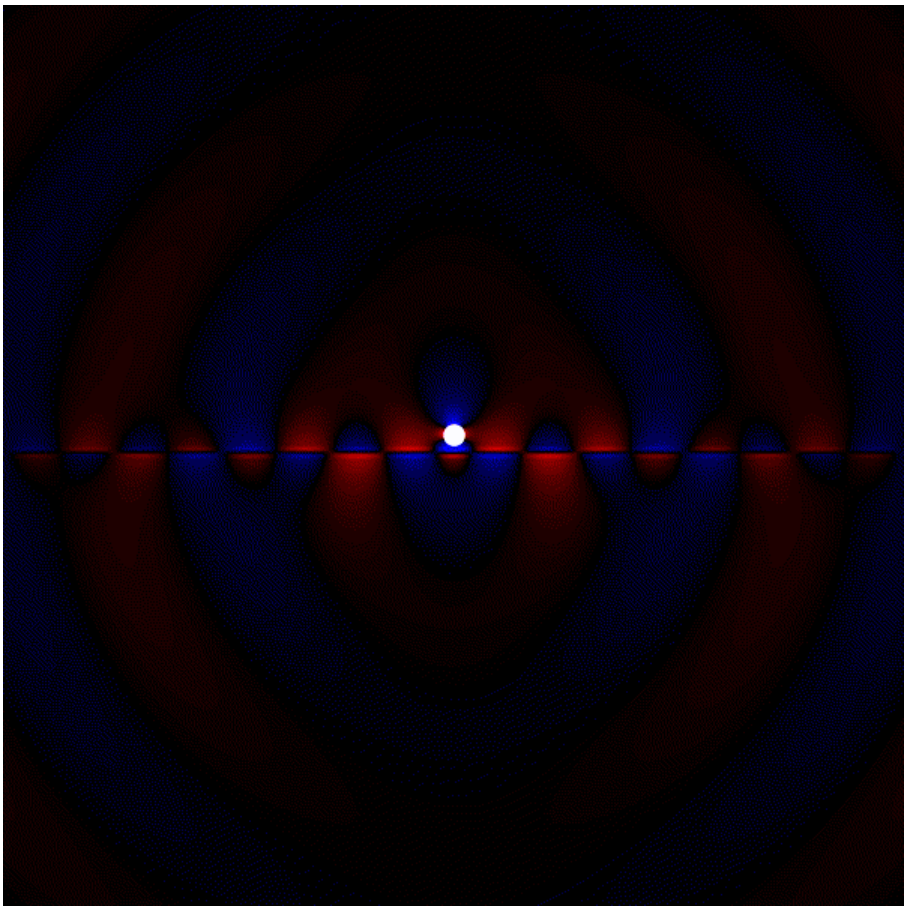

Fig. 2.2: Vertical dipole radition excites SPPs.

c-dipole

c-dipole SPP

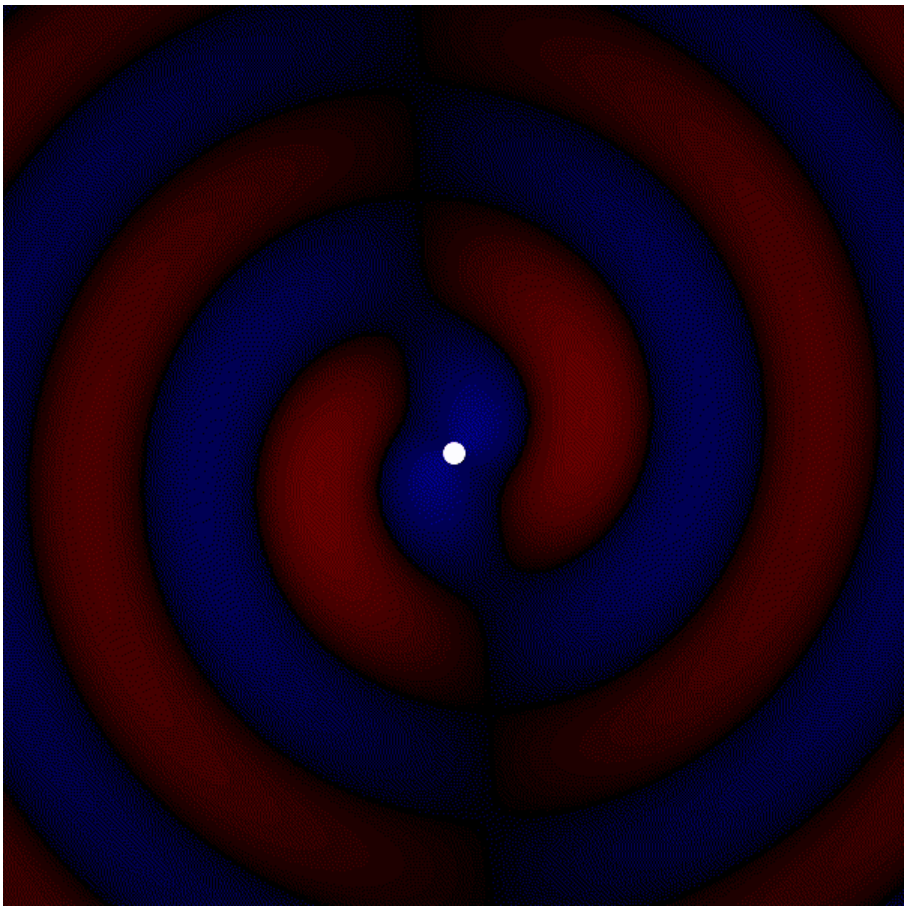

Fig. 2.3: Circular dipole radition.

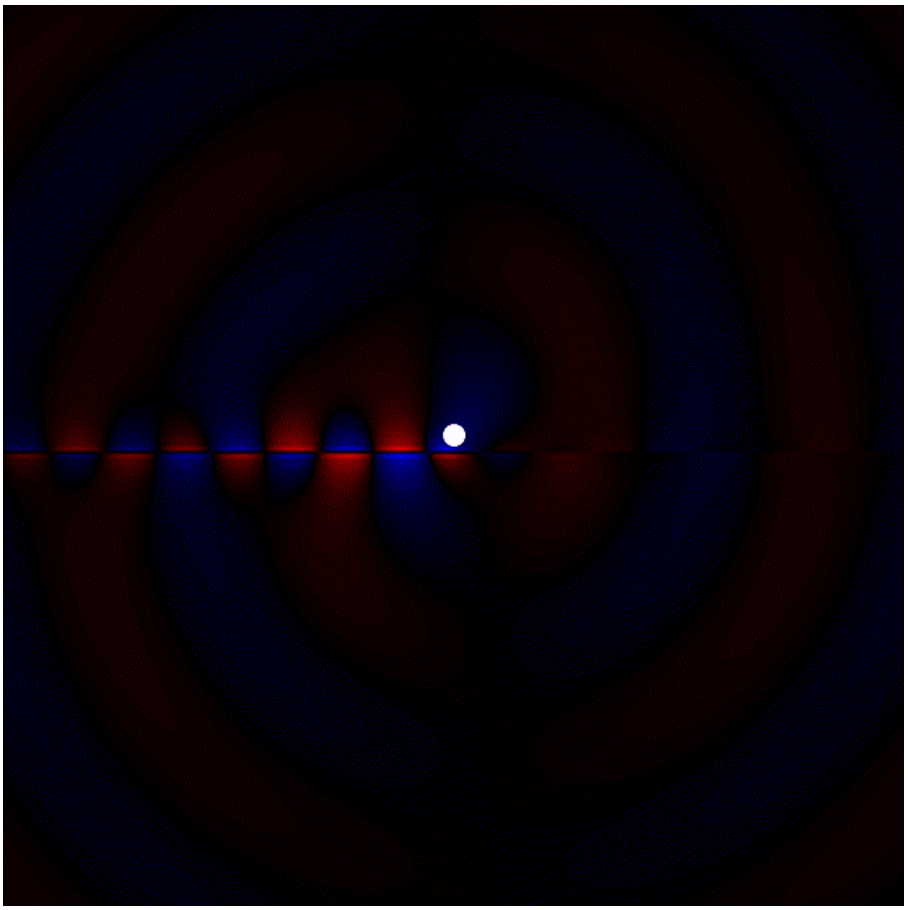

Fig. 2.4: Circular dipole radition excites SPPs.

PROOF INDEX

definition-0