Preliminaries

*1

Suppose that

A = { x : x $\in Nandxiseven\}$,
$B = \{x : x \in Nandxisprime\}$,
$C = \{x : x \in Nandxisamultipleof5\}$.

Then

A $\cap B = \{2\}$
$B \cap C = \{5\}$
$A \cup B = \{x : x \in Nandxisevenorxisprime\}$
$A \cap (B \cup C) = \{x : x \in Nandx = 2orxisamultipleof10\}$

*2

If $A = \{a, b, c\}, B = \{1, 2, 3\}, C = \{x\}, D = \emptyset$ then

A $\times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3), (c, 1), (c, 2), (c, 3)\}$
$B \times C = \{(1, x), (2, x), (3, x)\}$
$A \times B \times C = \{(a, 1, x), (a, 2, x), (a, 3, x), (b, 1, x), (b, 2, x), (b, 3, x), (c, 1, x), (c, 2, x), (c, 3, x)\}$
$A \times D = \emptyset$.

*3

Find an example of two nonempty sets A and B for which $A \times B = B \times A$.
Consider any nonempty set $A = B$.

*4

Prove $A \cup \emptyset = A$.

By definition $A \cup \emptyset = \{x : x \in A \vee x \in \emptyset\}$. Note that the second condition is always false and hence

Prove $A \cap \emptyset = \emptyset$.

This is very similar to the previous proof. By definition $A \cap \emptyset = \{x : x \in A \wedge x \in \emptyset\}$. Note that the second condition is

*5

Prove $A \cup B = B \cup A$.

This follows directly from the definition

Prove $A \cap B = B \cap A$.

This follows directly from the definition

*6

Prove $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

x $\in A \cup (B \cap C) \iff x \in A \vee x \in B \cap C$
$\iff x \in A \vee (x \in B \wedge x \in C)$
$\iff (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)$
$\iff x \in (A \cup B) \vee x \in (A \cup C)$
$\iff x \in (A \cup B) \cap (A \cup C)$

Therefore $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

*7

Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Let $x \in A \cap (B \cup C)$. Then $x \in A$ and $x \in B \cup C$. There are two cases. If $x \in B$ then $x \in A \cap B$ by definition. Similarl

Let $x \in (A \cap B) \cup (A \cap C)$. There are two cases. If $x \in A \cap B$ then clearly $x \in A \cap (B \cup C)$. Similarly, if $x \in A \cap C$ the

*8

Prove $A \subset B$ if and only if $A \cap B = A$.

Assume $A \cap B = A$. Then for all $a \in A$ it is true that $a \in A \cap B$ and thus $a \in B$. Hence $A \subset B$.

Assume $A \subset B$. Then for all $a \in A$ it is true that $a \in B$. Since $a \in A$ and $a \in B$ we know $A \subset A \cap B$. Clearly $A \cap B \subset$

*9

Prove $(A \cap B)' = A' \cup B'$.