Assignment 01

Exercise 1:

1.

1. Length - Ordinal
   Sweetness - Nominal
   Colour - Nominal

2. $P(\text{Banana}) = \frac{600}{1900}$

3. $P(\text{Long}) = \frac{550}{1900}$

4. $P(\text{Banana} | \text{Long}) = \frac{500/1900}{550/1900} = \frac{500}{550}$

5. $P(\text{Long} | \text{Banana}) = \frac{500/1900}{600/1900} = \frac{500}{600}$

6. $P(\text{Banana} | x) = \frac{P(\text{Medium}|\text{Banana}) \cdot P(\text{Sweet}|\text{Banana}) P(\text{Green}|\text{Banana}) \times}{P(\text{Banana})}$

$$= \frac{100}{600} \times \frac{500}{600} \times 0 \times \frac{600}{1900} = 0$$

$$P(\text{Papaya} | x) = \frac{200}{300} \times \frac{250}{300} \times \frac{150}{300} \times \frac{300}{1900} = 0.043$$

$$P(\text{Apple} | x) = \frac{100}{1000} \times \frac{800}{1000} \times \frac{300}{1000} \times \frac{1000}{19000} = 0.012$$

- Since $P(\text{Papaya}|x)$ is high, therefore the given fruit most likely belong to Papaya class.

Exercise 2:

1. To find the loss of the second class (ω_2) $l_2(x)$, we check how many patterns actually belonging to ω_1 (given by the integration of $p(x|\omega_1)$) is misclassified as ω_2. Hence, we used probabilities of the first class integrate it over the region falling in the second class.

2. a) $l_1 < l_2 \Rightarrow \lambda_{12} > \frac{2}{3}$

   b) $l_2 < l_1 \Rightarrow \lambda_{12} < \frac{2}{3}$

   c) $l_2 = l_1 \Rightarrow \lambda_{12} = \frac{2}{3}$

3. $l_1 = l_2$

$$\Rightarrow \quad \lambda_{21}\, p(x|\omega_2)\, P(\omega_2) = \lambda_{12}\, p(x|\omega_1) \cdot P(\omega_1)$$

$$\lambda_{21} \cdot \frac{1}{\sqrt{\pi}} \cdot \exp(-(x-1)^2)(1-p) = \lambda_{12} \cdot \frac{1}{\sqrt{\pi}} \exp(-x^2) \cdot p$$

Taking ~~not~~ log on both sides (ln)

$$+\ln \lambda_{21} - (x-1)^2 + \ln(1-p) = \ln \lambda_{12} - x^2 + \ln p.$$

$$-\cancel{x^2} - 1 + 2x + \cancel{x^2} = \ln \lambda_{12} - \ln \lambda_{21} + \ln p - \ln(1-p)$$

$$2x = \ln \frac{\lambda_{12}}{\lambda_{21}} + \ln \frac{p}{(1-p)} + 1$$

$$2x = \ln\left(\frac{\lambda_{12}}{\lambda_{21}} \cdot \frac{p}{(1-p)}\right) + 1$$

$$x = \frac{1}{2}\left(\ln\left(\frac{\lambda_{12}}{\lambda_{21}} \cdot \frac{p}{(1-p)}\right) + 1\right)$$