Exercise 1
Question (6.a)

$\langle w, \tilde{x} \rangle$

Unnormalized distance $d(\tilde{\phi}, H) \times \|w\|_2 = \langle w, \tilde{x} \rangle + w_0$ — (i)

Given $w^* = \sum_{i=1}^{N} \alpha_i^* T_i \tilde{x}_i$ (from script)

Using in (i)

$$\langle \sum_{i=1}^{4} \alpha_i^* T_i \tilde{x}_i , \tilde{x} \rangle + w_0^*$$

$$= \sum_{i=1}^{4} \alpha_i^* T_i \langle \tilde{x}_i , \tilde{x} \rangle + w_0^*$$

where $\tilde{x} = \phi(x)$

We know: $\boxed{k(x_i, x) = \langle \tilde{x}_i ; \tilde{x} \rangle = \langle \phi(x_i), \phi(x) \rangle}$

Thus $\sum_{i=1}^{4} \alpha_i^* T_i \langle \tilde{x}_i , \tilde{x} \rangle + w_0^* = \sum_{i=1}^{4} \alpha_i^* T_i k(x_i, x) + w_0^*$

b) $\|w\|_2 = \sqrt{\sum_{i=1}^{4}\sum_{j=1}^{4} T_i T_j \alpha_i^* \alpha_j^* k(x_i, x_j)}$

We know:

$$\|w\|_2 = \sqrt{\langle w, w \rangle}$$

$$= \sqrt{\langle \sum_{i=1}^{4} \alpha_i^* T_i \tilde{x}_i , \sum_{j=1}^{4} \alpha_j^* T_j \tilde{x}_j \rangle}$$

$$= \sqrt{\sum_{i=1}^{4}\sum_{j=1}^{4} \alpha_i^* \alpha_j^* T_i T_j \langle \tilde{x}_i , \tilde{x}_j \rangle}$$ (using the knowledge)

$$= \sqrt{\sum_{i=1}^{4}\sum_{j=1}^{4} \alpha_i^* \alpha_j^* T_i T_j \, k(x_i, x_j)}$$

Exercise 2

1) $$k\langle x_i, x\rangle = \langle \phi(x_i), \phi(x)\rangle = \langle \tilde{x}_i, \tilde{x}\rangle \quad (1)$$

$$\phi(x) = e^{-\gamma x^2}\left(1, \sqrt{\frac{2\gamma}{1!}}x, \sqrt{\frac{(2\gamma)^2}{2!}}x^2 \dots\right)$$

Using above in eq. (1).

$$\left\langle e^{-\gamma x_i^2}\left(1, \sqrt{\frac{2\gamma}{1!}}x_i, \sqrt{\frac{(2\gamma)^2}{2!}}x_i^2 \dots\right), e^{-\gamma x^2}\left(1, \sqrt{\frac{2\gamma}{1!}}x, \sqrt{\frac{(2\gamma)^2}{2!}}x^2 \dots\right)\right\rangle$$

performing dot product:

$$e^{-\gamma x_i^2} \cdot e^{-\gamma x^2}\left(1 + \sqrt{\frac{2\gamma}{1!}}x_i x + \sqrt{\frac{(2\gamma)^2}{2!}}x_i^2 x^2 + \sqrt{\frac{(2\gamma)^3}{3!}}x_i^3 x^3 \dots\right)$$

Taylor series

$$= e^{-\gamma(x_i^2 + x^2)} \cdot 2\gamma x_i x$$

$$e^{-\gamma x_i^2} \cdot \sqrt{\frac{2\gamma}{1!}} x_i \cdot$$

$$e^{-\gamma x^2} \sqrt{\frac{2\gamma}{1!}} x$$

$$= e^{-\gamma(x_i + x)} \cdot \left(\sqrt{\frac{2\gamma}{1!}}\right)^2 x_i x$$

$$= \dots$$

$$e^{-\gamma(x_i^2 + x^2)} \cdot \left(1 + \frac{2\gamma}{1!}x_i x + \frac{(2\gamma)^2}{2!}x_i^2 x^2 + \frac{(2\gamma)^3}{3!}x_i^3 x^3, \dots\right)$$

from taylor series:

$$= e^{-\gamma(x_i^2 + x^2)} e^{2\gamma x_i x} = e^{-\gamma(x_i^2 + x^2 - 2x_i x)}$$

$$= e^{-\gamma(x_i - x)^2} = e^{-\gamma\left(\sqrt{(x_i - x)^2}\right)^2} = e^{-\gamma \|x_i - x\|_2^2}$$

2. a)

Property	C small	C large
Margin	large	small
Complexity	simple	complex
Train accuracy	low	high

b)

Property	γ small	γ large
figure	A	B
Influence	small	high