Exercise 1

3 a) $a = 2, \quad b = 4.$ $\quad f(x) = a(1+x)^2 + bx$

	n_1	n_2	n_3	n_4	n_5	n_6
Value n_i	2	3	9	18	8	26
parent(n_i)	$\{n_5, n_2\}$	$\{n_3\}$	$\{n_4\}$	$\{n_6\}$	$\{n_6\}$	$\{\}$
Derivative $\frac{\partial f}{\partial n_i}$	$2an_2 + b$ $= 2a(1+x) + b$	$2 \cdot a \cdot n_2$ $= 2a(1+x)$	a	1	1	1

a)
$n_1 = x = 2$
$n_2 = 1 + n_1 = 3$
$n_3 = n_2^2 = 9$
$n_4 = a \cdot n_3 = 18$
$n_5 = b \cdot n_1 = 8$
$n_6 = n_4 + n_5 = 26$

b) $\frac{\partial f}{\partial n_6} = \frac{\partial n_6}{\partial n_6} = 1$ $\quad$ parent$(n_6) = \{\}$ $\quad (\because f = n_6)$

$\frac{\partial f}{\partial n_5} = \frac{\partial n_6}{\partial n_5} = \frac{\partial (n_4 + n_5)}{\partial n_5} = 0 + 1 = 1$ $\quad$ parent$(n_5) = \{n_6\}$

$\frac{\partial f}{\partial n_4} = \frac{\partial n_6}{\partial n_4} = \cancel{\frac{\partial n_6}{\partial n_5} \cdot \frac{\partial n_5}{\partial n_4}} = \frac{\partial (n_4 + n_5)}{\partial n_4} = 1 + 0 = 1$ $\quad$ parent$(n_4) = \{n_6\}$

$\frac{\partial f}{\partial n_3} = \frac{\partial n_6}{\partial n_3} = \frac{\partial n_6}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_3} = 1 \cdot a = a = 2$ $\quad$ parant$(n_3) = \{n_4\}$

$\frac{\partial f}{\partial n_2} = \frac{\partial n_6}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_3} \cdot \frac{\partial n_3}{\partial n_2} = 1 \cdot a \cdot 2 \cdot n_2 = 2a \cdot n_2 = 4 n_2 = 12$ $\quad$ parent$(n_2) = \{n_3\}$

$\cancel{\frac{\partial f}{\partial n_1} = \frac{\partial n_6}{\partial n_2} + \frac{\partial n_6}{\partial n_5} = 2 \cdot a \cdot n_2 + 1}$

$\frac{\partial f}{\partial n_1} = \frac{\partial n_6}{\partial n_2} \cdot \frac{\partial n_2}{\partial n_1} + \frac{\partial n_6}{\partial n_5} \cdot \frac{\partial n_5}{\partial n_1} = 12 \cdot 1 + 1 \cdot b = 12 + 4 = 16$

parent$(n_1) = \{n_2, n_5\}$

c) n_5 & (n_4, n_3, n_2) can be executed in parallel.