Exercise 1

1) $$L(p) = \prod_{k=1}^{3} p(1-p)^{n_k-1} = p^3(1-p)^5$$

2) $$\hat{p}_{ML} = \underset{p}{\operatorname{argmax}} \prod_{k=1}^{3} p(1-p)^{n_k-1}$$

Instead of maximising, we can maximize $\ln L(p)$

$$\hat{p}_{ML} = \underset{p}{\operatorname{argmax}} \ln L(p) = \underset{p}{\operatorname{argmax}} \ln \prod_{k=1}^{3} p(1-p)^{n_k-1}$$

Maximization: $$\frac{\partial \ln L(p)}{\partial p} = \frac{\partial \ln \prod p(1-p)^{n_k-1}}{\partial p} = 0$$

$$= \sum_{k=1}^{3} \frac{\partial \ln p(1-p)^{n_k-1}}{\partial p} = 0$$

$$= \sum_{k=1}^{3} \left(\frac{\partial \ln p}{\partial p} + \frac{\partial \ln (1-p)^{n_k-1}}{\partial p} \right) = 0$$

$$= \sum_{k=1}^{3} \left(\frac{1}{p} - \frac{(n_k-1)}{(1-p)} \right) = 0$$

$$= \left(\frac{1}{p} - \frac{1}{1-p} + \frac{1}{p} - 0 + \frac{1}{p} - \frac{4}{1-p} \right) = 0$$

$$\frac{3}{p} = \frac{5}{1-p} \qquad p = 3/8 = \hat{p}_{ML}$$

Exercise 2

1) a) h corresponds directly to the standard deviation of the gaussian function. Thus increasing 'h' ~~increasing~~ increases the variances of each gaussian function centered at each points.

b) As h increases, the ~~bin~~ histogram height decreases i.e. $\hat{p}(x)$ reduces for ~~each~~ each bin. Simultaneously, bin size increases as well i.e., more points get ~~are~~ aggregated.

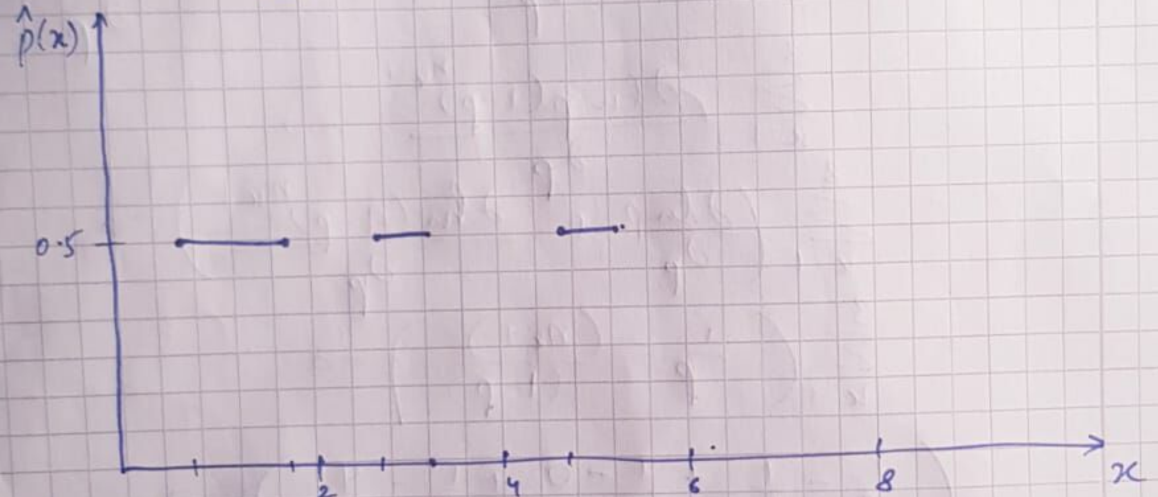
c) for low value of h (e.g. 0.2), we get peaks for each points. ie, gaussian function centred at each point. As h increases, the $\hat{p}(x)$ becomes more smooth ~~and~~ and wide (variances increases) thus aggregating more points. As more points are aggregated peaks reduces ~~and finally we get ...~~

2. $h = 0.35$ as the function is smooth and aggregates two points which are close.

3) a) $h = 0.5$

$$\hat{p}(x) = \frac{1}{2}\left(\begin{cases} 1, & 0.75 \le x \le 1.25 \\ 0, & \text{else} \end{cases} + \begin{cases} 1, & 1.25 \le x \le 1.75 \\ 0, & \text{else} \end{cases} + \begin{cases} 1, & 2.75 \le x \le 3.25 \\ 0, & \text{else} \end{cases} + \begin{cases} 1 & 4.75 \le x \le 5.25 \\ 0 & \text{else} \end{cases} \right)$$

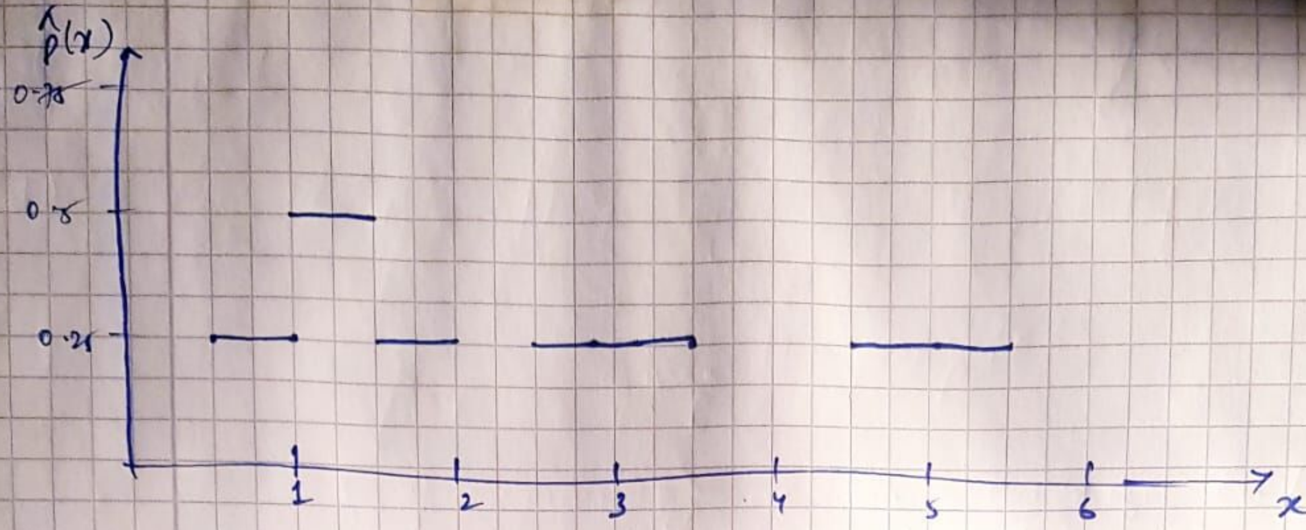
$$= \frac{1}{2} \begin{cases} 0 & 0 \le x < 0.75 \\ 1 & 0.75 \le x \le 1.75 \\ 0 & 1.75 < x < 2.75 \\ 1 & 2.75 \le x \le 3.25 \\ 0 & 3.25 < x < 4.75 \\ 1 & 4.75 \le x \le 5.25 \\ 0 & \text{else} \end{cases}$$



b) $h = 1.0$

$$\hat{p}(x) = \frac{1}{4}\left(\begin{cases} 1, & 0.5 \le x \le 1.5 \\ 0, & \text{else} \end{cases} + \begin{cases} 1, & 1 \le x \le 2 \\ 0, & \text{else} \end{cases} + \begin{cases} 1, & 2.5 \le x \le 3.5 \\ 0, & \text{else} \end{cases} + \begin{cases} 1, & 4.5 \le x \le 5.5 \\ 0, & \text{else} \end{cases} \right)$$

$$= \frac{1}{4} \begin{cases} 0, & 0 \le x < 0.5 \\ 1, & 0.5 \le x < 1 \\ 2, & 1.0 \le x \le 1.5 \\ 1, & 1.5 < x \le 2 \\ 0, & 2.0 < x < 2.5 \\ 1, & 2.5 \le x \le 3.5 \\ 0, & 3.5 < x < 4.5 \\ 1, & 4.5 \le x \le 5.5 \\ 0, & \text{else} \end{cases}$$



c) $h = 2.5$

$$\frac{1}{10}\left(\begin{cases}1, & -0.25 \le x \le 2.25\\ 0, & \text{else}\end{cases} + \begin{cases}1, & 0.25 \le x \le 2.75\\ 0, & \text{else}\end{cases} + \begin{cases}1, & 1.75 \le x \le 4.25\\ 0, & \text{else}\end{cases} + \begin{cases}1, & 3.75 \le x \le 6.25\\ 0, & \text{else}\end{cases} \right)$$

$$\hat{p}(x) = \frac{1}{10}\begin{cases} 1 & -0.25 \le x < 0.25 \\ 2 & 0.25 \le x \le 1.75 \\ 3 & 1.75 \le x \le 2.25 \\ 2 & 2.25 \le x \le 2.75 \\ 1 & 2.75 \le x \le 3.75 \\ 2 & 3.75 \le x < 4.25 \\ 1 & 4.25 \le x \le 6.25 \\ 0 & \text{else} \end{cases}$$