PATTERN RECOGNITION

Assignment 3

Marjana Tahmid

1017997

Mohammad Kalim Akram

1017776

Namratha Nandagudi

1018470

Exercise 1

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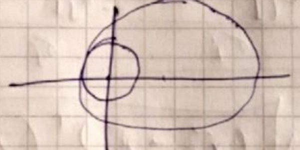
	μ_1	Σ_1	μ_2	Σ_2	Decision Surface	Slice
(i)	$\begin{pmatrix}0\\0\end{pmatrix}$	$\begin{pmatrix}1&0\\0&1\end{pmatrix}$	$\begin{pmatrix}2\\0\end{pmatrix}$	$\begin{pmatrix}3&0\\0&3\end{pmatrix}$	b	e
(ii)	$\begin{pmatrix}0\\0\end{pmatrix}$	$\begin{pmatrix}1&0\\0&1\end{pmatrix}$	$\begin{pmatrix}2\\0\end{pmatrix}$	$\begin{pmatrix}1&0\\0&3\end{pmatrix}$	c	f
(iii)	$\begin{pmatrix}0\\0\end{pmatrix}$	$\begin{pmatrix}2&0\\0&3\end{pmatrix}$	$\begin{pmatrix}2\\0\end{pmatrix}$	$\begin{pmatrix}6&0\\0&1\end{pmatrix}$	a	d

(i) mean of $\omega_1 = 0$

variance of $\omega_1 = \begin{matrix}1 & (x_1)\\ 1 & (x_2)\end{matrix}$, so decision boundary (b) seems correct. & the corresponding slice would be (e) because ω_2 is spread out.



(ii) mean of $\omega_1 = 0$; variance of $\omega_1 = \begin{matrix}1 & (x_1)\\ 1 & (x_2)\end{matrix}$ (Same in both

whereas of the variance of ω_2 is high in x_2 dim, thus it must be stretched in x_2. Therefore fig 'c' is apt decision boundary. fig f is apt for slice, as it ~~does~~ does not have same means and chances of ω_2 are high and less spr

(iii) mean of $\omega_1 = 0$; ω_1 is stretched in x_2 than x_1,

ω_2 is stretched in x_1. Thus, fig 'a' is apt as it depicts the high variation of ω_2 in x_1 dim.

fig 'd' is the slice for this, as their means corresponds.

Exercise 2

1.

a) when we look at the figure 2 nearest neighbor of (1,1200) appears to be (1.5,1000)

b) The result obtained by Euclidean distance does not match with the expected value as seen from the figure 2.

- The y axis values (Monthly income) are **relatively large** compared to the x axis values (Years of education)
- The Largest scaled feature would dominate other features
- if two data vectors have no attribute values in common, they may have a smaller distance than the other pair of data vectors containing the same attribute values

2.

a) After Standardizing there will be **no** units for dimensions. Both in the case of x and y dimensions.

b) **A fair comparison** can be done with various dimensions of different scales which will be converted to the dimensions with no unit after standardizing.

c)

L.H.S $\quad d_E(\tilde{x}_i, \tilde{x}_j) = \sqrt{(\tilde{x}_i - \tilde{x}_j)^T (\tilde{x}_i - \tilde{x}_j)}$

$$= \sqrt{(\tilde{x}_i - \tilde{x}_j \quad \tilde{y}_i - \tilde{y}_j)\begin{pmatrix}\tilde{x}_i - \tilde{x}_j \\ \tilde{y}_i - \tilde{y}_j\end{pmatrix}}$$

$$= \sqrt{(\tilde{x}_i - \tilde{x}_j)^2 + (\tilde{y}_i - \tilde{y}_j)^2}$$

$$= \sqrt{\left(\frac{x_i - \mu_x}{\sigma_x} - \frac{x_j - \mu_x}{\sigma_x}\right)^2 + \left(\frac{y_i - \mu_y}{\sigma_y} - \frac{y_j - \mu_y}{\sigma_y}\right)^2}$$

$$= \sqrt{\frac{(x_i - x_j)^2}{\sigma_x^2} + \frac{(y_i - y_j)^2}{\sigma_y^2}}$$

R.H.S. $\quad \sqrt{(x_i - x_j)^T S^{-1} (x_i - x_j)}$

$$= \sqrt{(x_i - x_j \quad y_i - y_j)^T S^{-1} \begin{pmatrix}x_i - x_j \\ y_i - y_j\end{pmatrix}}$$

$$= \sqrt{(x_i - x_j \quad y_i - y_j)\begin{bmatrix}1/\sigma_x^2 & 0 \\ 0 & 1/\sigma_y^2\end{bmatrix}\begin{bmatrix}x_i - x_j \\ y_i - y_j\end{bmatrix}}$$

$$= \sqrt{\left(\frac{x_i - x_j}{\sigma_x^2} \quad \frac{y_i - y_j}{\sigma_y^2}\right)\begin{pmatrix}x_i - x_j \\ y_i - y_j\end{pmatrix}}$$

$$= \sqrt{\frac{(x_i - x_j)^2}{\sigma_x^2} + \frac{(y_i - y_j)}{\sigma_y^2}}$$

L.H.S = R.H.S

3

a) The points which lies on the same elliptical path denotes equal distance from the mean.

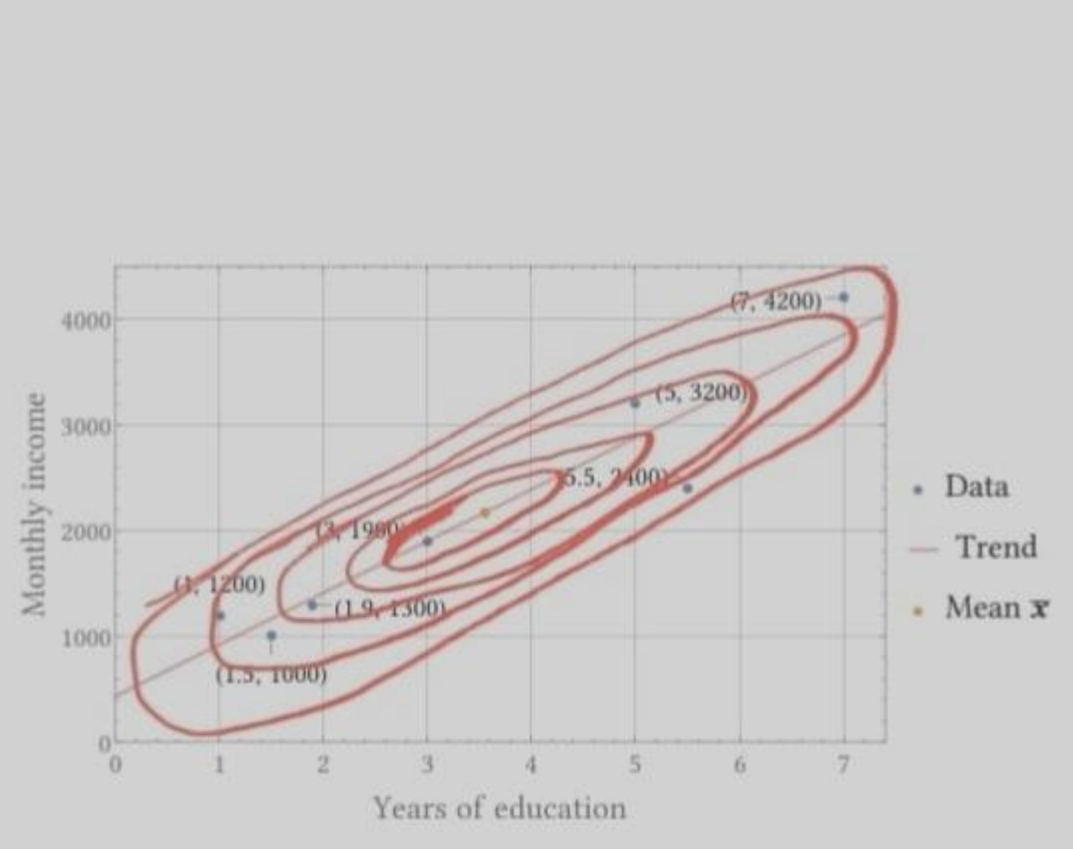
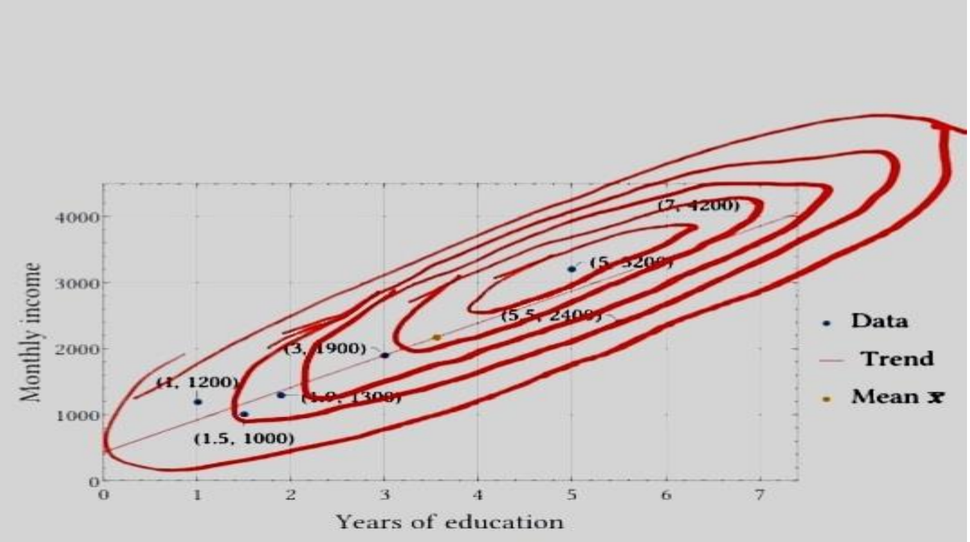


Figure 2: Example dataset which relates education to income (Table 2).

b) When the point (5,3200) acts as center then we can see that the point (7,4200) is the nearest neighbor as point(7, 4200) lies on the inner curve which means it is at a shorter distance(nearer) than the point (5.5,2400)



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