Week 14 Homework

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Generally:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Using the above for all questions below:

1) $f(x) = \frac{1}{(1-x)} = \frac{1}{(1-a)} + \frac{(x-a)}{(1-a)^2} + \frac{(1-a)^2}{(1-a)^3} + \frac{(1-a)^3}{(1-a)^4} + \cdots$

If a = 0 (Maclaurin Series),

$$f(x) = \frac{1}{(1-0)} + \frac{(x-0)}{(1-0)^2} + \frac{(x-0)^2}{(1-0)^3} + \frac{(x-0)^3}{(1-0)^4} + \cdots$$

$$f(x) = 1 + x + x^2 + x^3 + x^4 + \cdots$$

$$f(x) = \sum_{n=0}^{\infty} x^n, x \in (-1,1)$$

2) $f(x) = e^x = e^a + e^a(x-a) + e^a(x-a)^2 + e^a(x-a)^3 + \cdots$

If a = 0 (Maclaurin Series),

$$f(x) = e^0 + e^0(x-0) + e^0(x-0)^2 + e^0(x-0)^3 + \cdots$$

$$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}, x \in R$$

3) $f(x) = \ln(1+x) = \ln(1+a) + \frac{(x-a)}{(1+a)} - \frac{(x-a)^2}{2!(1+a)^2} + \frac{(x-a)^3}{3!2(1+a)^3} - \frac{(x-a)^4}{4!(3)(2)(1+a)^3} + \cdots$

If a = 0 (Maclaurin Series),

$$f(x) = \ln(1+0) + \frac{(x-0)}{(1+0)} - \frac{(x-0)^2}{2!(1+0)^2} + \frac{(x-0)^3}{3!2(1+0)^3} - \frac{(x-0)^4}{4!(3)(2)(1+0)^3} + \cdots$$

$$f(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

$$f(x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}, x \in (-1,1]$$

also

$$f(x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}, x \in (-1,1]$$