# Modelling the Analog Electronics

August 2, 2019

## 1 Low Pass Filter

The Minicircuit Low Pass Filter (LPF) has a bandwidth from $10MHz$ to $6GHz$. So, the first step was to model the regimes $0-10MHz$ and $6-20GHz$ for computation purposes only. For $0-10MHz$, we add the lowest gain $0.01dB(10MHz)$ to all frequencies below $10MHz$. For the region $6-20GHz$, we add a $\frac{0.02}{f}$ curve in the *log* space (to have an almost continuous slope for the gain curve) and then use linear interpolation for all the integer frequencies from $0-20000MHz$.

## 2 Logical Processing of Signal

We pass the incoming signal $s$ through a passive differentiator to generate the derivative signal. We will call this $dsig$. Then, we divide the signal into two parts. One will pass though another differentiator (active). We will call this signal $ddsig$. Anpther part from $dsig$ will pass through an inductor $j\omega L$ and then to an integrator to get back the original signal without the DC offset. We will call this signal $s'$. Then, we will perform the following operations (the upper case of signal names signify Fourier Transforms):

1. $dsig \times s' \rightarrow zc'$
2. $ddsig \times s' \rightarrow th'$

$zc'$ and $th'$ signify the zero crossing signal and threshold signal respectively(with noise). To cancel this noise, we will put the LPF modelled in section 1. This would yield the high-noise free $zc$ and $th$ respectively.

Now we can see some strong correlation between $zc$ and $th$. The idea is to find the zero crossings from the $zc$ signal when the $th$ crosses some threshold. The main challenge in this task is the varying peak amplitudes within the same window of data for pileup vs non-pileup regimes.

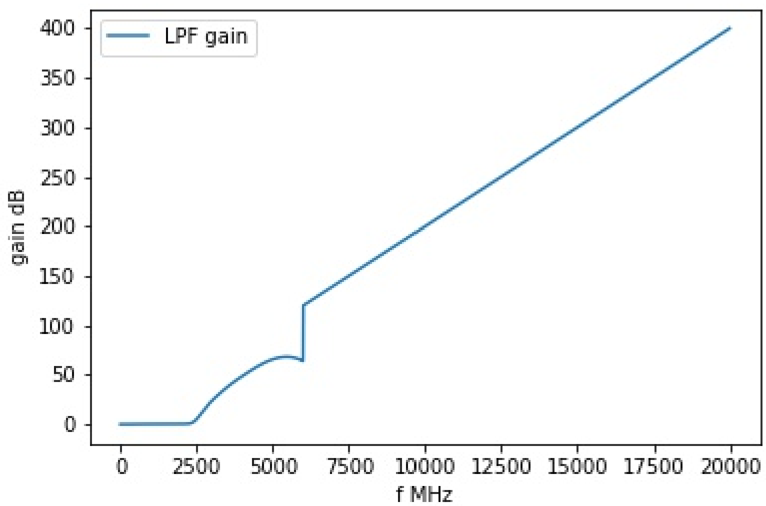



Figure 1: LPF

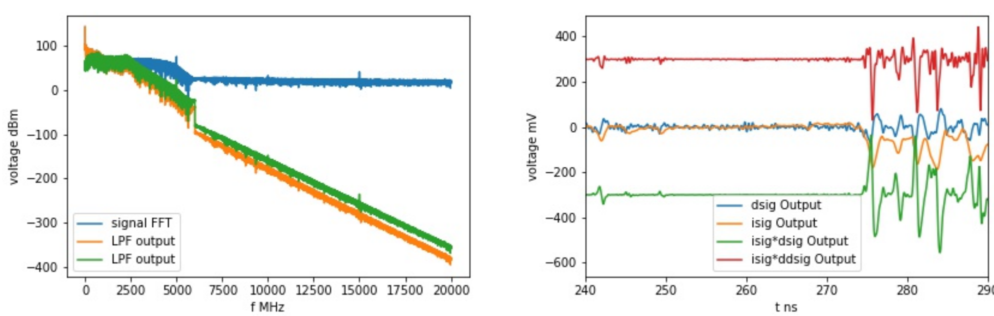



Figure 2: Signal spectrum and logical processing

# 3 Circuit Design and Challenges

For the logical processing of the signal, we need the derivative and integral of the signal. We can do this using an differentiator and integrator respectively. They can be both passive (using RC circuits) or active (using op-amps). Let us discuss each of them seperately, including the changes one can make in the code to use different versions of the $integrator\,()$ and $differentiator\,()$ functions.

## 3.1 Integrator

The passive integrator is a circuit with a transfer function:

$$H\left(\omega\right) = \frac{Z_c}{Z_c + R} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\frac{\omega}{\omega_0}}$$

where, $\omega_0 = \frac{1}{RC}$ .

For an practical active integrator (tackling the problem of the shift in input offset voltage for low frequency input), the transfer function is:

$$H\left(\omega\right) = -\frac{Z_f}{Z_i} = -\frac{R_f \mid\mid Z_c}{R_{in}} = -\frac{R_f}{R_{in}}\left(\frac{1}{1 + j\omega R_f C}\right) = A_{cl}\left(\frac{1}{1 + j\frac{\omega}{\omega_0}}\right)$$

where, $\omega_0 = \frac{1}{R_f C}, A_{cl} = -\frac{R_f}{R_{in}}$ .

In the code, to make a passive integrator from an active one, set $R_{in} = R_f = R$ and to make an ideal active integrator, set $R_f = 0$, for which the transfer function becomes:

$$H_{ideal}\left(\omega\right) = -\frac{Z_f}{Z_i} = -\frac{Z_c}{R_{in}} = -\frac{1}{j\omega R_{in} C} = j\frac{\omega_0}{\omega}$$

## 3.2 Differentiator

The passive differentiator is a circuit with a transfer function:

$$H\left(\omega\right) = \frac{R}{Z_c + R} = \frac{j\omega RC}{1 + j\omega RC} = j\frac{\omega}{\omega_0}\left(\frac{1}{1 + j\frac{\omega}{\omega_0}}\right)$$

where, $\omega_0 = \frac{1}{RC}$ .

For a practical active differentiator (tackling the problem of high frequency noise amplification), the transfer function is:

$$H\left(\omega\right) = -\frac{Z_f}{Z_i} = -\frac{R_f \mid\mid Z_{cf}}{R_{in} + Z_{cin}} = -\frac{j\omega R_f C_{in}}{1 + j\omega R_{in} C_{in}}\left(\frac{1}{1 + j\omega R_f C_f}\right) = -jA_{cl}\frac{\omega}{\omega_1}\frac{1}{\left(1 + j\frac{\omega}{\omega_1}\right)\left(1 + j\frac{\omega}{\omega_2}\right)}$$

where, $\omega_1 = \frac{1}{R_{in}C_{in}}, \omega_2 = \frac{1}{R_f C_f}, A_{cl} = \frac{R_f}{R_{in}}$ .

In the code, to make a passive differentiator from an active one, set $R_{in} = R_f = R, C_f = 0$ and to make an ideal active differentiator, set $R_f = 0$, for which the transfer function becomes:

$$H_{ideal}(\omega) = -\frac{Z_f}{Z_i} = -\frac{R_{in}}{Z_{cin}} = -j\omega R_{in}C_{in} = -j\frac{\omega}{\omega_0}$$

where, $\omega_0 = \frac{1}{R_{in}C_{in}}$ .

## 3.3 Band-pass Filters

We need to design some passive band-pass filters to address the correct band for the integrators and differentiators. This is just a regular RLC circuit with output measure across the resistor. If the lower and upper -3 dB cutoff frequencies are defined to be $f_1$ and $f_2$ respectively, then, the bandwidth and center frequency $f_c$ of the filter is given by:

$$BW = f_2 - f_1$$
$$f_c = \sqrt{f_1 f_2}$$

Choosing a standard value of C, we can find the values of R, L needed to design the circuit using the following formulas:

$$L = \frac{1}{4\pi^2 f_c^2 C}$$
$$R = L \times BW$$

Finally, the transfer function of the bandpass filter will be:

$$H(\omega) = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

## 3.4 Challenges in Real Circuits

There are several problems that are arising while designing the analog circuit for the processing of data as per Sec.2. Here is a list of concerns.

1. The signal we are dealing with is quite broadband $(0 - 2.5GHz)$ after applying the low pass filter to remove high frequency noises $(> 2.5GHz)$. So, to design integrators and differentiators in such a wide range requires extremely low capacitances (in the order of $10^{-3} - 10^{-6}$ pF) which doesn't make much sense to use (since the stray/parasitic capacitances in the circuit board can causes errors in the order of a few pFs).

2. Another concern is the bandwidth limitation of the amplifier ZX60-6013E-S+ from mini-circuits, which is $20 - 6000MHz$. So, frequencies below $20MHz$ practically by-passes the amplifier itself. Looking into the power spectrum of the signal, there appears to be a considerable portion of power in these frequencies which may distort the shape of the signal.

3. The saturation voltage of the amplifier ZX60-6013E-S+ is $12V$. Our signal, for example, is a few hundred mVs say $\sim 100 - 500mV$, then the maximum gain we can get from the amplifier without clipping the output voltage is $\sim 28 - 42dB$. So, while designing the circuits, we should keep in mind this figure to avoid saturation (working in $25dB$ maximum gain seems quite reasonable).

### 3.5 Approach to challenges

The apparent solution to the list of problems is not straight-forward but the idea is to analyse signals like most important signals in the algorithm like $s, ddsig, s', s' \times ddsig$ and find the band ($> 20MHz$) that is most useful in maintaining the shape of the signal. Once we figure that out, we come up with some pre-processing algorithm to retrieve the arrival times, on the basis of the slightly distorted signal.

## 4 Exploring the Real Circuit parameters

### 4.1 Theory

For an integrator to work properly, the minimum frequency component (that we care about) in the signal $f_s$ should be atleast $10 \times f_c$ where $f_c$ is the corner frequency of the integrator given by:

$$f_c = \frac{1}{2\pi R_f C}$$

The 0-dB frequency for this integrator is given by:

$$f_0 = \frac{1}{2\pi R_{in} C}$$

Similarly, for a diiferentiator, the maximum frequency component $f_s$ should be atleast $\frac{f_{c1}}{10}$ or $\frac{f_{c2}}{10}$ (whichever smaller), where:

$$f_{c1} = \frac{1}{2\pi R_{in} C_{in}}; f_{c2} = \frac{1}{2\pi R_f C_f}$$

The 0-dB frequencies here are given by:

$$f_{01} = \frac{1}{2\pi R_f C_{in}}; f_{02} = \frac{1}{2\pi R_{in} C_f}$$

### 4.2 Proposed solution

The integrator/differentiator we are trying to design has to encounter a very wideband signal, even after passing through the loss pass filter discussed in section (1), with a cutoff frequency of $2.2GHz$. Keeping all the other features of the mini-circuit components in mind, we propose to make multiple copies of the signal to subdivide into bands and pass each of them through band pass filters which are added in the end to get an honest wide band processing circuit output which is ready for further pre-processing. The details of the design is discussed in the next subsection.

## 4.3 Designing the Integrator

The integrator we are looking at the Bode plot of an integrator in figure (3) divided into three bands and each band has its specific characteristics. As per our discussion in sub-section 3.3, we want a maximum gain of $25dB$ and use a capacitor $C_{in} = 8.95pF$ for all the three sub-bands.

### 4.3.1 Sub-Band 1:

1. $(R_{in}, R_f) = (500\Omega, 8.89k\Omega)$
2. $(f_c, f_0) = (2MHz, 35.56MHz)$

### 4.3.2 Sub-Band 2:

1. $(R_{in}, R_f) = (28.7\Omega, 500\Omega)$
2. $(f_c, f_0) = (35.56MHz, 632.45MHz)$

### 4.3.3 Sub-Band 3:

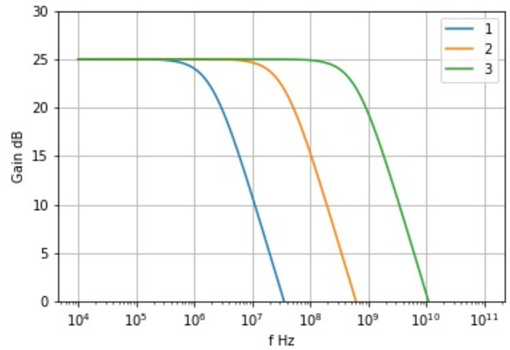
1. $(R_{in}, R_f) = (1.61\Omega, 28.7\Omega)$
2. $(f_c, f_0) = (632.45MHz, 11.2GHz)$ (Although the amplifier will see upto $6GHz$ only)

## 4.4 Designing the Differentiator

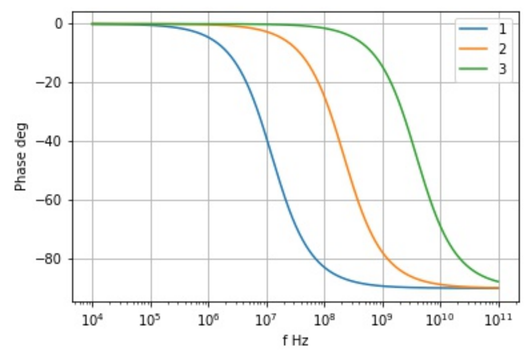
Similar to the previous sub-section, we are looking at the Bode plot of a differentiator in figure (4) divided into two bands and each band has its specific characteristics. As per our discussion in sub-section 3.3, we again want a maximum gain of $\sim 18dB$ and want to degenerate $f_{c1}$ and $f_{c2}$ into a single frequency $f_c$ (due to the disjoint nature of our design).

### 4.4.1 Sub-Band 1:

1. $(R_{in}, R_f) = (1\Omega, 17.78k\Omega)$
2. $(C_{in}, C_f) = (796pF, 44.77pF)$
3. $(f_0, f_c) = (11.24MHz, 200MHz)$




(a) Amplitude




(b) Phase

Figure 3: Integrator response

#### 4.4.2 Sub-Band 2:

1. $(R_{in}, R_f) = (1\Omega, 17.78\Omega)$
2. $(C_{in}, C_f) = (44.77pF, 2.52pF)$
3. $(f_0, f_c) = (200MHz, 3.56GHz)$

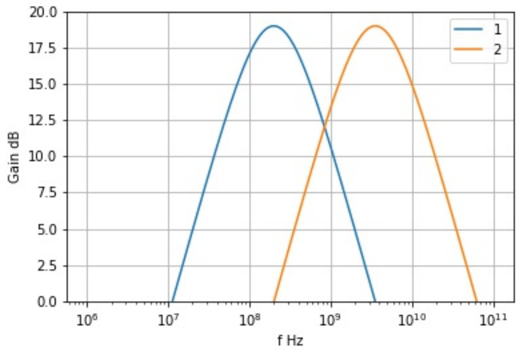
### 4.5 Designing the Band-pass Filters

By selecting the value of $C = 1pF$, we can design the following filters, as shown in figure (5). The labels 1,2,3 represent the three sub-bands for the integrator circuit discussed in sub-section 4.3 where as 4,5 represent the bands for the differentiator circuit. The values for $(L, R)$ in the same order as labels are listed as follows: $(11.9k\Omega, 356.1\mu H)$, $(672.3\Omega, 1.12\mu H)$, $(37.8\Omega, 3.57\mu H)$, $(2.13k\Omega, 11.3\mu H)$, $(119.5\Omega, 35.6\mu H)$.

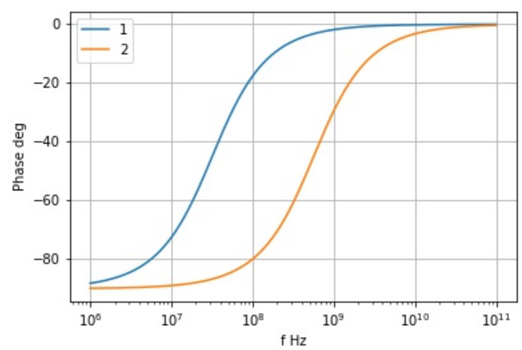
## 5 Exploring the real signal

### 5.1 Shape of the raw signal

Since the amplifier has a limited bandwidth $(20 - 6000MHz)$, the some frequency components in the incoming signal won't be seen by the amplifier. The low frequency components would be by-passed. This significantly reduces the signal strength without much changes in the signal shape, as evident from figure (5). This would allow us to perform similar kind of operations, that we were




(a) Amplitude




(b) Phase

Figure 4: Differentiator response

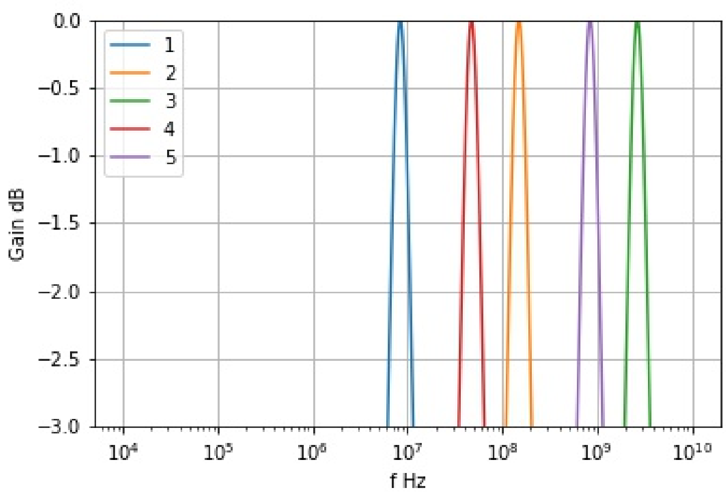



Figure 5: Different Band-pass filters

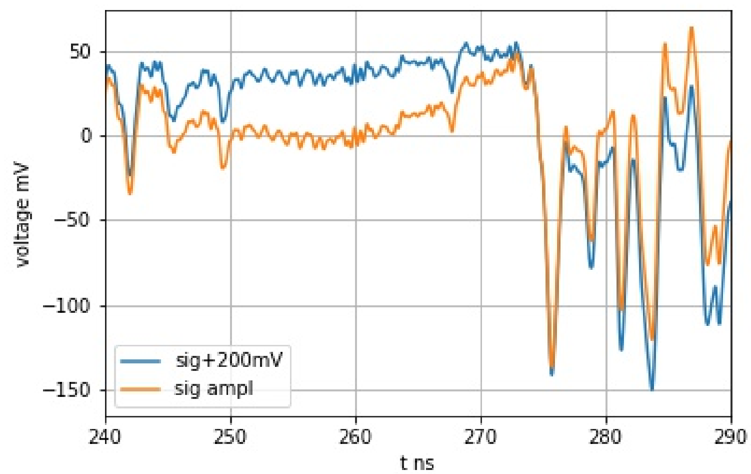



Figure 6: Signal seen by amplifier

attempting to do on the idea incoming signal itself. If needed, we can just add some d.c power to the signal with a feedback amplifier.

## 5.2 Processing the signal

Now the task is to apply all the designs developed in proper order to see what the processed output looks like and how close it is to the simulated one.