LectureNote2: CS229 (SPRING2022, Stanford)

Supervised Learning

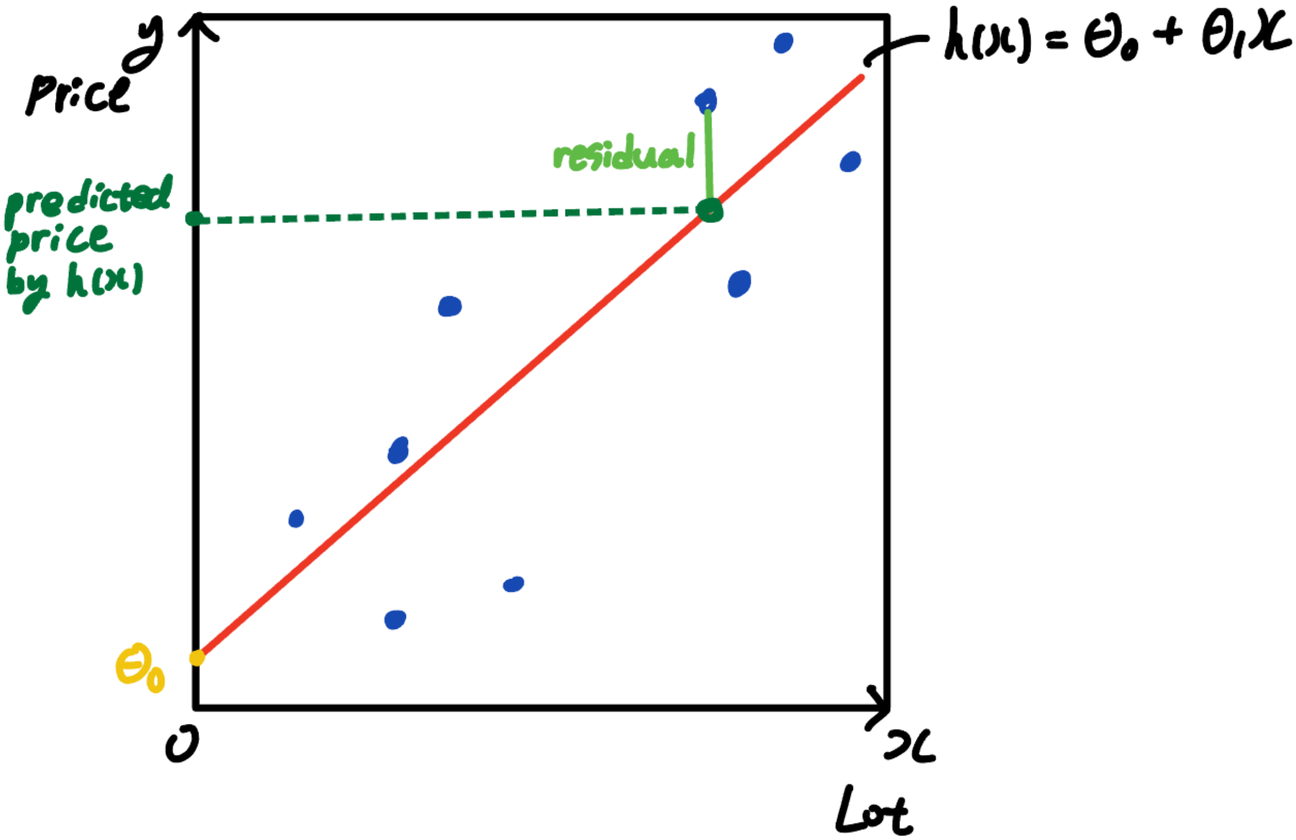
Set Up

- Prediction:
  - $h: \begin{matrix} x & \rightarrow & y \\ images & & cat \\ text & & hate\ speech? \\ housedata & & price \end{matrix}$

- Given: training set
  - $\{ (x^{(1)}, y^{(1)}), ..., (x^{(n)}, y^{(n)}) \}$
  - y = supervision

- Do: find **good** $h(hypothesis) : x \rightarrow y$
  - we care about **new** x's that are not in our training set
  - if y is *discrete* $\rightarrow$ Classification
  - if y is *continuous* $\rightarrow$ Regression

- How do we represent h?
  - $h(x) = \theta_0 + \theta_1 x_1$



- Generalization
  - Training Set

$$\begin{array}{ccccccc} & size(x_1) & bedroom(x_2) & lot\ size(x_3) & \dots & feature(x_d) & price(y) \\ x^{(1)} & 2104 & 4 & 45k & \dots & x_d^{(1)} & 400 \\ x^{(2)} & 2500 & 3 & 30k & \dots & x_d^{(2)} & 900 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x^{(n)} & x_1^{(n)} & x_2^{(n)} & x_3^{(n)} & \dots & x_d^{(n)} & y^{(n)} \end{array}$$

  - $(x^{(i)}, y^{(i)}) \leftarrow$ i-th training example
  - $\theta_{parameters} = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_d \end{bmatrix} \quad x^{(i)}_{features} = \begin{bmatrix} x_0^{(i)} \\ x_1^{(i)} \\ \vdots \\ x_d^{(i)} \end{bmatrix}$
  - $X_{train\ data} = \begin{bmatrix} -x^{(1)}- \\ -x^{(2)}- \\ \vdots \\ -x^{(n)}- \end{bmatrix} \in \mathbb{R}^{n(d+1)} \quad \vec{y} = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(n)} \end{bmatrix}$
    - $X_{train\ data}$ does not contain y's

    - +1 is the extra dimension x_0, the intercept term, whose value is set to be 1

- $h_\theta(x) = \theta_0 x_0 + \theta_1 x_1 + ... + \theta_d x_d \ = \sum_{j=0}^{d} \theta_j x_j \ (x_0 = 1)$, I want $h_\theta(x) \simeq y$

- $J(\theta)_{cost\ function} = \frac{1}{2} \sum_{i=1}^{n} (h_\theta(x^{(i)}) - y^{(i)})^2{}_{least\ squares}$ = $\frac{1}{2}(X\theta - \vec{y})^T(X\theta - \vec{y})$
    - $\frac{1}{2}$ is for convenience purpose (It cancels out with 2 when we take the derivative)
    - we only care about the minimizer, which is the θ that minimizes $J(\theta)$. The "actual value" of $J(\theta)$ is not important.
    - This process is called "Optimization"

- Gradient Descent with Matrix Operations
    - $\nabla_\theta J(\theta) = \nabla_\theta \frac{1}{2}(X\theta - \vec{y})^T(X\theta - \vec{y})$
      $= \frac{1}{2}\nabla_\theta \ ((X\theta)^T X\theta \ - \ (X\theta)^T\vec{y} \ - \ \vec{y}^T(X\theta) \ + \ \vec{y}^T\vec{y})$
      $= \frac{1}{2}\nabla_\theta \ (\theta^T(X^T X)\theta \ - \ \vec{y}^T(X\theta) \ - \ \vec{y}^T(X\theta))$
      $= \frac{1}{2}\nabla_\theta \ (\theta^T(X^T X)\theta \ - \ 2(X^T\vec{y})^T\theta)$
      $= \frac{1}{2}(2X^T X\theta - 2X^T\vec{y})$
      $= X^T X\theta - X^T\vec{y}$
    - we want $X^T X\theta - X^T\vec{y} = 0 \ \rightarrow \ X^T X\theta = X^T\vec{y}$
    - $\therefore \theta = (X^T X)^{-1} X^T\vec{y}$
    - if X is non-invertible, it means that there are redundant features, which can result in multicollinearity, or linearly dependent examples.

- Batch vs Stochastic Minibatch
    - Batch: the whole training set
    - Minibatch: randomly select a small subset B of the training data (B << n)
    - calculate a noisy estimation $\theta^{(t+1)} = \theta^{(t)} - \alpha_B \sum_{i \in B} (h_\theta(x^{(i)}) - y^{(i)})x^{(i)}$