LectureNote3: CS221 (FALL2021, Stanford)

More About Optimization

Stochastic Gradient Descent (SGD)

- gradient descent is slow
  - each iteration requires going over "all training examples" $\sum_{(x,y)\in D_{train}} Loss(x, y, w) \leftarrow$ expensive when have lots of data

- Algorithm: Stochastic Gradient Descent
  - initialize w = [0, ... ,0]
  - for t = 1, ... , T:
    - for $(x, y) \in D_{train}$
      - $w \leftarrow w - \eta \nabla_w Loss(x, y, w)$
  - instead of going through all training examples and performing **one** update, perform an update "after each example" $\rightarrow$ *frequent updates can lead to faster convergence*
  - in batch gradient descent, the gradient computation over the entire dataset can be computationally expensive, especially when the dataset is large. SGD reduces this computational burden by estimating the gradient using a single example or a small batch of examples.
    - this makes it particularly useful for large datasets and online learning scenarios, where new data is continuously available, and we need to update the model in real-time
  - stochastic gradient descent can escape local minima more effectively compared to batch gradient descent
    - it introduces noise into the optimization process by using a single example (or a random subset of data) to estimate the gradient, and the noise allows SGD to have a higher chance of escaping shallow local minima
  - trade off: each update is not high quality
    - It's not about *quality*, it's about **quantity**.

- Question: What should η be?
  - small η
    - pros: conservative, more stable
    - cons: slow, might get stuck in local minima
  - big η
    - pros: aggressive, faster
    - cons: might overshoot the minimum, leading to divergence or oscillation
- Strategies:
  - constant: η = 0.1
  - decreasing: η = $\frac{1}{\sqrt{number\ of\ updates\ made\ so\ far}}$

Group DRO

- Inequalities between different groups arise in machine learning because the goal of optimization is to minize the **average loss**

- Linear regression with groups

x	y	$group$
1	4	A
2	8	A
5	5	B
6	6	B
7	7	B
8	8	B

  - predictor $f_w(x) = w \cdot \phi(x)$ does not use the group information
  - Neither $TrainLoss(w)$ nor $Loss(w)$ uses group information

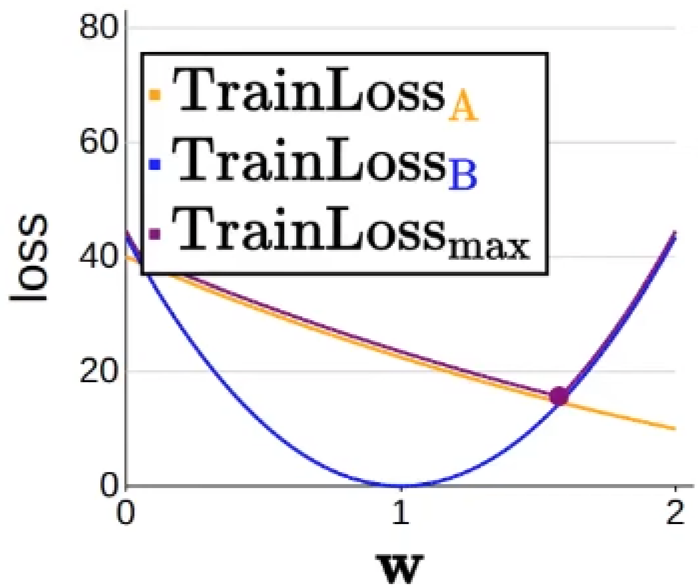
- Per-group loss
  - $TrainLoss_g(w) = \frac{1}{|D_{train}(g)|} \sum_{(x,y) \in D_{train}(g)} Loss(x, y, w)$

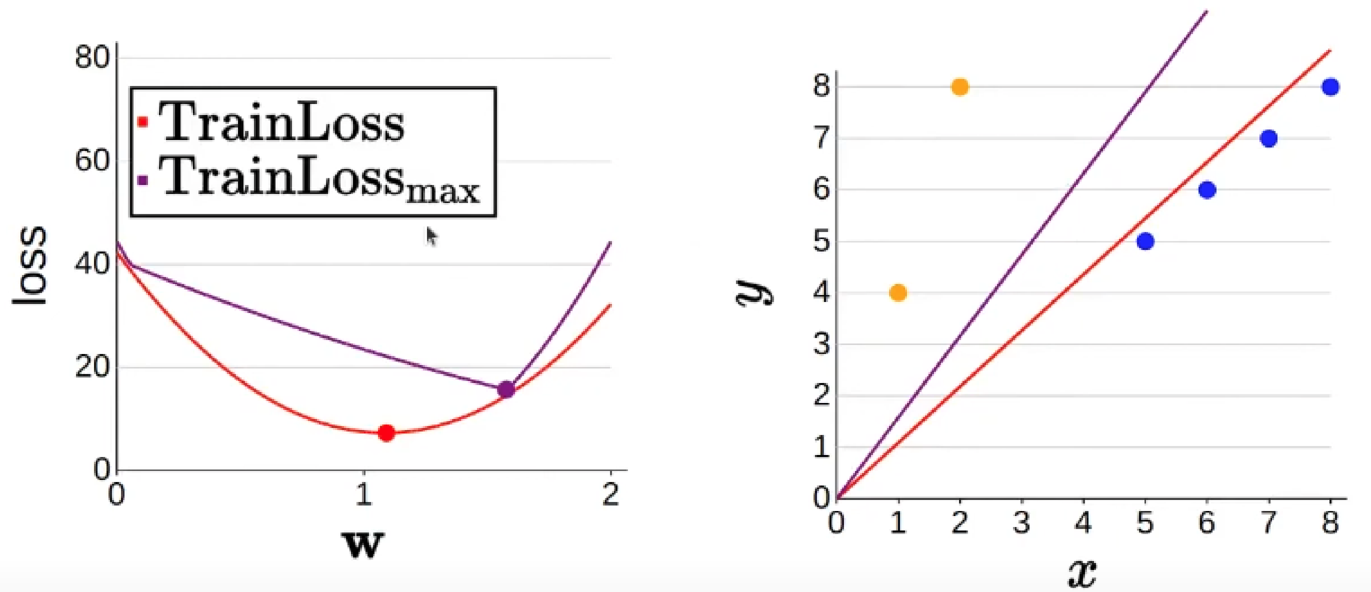
  - **Disparity** in loss between different groups
    - For w = 1
      - $TrainLoss_A(1) = 22.5$
      - $TrainLoss_B(1) = 0$

- Maximum group loss
  - $TrainLoss_{max}(w) = max_g\ TrainLoss_g(w)$

- $TrainLoss_{max}(1) = max(22.5, 0) = 22.5$

- Average group loss vs Maximum group loss





  - Standard learning: minimizer of average loss $w = 1.09$
    - linear regression can result in a biased line that tilts towards a larger size group
  - Group distributionally robust optimization (Group DRO): minimizer of maximum group loss $w = 1.58$
    - treats groups more equally regardless of the size of each group

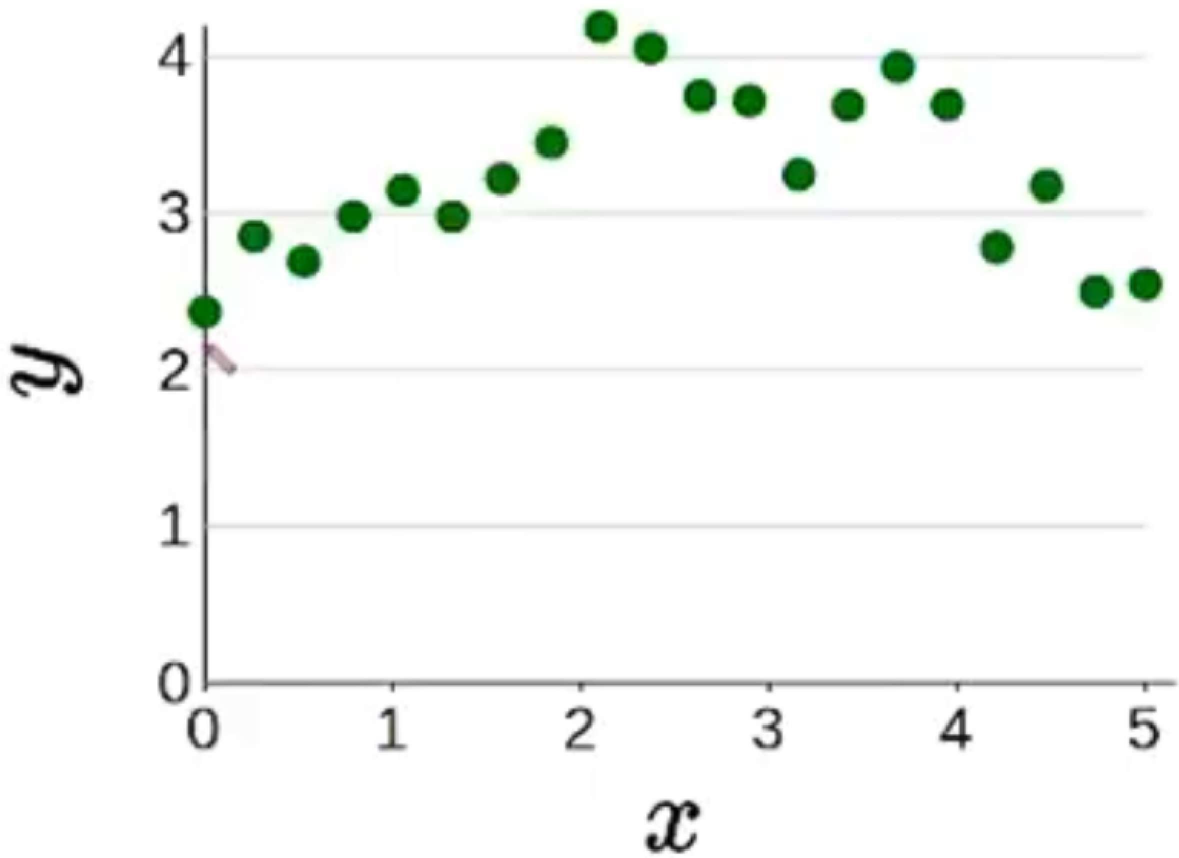
- Training via gradient descent: **minimize the maximum group loss**
  - $TrainLoss_{max}(w) = max_g \, TrainLoss_g(w)$
  - $\nabla TrainLoss_{max}(w) = \nabla TrainLoss_{g^*}(w)$

- $g^* = \underset{g}{argmax} TrainLoss_g(w)$
  - we cannot use SGD because it's a sum over terms, but this is a maximum over a sum

- Possible Issues: intersectinality? don't know groups? overfitting?

Non-Linear Features

- How do we fit a non-linear predictor? *change feature vector ϕ*



i. Qudratic predictors
   - $\phi(x) = [1, x, x^2]$
   - $F = \{f_w(x) = w \cdot \phi(x) : w \in \mathbb{R}^3\}$
   - ex) $f(x) = [2, 1, -0.2] \cdot \phi(x) = 2 + x - 0.2x^2$

ii. Piecewise constant predictors
   - partitioning the input space
   - $\phi(x) = [1[a < x \leq b], 1[c < x \leq d], 1[i < x \leq j], 1[k < x \leq l]]$
   - $F = \{f_w(x) = w \cdot \phi(x) : w \in \mathbb{R}^4\}$
   - ex) $f(x) = [1, 2, 4, 3] \cdot \phi(x)$

iii. Predictos with periodicity structure
   - $\phi(x) = [1, x, x^2, cos(3x)]$
   - $F = \{f_w(x) = w \cdot \phi(x) : w \in \mathbb{R}^4\}$

- **You can just throw in any features you want!**
- Key idea: non-linearity
  - Expressivity: score $w \cdot \phi(x)$ can be a **non-linear** function of x
  - Efficiency: score $w \cdot \phi(x)$ always a **linear** function of w

- Linear in What? **Non-linear predictors with linear machinery**
  - Prediction: $f_w(x) = w \cdot \phi(x)$
    - Linear in w? *Yes*
    - Linear in $\phi(x)$? *Yes*
    - Linear in x? **Not always!**
- This approach allows us to apply the principles, methods, and algorithms developed for **linear predictors** to more complex or **non-linear** problems by utilizing a linear model in a transformed feature space
- Linearity in vector
  i. Homogeneity: $f(\alpha\vec{v}) = \alpha f(\vec{v})$
  ii. Additivity: $f(\vec{v} + \vec{w}) = f(\vec{v}) + f(\vec{w})$
  - *The dot product operation* **preserves linearity** due to its inherent properties
    a. Homogeneity: $(\alpha v) \cdot w = \alpha(v \cdot w)$
    b. Additivity: $(v + u) \cdot w = (v \cdot w) + (u \cdot w)$

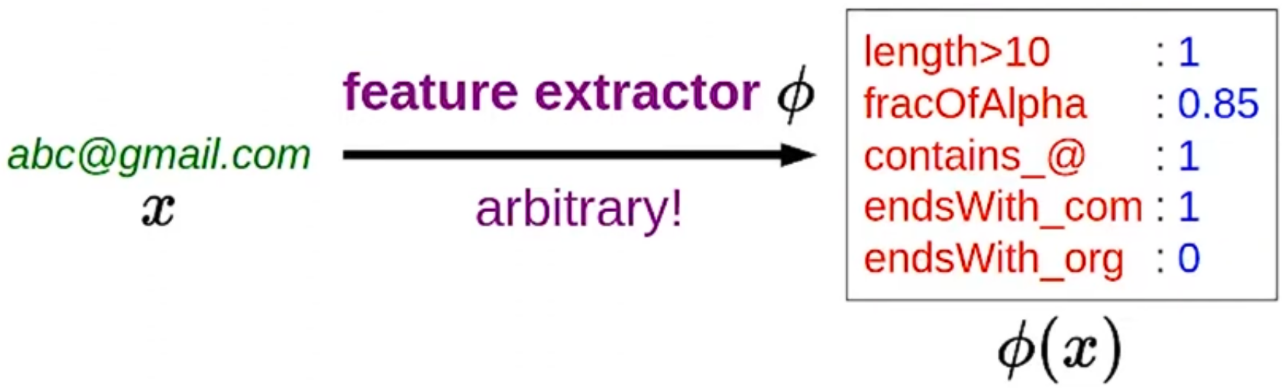
Non-Linear Classifier

- Quadratic classfiers
  - $\phi(x) = [x_1,\ x_2,\ x_1^2 + x_2^2]$
  - $f(x) = sign([2, 2, -1] \cdot \phi(x))$
  - $f(x) = \begin{cases} 1 & if f(x_1 - 1)^2 + (x_2 - 1)^2 \leq 2 \\ -1 & otherwise \end{cases}$
  - decision boundary is a circle
- Visualization in feature space (transforming feature space)

- Input space: $x = [x_1,\ x_2]$, decision boundary is a circle
- Feature space: $\phi(x) = [x_1,\ x_2,\ x_1^2 + x_2^2]$, decision boundary is a line (curve in this example)

Feature Templates

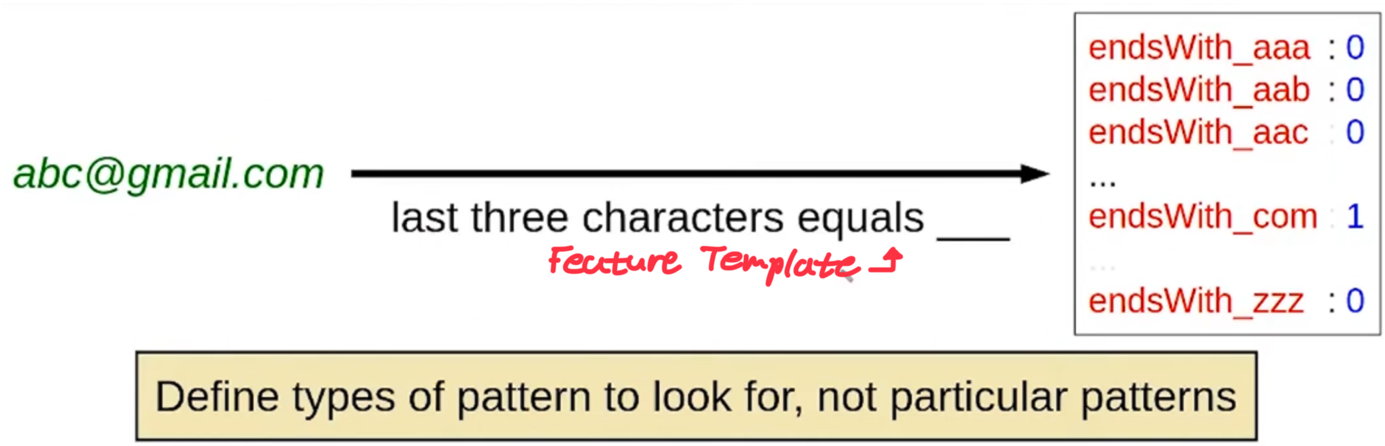
- $F = \{f_w(x) = sign(w \cdot \phi(x)) : w \in \mathbb{R}^d\}$
  - Feature extraction: choose F based on *domain knowledge*
  - Learning: choose $f_w \in F$ based on *data*
  - We want F to contain good predictors but not be too big
- Feature extraction with feature names
  - Question: what properties of x might be relevant for predicting y?
  - Feature extractor: Given x, produce set of (*feature name*, *feature value*) pairs



  - Prediction with feature names

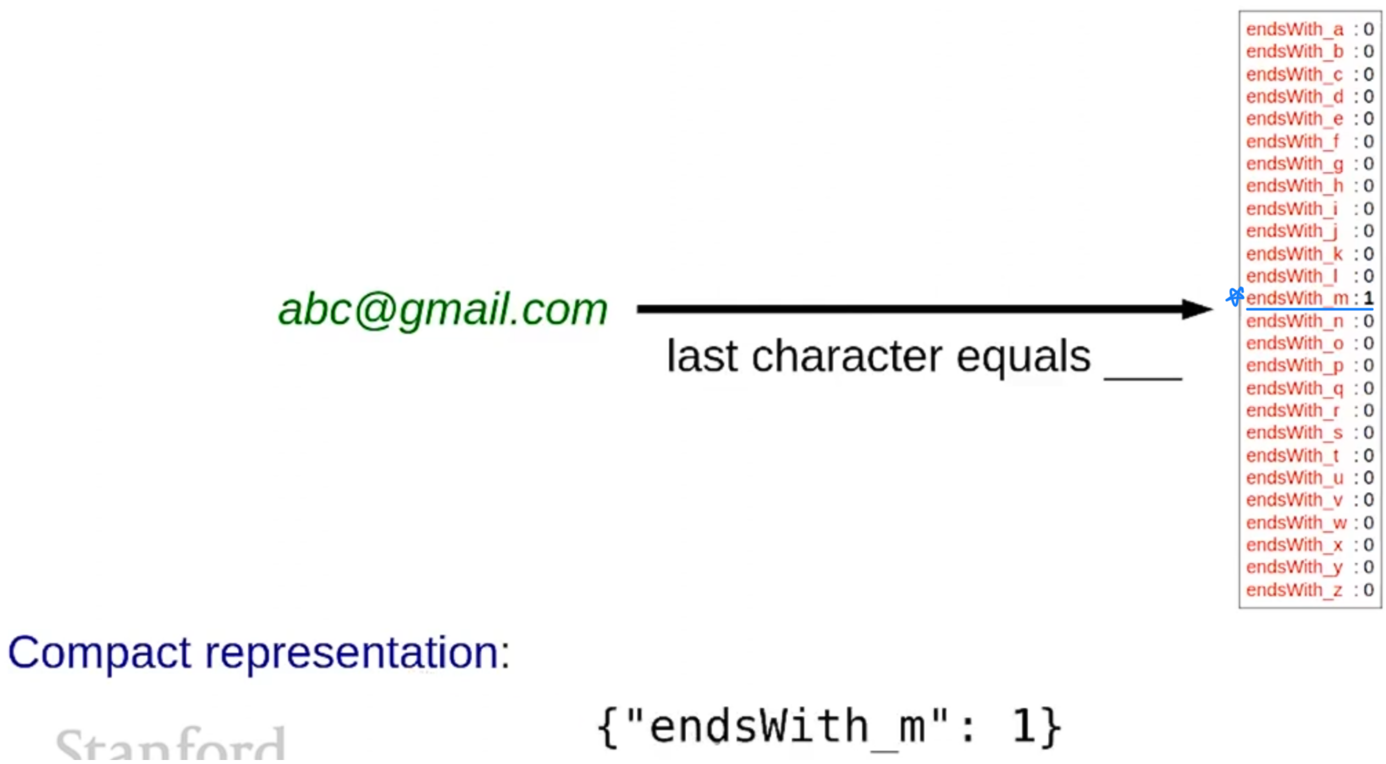
    - Score: weighted combination of features
      - $w \cdot \phi(x) = \sum_{j=1}^{d} w_j \phi(x)_j$
      - each feature is providing a vote
      - if $w_j \phi(x)_j$ is positivie $\rightarrow$ voting in favor of positive classification
      - if $w_j \phi(x)_j$ is negative $\rightarrow$ voting in favor of negative classification
      - the magnitude of w_j determines the strength of the vote

- Feature Templates: a group of features all computed in a similar way



- examples of feature templates
    a. Last three characters equal___
    b. Length greater than___
    c. Fraction of alphanumeric characters

- Sparsity in feature vectors: most feature values are zero



- Two feature vector implemntations
    i. Arrays (good for dense features)
    ii. Dictionaries (good for sparse features)

pixelIntensity(0,0)	0.8
pixelIntensity(0,1)	0.6
pixelIntensity(0,2)	0.5
pixelIntensity(1,0)	0.5
pixelIntensity(1,1)	0.8
pixelIntensity(1,2)	0.7
pixelIntensity(2,0)	0.2
pixelIntensity(2,1)	0
pixelIntensity(2,2)	0.1

fracOfAlpha	0.85
contains_a	0
contains_b	0
contains_c	0
contains_d	0
contains_e	0
...	
contains_@	1
...	

```
[0.8, 0.6, 0.5, 0.5, 0.8, 0.7, 0.2, 0, 0.1]
```

```
{"fracOfAlpha": 0.85, "contains_@": 1}
```