Government-Backed Financing and Aggregate Productivity*

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Abstract

Government-backed financing enhances firms' credit access, helping financially constrained firms but also prolonging the survival of low-productivity firms. These offsetting effects make the net effect of the policy on aggregate productivity ambiguous. I study the effects of government-backed financing by exploiting the Korean government's increase in government loans to firms from 2.25% to 3.12% of the GDP after 2017. I show that credit cost decreased more for firms eligible for government loans relative to ineligible firms. Moreover, eligible firms with higher pre-policy credit costs had larger post-policy increases in investment than eligible firms with lower pre-policy credit costs. However, I also find that the exit rate of unproductive eligible firms decreased the most following the policy. To quantify the effect on aggregate productivity, I build a heterogeneous-firm model with endogenous credit costs incorporating government and private loans. Over a span of 10 years, aggregate productivity decreases by 0.3%, explained by a 0.1% increase coming from higher investment by constrained firms and a 0.4% decrease attributed to the reduced exit rates among low-productive firms.

Keywords: Government-backed financing, financial friction, misallocation, zombie firms
JEL codes: E22, E44, E65

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1. Introduction

Government-backed financing policies, typically in the form of loan guarantees, direct loans, and financial assistance programs, are implemented worldwide to promote firms' investments and growth.[1] However, their net impact on aggregate productivity remains uncertain. These policies have been successful in facilitating funding for financially constrained yet productive firms, potentially enhancing overall productivity (Stiglitz 1993; Banerjee and Duflo 2014; Jiménez, Peydró, Repullo, and Saurina Salas 2018; Díez, Duval, and Maggi 2022).[2] But, they also allow less productive firms to persist, potentially reducing aggregate productivity (Caballero, Hoshi, and Kashyap 2008; Acharya, Lenzu, and Wang 2021; Faria-e-Castro, Paul, and Sánchez 2021).

I study the effects of a significant increase in government loans to firms, addressing the trade-off between increased investment by productive firms and lower exit of unproductive firms. I use a newly constructed dataset of the financial statement of Korean manufacturing firms, which includes data not only from operating firms but also from exiting firms, allowing me to better understand the trade-off. I exploit an unprecedented increase in government loans brought by a change of government in 2017. The government loans to firms rose from 2.25 % of GDP before 2017 to 3.12 % by 2019, as shown in Firugre 1. These loans are only for small-mid sized enterprises, are provided at fixed interest rates below market rates, and also require less collateral than private loans. [3]

[1] For example, Small Business Administration (SBA) Loans in the US, Canada Small Business Financing Program, Small and Medimum Enterprise Financing by Japan Finance Corporation in Japan.

[2] The role of financial constraints in distorting the allocation of capital has been widely studied and highlighted as a major factor in reducing aggregate productivity. See Buera, Kaboski, and Shin 2011, Khan, Senga, and Thomas 2014, Moll 2014, Midrigan and Xu 2014.

[3] To qualify as a small-mid sized enterprises, a firm's total assets must not exceed 380 million USD, and its three-year average annual sales should fall within the 60 to 120 million USD range, with sector-specific sales cutoffs. The government interest rate is, on average, 2.5% lower than the average interest rate on new loans offered by the bank, which stands at 3.7%, despite having a 1.2 percentage point higher delinquency rate than bank loans.

I find that government-backed financing helps financially constrained firms increase investment, thereby increasing aggregate productivity by 0.1% over a span of 10-year through improved capital allocation. However, government-backed financing also helps low-productive firms survive, thereby reducing aggregate productivity by 0.4% over a span of 10-year. In sum, government-backed financing decreases aggregate productivity by 0.3% over a span of 10-year.

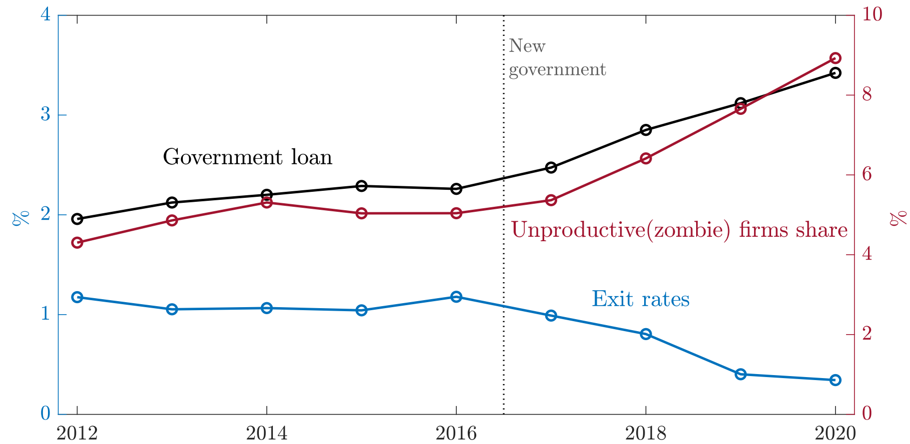
To arrive at these findings, I first document the effects of increased government loans on firms by employing a difference-in-differences analysis with a comparison of the three key outcome variables of eligible and ineligible firms: funding costs (proxied by firm specific credit spreads),[4] investment, and exit. In contrast to many other programs, such as the US Treasury's PPP and CEBA in Canada, the Korean policy did not change in response to a crisis or any unexpected shocks. Instead, it responded to the political platform of the new government, providing me with a clear setting to evaluate the impact of this policy. The primary policy goal of the new government was to create a favorable business environment for small-mid sized enterprises and to promote inclusive growth by leveling the playing field between large and small-mid sized enterprises.

I construct a dataset of Korean manufacturing firms by merging the financial statements of both exiting firms, collected from NICE (National Information & Credit Evaluation) using a list of exiting firms obtained from CRETOP, Korea Enterprise Data, and currently active firms, also sourced from NICE. The dataset covers manufacturing firms with assets over 9 million USD, subject to external audits. Among the sample firms, 84% are non-listed firms, while 86% are small-mid sized firms that are eligible for government loans.

I find that after the policy shift, the cost of funding decreased more for eligible firms than for non-eligible firms, controlling for the firms' financial states. I take this

[4] I measure cost of funding based on credit spread calculated by the deviation of interest rates a specific firm pays from the Korea corporate bond yield (3yr, AA-).

FIGURE 1. Government loans, exit rates and share of unproductive firms



Notes: Government loans, which represent loans granted to private non-financial enterprises as a percentage of GDP, are indicated on the left-hand side. Exit, which indicates business closures, is also represented on the left-hand side. A firm is identified as a zombie firm if its interest coverage ratio (i.e., the ratio of operating income to interest expenses, ICR) has been less than one or its operating profit is negative for at least three consecutive years and if it is at least 10 years old. Zombie shares are represented on the right-hand side.
Sources: National Information & Credit Evaluation (NICE), Bank of Korea flow of funds statistics, Author's calculation

as an indication of improved credit access for eligible firms. Moreover, eligible firms with higher pre-policy funding costs exhibited a 5-percentage-point greater increase post-policy than eligible firms with lower pre-policy funding costs, consistent with the findings of Banerjee and Duflo (2014) and Jiménez, Peydró, Repullo, and Saurina Salas (2018).

Moreover, improved credit access reduced firms' exits, with a more pronounced effect on unproductive firms. This led to a decrease in exit rates and a higher share of unproductive firms, which are labelled as zombie firms, as depicted in Figure 1 with the increasing trend in the government loans after the new government. Specifically, I classify firms as zombie firms if they are older firms persistently incapable of servicing their debt with their operating profit, in line with the definition of zombie firms widely used by the literature.[5] Additionally, I find that the exit rate of unproductive eligible firms decreased by 2.8 percentage points more than that of productive eligible firms. This finding aligns with the literature that studies the rise of zombie firms and credit misallocation resulting from government-subsidized loans (Acharya, Lenzu, and Wang 2021), as well as some features of financial intermediation such as forbearance lending (Tracey 2019) and relationship lending (Faria-e-Castro, Paul, and Sánchez 2021).

To quantify the aggregate effect of government-backed financing on productivity, I build a heterogeneous-firm model with government-back financing. The model extends Arellano, Bai, and Kehoe (2019) and features heterogeneous intermediate goods firms that produce homogeneous product using capital as an input. They can borrow from private creditors and the government. Firms with insufficient operating profit to repay debt default and exit.

Private creditors require to be compensated for holding defaultable debt, and so financial frictions arise endogenously, resulting in a borrowing constraint that is more restrictive for liquidity-strapped firms. Due to this friction, firms with less cash-on-hand tend to invest less compared to firms with more cash-on-hand, even when other financial factors such as size and profitability are taken account, as in Khan, Senga, and Thomas (2014). Firms with low cash-on-hand need to resort to higher borrowing to achieve the same level of investment as high cash-on-hand firms, resulting in a higher default risk and funding costs. Furthermore, these financial frictions heighten the vulnerability of firms with limited cash-on-hand, placing them at a greater risk of default and subsequent exit from the market.

Crucially, the government does not require compensation for default risk. Government loans are subsidized, have a fixed limit, and are partially contingent on

[5] I follow McGowan, Andrews, and Millot (2017), R. Banerjee and Hofmann (2018), Hong, Igan, and Lee (2021), and define a firm as a zombie firms if its interest coverage ratio (i.e. the ratio of operating income to interest expenses, ICR) has been less than one or its operating profit is negative for at least three consecutive years and if it is at least 10 years old.

firms' operating profits: if a firm is unable to fully repay, the government forgives a portion of the remaining debt. In this way, the loans relax firms' financial constraints. Additionally, the state-contingent nature of the government loans insures firms against idiosyncratic transitory shocks.

Government loans not only help firms increase investments but also assist firms that would have otherwise exited the market in surviving, leading to a higher prevalence of low-productive firms. This generates a general equilibrium effect similar to congestion externalities studied in Caballero, Hoshi, and Kashyap (2008), Acharya, Crosignani, Eisert, and Eufinger (2020), and Acharya, Crosignani, Eisert, and Steffen (2022).[6] As the loans enable low-productive firms to accumulate more capital and increase production, there is downward pressure on the equilibrium price of intermediate goods, reducing firms' profitability. This effect is not only damaging to the profitability of operating firms but also discourages potential entrants from entering the market.

I calibrate the model to match pre-policy aggregate moments of the Korean firm data, and measure the aggregate effect of government-backed financing. The calibrated model matches untargeted cross-sectional moments based on firms' net-income ratios. For my main exercises, I simulate the full transition of the economy after the introduction of government loans. Using this simulation, I replicate the regression analysis described above, examining the response of firms' investment and exit.

The model effectively captures the firm-level heterogeneity in responses to government loans in terms of investment and exit, closely mirroring the patterns documented with the data.Firms initially characterized by higher pre-policy credit spreads increase their investment by 4 percentage points more than firms with lower

[6]Contrary to empirical studies demonstrating a negative spillover effect of zombie firms on non-zombie firms, where the relative performance of healthy firms deteriorates as the fraction of zombies increases, Schivardi, Sette, and Tabellini (2020) argue that there is no causal relationship. They posit that the relative performance of non-zombie firms worsens due to aggregate shocks, leading to a larger share of zombie firms. In my model, there is no explicit negative spillover effect resulting from a larger share of unproductive (zombie) firms. Instead, improved access to credit expands production capacity, both intensively and extensively, exerting downward pressure on prices through general equilibrium effects.

pre-policy credit spreads in simulated data, while the data show a 5 percentage point higher increase. Additionally, the simulated data show that unproductive firms' exit rate decreases by 2.3 percentage points more than productive firms, whereas the data indicate a larger decrease of 2.8 percentage points for unproductive firms compared to productive firms. Indeed, the main trade-off of government loans on aggregate productivity through enhanced credit access is well summarized by these two heterogeneous responses.

I also look at the behavior of zombie firms to better understand the effect of government loans on productivity. In particular, I look at transition rates for firms becoming zombie firms. The calibrated model generates the observed transition probabilities in the data, even though it was not explicitly targeted. The general equilibrium effect induced by the government loans reduces firms' overall profitability and increase the share of cash-strapped firms. Normal firms are more likely to transition into zombie firms, and zombie firms are more likely to remain as zombie firms. These factors collectively lead to a greater share of zombie firms in the economy.

To quantify the aggregate effect on productivity, I decompose aggregate productivity into two components: capital allocation efficiency labelled as "intensive efficiency" and the composition of productivity labelled as "extensive efficiency".[7] As government loans assist firms with low cash-on-hand in increasing their investments, this leads to an improvement in intensive efficiency. However, the government's intervention changes the extensive margin and worsens the selection process, resulting in a decrease in extensive efficiency. The loss from extensive efficiency (0.4%) outweighs the gain from intensive efficiency (0.1%). Consequently, the economy experiences a decrease in productivity over 10 years by 0.3%.

[7]The intensive efficiency equals 1 when capital is distributed across firms in a way that equalizes the marginal product of input across firms.

2. Korean Policy and Firm Level Data

2.1. Korean government-backed financing policy

The Korean government has long provided financial support to small and medium-sized enterprises (SMEs) primarily through loan guarantee programs and government loans. In practice, various government ministries and agencies, as well as local governments, raise funds through budget allocation, borrowing from public funds, and bonds issuance to extend financial assistance to SMEs under favorable terms.

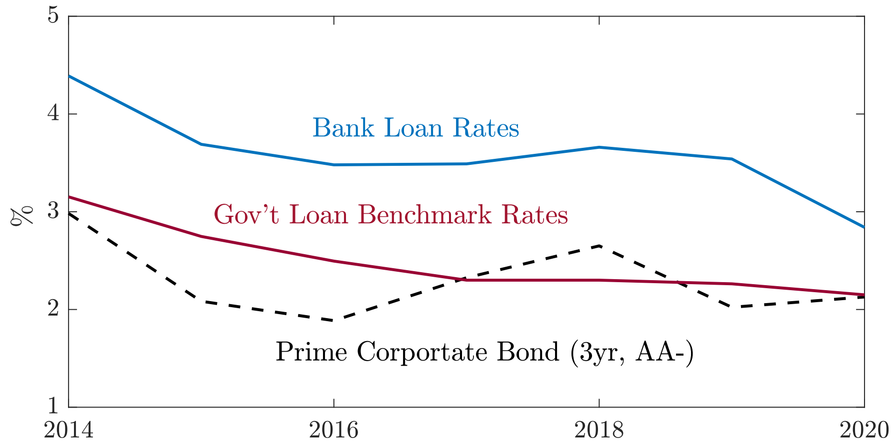
The change in government in 2017 brought a significant policy change, resulting in an unprecedented increase in government loans. This is reflected in the the upward trend of government loan amounts, as indicated in in Figure 1. Specifically, the loan amount was 2.25 % of GDP on average for 2014∼16, which rose to 2.85 % as of 2018, and 3.12 % as of 2019, respectively. The primary goal of the new government was to create a favorable business environment for SMEs and promote inclusive economic growth by leveling the playing field between large and small to medium-sized firms.[8]

Government loans target small-mid sized firms, whose status is determined by criteria defined in the legislation.[9] To qualify as a SME, a firm's total assets must not exceed 380 million USD, and its three-year average annual sales should fall within the 60 to 120 million USD range, with sector-specific sales cutoffs. Moreover, even if a firm meets the SME size benchmarks, it must maintain separation from ownership and management entities known as Chaebols, such as Samsung or Hyundai in Korea. Firms exceeding size requirements can access government loans for three years after they

[8] Promoting SMEs was included as major 100 policy tasks (released in 2017). Moreover, the Small and Medium Business Administration was promoted to ministry status in 2017, becoming the Ministry of SMEs and Startups.

[9] Government loan programs encompass a diverse array of types, ranging from initiatives addressing management challenges like cash shortages to those bolstering innovation and promoting exports. These programs involve a range of institutions, including government ministries, agencies, and local governments. The specific eligibility criteria can differ based on the involved institutions and specific programs. However, a common prerequisite for eligibility is SME status.

FIGURE 2. Benchmark rates of the government loans



Notes: The interest rate for a loan is determined by making adjustments relative to the benchmark interest rate, considering factors such as the firm's credit rating, the intended use of the funds, and the presence of collateral. Bank loan rates represent the average interest rates applied to newly issued loans to firms. Prime corporate bond rates are the yields of corporate bonds with a maturity of 3 years and a credit rating above AA-.
Sources: Korea SMEs and Startups Agency, Bank of Korea

exceed those requirements.

The government loans are commonly provided up to a specific limit at a fixed interest rate, lower than the market borrowing rates.[10] The interest rates for government loans are determined based on adjustments made around the benchmark interest rate presented in Figure 2. The adjustment is determined by taking into account factors such as the credit rating of the company, the purpose of the funds, and the presence of collateral.

2.2. Financial statements of Korean manufacturing firms

I construct dataset of financial statements from Korean manufacturing firms on operating and exiting firms. This inclusion of exiting firms allows me to observe their

[10]The government interest rate is, on average, lower than the average interest rate on new loans offered by the bank, despite having a higher delinquency rate than bank loans. There is no publicly available data on the delinquency rate of the government loans themselves. However, the government agency related to loan guarantee releases data on the delinquency rate for this loan guarantee, which is 3.7% lower than the delinquency rate for bank loans, which stands at 2.0%.

financial condition at the time of exit and provides a better understanding of the heterogeneous effects on firms' extensive margins to the increase in government loans. The dataset on active firms comes from the NICE (National Information & Credit Evaluation). I constructed the list of exiting firms based on information obtained from CRETOP, Korea Enterprise Data, and then collected firms' financial information from the NICE using the list.

The data covers manufacturing firms with assets over 9 million USD, subject to external audits and required to release their balance sheet information to the Financial Supervisory Commission.[11] Revenue of sample firms accounts for approximately 80% of total sales. Sample firms consist of 12,976 active firms and 1,593 exiting firms. The majority of sample firms are non-listed firms (12,857) and are small-medium firms (12,461) that are eligible for government loans. The categorization of non-listed firms and small-medium firms is determined based on each firm's status in the year 2020. [12] The indicator of small-mid sized firm is also subject to external audit. A firm classified as a small-mid sized firm is a firm which is officially confirmed to be eligible for government program for SMEs.

The key variables used for the analysis include credit spreads, investment, and an exit indicator. Credit spread is defined as the deviation of interest rates paid by a specific firm from the Korea corporate bond yield (3yr, AA-). The interest rates of a specific firm are calculated using the total amount of debt and the total amount of interest expenses paid for a specific year. Tangible asset growth is employed as a proxy for investment. The exit indicator denotes whether a firm has publicly announced its closure, excluding cases of merger with other firms.

[11] Firms are allowed to enter, exit, and pause reporting for several years during the sample period when their assets go below the threshold.

[12] Usually, only a small portion of SMEs undergo the transition to become large firms. Specifically, on average, this transition rate amounts to just 0.004% for the period between 2017 and 2019.

3. Effects of Government-Backed Financing

In this section, I document the effects of the Korean government's expansionary credit policy on firms' funding costs (subsection 3.1), investment (subsections 3.2), and exit (3.3). These findings help us understand the main trade-offs of the policy in terms of overall productivity through improved credit access. By examining the impact on funding costs, we can assess whether the policy improved firms' credit access. Analyzing the impact on investment allows us to determine whether firms that were financially constrained have increased their investments, which can potentially enhance aggregate productivity by optimizing capital allocation across firms. Furthermore, exploring the effect on firms' exit informs us about whether the policy led to the more survival of unproductive firms, which could potentially undermine aggregate productivity due to a worsened composition of firms' productivity.

To conduct this analysis, I employ a difference-in-difference approach. I compare changes in these three key outcome variables between eligible and ineligible firms, and also consider how financial characteristics within eligible firms may impact these outcomes. Unlike some other programs like the US Treasury's PPP and Canada's CEBA, which responded to crises or unexpected shocks, Korean government-backed financing was influenced by the political agenda of the new government. This feature provides a clear framework for evaluating the policy's impact through the difference-in-differences analysis.

3.1. Effect on funding costs

I first investigate the effect of government loans on firms funding cost, measured as credit spread calculated by the deviation of interest rates a specific firm pays from the Korea corporate bond yield (3yr, AA-). The objective is to ascertain whether the increased government loan after the new government reduced financing cost for eligible firms on average. For this analysis, I estimate the following equation using data from 2014 to 2019:[13]

$$\text{Spread}_{ist} = \alpha + \sum_{k \neq 2016} \beta^k \text{Year}_k D_{is}^{sme} + \gamma^x X_{ist-1} + \gamma_{st} + \gamma_i + \epsilon_{ist} \tag{1}$$

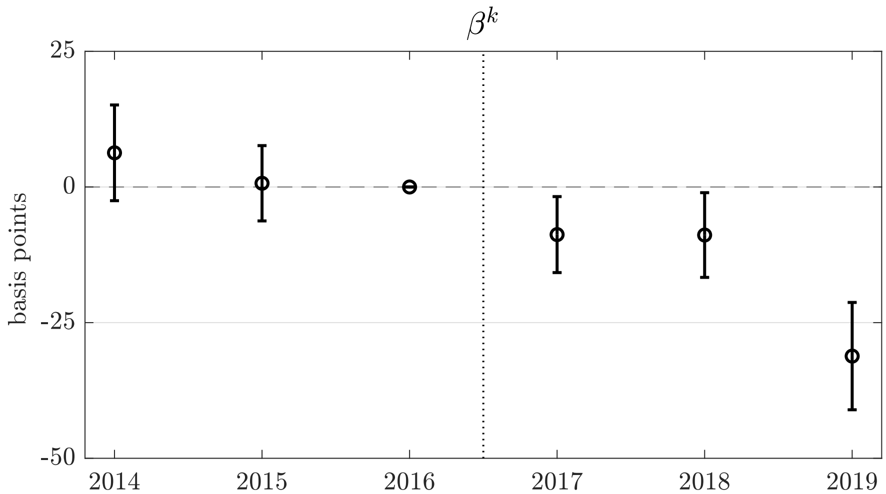
where Spread_{ist} is a firm i's credit spread in sector s for year t in basis points, Year_k is a dummy for year k, D_{is}^{sme} is an indicator for whether a firm is a SME, γ_{st} is sector-year interacted fixed effects, γ_i is a firm fixed effect, X_{ist-1} is a vector of firm specific controls including equity to asset ratio, debt to asset ratio, cash to asset ratio, operational profit to asset ratio. I drop the dummy for the year 2016 (one year before the new government).

The results, which can be summarized with the coefficient β^k in Figure 3, show that following the policy change, the spread for the eligible group decreased more significantly than for the non-eligible group. To be specific, the coefficient β^k represents the difference in the spread gap between SMEs and large firms for a specific year relative to the year 2016. Therefore, the spread gap between the eligible and non-eligible firms decreased after the policy change.

I next investigate the impact of eligibility for government loans on credit spread sensitivity to firms' indebtedness. The objective is to study whether the credit spreads given the same level of leverage decreased after the policy, and how this feature would depend on eligibility for the policy. Specifically I estimate the following equation for

[13] This sample period is used for all empirical analysis. This time frame was chosen in consideration of shifts in macroeconomic conditions within Korea. Years before 2014 were excluded due to substantial monetary easing measures that had already taken place in Korea. Furthermore, the year 2020 was omitted from the analysis due to the onset of the Covid-19 pandemic. During the 2014-2016 period characterized by a steady increase in government loans, the average key interest rate in Korea was 1.6%, whereas in the 2017-2019 period marked by a significant uptick in government loans under the new administration, the average key interest rate in Korea stood at 1.5%.

FIGURE 3. Effect on credit spread by eligibility



Note: These plots show a difference in the spread gap between SMEs and large firms for specific years relative to year 2016 with 90% confidence intervals. Estimates from equation 1 represented in basis points.

two sub-periods, Before (2014-16) and After (2017-19):

$$\text{Spread}_{ist} = \beta_0 \text{Debt Ratio}_{ist-1} + \beta_1 D^{sme}_{is} \text{Debt Ratio}_{ist-1} \text{After}_t + \beta_2 \text{Debt Ratio}_{ist-1} \text{After}_t + \gamma_{st} + \gamma_i + \epsilon_{ist} \quad (2)$$

where Debt ratio$_{ist}$ is firm i's debt to asset ratio in sector s for year t, After$_t$ is a dummy after the policy, and all other specifications are same with equation (1).

The results indicate that the sensitivity of credit spread to the debt ratio decreased for eligible firms post-policy, while there was no change for non-eligible firms, as presented in Table 1. Specifically, the estimate for coefficient β_0 indicates that a one-percentage-point increase in the debt ratio is associated with an average increase of 0.35 basis points in credit spread. β_2 indicates the change in credit spread sensitivity to the debt ratio for the non-eligible group (large firms) after the policy change. The non-significantly different from zero estimate of β_2 suggests that the policy did not have a significant impact on the sensitivity of credit spread to the debt ratio for non-eligible firms. On the other hand, the significantly negative β_1 indicates that the sensitivity of credit spread

TABLE 1. Credit Spread Sensitivity to Debt Ratio

		Spread
β_0	Debt Ratio	0.35***
		(0.10)
β_1	Debt Ratio $\times$ SME$\times$ After	-0.26***
		(0.10)
β_2	Debt Ratio $\times$ After	-0.04
		(0.13)

Notes: Estimates from equation 2 represented in basis points. Standard errors in parentheses are clustered by firms. *** $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

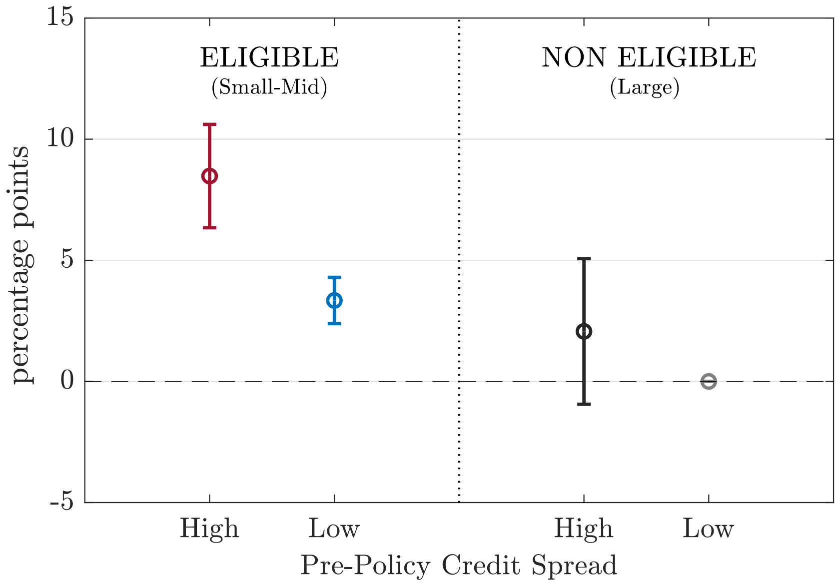
to the debt ratio decreased for eligible firms post-policy. Specifically, following the policy change, credit spreads for SMEs increased less by 0.26 basis points for each one-percentage-point increase in the leverage ratio, while there was no change for large firms.

I interpret these empirical results as an indication of improved credit access for eligible firms following the policy change. Next, I explore questions related to how this improved credit access has influenced investment and exit.

3.2. Effect on investment

I investigate whether the policy helps firms expand investment by reducing funding costs. To do this, I compare changes in investment among four groups categorized by eligibility and pre-policy credit spread. Firms with high pre-policy credit spreads, after controlling for other financial characteristics, may have faced higher funding costs for investments compared to firms with lower pre-policy credit spreads. Given that the policy shift improved credit access for eligible firms, it is expected that firms with higher funding costs, which may have been constrained in their ability to invest up to their efficient scale, would increase their investment relative to firms with lower

Figure 4. Effect of policy on investment



Note: The figure plots the coefficient β_1, β_2, and β_3 respectively from the equation 3 with 90% confidence intervals. Eligible and non eligible groups are divided by the small-mid sized enterprises indicator. Firms in high group are firms whose credit spread in 'Before' preiod (2014-2016) was in 10th percentile.

funding costs. Specifically, I estimate the following equation:

$$\begin{aligned}\text{Investment}_{ist} = \; & \beta_1 D^{sme}_{is} D^{High}_{is} \text{After}_t + \beta_2 D^{sme}_{is}(1 - D^{High}_{is})\text{After}_t \\ & + \beta_3 (1 - D^{sme}_{is}) D^{High}_{is} \text{After}_t + \gamma^x X_{ist-1} + \gamma_{st} + \gamma_i + \epsilon_{ist}\end{aligned} \tag{3}$$

where D^{High}_{is} is an indicator whether a firm i's mean spread of Before period is in the upper 10th percentile, X_{ist} is a vector of firm specific controls including log of tangible asset, and operating profit to asset ratio to control for firms' marginal benefit to investment.

The eligible group exhibited an average increase in investment and this increase was more pronounced among firms with high pre-policy credit spread. Specifically, firms that initially paid higher credit spreads increased their investment by 5 percentage points more than firms with low pre-policy credit spreads.[14] On the contrary, no significant effect was observed among the non-eligible group. The results are illustrated in Figure 4.

To assess robustness, I conducted an event study using the same specifications applying year dummies separately for SMEs and large firms. Furthermore, the model was estimated using pre-policy credit spread values directly, without the use of dummy indicators. The outcomes remain consistent with the previously outlined results and are provided in Appendix A1.

3.3. Effect on exit

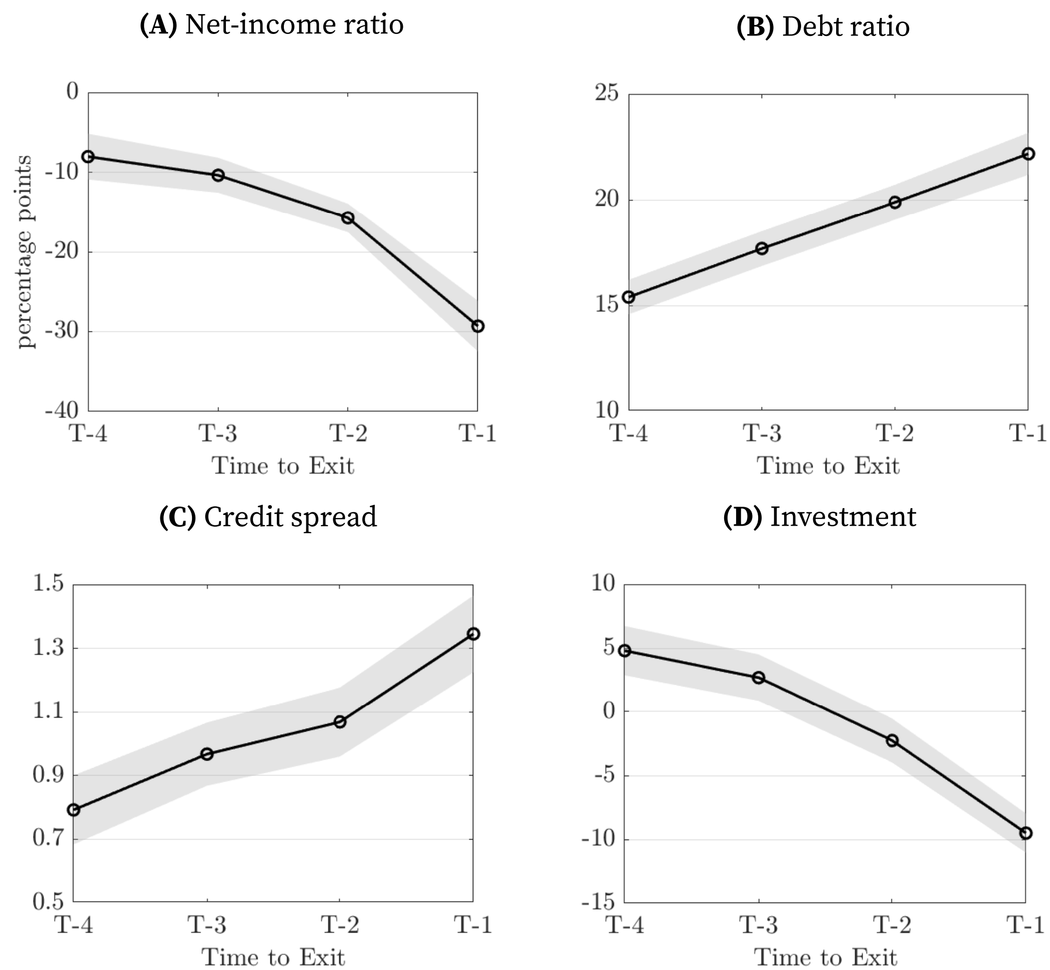
Firms' exit rates decreased from 1.4% during the Before periods (2014-2016) to 0.9% during the After periods (2017-2019), as presented in Figure 1. Importantly, the exit rates had remained stable during the periods preceding the change of policy. Government-backed financing reduces firms' exits by changing the extensive margin and mitigating the driving forces behind firms' exits. [15] To see the financial states forcing firms to exit, firms frequently undergo a cash shortage, an upsurge in debt ratio, increased credit spreads, decreased investment as they approach closure. This trend suggests that firms facing a sustained cash shortage face difficulties in obtaining financing, ultimately leading to their exit. See Figure 5.

Exit rates decreased particularly more among unproductive firms, which are often situated at the margin of potential exit. To classify these unproductive firms, I drew upon existing literature centered around zombie firms, a concept encompassing

[14]This finding aligns with the findings of Banerjee and Duflo (2014), who demonstrate that constrained firms tend to use government credit to expand production, while unconstrained firms primarily use it as a substitute for other borrowing. In my analysis, the pre-policy credit spread serves as an approximation of firms' financial constraints and suggests that more constrained firms increased their investment to a greater extent.

[15]This trend is consistent with the findings documented by Caballero, Hoshi, and Kashyap (2008), Acharya, Crosignani, Eisert, and Steffen (2022), and Faria-e-Castro, Paul, and Sánchez (2021), who show that improved credit access decreases firms' exits.

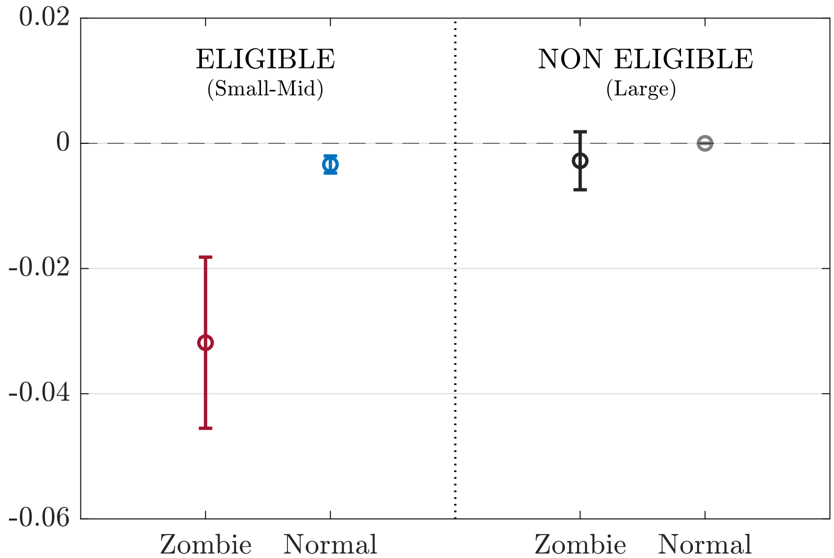
FIGURE 5. Financial state before exit



Notes: These plots show the relative financial state of firms with specific distance to exit. Specifically, those are series of coefficient of $y_i = \alpha + \sum_{k=1}^{4} \beta_k D_i^{T-k} + \epsilon_i$, where D_i^{T-k} is an indicator whether a specific firm i closes down and exits after k periods. The shaded area indicates the 90% confidence interval.

companies that have barely survived thanks to government financial assistance. Following McGowan, Andrews, and Millot (2017), R. Banerjee and Hofmann (2018), and Hong, Igan, and Lee (2021), I define zombie firms as those whose interest coverage ratio (ICR), i.e., the ratio of operating income to interest expenses, has remained below one, or those that have sustained negative operating profits for a minimum of three consecutive years, provided they are at least 10-year old. The share of zombie firms is depicted in Figure 1. We can observe that the share of zombie firms, which was stable

FIGURE 6. Effect of policy on exit probability



Note: The figure plots the coefficient β_1, β_2, and β_3 respectively from the equation 3 with 90% confidence intervals. Eligible and non eligible groups are divided by the small-mid sized enterprises indicator.

prior to the policy change, experienced a significant increase following the policy implementation.

I also estimate the following regression to assess whether the decrease in exit rates was more pronounced among unproductive firms, using four groups categorized based on eligibility for the policy and a zombie indicator in the previous year:

$$\text{Exit}_{it} = \beta_1 D_i^{sme} D_{it-1}^{Zombie}\text{After}_t + \beta_2 D_i^{sme}(1-D_{it-1}^{Zombie})\text{After}_t + \beta_3(1-D_i^{sme})D_{it-1}^{Zombie}\text{After}_t + \gamma_t + \epsilon_{it} \tag{4}$$

The eligible firms exhibited an average decrease in exit rates, and importantly, this decrease in exit rates was particularly larger for unproductive firms, specifically those categorized as zombie firms. By contrast, there was no significant change in exit rates within the non-eligible firms. See the result depicted in Figure 6.

Furthermore, I explored the policy's effects in relation to exit rates and credit spreads.

Firms with initially high pre-policy credit spreads were more likely to exit in the Before period, which dampens the magnitude of change in their exit rates. To provide a more comprehensive analysis, I adopted a three-year average credit spread instead of pre-policy credit spread. I categorized firms into four groups based on their eligibility and their lagged three-year mean credit spread. This analysis is aimed to document how the policy influenced the exit threshold associated with credit spreads, not the treatment effect on a specific group. The result shows no discernible effect on the non-eligible group, a decrease in exit rates among the eligible group, and a particularly pronounced decrease in exit rates among those firms that historically paid high credit spreads. For a detailed results, refer to Appendix A2.

4. Model

I develop a heterogeneous firm model in order to interpret this cross-sectional evidence and study its aggregate implications, mainly based on Arellano, Bai, and Kehoe (2019).

Time is discrete and infinite. There is no aggregate uncertainty, and in the following Sections, I study the how the economy would response to the introduction of the government loan. There are continuums of final goods firms, intermediate goods firms, private creditors, and the government. The final goods firms convert homogeneous intermediate goods into a final good and sell them at price 1. The intermediate goods firms are competitive and produce homogeneous product using capital as an input. They can borrow to finance their operating costs from private creditors and the government when the government loan is in place. In case it is infeasible for firms to pay the operating costs and debts, firms default and exit market with zero value.

Before formally describing the economy, I provide a brief overview of the timeline. At the beginning of each period, intermediate goods firms are subject to two idiosyncratic shocks: one is persistent while the other is i.i.d, which determine their production.

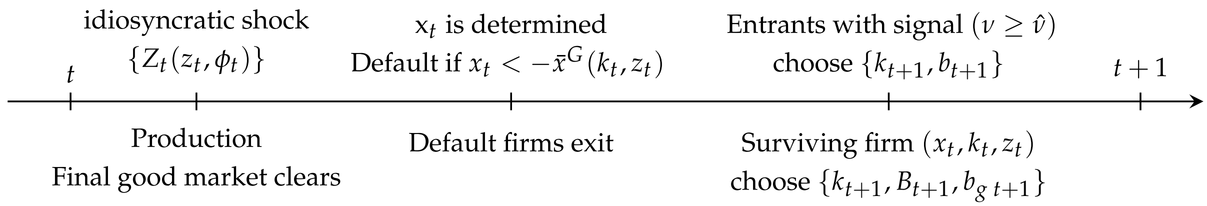


FIGURE 7. Timeline

Firms sell their outputs to final goods firms and the final goods market clears. The level of cash-on-hand is determined by revenue, operating costs and debt. Based on their cash-on-hand levels, feasibility to continue operating is determined. At the end of the period, potential entrants receive a signal about their productivity in the following period. Surviving incumbent firms and entering firms then make decisions on borrowing and capital. The timeline of the economy is summarized in Figure 7.

4.1. Final Good Firms

The final good firms produce final goods y_F using Y as an input to maximize,

$$\max_{Y} \underbrace{\overline{z} Y^{\alpha_y}}_{y_F} - pY \tag{5}$$

where, p is the price of intermediate good and $\overline{z}$ is the average productivity of intermediate good firms, which are both endogenously determined by intermediate good firms' decisions. The total is determined not only by the quantity of intermediate good Y but also by the productivity composition of intermediate good firms $\overline{z}$, which I formally describe in subsection 4.2. First order condition gives the demand function for intermediate goods,

$$p = \overline{z} \alpha_y Y^{\alpha_y - 1}. \tag{6}$$

4.2. Intermediate Goods Firms

Environment Intermediate goods firm produce a homogeneous product y_t using capital k_t, and sell it to final good firms at price p. They face two types of idiosyncratic productivity shocks. One is persistent, z_t that follows $AR(1)$ process

$$\log z_t = \rho_z \log z_{t-1} + \sigma_z \varepsilon_{z,t} \tag{7}$$

where the innovation $\varepsilon_{z,t} \sim N(0,1)$ are independent across firms, and independent of ϕ_t which is the other shock that is i.i.d. The productivity in period t is determined as $z_t \exp(\phi_t)$. The production also requires an operating cost, which consists of two components: a fixed cost f, and a cost proportional to the capital stock $f_k k_t$. As a result, firms' operating profit is:

$$\pi_t = p z_t \exp(\phi_t) k_t^{\alpha} - f - f_k k_t \tag{8}$$

Government loans and default rule Government loans have two distinct features. Firstly, the government extends loans to firms at risk-free rate, $q_g = \beta$, up to a specific limit. Secondly, the government's payment is contingent on firms' cash shortage. Cash shortage is determined as the sum of cash-on-hand after full repayment and the maximum funds a firm can raise. Specifically, when firms fully repay their debt, the cash-on-hand is determined as follows:

$$x_t^{FR}(k_t, \overbrace{B_t}^{b_t+b_{gt}}, z_t, \phi_t) = (1-\tau)\, p z_t \exp(\phi_t)\, k_t^{\alpha} - (f + f_k k_t) - b_t - b_{gt} + \tau(\delta k_t + r_f(b_t + b_{gt})) \tag{9}$$

The cash-on-hand with full repayment is calculated by starting with the after-tax revenue and subtracting the operating costs, repayments on private and government loans. Additionally, the calculation takes into consideration the tax benefits associated with the depreciation of capital and debt.[16]

The maximum fund a firm can raise by borrowing from private lender and the government, and capital disposal is as follows:

$$\begin{aligned} \overline{x}^G(k,z) &= \max_{k_{t+1},b_{t+1},b_{gt+1}} q(k_{t+1},b_{t+1},b_{gt+1},z_t)b_{t+1} + q_g b_{gt+1} - \psi\left(k_t,k_{t+1}\right) \\ s.t. \quad & b_{g\,t+1} \leq \overline{b_g} \end{aligned} \tag{10}$$

where $\psi(k_t, k_{t+1})$ is capital investment and associated adjustment cost:

$$\psi(k_t,k_{t+1}) = \begin{cases} (k_{t+1} - (1-\delta)k_t) + p_k^+ \frac{(k_{t+1}-(1-\delta)k_t)^2}{2(1-\delta)k_t} & \text{if} \quad k_{t+1} - (1-\delta)k_t \geq 0 \\ \\ (k_{t+1} - (1-\delta)k_t) + p_k^- \frac{(k_{t+1}-(1-\delta)k_t)^2}{2(1-\delta)k_t} & \text{if} \quad k_{t+1} - (1-\delta)k_t < 0 \end{cases} \tag{11}$$

b_{t+1} is borrowing from private lender, and q_t is the private debt price, which is discussed in the following subsection 4.3. b_{gt+1} is borrowing from the government, and q_g is the debt price of the government loan.

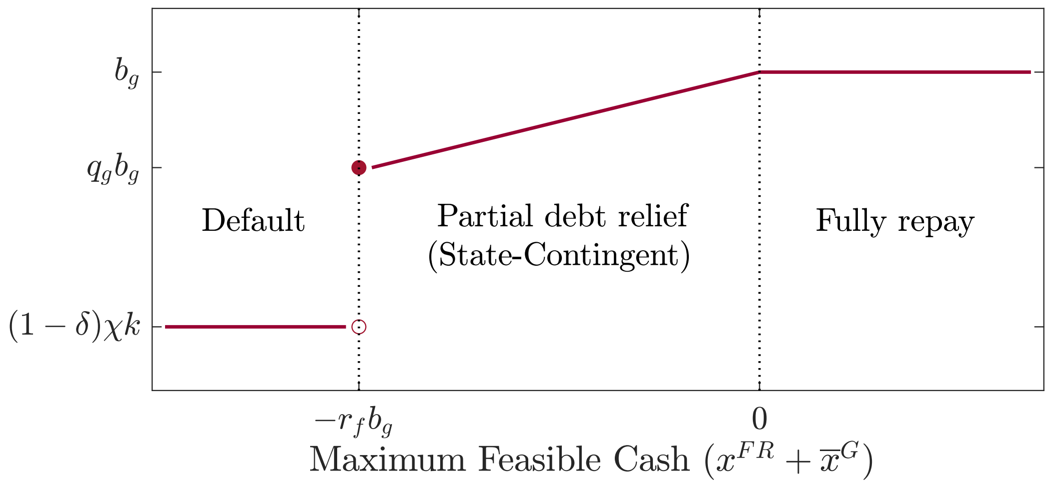
If $x_t^{FR} + \overline{x}^G(k,z) < 0$, then this firm experiences cash-shortage in period t. Given that a firm has borrowed b_g, the government payment from the firm depends on the cash shortage as follows:

$$\text{Payment} = \begin{cases} b_{gt} & \text{if} \quad x_t^{FR} + \overline{x}^G(k_t,z_t) \geq 0 \\ b_{gt} + x_t^{FR} + \overline{x}^G(k_t,z_t) & \text{if} \quad -\left(1-q_g\right) b_{g\,t} \leq x_t^{FR} + \overline{x}^G(k_t,z_t) < 0 \\ \max\left[b_{gt}, (1-\chi)k_t\right] & \text{if} \quad x_t^{FR} + \overline{x}^G(k_t,z_t) < -\left(1-q_g\right) b_{g\,t} \end{cases} \tag{12}$$

If a firm does not experience any cash shortage as in the first case, the firm repays

[16]The assumption of tax benefit of debt is common in the financial frictions literature. (See Covas and Haan 2011, Jermann and Quadrini (2012), Begenau and Salomao (2019)) This feature makes debt more attractive and slows down the rate at which firms grow out of financial frictions. Here I subtract the risk-free rate for tractability reasons following Xiao (2020).

FIGURE 8. Payment to government



the government in full. If a firm's cash shortage is less than the interest payment on the government loan, as in the second case, the government alleviates the debt by an insufficient amount. The government receives less by the value of the cash shortage. If a firm's cash shortage exceeds the interest payment on the government loan, as in the third case, the firm defaults and exits. The government obtains priority for the seized capital after deducting the default cost, $\chi(1-\delta)k_{t+1}$. Firms' default set and contingent payment to government is depicted in Figure 8.

Let's denote $\tilde{\phi}^G(k_t, B_t, z_t)$ the cutoff that determines the full repayment, such that $x_t^{FR}(k_t, B_t, z_t, \tilde{\phi}^G) + \overline{x}^G(k_t, z_t) = 0$, and $\hat{\phi}^G(k_t, B_t, z_t)$ the cutoff that determines defaults, such that $x_t^{FR}(k_t, B_t, z_t, \hat{\phi}^G) + \overline{x}^G(k_t, z_t) = -\left(1-q_g\right) b_g$. Given (k_t, B_t, z_t) the firm's cash-on-hand will vary by the realization of ϕ as follows:

$$x_t(k_t, B_t, z_t, \phi_t) = \begin{cases} (1-\tau)\, p z_t \exp(\phi_t)\, k_t^{\alpha} - (f + f_k k_t) - B_t + \tau(\delta k_t + r_f B_t) & \text{if} \quad \tilde{\phi}^G \leq \phi \\ (1-\tau)\, p z_t \exp\left(\tilde{\phi}^G\right) k_t^{\alpha} - (f + f_k k_t) - B_t + \tau(\delta k_t + r_f B_t) & \text{if} \quad \hat{\phi}^G \leq \phi < \tilde{\phi}^G \\ \text{Default} & \text{if} \quad \phi < \hat{\phi}^G \end{cases} \tag{13}$$

Government loans decrease default sets by increasing the maximum amount of funds a firm can raise, $\overline{x}^G(k,z)$ in two ways: firstly it directly increases $\overline{x}^G(k,z)$ by lending at risk-free rate up to some limit and secondly, the subsidized nature of the government loan changes the private debt price schedule q, which also increases $\overline{x}^G(k,z)$. One of the key assumptions is that government loans are not available to potential entrants. A mass of actual entrants is not affected by government loans.

Recursive Problem The idiosyncratic state of a firm, (x,k,z), records its cash-on-hand x_t, the current capital stock k_t, the current persistent idiosyncratic shock z_t. The dynamic problem of surviving firm (x,k,z) consists of choosing total loan B', government loan b'_g, and next period's capital k'. Given the choice for total loan B' and government loans b'_g, the firm's choice for private loan is determined as $B'-b'_g$. The value of surviving firm (x,k,z) is as follows:

$$
\begin{aligned}
V(x,k,z) = \max_{k',B',b'_g} d &+ \beta \sum_{z'} \pi\left(z' \mid z\right) \left[\int_{\phi'>\tilde{\phi}^G} V\left(x'\left(k',B',z',\phi'\right),k',z'\right) d\Phi\left(\phi'\right)\right] \\
&+ \underbrace{\beta \sum_{z'} \pi\left(z' \mid z\right) \left[\left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G\right) V\left(x'\left(k',B',z',\tilde{\phi}^G\right),k',z'\right)\right]}_{\text{Value from government's debt relief}} \\
\text{s.t} \quad & d = x - \psi\left(k,k'\right) + q\left(k',b',b'_g z\right)\left(B'-b'_g\right) + q_g b'_g \geq 0 \\
& x(k',b',b'_g,z',\phi') = (1-\tau)\, pz' \exp(\phi') k'^{\alpha} - f_k k' - f - B' + \tau\left(\delta k + r_f B'\right) \qquad (14) \\
& \tilde{\phi}^G\left(k',B',b'_g,z'\right) = \log\left(\frac{-\overline{x}^G\left(k',z'\right) + f + f_k k' + B' - \tau\left(\delta k + r_f B'\right)}{(1-\tau)\, pz' k'^{\alpha}}\right) \\
& \hat{\phi}^G\left(k',B',b'_g,z'\right) = \log\left(\frac{-\overline{x}^G\left(k',z'\right) + f + f_k k' + B' - \left(1-q_g\right) b'_g - \tau\left(\delta k + r_f B'\right)}{(1-\tau)\, pz' k'^{\alpha}}\right) \\
& b'_g \leq \overline{b_g}, \quad b'_g \leq B'
\end{aligned}
$$

$\overline{x}^G(k,z)$ is defined in equation (10). The constraints in the last line indicate that the borrowing from the government capped by the limit $\overline{b_g}$, and the non-negative borrowing from private creditor $B' - b'_g \geq 0$ respectively. The value when the government loans is not in place is determined with $b_g = 0$.

Firm Entry I model firm entry in line with Clementi and Palazzo (2016). Every period there is a constant mass $M > 0$ of prospective entrants, each of which receives a signal ν about their productivity, with $\nu \sim Q(\nu)$. Conditional on entry, the distribution of the idiosyncratic shock z in the first period of operation is $G(z \mid \nu)$, strictly decreasing in ν. Firms have to pay an entry fee ($c_e > 0$) so not all firms find it optimal to enter.

Entrants only start operating in the period after the entry decision, but must make decision today on capital they want to start operating in the following period given starting capital k_e [17]. Entrants need to raise funds for capital investment and related adjust cost through issuing debt. The value function of the potential entrant with signal ν is

$$\begin{aligned}
V^e(\nu) &= \max_{k',b'} \beta \sum_{z'} \int_{\phi' > \hat{\phi}} V\left(x'\left(k', b', z', \phi'\right), k', z'\right) d\Phi\left(\phi'\right) dG\left(z' \mid \nu\right) \\
\text{s.t} \quad & -\psi(k_e, k') + q^e(k', b', \nu)b' \geq 0 \\
& x(k', b', z', \phi') = (1-\tau)\, pz' \exp(\phi') k'^{\alpha} - f_k k' - f - b' + \tau\left(\delta k' + r_f b'\right) \\
& \hat{\phi}\left(k', b', z'\right) = \log\left(\frac{-\overline{x}\left(k', z'\right) + f + f_k k' + b' - \tau\left(\delta k' + r_f b'\right)}{(1-\tau)\, pz' k'^{\alpha}}\right)
\end{aligned} \tag{15}$$

where $q^e(b', k', \nu)$ is debt price given debt b', capital k', signal ν about productivity z'. Potential entrants make decision over private loan and capital since they cannot access to government loans. Furthermore the default and exit cutoff $\hat{\phi}$ is determined by the

[17] Firm entry in my model is equivalent to a decision to grow its size and to be subject to external audits, to be consistent with the data. Therefore it is natural for entrants starting with some initial capital. I calibrate the parameters such that I match the relative average size of entrants to incumbents' average size in the data.

maximum fund without government loans,

$$\overline{x}(k,z) = \max_{k',b'} q(k',b',0,z)b' - \psi(k,k') \tag{16}$$

This indicates the government loans is accessible only after the potential entrants enter and survive. The surviving firms will have state (x,k,z), and then will be allowed to access to government loans.

An entrant invests and starts operating if and only if the value of entry exceeds the entry fee, i.e $V^e(\nu) \geq c_e$. Since an incumbent's value $V(x,k,z)$ is weakly increasing in the transitory productivity z and the conditional distribution $G(z \mid \nu)$ is strictly decreasing in ν. Accordingly $V^e(\nu)$ is strictly increasing in the signal ν. This means that there will be a threshold for the signal, denoted by $\hat{\nu}$, such that potential entrants will enter if and only if they receive a signal greater than or equal to $\hat{\nu}$,

$$V^e(\hat{\nu}) = c_e \tag{17}$$

4.3. Private creditor

The private creditor is perfectly competitive. The debt price adjusts to reflect the probability of default and is determined by equating the expected return from providing a loan to the lender's funding costs.

Incumbents The debt price of incumbent firms with capital k', total debt B', government loan b'_g, and productivity z is determined as follows:

$$q\left(k',B',b'_g,z\right) = \beta \sum_{z'} \left[\left(1-\Phi\left(\hat{\phi}^G\right)\right) + \Phi\left(\hat{\phi}^G\right) R^G(B',b'_g,k')\right] \pi(z' \mid z) \tag{18}$$

where,

$$\hat{\phi}^G\left(k', B', b'_g, z'\right) = \log\left(\frac{-\bar{x}^G\left(k', z'\right) + f + f_k k' + B' - \left(1 - q_g\right)b'_g - \tau\left(\delta k + r_f B'\right)}{(1-\tau)\, p z' k'^{\alpha}}\right) \tag{19}$$

Upon default, the government takes the priority over the seized firm's capital after deducting default cost. The private lenders takes the remaining capital and should pay a fixed cost of the firm's default. Then the recuperation rate of private loan is as follows,

$$R^G(B', b'_g, k') = \min\left(1, \max\left(0, \frac{\chi(1-\delta)k' - b'_g - \eta}{B' - b'_g}\right)\right) \tag{20}$$

Debt price without the government loan is determined with $b_{g\ t+1} = 0$.

Entrants Similarly, the debt price of entering firms with capital k', debt b', and signal about the productivity ν is as follows:

$$q_e\left(k', b', \nu\right) = \beta \sum_{z'} \left[\left(1 - \Phi\left(\hat{\phi}\right)\right) + \Phi\left(\hat{\phi}\right) R\left(b', k'\right)\right] dG\left(z' \mid \nu\right) \tag{21}$$

where,

$$\hat{\phi}\left(k', b', z'\right) = \log\left(\frac{-\bar{x}\left(k', z'\right) + f + f_k k' + b' - \tau\left(\delta k' + r_f b'\right)}{(1-\tau)\, p z' k'^{\alpha}}\right) \tag{22}$$

$$R(b', k') = \min\left(1, \max\left(0, \chi\frac{(1-\delta)k'}{b'} - \eta\right)\right) \tag{23}$$

4.4. Stationary recursive equilibrium

The stationary recursive equilibrium for the economy consists of (i) policy and value functions of incumbent firms $\left\{B'(x,k,z), b'_g(x,k,z), k'(x,k,z), V(x,k,z)\right\}$; (ii) policy and value functions of entering firms $\left\{b'(\nu), k'(\nu), V(\nu)\right\}$; (iii) the bond price schedule

$q^G\left(B', b'_g k', z\right)$, $q^e\left(b', k', \nu\right)$; (iv) price of final good p, demand for final good $y_f(p)$, average productivity of intermediate good firms $\bar{z}$, and mass of entrants ; (v) a stationary measure μ such that: (1) policy and value functions of intermediate goods firms solve firm's problem (2) price of debt from private lenders is determined competitively; (3) final good market clears; (4) the cross-sectional distribution $\mu(x, k, z)$ is stationary.

Here I specify the equilibrium conditions.

Aggregate production of intermediate good satisfies

$$Y = \sum_z \int_\phi z \exp(\phi) \int_{x_{-1}, k_{-1}, z_{-1}} k(x_{-1}, k_{-1}, z_{-1})^\alpha \mu_{-1}\left(x_{-1}, k_{-1}, z_{-1}\right) d\Phi(\phi) \pi\left(z \mid z_{-1}\right) \tag{24}$$

This condition means that the outputs of defaulting firms are included in the total output, and the total production only depends on the previous distribution of firms. Accordingly, final good output satisfies $y_f = \bar{z} Y^{\alpha_y}$

Average productivity of intermediate good firms $\bar{z}$ is

$$\bar{z} = \sum_{z_i} z_i w(z_i) \tag{25}$$

where, $w(z_i)$ is a share of output produced by firms whose productivity is z_i:

$$w(z_i) = \int_\phi \frac{\int_{x_{-1}, k_{-1}, z_{-1}} z_i \exp(\phi) k\left(x_{-1}, k_{-1}, z_{-1}\right)^\alpha \mu_{-1}\left(x_{-1}, k_{-1}, z_{-1}\right) d\Phi(\phi) \pi\left(z_i \mid z_{-1}\right)}{Y} \tag{26}$$

Market clearing in the final goods market requires that total consumption equals to final good output, less the investment, the associated adjustment cost, and loss of resources from defaults:

$$\begin{aligned} C = {} & y_f - \int_{x,k,z} \psi\left(k, k'(x, k, z)\right) d\mu(x, k, z) \\ & - \int_{x_{-1}, k_{-1}, z_{-1}} \sum_z \int_{\phi < \hat{\phi}^G} \left[\eta - (\chi(1-\delta) k\left(x_{-1}, k_{-1}, z_{-1}\right)) d\Phi(\phi)\right] \pi\left(z \mid z_{-1}\right) d\mu_{-1}\left(x_{-1}, k_{-1}, z_{-1}\right) \end{aligned} \tag{27}$$

Specifically, the first term in equation (27) is final good output, and the second term is investment and related adjustment cost. The last term is related with firms' default. Firms with a previous state (x_{-1}, k_{-1}, z_{-1}) default given their choice for capital, debt and realized productivity z and ϕ. In this case the depreciated capital returns to the defaulting firm, and is used to repay to private lenders or the government after deducting cost related with default, $(1-\chi)\delta k + \eta$.

Finally, let $\mu(x, k, z)$ be the steady state distribution of firms with cash-on-hand x, capital k, and persistent productivity z. This distribution satisfies the following law of motion:

$$\mu(x', k', z') = \int \Lambda(x', k', z', x, k, z)\mu(x, k, z) + M \int_{\nu \geq \hat{\nu}} \Lambda^e(x', k', z', \nu) dQ(\nu) \tag{28}$$

The first term in the law of motion is determined by incumbent firms. To understand this term, we need to consider the probability that an incumbent firm with a particular state (x, k, z) transitions to a different state (x', k', z'), which is denoted by $\Lambda(x', k', z', x, k, z)$. The transition probability $\Lambda(x', k', z', x, k, z) = \pi(z' \mid z)d\Phi(\phi')$ if, at that state (x, k, z), the decision rules $k' = k'(x, k, z)$ and $B' = B'(x, k, z)$ together with ϕ' produce the particular level of cash-on-hand x'. The determinants of x' is defined in equation (13). It is important to note that $\phi' \geq \hat{\phi}^G$ specified in equation (14), so that the firm does not default. If any of these conditions do not hold, then $\Lambda(x', k', z', x, k, z) = 0$.

The second term in the transition function comes from new entrants. Similar to the case of incumbent firms, conditional on receiving a signal about the productivity, with which their value of entering is greater than the entry cost, i.e. $\nu \geq \hat{\nu}$, where $V^e(\hat{\nu}) = c_e$, the probability that a new entrant with a signal ν transits to (x', k', z') is given by $\Lambda^e(x', k', z', \nu)$. The transition probability $\Lambda^e(x', k', z', \nu) = \pi(z' \mid \nu)d\Phi(\phi')$ if, given the signal ν, the decision rules $k' = k'(\nu)$ and $b' = b'(\nu)$ together with ϕ' produce the particular level of cash-on-hand x'. Here, $\phi' \geq \hat{\phi}$ specified in equation (15), so that the firm can survive. The default cutoff $\hat{\phi}$ is the cut-off without the government loans, because the government loans are available only after the potential entrants enter and survive and become a incumbent. If any of these conditions do not hold, then $\Lambda^e(x', k', z', \nu) = 0$.

4.5. Firm's decision

Here, I characterize firms' decisions as follows. I begin by analyzing firms' decision to borrow from the government. Next, I will characterize the decision rule as a function of their cash-on-hand, which is associated with a nonnegative equity payout constraint. Lastly, I will explain firms' optimal choices for capital and borrowing based on the first-order condition of Bellman equation (14).

Decision on borrowing from the government Proposition 1 characterizes the decision on how much to borrow from the government.

PROPOSITION 1. *Given a choice for total debt and capital $\{B', k'\}$, if the total debt can be financed only by the government loan, $B' - \overline{b_g} \leq 0$, a firm will borrow only from the government $b'_g = B'$, and if the total debt cannot be financed only by the government loan due to the limit on the government loan, firm's borrowing from the government $b'_g = \overline{b_g}$*

PROOF. See appendix □

Intuitively, given total debt and capital, firms' value is strictly increasing by substituting private loans with government loans. By the Proposition 1, we can define firms' problem as a choice over total debt and capital, and the debt composition between a private loan and a government loan is determined by the level of total debt.

Decision rules associated with nonnegative equity payout constraint The firms' decision rules are characterized in more detail as a function of cash-on-hand as follows:

PROPOSITION 2. *The optimal decision of a surviving firm with cash-on-hand x, persistent productivity z, and capital k is characterized by one of the following three cases:*

(1) ***Default*** *: there exists a threshold $\underline{x}(k,z)$ such that firms with $x < \underline{x}(k,z)$ default since it is infeasible for these firms to satisfy the non-negativity equity payout constraint.*

(2) ***Unconstrained*** *: there exists a threshold $\hat{x}(k,z)$ such that the firm is financially unconstrained if $x > \hat{x}(k,z)$, i.e., the nonnegative equity payout constraint is slack. The bond price, capital, and total borrowing do not vary with cash-on-hand, whereas equity payouts increase one for one with cash-on-hand.*

(3) ***Constrained*** *Firms with cash-on-hand $x \in [\underline{x}(k,z), \hat{x}(k,z)]$ are financially constrained, i.e., the nonnegative equity payout constraint is binding. The equity payout is zero.*

PROOF. See appendix $\square$

Decision for capital and total borrowing Here I characterize firms' decisions mainly based on firms' first-order conditions for capital and debt. The first-order condition with respect to capital is:

$$\beta \sum_{z'} \pi(z' \mid z) \left[\int_{\phi' \geq \tilde{\phi}^G} \frac{\partial V(x'(k',B',z',\phi'),k',z')}{\partial k'} d\Phi(\phi') + \left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G)\right) \frac{\partial V\left(x'(k',B',z',\tilde{\phi}^G),k',z'\right)}{\partial k'} \right]$$
$$= (1+\eta(x,k,z)) \left[\frac{\psi(k,k')}{\partial k'} - \frac{\partial q}{\partial k'} \left(B' - \overline{b_g}\right) \right] - \beta \sum_{z'} \pi(z' \mid z) \left(-\frac{\partial \hat{\phi}^G}{\partial k'} \right) \phi(\hat{\phi}^G) V\left(x'(k',B',z',\tilde{\phi}^G),k',z'\right) \tag{29}$$

Where η is the multiplier associated with the nonnegative equity payout conditions, and derivative of value function with respect to capital can be derived using the envelope condition:

$$\frac{\partial V(x',k',z')}{\partial k'} = (1+\eta(x',k',z')) \left(pz' \exp(\phi')\alpha k'^{\alpha-1} - f_k - \frac{\partial \psi(k',k''(x',k',z'))}{\partial k'} \right) \tag{30}$$

The optimal choice for capital is determined at which the expected marginal benefit is equated to the expected marginal cost. The expected marginal benefit of capital indicated in the left-hand side of equation (29), consists of two terms. The first term captures the marginal product in future states where the firm fully repays, and the second term captures the marginal product in future states where the firm gets partial debt relief from the government. The expected cost, given by the right-hand side of equation (29), equals the investment and related adjustment cost, which is captured as the first term, and a wedge, which is captured by the remaining terms in the right-hand side. The first term of the wedge comes from the increase in the bond price from investing an extra unit of capital. The second term of the wedge comes from the gain associated with a decrease in default risk with an additional unit of capital This term is proportional to the firm's future value evaluated at default cutoff $V\left(x'\left(k', B', z', \tilde{\phi}^G\right), k', z'\right)$, probability of the cutoff $\phi(\tilde{\phi}^G)$, and $-\frac{\partial \hat{\phi}^G}{\partial k'}$, which captures how the cutoff changes with capital. Since the default cutoff decreases with capital (higher probability to repay with a higher capital), the marginal cost of capital is the investment cost net of gains from increased repayment probability and debt price.

The first-order condition with respect to new borrowing is as follows:

$$\beta \sum_{z'} \pi(z' \mid z) \left[\int_{\phi' \geq \tilde{\phi}^G} \left(1 + \eta'\left(x'(k', B', z', \phi'), k', z'\right)\right) d\Phi(\phi') + \left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G)\right) \left(1 + \eta'\left(x'(k', B', z', \tilde{\phi}^G), k', z'\right)\right) \right]$$
$$+ \beta \sum_{z'} \pi(z' \mid z) \left(\frac{\partial \hat{\phi}^G}{\partial B'}\right) \phi(\hat{\phi}^G) V\left(x'(k', B', z', \tilde{\phi}^G), k', z'\right) = (1 + \eta(x, k, z)) \left[q + \frac{\partial q}{\partial B'}\left(B' - \overline{b_g}\right)\right] \tag{31}$$

The optimal level of new borrowing equates the marginal benefit of new borrowing to the expected marginal cost. Borrowing one more unit gives a direct increase in current resources of q and leads to a fall in the price of existing debt, giving a total change in current resources of $q + \frac{\partial q}{\partial B'}\left(B' - b_g\right)$. Notice that the fall in the debt price only applies to the debt from the private creditor $B' - b_g$ since the government loans do not require the compensation for default risks. These resources help relax the nonnegtive equity payout condition, hence are valued at the multiplier η. The marginal cost of borrowing, given by the left-hand side of the equation (31), consists of three terms. The first term reflects the cost of repaying full repayment states and the second term captures the cost of repaying in states with the government's partial debt relief. These terms are weighted by the shadow price of cash-on-hand in those states, $1+\eta'$. The last term is the loss in value from the default.

4.6. Micro level policy effects: investment and exit

The effects of the introduction of government loans can be divided into two parts. Firstly, the policy affects the feasibility of individual firms to continue operating, as well as their decisions regarding leverage and investment. These responses generate the general equilibrium effect by changing the price at intermediate firms sell their products. Here, I am going to explain how the introduction of government loans change firms investment exit behavior given the price is fixed (no general equilibrium effect).

To illustrate the economic mechanisms through which government loans impact firms' investment decisions, let's consider a case where firms are constrained as in Proposition 2 and there is no capital adjustment cost. Since a firm is constrained, i.e. non negative dividend payout condition is binding, additional capital is associated with additional borrowing. Assume that the firm's total borrowing exceeds the government loan limit, and then the firm borrows up to the limit from the government, as shown in Proposition 1. By substituting equation (31) into equation (29), we can derive the optimality for capital.

The marginal benefit curve, left hand of equation (32), is downward sloping due to diminishing returns to capital. The curve of marginal cost, right hand of equation (32), is flat at $\frac{1}{\beta}$ when capital can be financed without incurring default risk. However it becomes upward-sloping when the borrowing required to finance capital creates default risk, as debt price q decrease with borrowing and the debt price elasticity ϵ increases as well.

$$\frac{\sum_{z'} \pi(z' \mid z) \left[\int_{\phi'>\tilde{\phi}^G} MPK(k', B', z', \phi') d\Phi(\phi') + \left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G)\right) MPK(k', B', z', \tilde{\phi}^G) + \left(-\frac{\partial \hat{\phi}_g}{\partial k'}\right) \phi(\hat{\phi}^G) \tilde{V}\right]}{\sum_{z'} \pi(z' \mid z) \left[\Delta + \frac{\partial \hat{\phi}^G}{\partial B'} \phi(\hat{\phi}^G) \tilde{V}\right]}$$

$$= \frac{1 - \frac{\partial q}{\partial k'} \left(B'(x, k', z) - b_g\right)}{q(1-\epsilon)}$$

$$\text{where,} \quad \epsilon = -\frac{\partial q}{\partial B'} \frac{\left(B' - b_g\right)}{q}$$

$$MPK(k', B', z', \phi') = \left(1 + \eta'(x', k', z')\right) \left[pz' \exp\left(\phi'\right) \alpha k'^{\alpha-1} - f_k + (1-\delta)\right]$$

$$\tilde{V} = V\left(x'\left(k', B', z', \tilde{\phi}^G\right), k', z'\right)$$

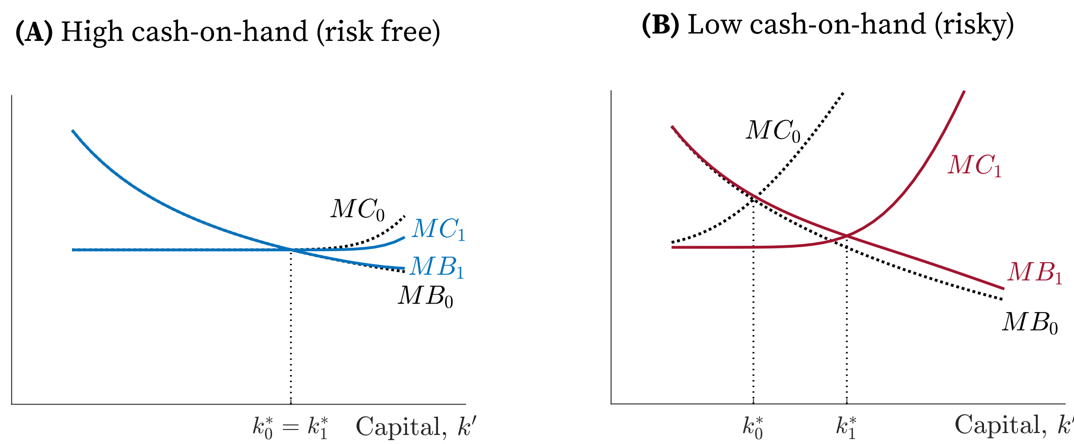
$$\Delta = \int_{\phi' \geq \tilde{\phi}^G} (1 + \eta'(x'(k', B', z', \phi'), k', z') d\Phi(\phi')$$

$$+ \left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G)\right) (1 + \eta'(x'(k', B', z', \tilde{\phi}^G), k', z') + \left(\frac{\partial \hat{\phi}^G}{\partial B'}\right) \phi(\hat{\phi}^G) \tilde{V} \tag{32}$$

In Figure 9, I plot the marginal benefit and marginal cost schedules as a function of tomorrow's capital holding k' for two types of firms: those with high cash-on-hand in the left panel and those with low cash-on-hand in the right panel. These two types of firms share the same values for today's capital k and productivity z. The key distinction between these firms lies in the fact that low cash-on-hand firms need to resort to a high level of debt in order to maintain the same level of capital for tomorrow. Consequently, it can be inferred that low cash-on-hand firms are required to pay a higher interest rate to retain the same amount of capital compared to high cash-on-hand firms.

In the initial equilibrium without government loans plotted with black dashed lines, high cash-on-hand firm's marginal cost and benefit curve intersects where the marginal cost curve is flat since the firm can finance their optimal level of capital

FIGURE 9. Marginal Benefit and Marginal Cost to Investment



Notes: Responses of risk free and risky firms to government loans are presented as shifts of marginal benefit and marginal costs curves as a function of capital investment. Two types of firm share same level of productivity and capital. Left panel is for a firm with high cash-on-hand (risk free) and right panel is is firm with low cash-on-hand (risky). The black dashed lines plot the curves without government loans, and blue (risk free) and red (risky) solid lines plot the curves with government loans given the intermediate good price fixed.

without incurring default risk such that

$$\frac{1}{\beta} = \sum_{z'} \pi(z' \mid z) \int_{\phi'} \left[pz' \exp\left(\phi'\right) \alpha k'^{\alpha-1} - f_k + (1-\delta) \right] d\Phi(\phi')$$

On the contrary, the marginal cost and benefit curve of the low cash-on-hand firms intersects where the marginal cost curve is upward sloping. Due to endogenous borrowing constraint arising from limited commitment, low cash-on-hand firms hold less capital than high cash-on-hand firms. In frictionless economy, the level of capital is not determined by the level of cash-on-hand.

When government loans are introduced with p fixed, which are indicated as solid lines, the marginal cost curve becomes flatten as firms are able to finance capital with less default risk. The marginal benefit, left hand of equation (32), increases as default risk becomes lower. In the region where the marginal cost curve is flat, the marginal benefit does not change with the government loans. In the region with positive default risk without government loans, the marginal benefit curve shifts up as the default probability given same choice is lower. Therefore, high cash-on-hand firm's new equilibrium stays same since the marginal cost curve was flat in state without government loans. Low cash-on-hand firm's new equilibrium is set at a higher capital investment. The risk free firms and risky firms only differ in their cash-on-hands, which implies that the risky firms are hit by bad shock, low ϕ, and end up having low cash-on-hand. The government loans help such firms with cash shortage not to reduce their investment much.

Turning our attention to the impact on the exit margin, Figure 8 shows that the firms rescued from the default and exit are precisely those with a cash shortage. Firms with limited cash-on-hand are more likely to experience this cash-shortage, and are more likely to be receive partial debt relieve and saved from exit thanks to the government loans.

In summary, the government loans help low cash-on-hand firms increase their investment and also help low-cash-on hand firms survive from defaulting. Cash shortages among firms can stem from diverse sources such as low capital size, low persistent productivity, or bad transitory shock. Depending on the underlying cause of a firm's low cash-on-hand, their investment response or change in exit margin will vary. This means that firms' response to the policy will vary depending on their specific characteristics, as we have observed in the data. The goal of the quantitative work is to discipline the model and study the aggregate implication of the policy.

5. Quantitative Analysis

This section explores the quantitative implications of the model. I discuss how firms' policy functions respond to the introduction of government loans. I also compare the model implications against empirical observation we discussed in previous sections.

To get these results, I first solve the model without government loans and calibrate it in a way that the moments generated from the steady states of the economy without government loans match the pre-policy aggregate moments of Korean firm data from 2010 to 2017. Next, I introduce government loans to the calibrated model. Beginning with the steady state without government loans, I first determine the new steady state of the economy with government loans. Then, I find the transition path between the two economies. Using the equilibrium price path and model solutions for policy functions, I simulate the economy over a 3-year period following the introduction of government loans to mimic the data. I construct panels of simulated firms based on this simulation, and further details are provided in Appendix A6.

5.1. Functional forms and Parameterization

Functional forms The i.i.d idiosyncratic productivity shock ϕ is log normally distributed, with mean 0 and and standard deviation σ_ϕ. The distribution of signals for the entrants is Pareto. I posit that $\nu \geq \underline{\nu} > 0$ and that $Q(\nu) = 1 - (\underline{\nu}/\nu)^{\xi}$, $\xi > 1$. The realization of the idiosyncratic productivity in the first period of operation follows the process $\log z = \rho_z \log \nu + \sigma_z \varepsilon_{z,t}$, where $\varepsilon_{z,t} \sim N(0, 1)$. I set $\underline{\nu} = \exp\left(\frac{-4.5\sigma_z}{sqrt(1-\rho_z^2)}\right)$.

Parameterization I classify the parameters into two groups: those that are exogenously assigned and those that are chosen to match aggregate moments of Korean firm data. Each period reflects one year. Table 2 reports the parameter values.

There are 7 fixed parameters. The discount factor, β is set to be 0.97, so that the annual interest rate is 3%. The share of capital α is set to be 0.3, and the annual depreciation rate δ is set to be 10%. The tax rate τ is set to be 0.275 based on Korea's corporate tax rate. Following Xiao (2020), I set the recuperation rate of bond χ to be 0.47. Using the estimates of Foster, Haltiwanger, and Syverson (2008), I set the serial correlation of the firm-level productivity shock ρ_z to 0.9. The parameter that captures the return to scale of final good producer, α_y, is set to be 0.85, consistent with the range of estimates in Atkeson and Kehoe (2005). The mass of potential entrants M is normalized to 1.

The rest of 10 parameters are set to match pre-policy aggregate moments from 2010 to 2016.[18] I calculated cross-sectional moment following Ocampo and Robinson (2022). The detailed definition of the moments calculated from data and model is presented in Appendix A7. The first five moments relate to incumbent firms' investment, borrowing, and exits. These moments include the mean investment of all incumbent firms, as well as the mean investment of firms whose net-income asset ratio is above and below the median. Additionally, mean credit spreads, exit rates are considered. The next three moments pertain to entrants, including relative median size, relative TFP of entrants, age 1 firms' mean investment.[19] To maintain consistency with the data, entrants in the model are defined as firms that survive after experiencing transitory shocks. While relatively small, these firms have higher productivity than incumbent firms.[20] The last two moments are related with states at exit. Specifically mean net-income asset ratio and relative TFP at exit is considered. The moments from data and model is presented in Table 3. All of relative moments are calculated with a moment relative to unconditional average or median.

The model performs relatively well in matching key moments in the distribution of firms' financial states. It generates a similar mean investment level and effectively captures the heterogeneity of investment depending on firms' net-income ratios.

[18] I calculated aggregate moments using sample periods from 2010 to 2016 to have more data on firms that exit and age 1 firms instead of using years from 2014 to 2016, which are used for empirical analysis in Section 3.

[19] Using data each firm TFP calculated sales to average of current total asset size and previous total asset size. Age 1 firm does not have the previous total asset size, and I use mean of age 2 firms TFP as a target moment.

[20] Foster, Haltiwanger, and Syverson (2008, 2016) found that entrants, despite exhibiting similar levels of technical efficiency as incumbents, often faced lower demand schedules and charged lower prices. However, conditional on survival, entrants tended to display greater total factor productivity as demand schedules shifted outward.

Table 2. Parameterization

Description	*Parameter*	*Source*
Fixed parameters		
Discount rate	$\beta = 0.97$	Annual interest rate 3%
Share of capital	$\alpha = 0.3$	Standard business cycle models
Depreciation	$\delta = 0.1$	Standard business cycle models
Tax rate	$\tau = 0.275$	Korea's corporate tax rate
Bond recovery rate	$\chi_k = 0.47$	Xiao (2020)
Persistence of z	$\rho_z = 0.9$	Foster, Haltiwanger, and Syverson (2008)
Returns to scale	$\alpha_y = 0.85$	Atkeson and Kehoe (2005)
Fitted parameters from moment matching		
Volatility of z	$\sigma_z = 0.1$	
Volatility of ϕ	$\sigma_\phi = 0.13$	Internally calibrated
Invest adj cost	$p_k^+ = 1.8$	Internally calibrated
Dis-invest adj cost	$p_k^- = 2.8$	Internally calibrated
Fixed operating cost	$f = 0.52$	Internally calibrated
Proportional cost	$f_k = 0.07$	Internally calibrated
Default cost	$\chi = 0.2$	Internally calibrated
Entry cost	$c_e = 3.2$	Internally calibrated
Initial capital	$k_e = 0.2$	Internally calibrated
Pareto exponent	$\xi = 3.2$	Internally calibrated
Government loans	$\bar{b}_g = 0.134$	

Moreover, the model aligns well with the mean credit spread and exit rates observed in the data. The model also replicates moments related to entrants that are in line with empirical observations; entrants tend to be smaller, more productive, and invest more than average firms. Additionally, the model accurately reflects the fact that firms tend to have less cash and lower productivity at exit.

Lastly, the limit of government loan $\overline{b_g}$ is set to align with the change in exit rates over 3 years as observed in the data. Following the policy introduction, the model reflects a decrease in exit rates by 0.5 percentage points, whereas the data shows a decrease of 0.4 percentage points.

TABLE 3. Targeted moments

Description	*Data*	*Model*
Incumbents		
Mean investment	0.11	0.11
Mean investment ($\frac{x}{k}$ < median)	0.06	0.07
Mean investment ($\frac{x}{k} \geq$ median)	0.15	0.14
Mean spread (%p)	1.46	1.61
Exit rates (%)	1.10	1.12
Entrants		
Median relative size at enter	0.16	0.17
Mean relative TFP at enter	1.81	1.55
Age 1 firms' mean investment	0.43	0.46
Firms that exit		
Mean net-income asset ratio at exit	-0.27	-0.30
Mean relative TFP at exit	0.61	0.59

5.2. Model performance: pre-policy moments

Table 4 presents the cross-sectional moments based on firms' net-income ratios, which were not explicitly targeted. I use cash-on-hand to capital ratio for the model moments and net-income to asset ratio for the data moments. These moments are calculated using the steady-state distribution of firms and policy functions, following the approach outlined by Ocampo and Robinson (2022). The model performs relatively well in generating cross-sectional moments, with the exception of the credit spread. The model effectively captures firms' heterogeneity based on their net-income ratios. Firms with higher net-income ratios tend to invest more, exhibit lower spreads, and have a lower likelihood of exiting. However, in the model, the dispersion of credit spreads is larger than that observed in the data. An intriguing observation pertains to the characteristics related to firms' size. Firms with lower net-income ratios tend to be larger, as smaller firms struggle to survive with low net-income ratios. The variance in

TABLE 4. Untargeted moments: Distribution by net-income asset ratio ($\frac{x}{k}$)

	Net-income asset ratio			
Moments	[0,25]	[25,50]	[50,75]	[75,100]
Data				
Net-income asset ratio	-0.10	0.02	0.06	0.16
Investment	0.05	0.06	0.11	0.19
Spread	1.83	1.61	1.30	1.08
Exit rate (%)	3.49	0.84	0.23	0.09
Log size (Relative)	1.00	0.98	0.92	0.78
Std of log size (Relative)	1.00	0.85	0.95	1.09
Model				
Net-income asset ratio	-0.10	0.02	0.12	0.31
Investment	0.06	0.09	0.12	0.17
Spread	6.78	0.36	0.10	0.05
Exit rates (%)	4.66	0.33	0.08	0.05
Log size (Relative)	1.00	0.97	0.95	0.60
Std of log size (Relative)	1.00	0.71	0.61	1.14

Notes: Moments calculated based on firms policy functions and steady state distribution without government loans following Ocampo and Robinson (2022). For example, exit rate of firms with first quartile net-income asset ratio is calculated as $E\left[\mathbf{1}(\text{exit}) \mid x^k \in Q_1\right] = \frac{\int_{x,k,z} \int_{\phi' \leq \hat{\phi} G(k'(x,k,z),B'(x,k,z),z')} \mathbb{1}\left(x^k \in Q_1\right) \pi(z'|z) \mu(x,k,z)}{\int_{x,k,z} \mathbb{1}\left(x^k \in Q_1\right) \mu(x,k,z)}$.

size is more pronounced for firms with net-income ratios below the first quartile and above the third quartile, as compared to firms within the interquartile net-income ratio range.

In Appendix A8 I present firms behaviors in steady state, which were obtained from simulation (not calculation based on steady state distribution). I show that the model generates the negative correlation between age and investment as observed in the data. I also present the financial states before exit. The model replicates well financial states before firms exit as observed in data: continued cash-shortage, higher leverage, higher spread, lower investment compared to firms that never exit or are far from the exit.

5.3. Quantitative exercises: policy effects

In this section, I first present the changes in firms' policy functions related to investment and survival rates resulting from the introduction of government loans, within a partial equilibrium framework given the price p fixed. Secondly, I compare the properties of zombie firms between the model and the data. To achieve this, I define zombie firms based on a simulated panel of firms that mimics the definition of zombie firms used in empirical analysis. I also provide the transition probabilities for normal (non-zombie) firms and zombie firms, along with the changes in these transition probabilities after the policy shift. Lastly, I replicate the regression presented in section 3 using a simulated panel of firms.

Firms decision rules and credit spread schedules Figure 10 plots policy functions for a firm with a median level of capital k and productivity z as a function of cash-on-hand levels. The left panel displays the survival probability and the right panel displays the optimal investment. The survival probability is obtained from firms' policy function as follows,

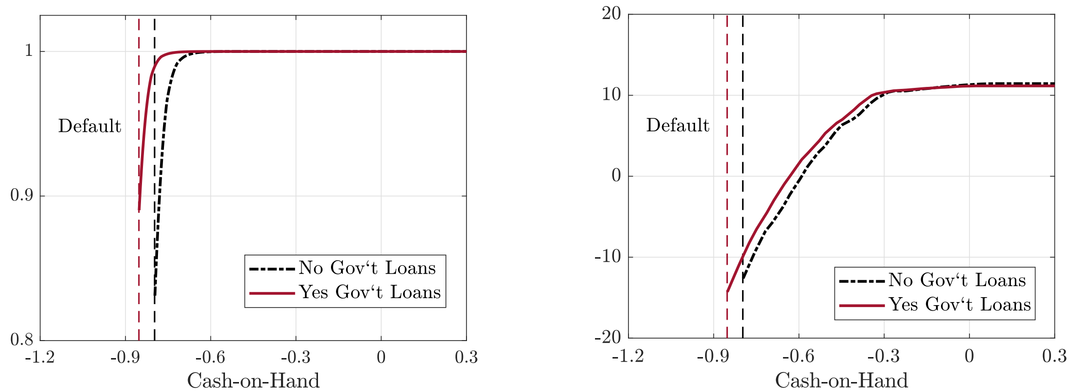
$$\sum_{z'} \left[1 - \Phi\left(\hat{\phi}^G\left(k'(x,k,z), B'(x,k,z), b'_g(x,k,z), z'\right)\right)\right] \pi(z' \mid z).$$

In each panel, I plot policy functions in economy with and without the government loans with p being fixed to pre-policy steady state level. The black dashed lines indicate policy functions for the economy without the government loans and the red solid lines indicate those for the economy with the government loans.

Let's first examine the survival probability. Firms with cash-on-hand x lower than $-\bar{x}^G$ should default, which is indicated by a vertical line. Firms with lower cash-on-hand x need to choose higher debt and, consequently, are less likely to survive. With the introduction of government loans, firms that would have defaulted without the

FIGURE 10. Survival Probability and Investment by Cash-on-Hand

(A) Survival Probability　　　**(B)** Investment (%)



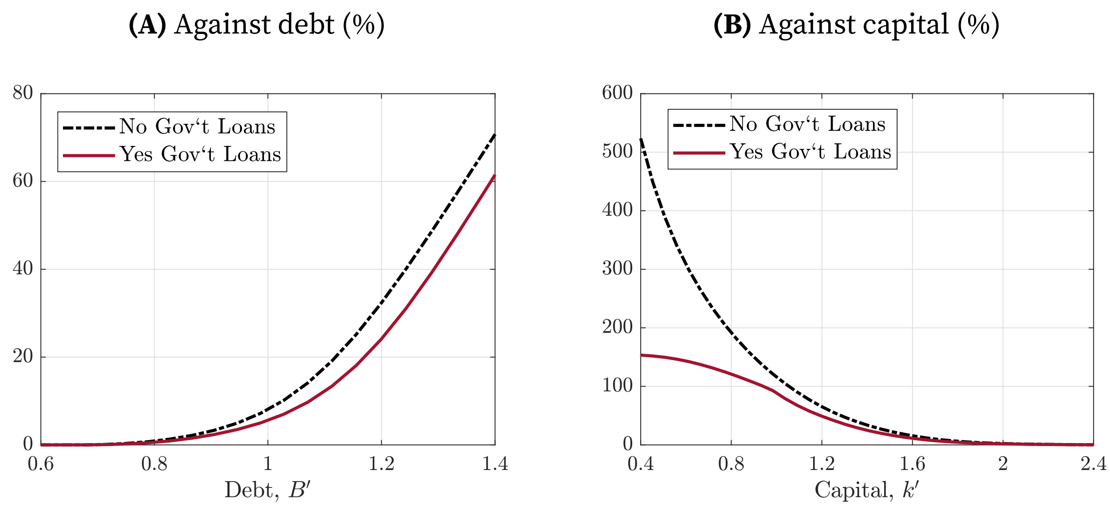
Notes: These plots display the survival probability and policy function for investment of firms with median capital k and productivity z with respect to the level of cash-on-hand x for the economy, both with and without government loans, while keeping p fixed at the steady-state level without government loans. Survival probability represents the likelihood of survival given firms' optimal choices regarding capital and debt., i.e. $\sum_{z'}\left[1-\Phi\left(\hat{\phi}^G\left(k'(x,k,z), B'(x,k,z), b_g'(x,k,z), z'\right)\right)\right]\pi(z' \mid z)$.

government loans now survive. This is why the vertical line moves rightward with introduction of government loans, representing an increased repayment threshold. Furthermore, given the same level of cash-on-hand, firms are more likely to survive when government loans are available.

Turning to the policy function for investment, firms with a lower cash-on-hand tend to invest less, given the same level of capital and productivity. This is because lower cash-on-hand is associated with higher debt, higher default risk, and a higher marginal cost of investment. With the introduction of government loans, firms can rely less on debt to finance the same level of investment. As a result, investment increases for firms with lower cash-on-hand. However, there is no change when cash-on-hand is high, as government loans do not alter their financing costs in this case.

Figure 11 compares the credit spread between two economies, one with government loans and the other without. The left panel plots the credit spread schedule with respect to debt, given tomorrow's fixed capital choice and the same

FIGURE 11. Credit Spread Schedule



Notes: These plots display credit spread schedule with persistent productivity to be fixed at average.

productivity. With the introduction of government loans, there is a reduction in the credit spread elasticity. In the right panel, the credit spread schedule is presented with respect to tomorrow's capital, assuming a fixed amount of debt and the same productivity. It's observed that, when firms hold more capital for tomorrow with the same debt and productivity, their credit spread decreases. However, with the introduction of government loans, firms holding less capital can borrow at a lower rate given the same amount of debt and same productivity.

Zombie firms I define zombie firms in the model, mimicking the definition applied in empirical analysis. In the model, firms are classified as zombie firms if their cash-on-hand is negative for three consecutive years and they are at least ten years old.[21] In Table 5, I present the properties of zombie firms based on both data and model simulations.

[21] Debt in the model is a one-period bond, and we cannot directly apply the concept of debt service from the data to the model. In the model, negative cash-on-hand indicates that firms are unable to cover their debt obligations solely from their operational profits, which corresponds to a similar definition used in the data. Furthermore, cash-on-hand in the data can be matched with net-income. On average, the net-income of firms with an interest coverage ratio less than 1 is negative in the data.

Table 5. Untargeted Moment Related with Zombie Firms

	Data	Model		Data	Model
Share of zombie firms	5.1	8.0	Debt to Asset Ratio^{*+}	9.7	10.1
Δ zombie share*	2.5	4.1	Profitability^{*+}	-11.2	-15.5
Log Size^{+}	115.2	111.0	Investment^{*+}	-12.2	-7.1

Notes: All figures with a "*" symbol are measured in percentage points, while all figures without the symbol are measured in percentage. The variables denoted with a "+" symbol indicate the mean difference between zombie and non-zombie firms.

The properties of zombie firms observed in simulated firms are consistent with the data, and the model effectively captures the differences from normal firms, similar to what is observed in the data. In the data, the average share of zombie firms in the years before the policy shift was 5.1%, and it increased by 2.5 percentage points after the policy change. In the model, the average share of zombie firms in the pre-policy steady state is 8.0%, and it increases by 4.1 percentage points over the three years after the introduction of government loans. I compare the relative mean differences between zombie firms and normal firms to validate my model. In the data, we observe that zombie firms are relatively larger, highly leveraged, less profitable, and invest much less compared to normal firms.

Heterogeneous response to the policy: data vs model Based on the calibrated model, my first step is to investigate whether the model generates predictions that align with the findings in the data regarding firm-level responses in terms of investment and exit to an increase in government loans. Using the panels of simulated firms, I replicate the specification outlined in the section 3.2 and 3.3 on the pooled sample.

To compare the heterogeneous response of investment to government loans between the model and the data, I replicate the specification in equation 3. Specifically I regress the growth rates of capital on variables including a dummy variable indicating whether a firm's mean credit spread was high in the three years leading up to the introduction of government loans D_i, and the interaction of this high credit spread dummy with the period after the policy was implemented (specifically, three years after the introduction of government loans) D_i^{High} After $_t$. Additionally, I include lagged log capital size, lagged profitability (defined as the ratio of operational profit to capital), and year fixed effects to control the general equilibrium effects. The specification is as follows,

$$\text{Investment}_{it} = \alpha_1 D_i^{\text{High}} \text{ After}_t + \gamma^x X_{it-1} + \gamma_t + \gamma^h D_i^{\text{High}} + \epsilon_{it} \tag{33}$$

where D_i^{High} is an indicator whether a firm's mean spread of Before period is the upper 10th percentiles. This specification aligns with the approach used in empirical findings. The primary difference is that there is only 2 groups in the model specification while there are 4 groups in the model specification. This is because all firms are eligible for the government loans in the model, and there is two groups by the mean credit spread in periods before the introduction of the government loans. The other difference is the omission of firm fixed effects in the model. Furthermore, the model specification includes year fixed effects due to the one-industry nature of the model's economy, while the data specification includes year-sector fixed effects.

Firms initially characterized by higher pre-policy credit spreads increase their investment by 4 percentage points more than firms with lower pre-policy credit spreads in simulated data, while the data show a 5 percentage point higher increase. To be specific, the coefficient that capture the heterogeneous response of investment based on pre-policy credit spread is $\beta_2 - \beta_1$ for data, α_1 for model. You can find the detailed results in Table 6a.

Similarly, I compare the heterogeneous response of exit to government loans between the model and data. Specifically, I estimate the following equation based on

Table 6. Heterogeneity in firms response to the policy

(a) Investment

Δ Investment	
Data ($\beta_2 - \beta_1$)	Model (α_1)
5.14	4.02
[2.69 7.58]	(0.28)

(b) Exit

Δ Probability to exit	
Data ($\beta_2 - \beta_1$)	Model (α_1)
-0.028	-0.023
[−0.012 − 0.045]	(0.009)

Notes: The data estimates come from the equation 3 for investment and equation 4 for exit probability, with the 95% confidence interval presented in brackets. The model estimate are from equation 33 for investment and equation 34 for exit probability, with the standard error in parentheses. Investment estimates are in percentage points, while exit probability estimates are in probability terms.

simulated firms,

$$\text{Exit}_{it} = \alpha_1 D^{\text{Zombie}}_{it-1} \ \text{After}_{t} + \gamma_z D^{\text{Zombie}}_{it-1} + \gamma_t + \epsilon_{it} \tag{34}$$

In the model, the exit rate of unproductive firms decreases more by 2.3 percentage points compared to productive firms, whereas in the data, it decreases by a greater margin of 2.8 percentage points. Specifically, the coefficient that captures the relative change in exit rates based on the indicator for unproductive (zombie) firms is denoted as $\beta_2 - \beta_1$ for the data and α_1 for the model. The results can be found in Table 6b.

I also analyze the transition probabilities of firms' statuses, which can be categorized as either zombie firms or normal firms (non-zombie firms). In the following year, a firm can transition to being a zombie or normal firm, or it can exit. Figure 12 compares these transition probabilities for the years preceding the government loans between the model and the data. The calibrated model accurately captures the observed transition probabilities in the data, even though it was not explicitly targeted during calibration.

To see the changes in transition probabilities, following the policy introduction, normal firms are more likely to transition into zombie firms, and zombie firms are more likely to remain as zombie firms. Furthermore, both zombie firms and normal firms exhibit reduced exit rates, but the decline in exit rates is more pronounced for zombie

FIGURE 12. Transition Probability: Pre-Policy

(A) Normal firms **(B)** Zombie firms

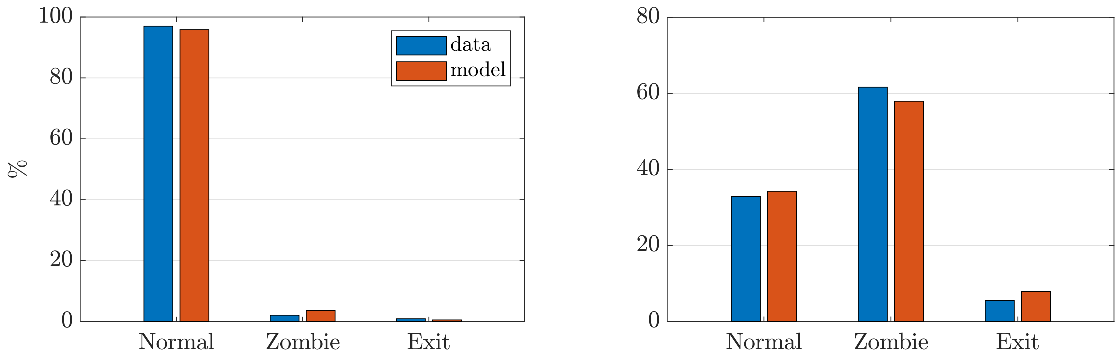
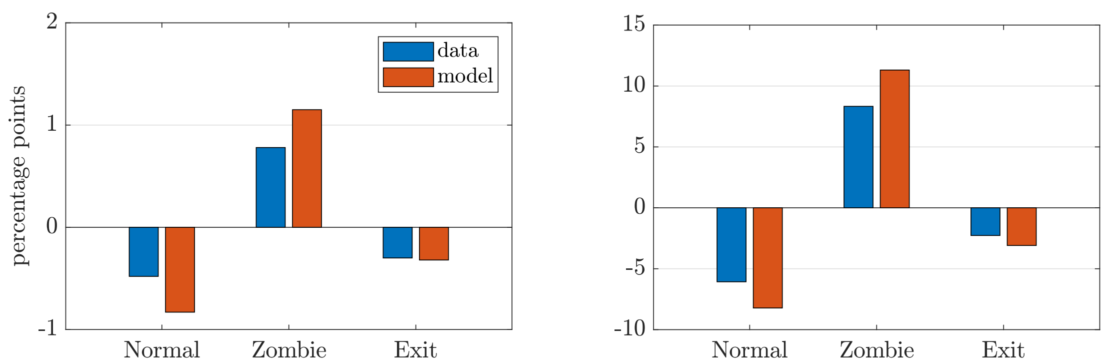


FIGURE 13. Change in Transition Probability

(A) Normal firms **(B)** Zombie firms



firms. These observed patterns are effectively captured by the model, both qualitatively and quantitatively, presented in Figure 13. Normal firms are more likely to become zombie firms because a greater number of firms are in operation, leading to congestion, which adversely affects normal firms' profitability. Similarly, zombie firms are more likely to persist in their status due to reduced exit rates and continued low profitability, largely stemming from the congestion effect.

6. Aggregate implication

My empirical and quantitative results suggest that government loans play a dual role. They help financially constrained firms increase their investments, thereby enhancing aggregate productivity through more efficient capital allocation. Simultaneously, these loans assist less productive firms in surviving, potentially influencing overall productivity. To quantify these two offsetting effects, I decompose aggregate productivity into two components: capital allocation efficiency, following the framework proposed by Hsieh and Klenow (2009), and the composition of productivity, which is related with selections.

First, I define the maximum level of output that a planner could achieve by reallocating fixed quantities of factors across a fixed mass of firm as follows,

PROPOSITION 3. *In an economy where a planner can freely reallocate capital across firms to maximize production, for a given mass of firms (M) and total capital (K), aggregate production is given by* $Y^* = M^{1-\alpha}\mathbf{E}\left[\tilde{z}^{\frac{1}{1-\alpha}}\right]^{1-\alpha} K^{\alpha}$, *where* $M = \int d\mu(x_{-1}, k_{-1}, z_{-1})$, $K = \int k(x_{-1}, k_{-1}, z_{-1}) d\mu(x_{-1}, k_{-1}, z_{-1})$, $\tilde{z} = \sum_z z\pi\left(z \mid z_{-1}\right)$.

PROOF. See appendix □

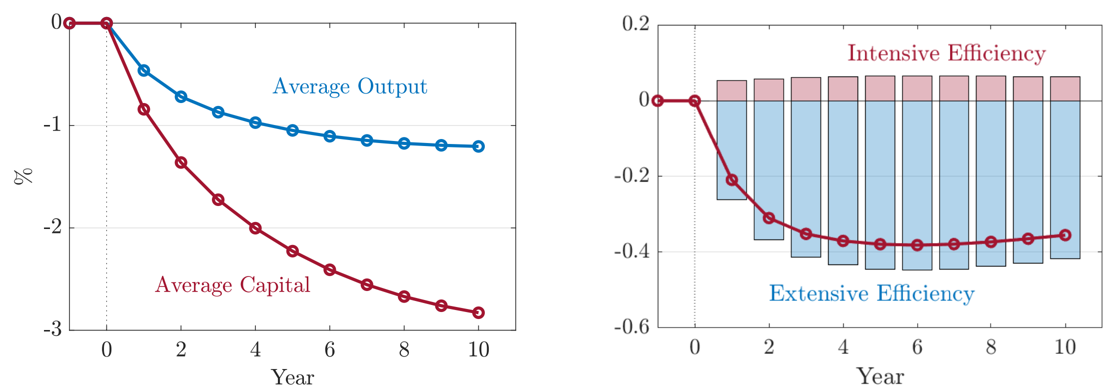
As a direct corollary the output in the decentralized economy can be decomposed as follows,

$$Y = \underbrace{M^{1-\alpha}}_{\text{Size effect}} \times \overbrace{\underbrace{\mathbf{E}\left[\tilde{z}^{\frac{1}{1-\alpha}}\right]^{1-\alpha}}_{\text{Ext. efficiency}} \times \underbrace{\frac{Y}{Y^*}}_{\text{Int. efficiency}}}^{\text{Aggregate Productivity}} \times \underbrace{K^{\alpha}}_{\text{Capital qtys.}} \tag{35}$$

The aggregate (average) TFP depends on two components. The first term reflects the composition of productivity, associated with the extensive margin and selection. The second term represents capital allocation efficiency, in line with the framework proposed by Hsieh and Klenow (2009). This term equals 1 when capital is distributed

FIGURE 14. Transition path over 10 years

(A) Average Firm Size **(B)** Aggregate Productivity



Notes: The figures indicate the percentage deviation from the steady state without government loans after the introduction of government loans in year 0 over 10 years.In the right panel, the red line represents the sum of changes in intensive and extensive efficiency, which is the net change in aggregate productivity

across firms in a way that equalizes the marginal product of input across firms. I label the first term as "extensive efficiency" and the second term as "intensive efficiency." [22]

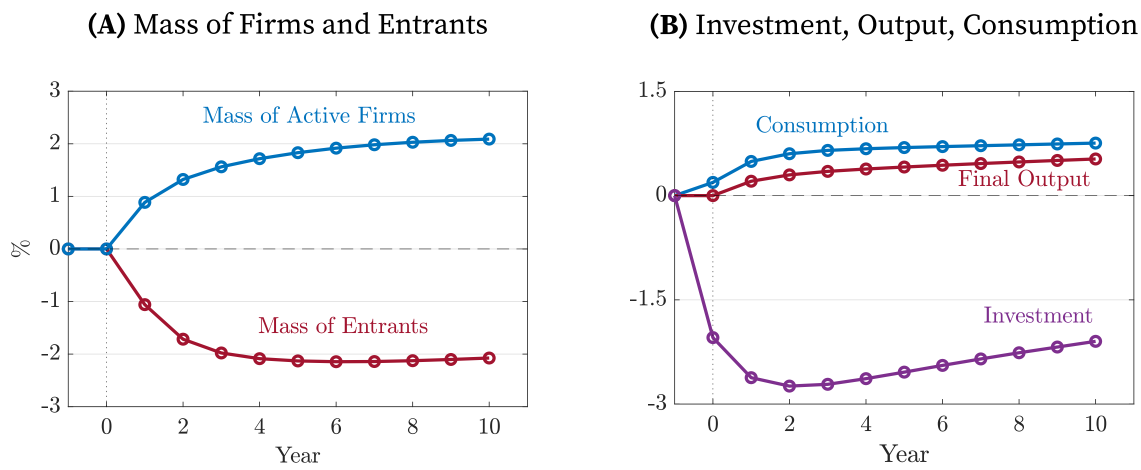
Figure 14 illustrates the transition path over 10 years following the introduction of government loans. The left panel displays the average output and capital.[23] Due to the general equilibrium effect, firms' size decreases on average.

The right panel shows the aggregate productivity decomposed into intensive and extensive efficiency. As government loans assist firms with low cash-on-hand in increasing their investments, this leads to an improvement in intensive efficiency. However, concurrently, the government's intervention alters the extensive margin and worsens the selection process, resulting in a decrease in extensive efficiency. The

[22] 1 minus intensive efficiency indicates the intensive inefficiency that indicate how the economy is far from the efficient level of output. The calibrated model suggests the intensive inefficiency is 1.1% in steady state of economy without government loans. This finding aligns with Midrigan and Xu (2014), who observed that the Korean manufacturing sector's TFP losses due to intensive inefficiency (marginal product of capital dispersion) ranged from 0.3% to 2.1% based on data from the years 1991 to 1999.

[23] Average output is calculated as $\frac{\int_{x_{-1},k_{-1},z_{-1}} \sum_z \int_\phi z\exp(\phi)k(x_{-1},k_{-1},z_{-1})^\alpha \pi(z|z_{-1})d\mu_{-1}(x_{-1},k_{-1},z_{-1})}{\int_{x_{-1},k_{-1},z_{-1}} d\mu_{-1}(x_{-1},k_{-1},z_{-1})}$, and average capital is calculated $\frac{\int_{x_{-1},k_{-1},z_{-1}} k(x_{-1},k_{-1},z_{-1})d\mu_{-1}(x_{-1},k_{-1},z_{-1})}{\int_{x_{-1},k_{-1},z_{-1}} d\mu_{-1}(x_{-1},k_{-1},z_{-1})}$.

FIGURE 15. Transition path over 10 years



Notes: The figures indicate the percentage deviation from the steady state without government loans after the introduction of government loans in year 0 over 10 years.

simulated results indicate that the loss from extensive efficiency (-0.4%) outweighs the gain from intensive efficiency (0.1%). In sum, aggregate productivity decreases by 0.3% over the 10-year period.

Figure 15 illustrates the transition path of aggregate variables. The left panel displays the mass of active firms and entrants. The lowered prices, due to general equilibrium effects, discourage potential entrants from entering the market. Exit rates of incumbent firms decrease because the impact of government loans outweighs the general equilibrium effect, leading to a larger mass of active firms in the economy.

The right panel shows the paths of consumption, final output, and investment. In period 0, firms reduce their investment because they anticipate lower prices in the following years, while the level of final output remains the same since it is determined in the previous year. Furthermore, with the introduction of government loans in year 0, fewer firms exit, resulting in lower default costs. Therefore, consumption increases in period 0. Over the course of 10 years, final output increases due to a larger mass of operating firms, even though per-firm production is smaller. This increase in final output, coupled with reduced investment, leads to higher consumption.

TABLE 7. Steady State Comparison

	Δ		Δ		Δ
Productivity	-0.3	Active Firms	+2.6	Capital	-0.4
(Intensive)	+0.1	Entrants	-2.2	Final output	+1.1
(Extensive)	-0.4			Consumption	+1.2

Notes: The percentage changes from the steady state without government loans to the new steady state with government loans.

In Table 7, I present the percentage changes between two steady states: one with government loans and one without. Aggregate productivity decreases by 0.3%. Entrants decrease by 2.2%, but the mass of operating firms increases by 2.6% due to lowered exit rates. The increase in the mass of operating firms results in higher final output by 1.1%. General equilibrium effects lead to lower capital. The combination of higher final output, lower exit rates, and reduced capital levels leads to higher consumption by 1.2%.

7. Conclusion

In this paper, I study the effect of government-backed financing policy on aggregate productivity, addressing the trade-off of the policy that has been broadly studied. For this research question, I exploit an extensive panel dataset of Korean manufacturing firms and the Korean government's policy shift, which significantly increased government loans after 2017.

Based on policy eligibility, I document effects on three key variables: credit spread, investment, and exit rates. Firstly, the credit spread of the eligible group decreased more than that of the non-eligible group after the policy shift, suggesting improved credit access for the eligible group. Secondly, eligible firms with higher pre-policy credit spreads exhibited greater post-policy increases in investment. Lastly, the exit rate of unproductive eligible firms decreased more following the policy. These findings capture the main trade-off of government loans on aggregate productivity through enhanced firms' credit access.

To quantify these two off-setting effect, I build a heterogeneous-firm model incorporating both government and private loans. My calibrated model generates heterogeneous responses to government loans in terms of investment and exit, consistent with the data. Over a span of 10 years, aggregate productivity experiences a decrease of 0.3%. The gain resulting from increased investment by constrained firms is 0.1%, while the loss due to a decreased exit rate among unproductive firms is 0.4%.

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Appendix

Government-Backed Financing and Aggregate Productivity

Jihyun Kim

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A1. Credit spread and investment

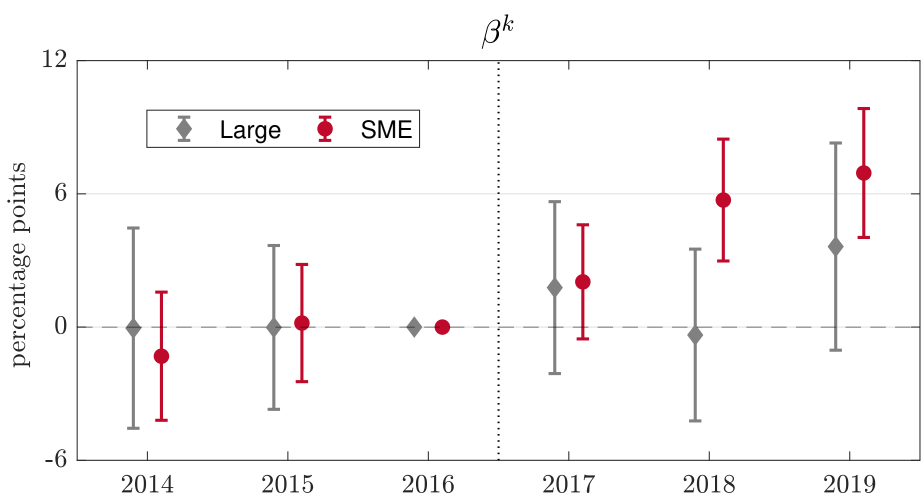
Event study I conducted event study analysis based on the same specification as equation 3, with the only difference being the use of year dummy variable instead of After dummy. Specific specification is as follows:

$$\text{Investment}_{ist} = \sum_{k \neq 2016} \beta^k \text{Year}_k D_i^{\text{High}} + \gamma^x X_{\text{ist}-1} + \gamma_{st} + \gamma_i + \epsilon_{\text{ist}} \tag{A1}$$

I conducted separate estimations of equation A1 for both the eligible group (SMEs) and the non-eligible group (large firms). The coefficient β_k, depicted in the plotted figures, represents the difference in investment between groups with low and high pre-policy credit spreads, relative to the year 2016. In the figures, grey diamonds represent large firms and red circles represent SMEs.

For the non-eligible group, there was no discernible shift in the investment difference between low and high pre-policy credit spread groups. Conversely, within the eligible group, there existed no significant difference in investment between these groups before the policy alteration. However, following the policy changes, firms with high pre-policy credit spreads exhibited a significant increase in investment.

FIGURE 1. Investment response by pre-policy credit spread



Notes: These plots show a difference in the investment between high pre-policy credit spread and low pre-policy credit spread firms for specific years relative to year 2016, separately for SMEs and large firms. Standard errors are clustered by firms. 90% confidence intervals are shown.

Continuous pre-policy credit spread The following equation shares the same specifications as equation 3, with the only difference being the use of the pre-policy credit spread, represented by the mean credit spread before the policy change, instead of a dummy indicator. The outcomes are presented in Table A1, consistently suggesting that investments increased more significantly among firms with higher pre-policy credit spreads within the eligible group. Conversely, no significant effects were observed among non-eligible firms.

$$\text{Investment}_{ist} = \beta_1 D_{is}^{sme} \text{Before CR}_{is}\text{After}_t + \beta_2 \text{Before CR}_{is}\text{After}_t + \gamma^x X_{ist-1} + \gamma_{st} + \gamma_i + \epsilon_{ist} \tag{A2}$$

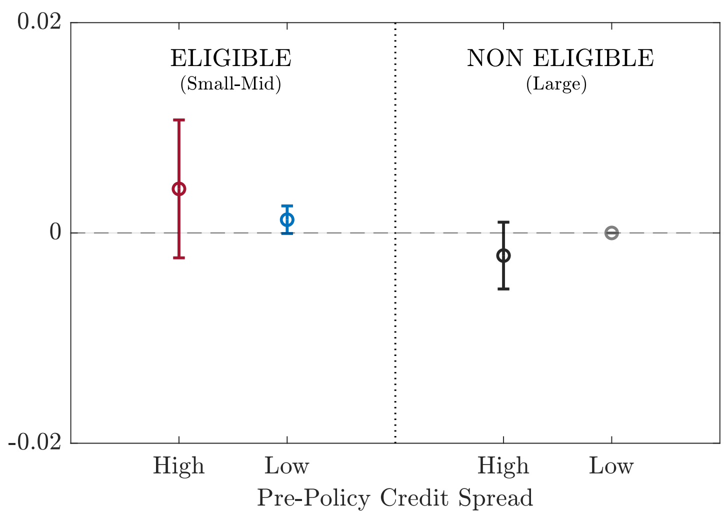
Table A1. Heterogeneous response of investment by pre-policy credit spread

	Investment(pp)
Before CR × SME× After (β_1)	1.32*** (0.29)
Before CR × After (β_2)	0.05 (0.26)

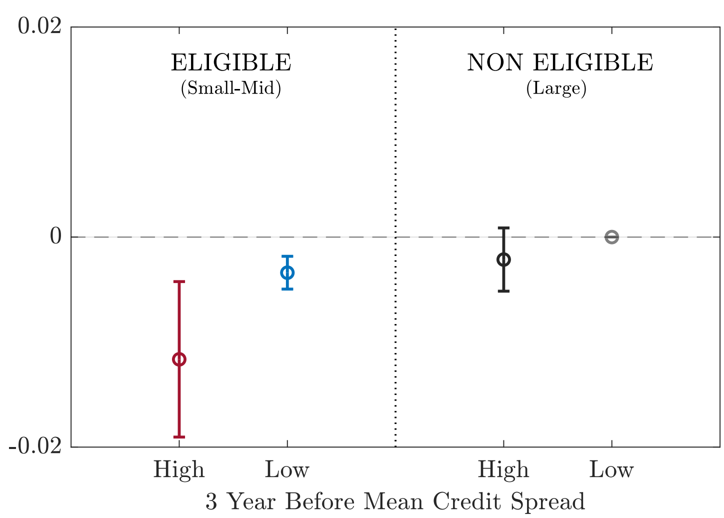
A2. Credit spread and exit rates

Firms with initially high pre-policy credit spreads were more likely to exit in the 'Before' period, thereby dampening the magnitude of change in their exit rates. To delve into this phenomenon further, I replicated the analysis using the three-year before average credit spread. This approach enables a closer examination of how the policy influenced the exit threshold concerning credit spreads, rather than treatment impact on a specific group. The outcomes, as illustrated in the following figure, are as follows: no discernible effect on the non-eligible group (large firms), while firms that, on average, maintained higher credit spreads experienced more pronounced decrease in terms of exit rates.

(A) Change in exit probability by pre-policy credit spread



(B) Change in exit probability by 3 year before mean credit spread



A3. Effect of government loans on exit rates and zombie share by eligibility

$$Y_{it} = \alpha + \sum_{k \neq 2016} \beta^k \text{Year}_k D_i^{sme} + \gamma_t + \epsilon_{it} \tag{A3}$$

FIGURE 3. Exit Rates

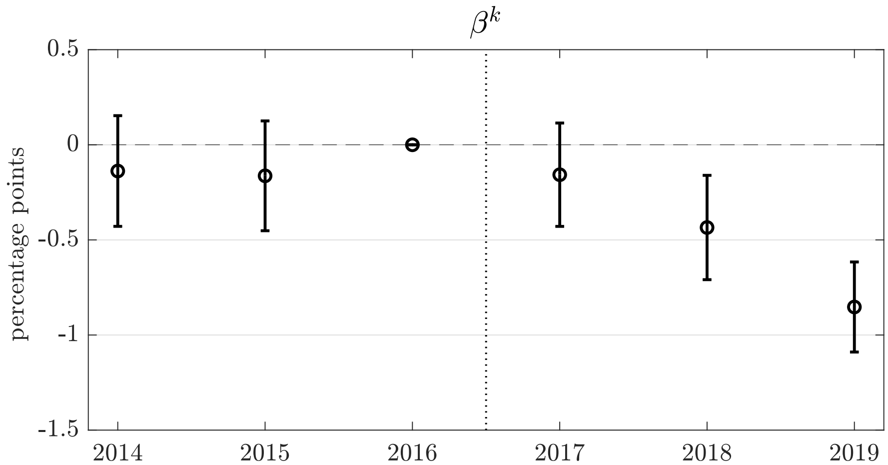
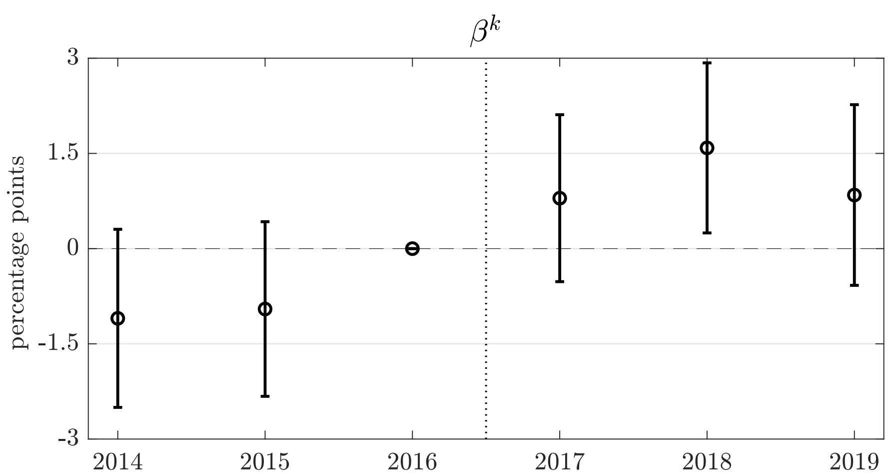


FIGURE 4. Zombie Share



A4. Exposure analysis: Bartik (1991)

To validate the model prediction, I performed the sector (industry) level regression using regional data based on Bartik exposure analysis. Given government loans in period t, sector s is assumed to have a higher exposure to the policy if a sector s had a higher share of small-mid enterprises in region r whose output share was relatively higher before the policy shift. Specifically, the exposure to the policy is calculated as follow,

$$\text{Exposure to Gov' Loan}_{st} = \sum_{r=1}^{13} \underbrace{\frac{\text{number of SMEs}_{sr}}{\text{number of firms}_{sr}}}_{\text{SMEs share in } r \text{ region } s \text{ industry}} \times \overbrace{\underbrace{\frac{\text{total output}_r}{\text{total output}}}_{\text{output share in region } r} \times Gov_t}^{Shock} \tag{A4}$$

Using the exposure, I conduct a following panel regression:

$$y_{st} = \beta \text{Exposure to Gov' Loan}_{st} + \gamma_t + \gamma_s + \epsilon_{st} \tag{A5}$$

The result shows that the increase in government loans decrease firms' exit and investment but increase the share of zombie firms, which are consistent with the model prediction.

TABLE A2. Aggregate Effects with a Reduced Form

	Exit rates	Investment	Zombie shares	Δ TFP
β	-0.009**	-0.064**	0.027*	-0.002*
	(0.003)	(0.013)	(0.025)	(0.001)

A5. Stationary Recursive Equilibrium of the Economy with Government Loans

The stationary recursive equilibrium for this economy consists of (i) policy and value functions of incumbent intermediate goods firms $\{B'(x,k,z), k'(x,k,z), V(x,k,z)\}$; (ii) policy and value functions of entering firms $\{b'(\nu), k'(\nu), V(\nu)\}$; (iii) the bond price schedule $q(B',k',z)$, $q^e(b',k',\nu)$; (iv) price of final good p, demand for final good $y_f(p)$, average productivity of intermediate good firms $\overline{z}$, and mass of entrants ; (v) a stationary measure μ such that:

(1) Given p, the function of $B'(x,k,z)$, $k'(x,k,z)$ solve the problem of incumbent firms, and $V(x,k,z)$ is the associated value function,

$$\begin{aligned} V(x,k,z) = \max_{k',B',b'_g} d + \beta \sum_{z'} \pi(z' \mid z) \left[\int_{\phi' > \tilde{\phi}^G} V\left(x'\left(k', B', z', \phi'\right), k', z'\right) d\Phi\left(\phi'\right) \right] \\ + \beta \sum_{z'} \pi(z' \mid z) \left[\left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G \right) V\left(x'\left(k', B', z', \tilde{\phi}^G\right), k', z'\right) \right] \end{aligned} \tag{A6}$$

subject to (A7) - (A11)

$$d = x - c\left(k, k'\right) + q\left(k', B', b'_g, z\right)(B' - b'_g) + q_g b'_g \geq 0 \tag{A7}$$

$$x(k', B', z', \phi') = (1-\tau)\, pz' \exp(\phi') k'^{\alpha} - f_k k' - f - B' + \tau\left(\delta k + r_f B'\right) \tag{A8}$$

$$\tilde{\phi}^G\left(k', B', b'_g, z'\right) = \log\left(\frac{-\overline{x}^G\left(k', z'\right) + f + f_k k' + B' - \tau\left(\delta k + r_f B'\right)}{(1-\tau)\, pz' k'^{\alpha}} \right) \tag{A9}$$

$$\hat{\phi}^G\left(k', B', b'_g, z'\right) = \log\left(\frac{-\overline{x}^G\left(k', z'\right) + f + f_k k' + B' - (1-q_g)\, b'_g - \tau\left(\delta k + r_f B'\right)}{(1-\tau)\, pz' k'^{\alpha}} \right) \tag{A10}$$

$$\overline{x}^G(k,z) = \max_{k',B',b'_g} q(k', B', b'_g, z)(B' - b'_g) + q_g b'_g - c\left(k, k'\right) \tag{A11}$$

$$b'_g \leq \overline{b_g}, \quad b'_g \leq B' \tag{A12}$$

(2) Given p, the function of $b'(\nu)$, $k'(\nu)$ solve the problem of entering firms, and $V(\nu)$ is the associated value function,

$$V^e(\nu) = \max_{k',b'} d + \beta \sum_{z'} \int_{\phi' > \hat{\phi}} V\left(x'\left(k', b', z', \phi'\right), k', z'\right) d\Phi\left(\phi'\right) dG\left(z' \mid \nu\right) \tag{A13}$$

subject to (A14) - (A17)

$$d = -\psi(k_e, k') + q^e(k', b', \nu)b' \geq 0 \tag{A14}$$

$$x\left(k', b', z', \phi'\right) = (1-\tau)\, pz' \exp\left(\phi'\right) k'^{\alpha} - f_k k' - f - b' + \tau\left(\delta k' + r_f b'\right) \tag{A15}$$

$$\hat{\phi}\left(k', b', z'\right) = \log\left(\frac{-\bar{x}\left(k', z'\right) + f + f_k k' + b' - \tau\left(\delta k' + r_f b'\right)}{(1-\tau)\, pz' k'^{\alpha}}\right) \tag{A16}$$

$$\overline{x}(k, z) = \max_{k', b'} q(k', b', 0, z)b' - \psi(k, k') \tag{A17}$$

(3) The bond price schedule ensures that lenders break even,

$$q\left(k', B', b'_g, z\right) = \beta \sum_{z'} \left[\left(1 - \Phi\left(\hat{\phi}^G\right)\right) + \Phi\left(\hat{\phi}^G\right) R^G\left(B', b'_g, k'\right)\right] \pi\left(z' \mid z\right) \tag{A18}$$

where,

$$\hat{\phi}^G\left(k', B', b'_g, z'\right)\right) = \log\left(\frac{-\overline{x}^G\left(k', z'\right) + f + f_k k' + B' - \left(1 - q_g\right) b'_g - \tau\left(\delta k + r_f B'\right)}{(1-\tau)\, pz' k'^{\alpha}}\right) \tag{A19}$$

$$R^G(B', b'_g, k') = \min\left(1, \max\left(0, \frac{\chi(1-\delta)k' - b'_g - \eta}{B' - b'_g}\right)\right) \tag{A20}$$

$$q_e\left(k', b', \nu\right) = \beta \sum_{z'} \left[\left(1 - \Phi\left(\hat{\phi}\right)\right) + \Phi\left(\hat{\phi}\right) R\left(b', k'\right)\right] dG\left(z' \mid \nu\right) \tag{A21}$$

where,

$$\hat{\phi}\left(k', b', z'\right) = \log\left(\frac{-\bar{x}\left(k', z'\right) + f + f_k k' + b' - \tau\left(\delta k' + r_f b'\right)}{(1-\tau)\, pz' k'^{\alpha}}\right) \tag{A22}$$

$$R(b', k') = \min\left(1, \max\left(0, \chi\frac{(1-\delta)k'}{b'} - \eta\right)\right) \tag{A23}$$

(4) The aggregate production of intermediate good satisfies

$$Y = \sum_z \int_\phi z \exp(\phi) \int_{x_{-1}, k_{-1}, z_{-1}} k(x_{-1}, k_{-1}, z_{-1})^{\alpha} \mu_{-1}\left(x_{-1}, k_{-1}, z_{-1}\right) d\Phi(\phi) \pi\left(z \mid z_{-1}\right) \tag{A24}$$

(5) Average productivity of intermediate good firms $\overline{z}$ is

$$\overline{z} = \sum_{z_i} z_i w(z_i) \tag{A25}$$

where, $w(z_i)$ is a share of output produced by firms whose productivity is z_i:

$$w(z_i) = \int_{\phi} \frac{\int_{x_{-1},k_{-1},z_{-1}} z_i \exp(\phi) k\left(x_{-1}, k_{-1}, z_{-1}\right)^{\alpha} \mu_{-1}\left(x_{-1}, k_{-1}, z_{-1}\right) d\Phi(\phi) \pi\left(z_i \mid z_{-1}\right)}{Y} \tag{A26}$$

(6) p clears final good market.

$$y_f(p) = \overline{z} Y^{\alpha_y} \tag{A27}$$

(7) The cross-sectional distribution of μ is a stationary measure of firms consistent with the firms decision rules and the law of motion for the stochastic variable.

A6. Solution Algorithm

I first discretize the idiosyncratic productivity shock z using Rouwenhorst method. The discretized shocks consist of 11 productivity points, and associated transition matrices $\pi(z' \mid z)$. The idiosyncratic state x is discretized into 15 endogenous grids that depend on the firm's state $\{k, z\}$. I use 50 points for capital, and 101 points for borrowing. The state space for the firm's problem has $\#x \times \#k \times \#z \times = 8,250$ grid points. The resulting array for bond price schedule, $q(k', b', z)$ has $\#k' \times \#b' \times \#z \times = 55,000$ grid points. I also discretize the i.i.d productivity shock ϕ into 101 points using Gaussian quadrature method and use it to evaluate the integrals in the debt price and the firm's continuation value.

I solve the model with two loops: an inner and an outer loop. In the inner loop, there are two separate procedures. Taking as given the price , p, I first find the default cut-off of cash-on-hand and associated debt price schedules. Next, given the found default cut-off and debt price schedules, I find the value function and related policy functions by iteratively solving each firms optimization problem until the value function converges. In the outer loop, taking as given the converged decisions from the inner loop, I start with a distribution of firms $\mu(x, k, s)$ and iterate until the distribution converges. Using a bisection search, I determine the price that clears the final good market.

A6.1. Debt price schedules

Given price p, I first construct maximum level of fund that firm (k, z) can raise, $\overline{x}(k, z)$ and bond price schedule $q(k', b', z)$. I start with an initial guess of $\overline{x}^0(k, z)$. Given $\overline{x}^0(k, z, z_\mu, S)$ I construct the associated default cut-off,

$$\hat{\phi}_0^G\left(k', B', z'\right) = \log\left(\frac{-\overline{x}_0^G\left(k', z'\right) + f + f_k k' + B' - \left(1 - q_g\right)\overline{b_g} - \tau\left(\delta k + r_f B'\right)}{(1-\tau)\, p z' k'^{\alpha}}\right) \quad \text{(A28)}$$

and the associated full repayment cut-off,

$$\tilde{\phi}_0^G\left(k', B', z'\right) = \log\left(\frac{-\overline{x}_0^G\left(k', z'\right) + f + f_k k' + B' - \tau\left(\delta k + r_f B'\right)}{(1-\tau)\, p z' k'^{\alpha}}\right) \quad \text{(A29)}$$

and the associated bond price schedule,

$$q_0\left(k', B', z\right) = \beta \sum_{z'} \left[\left(1 - \Phi\left(\hat{\phi}_0^G\right)\right) + \Phi\left(\hat{\phi}_0^G\right) R^G\left(B', k'\right)\right] \pi\left(z' \mid z\right) \tag{A30}$$

Here the debt price can be determined only if $B' > \overline{b_g}$. Otherwise the firm will finance their debt only via the government loan by the Proposition 1.

In the first step of the iteration, I update the $\bar{x}^1(k, z)$ using

$$\overline{x}_1^G(k, z) = \max_{k', B'} q_0\left(k', B', \overline{b_g}, z\right)\left(B' - \overline{b_g}\right) + q_g \overline{b_g} - \psi\left(k, k'\right)$$

Using updated $\overline{x}^1(k, z)$, I construct the associated default cutoff of productivity shock $\hat{\phi}_1^G\left(k', B', z'\right)$ and bond price schedule $q_1\left(k', B', z\right)$ using the analogs of A28, A30.

I continue this process iteratively until the constructed sequence of $\overline{x}_n^G(k, z)$ converge. I then record the associated array of default cutoff $\hat{\phi}_n^G\left(k', B', z'\right)$, full repayment cutoff $\tilde{\phi}_n^G\left(k', B', z'\right)$ and bond price schedule $q_n\left(k', B', z\right)$, which I hold fixed during each iteration of the firm decision rules. Using the bond price schedule, I construct the price schedule with respect to total debt as follows:

$$Q(k', B', z) = q_g \frac{\overline{b_g}}{B'} + q(k', B', z) \frac{B' - \overline{b_g}}{B'}$$

If $B' < \overline{b_g}$, the debt price equals to q_g.

A6.2. Inner Loops: Firm decisions rules

Given price p, I solve for the decision rules iterating over value functions. I iterate on a set of arrays of grid $\{X(k, z)\}$ that varies with (k, z)

$$X(k, z) = \{x_1, \ldots, x_N\}$$

(1) Given an initial guess for value function $V^0(x, k, z), V_{nb}^0(k, z)$, and for the set of arrays of grids $\left\{X^0(k, z)\right\}$, I solve for the cutoff $\hat{x}_1(k, z)$ by solving for $\hat{k}'(k, z)$ and $\hat{B}'(k, z)$, which is firms optimal decision when the nonnegative equity payout constraint does not bind, following Arellano, Bai, and Kehoe (2019). Specifically, I find $\hat{x}_1(k, z)$ by first solving a "relaxed" version of the firm's problem, where I drop the non-negative equity payout condition for the current period only. The associated decision for

capital and borrowing is denoted by $\hat{k}(k, z)$ and $\hat{B}(k, z)$, which can be obtained by solving the following problem,

$$\max_{k', B'} -c(k, k') + Q(k', B', z)B' + \beta \sum_{z'} \pi (z' \mid z) \left[\int_{\phi' > \tilde{\phi}^G} V_0 (x' (k', B', z', \phi'), k', z') \, d\Phi (\phi') \right]$$
$$+ \beta \sum_{z'} \pi (z' \mid z) \left[\left(\Phi \left(\bar{\phi}^G \right) - \Phi \left(\hat{\phi}^G \right) V_0 \left(x' \left(k', B', z', \bar{\phi}^G \right), k', z' \right) \right] \tag{A31}$$

Then the level of cash-on-hands, where the nonnegative equity payout constraint does not bind is

$$\hat{x}^{n+1}(k, z) = \psi(k, \hat{k}'(k, z)) - Q(\hat{k}'(k, z), \hat{B}'(k, z), z)\hat{B}'(k, z) \tag{A32}$$

Construct the grid $\left\{X^{n+1}(k, z)\right\} = \left\{x_1^{n+1}, x_2^{n+1}, \ldots, x_N^{n+1}\right\}$ by setting

$$x_1^{n+1} = -\bar{x}(k, z) \quad \text{and} \quad x_N^{n+1} = \hat{x}^1(k, z)$$

That is, we know that if the cash-on-hand x is so low, $x + \bar{x}(k, z) < 0$, even with the maximum funds raised by borrowing and disposing of capital, the associated dividends $d = x + \bar{x}(k, z)$ is negative and the firm will default. We also know that if the cash-on hand x is sufficiently high, so that $x \geq \hat{x}(k, z)$, the optimal decisions will be given by the nonbinding level of capital $\hat{k}(k, z)$ and borrowing $\hat{b}(k, z)$ because the decision is not affected by nonnegative equity payout condition. I then choose a set of intermediate points $\{x_2, \ldots, x_{N-1}\}$. Therefore, along with the value function $V(x, k, z)$ the endogenous grid $X = \{x_1, \ldots, x_N\}$ is updated in each iteration of the loop. Here's the specific steps.

a. First I construct the value for each choice $\{k', B'\}$ over the grid points such that

$$W_0(k', B', z) = \sum_{z'} \pi (z' \mid z) \left[\int_{\phi' > \tilde{\phi}^G} V_0 (x' (k', B', z', \phi'), k', z') \, d\Phi (\phi') \right]$$
$$+ \sum_{z'} \pi (z' \mid z) \left[\left(\Phi \left(\bar{\phi}^G \right) - \Phi \left(\hat{\phi}^G \right) V_0 \left(x' \left(k', B', z', \bar{\phi}^G \right), k', z' \right) \right] \tag{A33}$$

The value off the grid over x is calculated using linear interpolation.

b. Then I find the optimal options over the grids that maximize

$$-\psi(k,k') + Q(k',B',z)B' + W_0(k',B',z)$$

c. Using the solution as a initial guess, I solve the optimization problem using Powell's method.[24]

For calculation of default and full repayment cutoff, $\overline{x}^G$ off the grid points over k were calculated using linear interpolation. Value off the grid points for k', and x' are also calculated using linear interpolation. For example let's assume $k' \in [k_{i-1}, k_i]$. Then we can calculate $\overline{x}^G(k,z)$ which is off grid of capital using linear interpolation, and accordingly we can calculate $\tilde{\phi}^G(k',B',z')$ and $\hat{\phi}^G(k',B',z')$. For given shocks z' and ϕ', $x'(k',B',z',z')$ can be calculated using equation A8. Using this, I calculate $V_0(x',k_{i-1},z')$ and $V_0(x',k_i,z')$ by interpolating between x grid points based on the grid $\left\{X^0(k,z)\right\}$, respectively for k_{i-1} and k_i. Then I interpolate between two capital grid points. Furthermore when $x > x_N^0$, $V_0(x',k_i,z') = x' + V_0^{nb}(k',z')$.

(2) Solve for decisions at the intermediate points and find policy function, $\{k'(x,k,z), b'(x,k,z)\}$. At these nodes, since the non equity payout constraint is binding, for each (x,k,z) I can solve for b' off-grid given k' from the following equation,

$$x - \psi\left(k,k'\right) + Q\left(k',B',z\right)B' = 0$$

Then using the condition, I can find optimal choice for capital and borrowing in a previously outlined way.

(3) I update the value function to V^{n+1} using

$$V^{n+1}(x,k,z) = x - \psi(k,k') + Q(k',B',z)B' + \underbrace{\beta \sum_{z'} \int_{\phi'>\hat{\phi}(k',b'z')} V^n(x'(k',B',z',\phi'),k',z')}_{W(k',B',z)} \quad \text{(A34)}$$

- Policy functions for firms with binding NEP : $k' = k'(x,k,z)$, $B' = B'(x,k,z)$
- Policy functions for firms with non-binding NEP : $k' = \hat{k}'(k,z)$, $B' = \hat{B}'(k,z)$

[24]Specifically I use the fminsearch subroutine from the source codes accompanying the book Fehr, H. & Kindermann, F. (2018). Introduction to Computational Economics using Fortran. Oxford: Oxford University Press.

(4) Iterate until the value functions $W^n\left(k', b', z\right)$ on grid for capital, debt, and productivity converge.

A6.3. Outer loop: Stationary distribution and equilibrium price

In an outer loop, I update the price p based on stationary cross-sectional distribution.

I use a histogram-based approach to tracking the cross-sectional distribution following Young (2010). I use grids for net-income ratio, $\frac{x}{k}$ which is denoted by x_k, rather than cash-on-hand, x, itself. I use 101 grids point for net-income ratio, ranged from -2 to 2. I use denser grids for k, such that $n_k^o = 80$.

I simulate firms on a discretion grid of $\#x^k \times \#k \times \#z \times = 88,880$. Since there is a finite number of grid points for x^k and k, first I need to allocate the mass of any x' and k' to points on the x^k-grid and k-grid. Specifically, I allocate the mass of firms with any x' and k' to the bracketing interval $\left[x_{i-1}^k, x_i^k\right]$ on the x^k-grid in proportion to how close $x^k = \frac{x'}{k'}$ is to each side of the interval. Specifically, let $\omega_{xk}\left(x_i^k, x^k\right)$ be the probability that the choice of x^k is assigned to x_i :

$$\omega_{xk}\left(x_i^k, x^k\right) = \frac{x^k - x_{i-1}^k}{x_i^k - x_{i-1}^k} \quad \text{and} \quad \omega_{xk}\left(x_{i-1}^k, x^k\right) = 1 - \omega_x\left(x_i^k, x^k\right)$$

and $\omega_{xk}\left(x_i^k, x^k\right) = 0$ if $x^k \notin \left[x_{i-1}^k, x_i^k\right]$.

The same idea goes for k'. Let $\omega_k\left(k_j, k'\right)$ be the probability that the choice of k' is assigned to k_j :

$$\omega_k\left(k_j, k'\right) = \frac{k' - k_{j-1}}{k_j - k_{j-1}} \quad \text{and} \quad \omega_k\left(k_{j-1}, k'\right) = 1 - \omega_k\left(k_j, k'\right)$$

and $\omega_k\left(k_j, k'\right) = 0$ if $k' \notin \left[k_{j-1}, k_j\right]$.

I update the distribution as follows until the distribution $\mu(x, k, z)$ converges.

$$
\begin{aligned}
&\mu'(x_i^k, k_j, z') = \\
&\sum_{x_i^k, k_j, z} \int_{\phi' \geq \tilde{\phi}^G} \omega_{xk}\left(x_i, x^k\left(k'(x,k,z), B'(x,k,z), z', \phi'\right)\right) \omega_k\left(k_j, k'(x,k,z)\right) d\Phi(\phi')\pi(z' \mid z)\mu(x_i^k, k_j, z) \\
&+ \sum_{x_i^k, k_j, z} \left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G)\right) \omega_{xk}\left(x_i, x^k\left(k'(x,k,z), b'(x,k,z), z', \tilde{\phi}^G\right)\right) \omega_k\left(k_j, k'(x,k,z)\right) \pi(z' \mid z)\mu(x_i^k, k_j, z) \\
&+ M \int_{\nu \geq \hat{\nu}} \int_{\phi' \geq \hat{\phi}(k', b', z')} \omega_x\left(x_i, x'\left(k'(\nu), b'(\nu), z', \phi'\right)\right) \omega_k\left(k_j, k'(\nu)\right) d\Phi(\phi') H(z' \mid \nu) dG(\nu)
\end{aligned}
\tag{A35}
$$

where,

$$
x^k = \frac{x'(k'(x,k,z), B'(x,k,z), z', \phi')}{k'(x,k,z)}
$$

$$
x'(k', B', z', \phi') = (1-\tau)\, pz' \exp(\phi') k'^{\alpha} - f_k k' - f - B' + \tau\left(\delta k + r_f B'\right)
$$

$$
\tilde{\phi}^G\left(k', B', b'_g, z'\right) = \log\left(\frac{-\overline{x}^G\left(k', z'\right) + f + f_k k' + B' - \tau\left(\delta k + r_f B'\right)}{(1-\tau)\, pz' k'^{\alpha}}\right)
$$

$$
\hat{\phi}^G\left(k', B', b'_g, z'\right) = \log\left(\frac{-\overline{x}^G\left(k', z'\right) + f + f_k k' + B' - \left(1 - q_g\right) b'_g - \tau\left(\delta k + r_f B'\right)}{(1-\tau)\, pz' k'^{\alpha}}\right)
$$

Given the converged distribution $\mu(x^k, k, z)$, I calculate the excess demand $ED(p)$,

$$
ED(p; \mu) = \left[\frac{\overline{z}(\mu)\alpha_y}{p}\right]^{\frac{1}{1-\alpha_y}} - Y(\mu)
$$

where,

$$
Y(\mu) = \sum_{z'} \int_{\phi} \sum_{x_i^k, k_j, z} z' \exp(\phi) k'\left(x_i^k k_j, k_j, z\right)^{\alpha} \mu\left(x_i^k, k_j, z\right) d\Phi(\phi) \pi\left(z' \mid z\right)
$$

$$
\overline{z}(\mu) = \sum_{z_i} z_i \underbrace{\frac{\int_{\phi} \sum_{x_i^k, k_j, z} z_i \exp(\phi) k\left(x_i^k k_j, k_j, z\right)^{\alpha} \mu\left(x_i^k, k_j, z\right) d\Phi(\phi) \pi\left(z_i \mid z\right)}{Y}}_{\text{Share of output produced by firms with } z_i}
$$

I use a bisection search to determine the price that clears the final good market.

Specifically, I choose two prices, p_l and p_h, such that excess demand $ED(p_l) > 0$ and $ED(p_h) < 0$. Set $p^1 = \frac{p_l + p_h}{2}$. If $ED(p^1) < 0$ then update price as $p^2 = \frac{p_l + p^1}{2}$, and if If $ED(p^1) > 0$ then update price as $p^2 = \frac{p^1 + p^h}{2}$. I iterate the procedure until the price p^n converges.

A6.4. Transition path

I use the following algorithm to compute the transition path between two states with and without government loans.

(1) Calculate the stationary equilibrium with and without government loans. Save the associated policy functions and value functions of firms, the equilibrium price in two economies, and the distribution of the steady state in the economy without government loans.

(2) Assume that the economy transitions to the new steady state over a period of $T = 200$ years. Government loans are introduced at the beginning of period 1, specifically before decisions on default and firms' decisions regarding capital and borrowing, but after the realization of persistent and transitory productivity shocks. Guess the path for price $\{\mathbb{P}\}^0$, which is the vector of price from period 1 to T.

(3) Taking the paths of price as given, I calculate the full transition path by iterating the following steps:

   a. Solve for policy and value functions over the transition for $t = T-1, T-2, \ldots, 1$ by iterating backward. Specifically, I derive the policy and value functions for period t by using the value functions derived from period $t+1$:

$$\begin{aligned} V^t(x,k,z) = \max_{k',B',b'_g} d + \beta \sum_{z'} \pi\left(z' \mid z\right) \left[\int_{\phi' > \tilde{\phi}^G} V^{t+1}\left(x'\left(k',B',z',\phi'\right),k',z'\right) d\Phi\left(\phi'\right) \right] \\ + \beta \sum_{z'} \pi\left(z' \mid z\right) \left[\left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G \right) V^{t+1}\left(x'\left(k',B',z',\tilde{\phi}^G\right),k',z'\right) \right] \end{aligned} \tag{A36}$$

   b. Compute the evolution of firms distribution over the transition for $t = 2, 3, \ldots, T$ by iterating forward. Specifically, I update the firms distribution for period t from the firms distribution for period $t-1$:

$$\mu'_t(x_i^k, k_j, z') =$$

$$\sum_{x_i^k, k_j, z} \int_{\phi' \geq \tilde{\phi}^G} \omega_{xk} \left(x_i, x^k \left(k'(x,k,z), B'(x,k,z), z', \phi'\right)\right) \omega_k \left(k_j, k'(x,k,z)\right) d\Phi(\phi')\pi(z' \mid z)\mu_{t-1}(x_i^k, k_j, z)$$
$$+ \sum_{x_i^k, k_j, z} \left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G\right) \omega_{xk} \left(x_i, x^k \left(k'(x,k,z), b'(x,k,z), z', \tilde{\phi}^G\right)\right) \omega_k \left(k_j, k'(x,k,z)\right) \pi(z' \mid z)\mu_{t-1}(x_i^k, k_j, z)$$
$$+ M \int_{\nu \geq \hat{\nu}} \int_{\phi' \geq \hat{\phi}(k', b', z')} \omega_x \left(x_i, x'\left(k'(\nu), b'(\nu), z', \phi'\right)\right) \omega_k \left(k_j, k'(\nu)\right) d\Phi(\phi') H(z' \mid \nu) dG(\nu) \tag{A37}$$

Here, note that the cash-on-hand cutoff for default and being unconstrained varies with the equilibrium price, and the cutoff for potential entrants' signal to enter also varies with the equilibrium price.

c. Given the firms' distribution in each period t, I calculate the excess demand for each period as follows,

$$ED_t = \left[\frac{\overline{z}_t \alpha_y}{p_t}\right]^{\frac{1}{1-\alpha_y}} - Y_t$$

where,

$$Y_{t+1} = \sum_{z_{t+1}} \int_{\phi} \sum_{x_i^k, k_j, z} z_{t+1} \exp(\phi) k_{t+1} \left(x_i^k k_j, k_j, z\right)^{\alpha} \mu_t \left(x_i^k, k_j, z\right) d\Phi(\phi)\pi\left(z_{t+1} \mid z\right)$$

$$\overline{z}_{t+1} = \sum_{z_i} z_i \underbrace{\frac{\int_{\phi} \sum_{x_i^k, k_j, z} z_i \exp(\phi) k_{t+1} \left(x_i^k k_j, k_j, z\right)^{\alpha} \mu_t \left(x_i^k, k_j, z\right) d\Phi(\phi)\pi\left(z_i \mid z\right)}{Y_{t+1}}}_{\text{Share of output produced by firms with } z_i}$$

c. Calculate the vector of market-clearing price path $\{P\}^*$ such that $p_t^* = Y_t^{\alpha_y - 1}\overline{z}\alpha_y$. If the difference between the vector of market-clearing price path and $\{P\}^n$ is smaller than a pre-specified tolerance then stop. Otherwise I update the price vector $\{P\}^{n+1} = \lambda\{P\}^n + (1-\lambda)\{P\}^*$, and go back to the step a.

(4) Once the market-clearing price is found, I calculate the firms policy and value functions, and distributions over the transitions.

A6.5. Simulation and replication procedure

The simulation procedure aims to compare the heterogeneous responses of firms simulated from the model over the three years following the introduction of government loans and data. I simulate the economy for $T = 500$ periods and introduce government loans at $t = T - 2$. Policy functions and the equilibrium price remain the same from period 1 to T-3, while these policy functions and equilibrium price will transition from $t = T - 2$ to $t = T$. I start with $N = 5000$ firms, and every period, some firms exit, while the surviving firms make choices regarding capital and debt. Additionally, every period, firms enter based on the signals they receive from a pool of $N = 5000$ potential entrants. I construct the panel data of firms by discarding the first 450 years and replicate the estimation based on data using the constructed panel.

A7. Definition of moments: Data and Model

(1) Net-income ratio

- Data : $\frac{\text{Net income}}{\text{Total asset}}$
- Model : $\frac{\mathrm{x}_t}{\mathrm{k}_t}$

(2) Investment

- Data: $\frac{2\times[\text{Tangible Asset}_{t+1}-\text{Tangible Asset}_t]}{\text{Tangible Asset}_{t+1}+\text{Tangible Asset}_t}$
- Model: $\frac{2\times[k_{t+1}-(1-\delta)k_t]}{k_{t+1}+(1-\delta)k_t}$

(3) Spread

- Data: $\frac{2\times\text{Interest expense}_t}{\text{Total debt}_t+\text{Total debt}_{t-1}}\times 100$ – Korean corporate bond rate (AA- 3yr)
- Model: $\left(\frac{1}{q_t}-\frac{1}{\beta}\right)\times 100$

(4) Size

- Data: Tangible asset size
- Model: Capital size

(5) Profitability

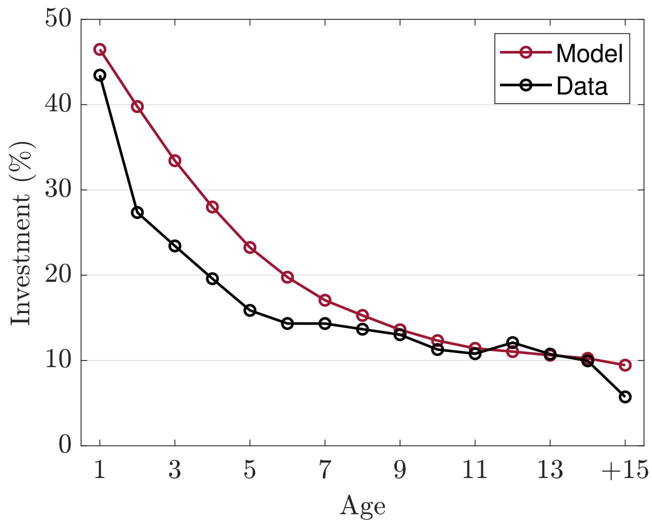
- Data: Operational profit / Total asset
- Model : $\frac{pz_t\exp(\phi_t)k_t^{\alpha}-f-f_k k_t}{k_t}$

(6) Individual firm's TFP (revenue-based)

- Data: $\frac{\text{Sales}_t\times 2}{\text{Total asset}_{t-1}+\text{Total asset}_t}$
- Model: $z_t\exp(\phi_t)$

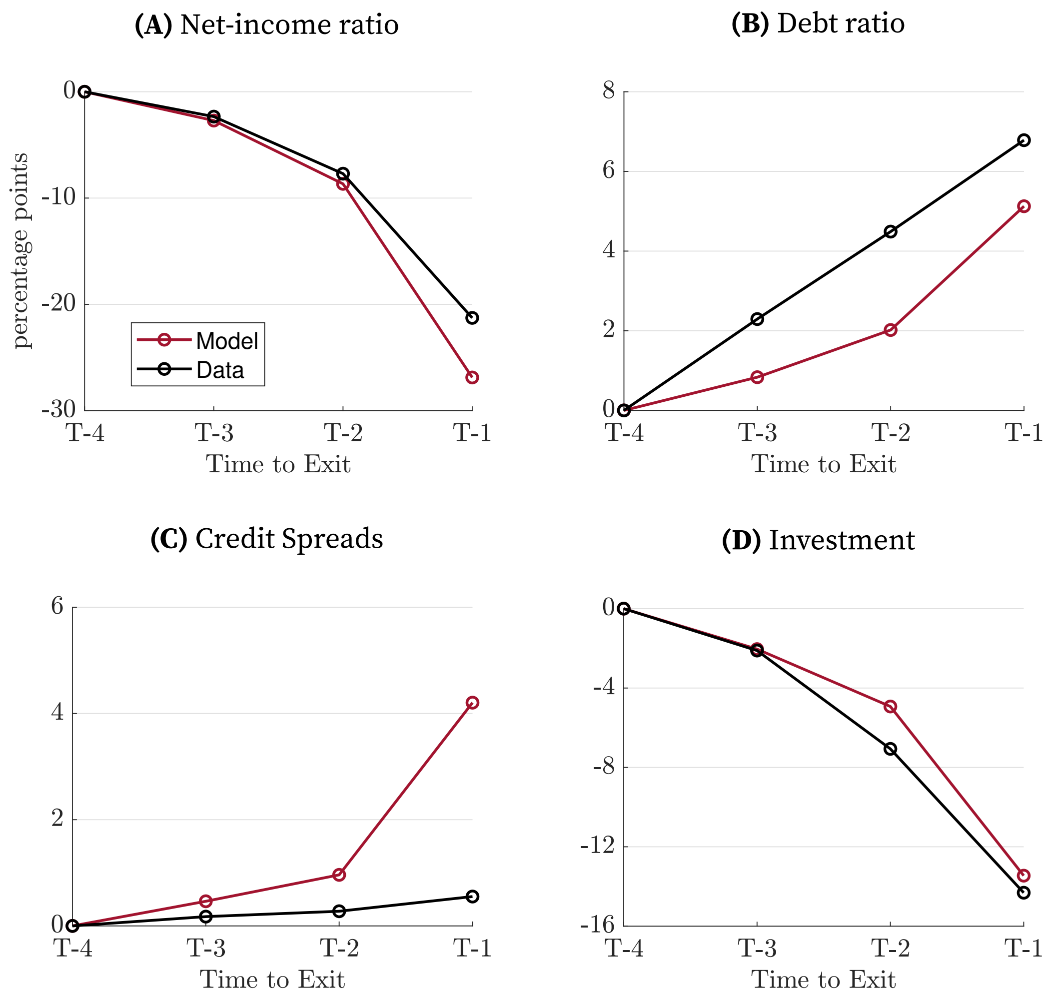
A8. Model performance: Untargeted

FIGURE 5. Investment by Age: Data vs Model



Notes: The figure shows investment by firms age.

FIGURE 6. Financial States Before Firm Exits: Data vs Model



Notes: These plots show the relative financial state of firms with specific distance to exit based on data and simulated firms.

A9. Proofs and Derivation

A9.1. Proof of Proposition 1

The proposition can be verified by confirming that firms' value strictly increases when substituting private loans with government loans given the same total debt amount. Given a choice for a total debt, denoted as B', a firm will borrow $b'_g = \overline{b_g}$ from the government if the selected total debt exceeds the government limit; otherwise, the firm will solely borrow from the government.

The derivative of firms value function in equation 14 with respect to b'_g given B' stays same,

$$\frac{V(x,k,z)}{b'_g} = \underbrace{(q_g - q) + \frac{\partial q}{\partial b'_g}(B' - b'_g)}_{\zeta} + \beta \sum_{z'} \pi(z' \mid z) \underbrace{\left(-\frac{\hat{\phi}^G}{b'_g}\right)}_{>0} V\left(x'\left(k', B', z', \tilde{\phi}^G\right), k', z'\right) \quad \text{(A38)}$$

The value of substituting private loans with government loans comes from two aspects: one arises from an increase in debt price, given the same borrowed debt amount (higher funding capacity), and the other comes from a reduced default probability, given the same borrowed debt amount.

Using the following conditions,

$$q\left(k', B', b'_g, z\right) = \beta \sum_{z'} \left[\left(1 - \Phi\left(\hat{\phi}^G\right)\right) + \Phi\left(\hat{\phi}^G\right) R^G(B', b'_g, k')\right] \pi(z' \mid z)$$

where,

$$\hat{\phi}^G\left(k', B', b'_g, z'\right) = \log\left(\frac{-\bar{x}^G\left(k', z'\right) + f + f_k k' + B' - (1-\beta)\, b'_g}{p z' k'^{\alpha}}\right)$$

$$R^G(B', b'_g, k') = \min\left(1, \max\left(0, \frac{\chi(1-\delta)k' - b'_g - \eta}{B' - b'_g}\right)\right)$$

We can derive,

$$\frac{\partial q}{\partial b'_g} = \beta \sum_{z'} \left[\left(-\frac{\hat{\phi}^G}{\partial b'_g}\right) \Phi'(\hat{\phi}^G)\left(1 - R^G\right) + \Phi(\hat{\phi}^G)\frac{\partial R^G}{\partial b'_g}\right] \pi(z' \mid z)$$

Now we have,

$$\zeta = \underbrace{\beta \sum_{z'} \left[\Phi \left(\hat{\phi}^G \right) (1 - R^G) \right] \pi(z' \mid z)}_{q_g - q} + \beta \sum_{z'} \left[\left(-\frac{\hat{\phi}^G}{\partial b'_g} \right) \Phi'(\hat{\phi}^G) \left(1 - R^G \right) (B' - b_{g'}) - \Phi(\hat{\phi}^G)(1 - R^G) \right] \pi(z' \mid z)$$

$$= \beta \sum_{z'} \left[\left(-\frac{\hat{\phi}^G}{\partial b'_g} \right) \Phi'(\hat{\phi}^G) \left(1 - R^G \right) (B' - b_{g'}) \right] \pi(z' \mid z) \geq 0 \tag{A39}$$

Therefore, $\frac{V(x,k,z)}{b'_g} > 0$.

A9.2. Proof of Proposition 2

Default: Firms only default if there is no feasible set that satisfies the non-negative dividends payout condition, i.e.,

$$\nexists \left(k', B' \right) \text{ such that } x - \psi \left(k, k' \right) + q \left(k', b', b'_g z \right) \left(B' - b'_g \right) + q_g b'_g \geq 0$$

Then we can define the default threshold on x such that $x + \overline{x}^G(k, z) < 0$, where $\bar{x}^G(k, z) = \max_{k',B'} q \left(k', B', \overline{b_g}, z \right) (B' - \overline{b_g}) + q_g \overline{b_g} - \psi \left(k, k' \right)$, which indicates the maximum level of fund a firm can raise with debt financing and capital disposal. If $x + \overline{x}^G(k, z) < 0$, then there is no a feasible set for a firm to satisfy the non-negative dividends payout condition. This is because a firm cannot avoid a negative dividend level even after maximizing their fund. Therefore if $x < \underline{x}(k, z) = -\overline{x}^G(k, z)$, firms default.

Unconstrained: For $x > \underline{x}(k, z) = -\overline{x}^G(k, z)$, we can construct a threshold $\hat{x}(k, z)$ such that the firms' choice for (k', B') does not depend on firms level of cash-on-hand, x, by solving a relaxed version of the firm's problem following Arellano, Bai, and Kehoe (2019), Ottonello and Winberry (2020). Specifically, we can solve the relaxed version of the probelm by dropping the nonnegative equity payout constraint for the current period only as follows,

$$\max_{k',B'} -\psi \left(k, k' \right) + q \left(k', b', b'_g z \right) \left(B' - b'_g \right) + q_g b'_g + \sum_{z'} \pi \left(z' \mid z \right) \left(\left[\int_{\phi' > \tilde{\phi}^G} V \left(x' \left(k', B', z', \phi' \right), k', z' \right) d\Phi \left(\phi' \right) \right] + \left[\Phi \left(\tilde{\phi}^G \right) - \Phi \left(\hat{\phi}^G \right) \right] \tilde{V} \right) \tag{A40}$$

where $\tilde{V} = \left(x'\left(k', B', z', \tilde{\phi}^G\right), k', z'\right)$

Then we can construct the threshold

$$\hat{x}(k,z) = \psi\left(k, \hat{k}'(k,z)\right) - q\left(\hat{k}'(k,z), \hat{B}'(k,z), z\right)\left(\hat{B}'(k,z) - \overline{b_g}\right) + q_g \overline{b_g}$$

where $(\hat{k}, \hat{B}')$ is the solution for the problem in equation A40. Note that cash-on-hand x enters simply as an additive constant in the objective function (the value of constrained firms is the sum of value in equation A40 and cash-on-hand x) and not in any constraint. Therefore, the solution for the relaxed problem does not depend on the level of cash-on-hand. If firms' cash-on-hand is above the threshold, then the dividends increases one for one with the cash-on-hand and the choice for capital and borrowing does not vary with the cash-on-hand.

Constrained: I will show that firms with $x \in [\underline{x}](k,z), \hat{x}(k,z)$ pay zero dividends, $d = 0$. The optimality condition for B' is as follows,

$$\beta \sum_{z'} \pi(z' \mid z)\left[\int_{\phi' \geq \tilde{\phi}^G} \left(1 + \eta'\left(x'(k', B', z', \phi'), k', z'\right)\right) d\Phi(\phi') + \left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G)\right)\left(1 + \eta'\left(x'(k', B', z', \tilde{\phi}^G), k', z'\right)\right)\right]$$
$$+ \beta \sum_{z'} \pi(z' \mid z)\left(\frac{\partial \hat{\phi}^G}{\partial B'}\right)\phi(\hat{\phi}^G) V\left(x'(k', B', z', \tilde{\phi}^G), k', z'\right) = (1 + \eta(x,k,z))\left[q + \frac{\partial q}{\partial B'}\left(B' - \overline{b_g}\right)\right] \tag{A41}$$

I will show that if a constrained firm pays a positive dividend, i.e., $\eta(x,k,z) = 0$ (the non-negative dividends payout condition slacks), then this leads to a contradiction.

Let's first consider a case of a firm with a zero probability of default in the next period. In this case, we can write the condition in equation A41 as follows,

$$\beta + \beta \sum_{z'} \pi(z' \mid z)\left[\int_{\phi'} \eta'(x'(k', B', z', \phi'), k', z') d\Phi(\phi')\right] = \beta$$

Since the firm is constrained, which implies a positive debt, $\eta'(x'(k', B', z', \phi'), k', z') > 0$ for some positive mass of realizations of z' and ϕ', which results in a contradiction.

Next, let's consider a case of a firm with a positive probability to default in the next period. In this case, using the following condition,

$$q + \frac{\partial q}{\partial B'}\left(B' - \overline{b_g}\right) = \beta \sum_{z'}\left\{\left[1 - \Phi\left(\hat{\phi}^G\right)\right] + \left[\left(-\frac{\partial \hat{\phi}^G}{\partial B'}\right)\phi(\hat{\phi}^G)\left(B' - \overline{b_g}\right)\left(1 - R^G\right)\right]\right\}\pi(z' \mid z)$$

we can write the condition in equation A41 as follows,

$$\beta \sum_{z'} \pi(z' \mid z) \left[\int_{\phi' \geq \tilde{\phi}^G} \eta' \left(x'(k', B', z', \phi'), k', z' \right) d\Phi(\phi') + \left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G) \right) \eta' \left(x'(k', B', z', \tilde{\phi}^G), k', z' \right) \right]$$
$$+ \beta \sum_{z'} \pi(z' \mid z) \left(\frac{\partial \hat{\phi}^G}{\partial B'} \right) \phi(\hat{\phi}^G) \left[\left(B' - \overline{b_g} \right) \left(1 - R^G \right) + V \left(x'(k', B', z', \tilde{\phi}^G), k', z' \right) \right] = 0$$

Since the firm is constrained $\eta'(x'(k', B', z', \phi'), k', z') > 0$ for some positive mass of realizations of z' and ϕ' and $\left[\left(B' - \overline{b_g} \right) \left(1 - R^G \right) + V \left(x'(k', B', z', \tilde{\phi}^G), k', z' \right) \right] > 0$, which results in a contradiction.

A9.3. Proof of Proposition 3

Given capital stock K, and a measure of firms $\mu(x, k, z)$ with mass M, the planner's problem can be defined using equation 24,

$$\max_{k(x_{-1}, k_{-1}, z_{-1})} \int_{x_{-1}, k_{-1}, z_{-1}} \sum_z z k \left(x_{-1}, k_{-1}, z_{-1} \right)^{\alpha} \pi \left(z \mid z_{-1} \right) d\mu_{-1} \left(x_{-1}, k_{-1}, z_{-1} \right)$$

$$\text{s.t.} \int k(x_{-1}, k_{-1}, z_{-1}) \, \mathrm{d}\mu(x_{-1}, k_{-1}, z_{-1}) \leq K.$$

The first order condition is given by

$$k(x_{-1}, k_{-1}, z_{-1}) = \left(\frac{\alpha}{\lambda_k} \right)^{\frac{1}{1-\alpha}} \tilde{z}^{\frac{1}{1-\alpha}}$$

where, $\tilde{z} = \sum_z z\pi \left(z \mid z_{-1} \right)$ is the conditional expected productivity given today's productivity, and λ_k is the Lagrangian multiplier for the resource constraint.

Integrating the equation, we have

$$\underbrace{\int k(x_{-1}, k_{-1}, z_{-1}) d\mu(x_{-1}, k_{-1}, z_{-1})}_{K} = \left(\frac{\alpha}{\lambda_k} \right)^{\frac{1}{1-\alpha}} \int \tilde{z}^{\frac{1}{1-\alpha}} d\mu(x_{-1}, k_{-1}, z_{-1})$$

Then the solution to the planner's problem is as follows,

$$k(x_{-1}, k_{-1}, z_{-1}) = \frac{\tilde{z}^{\frac{1}{1-\alpha}}}{\int \tilde{z}^{\frac{1}{1-\alpha}} d\mu(x_{-1}, k_{-1}, z_{-1})} K$$

Notice that the planner's allocation does not depend on the level of cash-on-hands x, and the planner equates the marginal product of capital across all firms.

The maximum output that can be achieved with reallocation resource across firms, Y^* is defined as in Proposition 2,

$$Y^* = K^{\alpha} \left[\int \tilde{z}^{\frac{1}{1-\alpha}} d\mu(x_{-1}, k_{-1}, z_{-1}) \right]^{1-\alpha}$$

Accordingly, we can write the output as follows,

$$Y = M^{1-\alpha} \times K^{\alpha} \times \left[\frac{\int \tilde{z}^{\frac{1}{1-\alpha}} d\mu(x_{-1}, k_{-1}, z_{-1})}{M} \right]^{1-\alpha} \times \frac{Y}{Y^*}$$

where, $M = \int d\mu(x_{-1}, k_{-1}, z_{-1})$