

Government-Backed Financing and Aggregate Productivity

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Government-Backed Financing

Loan guarantees, direct loans, debt relief

- Used worldwide mainly to promote growth of **small medium sized enterprises** (SMEs)
 - Intended to fill the financing gap between large firms and SMEs (OECD)

These policies often change in response to crisis episodes

No consensus about these policies' effects on aggregate productivity

- Benefit: Help financially constrained yet productive firms grow
Stiglitz (1993), Banerjee and Duflo (2014), Jiménez, Peydró, Repullo and Saurina Salas (2018)
- Cost: Help low-productivity firms survive (often called zombie firms)
Tracey (2019), Acharya, Crosignani, Eisert and Steffen (2021), Faria-e-Castro, Paul and Sánchez (2021)

What I Do

1. Exploit an increase in government loans to firms: **1pp of GDP over 3 years**
 - Expansion after 2017: newly elected government's policy agenda to promote SMEs
2. **Document policy effects** using new data
 - Data: **panel of audited financial statements** of Korean manufacturing firms (14,569)
 - Active + exiting firms (financial state at exit)
 - Policy eligibility: small-mid sized enterprises (SMEs)
3. Quantify aggregate effect using a heterogeneous-firm model
 - Arellano, Bai and Kehoe (2019), Ottonello and Winberry (2020)
 - **Endogenous borrowing costs** - Study **transitions** after the introduction of government loans

Firm-level policy effects based on difference-in-difference regression

- **Borrowing costs** of eligible firms decreased more relative to non-eligible firms
- **Investment** increased more for eligible firms with high pre-policy borrowing costs
- **Exit rates** decreased most among eligible low-productivity firms

Key trade-off of government-backed financing

- \uparrow investment of constrained firms - \downarrow exit of low-productivity firms

Aggregate productivity: -0.3% (over 10-year)

- Capital allocation: \uparrow investment of constrained firms (+0.1%)
- Active firms' composition: \downarrow exit of low-productivity firms (-0.4%)

Firm dynamics and financial frictions

Buera, Kaboski and Shin (2011), Moll (2014), Midrigan and Xu (2014)

Gopinath, Kalemli-Özcan, Karabarbounis and Villegas-Sanchez (2017), Arellano, Bai and Kehoe (2019)

Credit misallocation generated from subsidized loans

Caballero, Hoshi and Kashyap (2008), Tracey (2019),

Acharya, Crosignani, Eisert and Steffen (2021), Faria-e-Castro, Paul and Sánchez (2021)

Government's intervention in credit market

Banerjee and Duflo (2014), Jiménez, Peydró, Repullo and Saurina Salas (2018), Crouzet and Tourre (2021)

Contribution built on literature emphasizing financial friction

- Provide empirical evidence that suggests subsidized loans distort the selection
- Study policy effects outside crisis episodes, capturing the main trade-off

Korean Policy and Data

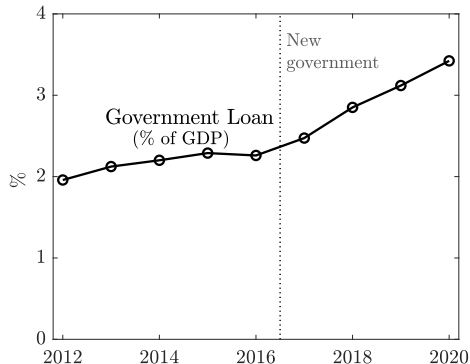
Government Loans in Korea

Eligibility: small-mid sized enterprises

1. Cutoff defined by the law
 - Total asset: 380 Mil USD (Top 3 %)
 - 3-year average sales: 60-120 Mil USD (varies by sectors)
2. Not affiliated with large conglomerate, *Chaebol* (e.g. Samsung)

Key features:

1. lower interest rates [Compare](#)
2. extended up to a fixed limit
3. partial debt relief during cash-shortages



Data: financial statements of Korean manufacturing firms

- Manufacturing firms with assets over 9 million USD subject to **external audits**
 - Revenue of sample firms $\approx 80\%$ of total sales
- Large (2,108) + **SMEs** (12,461) : **eligibility for the policy**
- Active firms (12,976) + **Exiting firms** (1,593): **observe financial state at exit**
- Key variables: sales, operating profit, interest expense, total debt, tangible assets

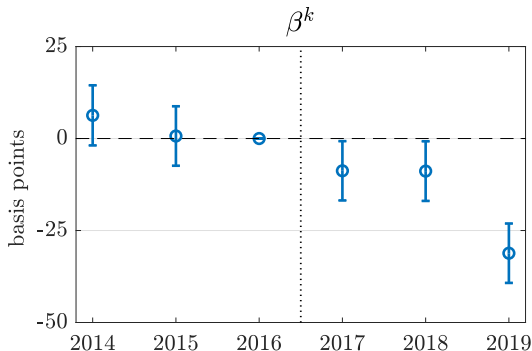
Difference-in-difference: borrowing costs (credit spread), investment, exit

- Before (2014-16) and After (2017-19) \times Eligibility (status in 2020) BOK Keyrates
- Credit spread = $\frac{\text{interest expense}}{\text{total debt}}$ - prime rate - Investment = Δ tangible asset

Firm Level Policy Effects

Empirical Fact 1: Borrowing Costs

$$\underbrace{\text{Credit Spread}_{ist}}_{\text{firm } i \text{ in sector } s \text{ of year } t} = \sum_{\substack{k=2014 \\ k \neq 2016}}^{2019} \beta^k D_t^k \underbrace{D_{is}^{\text{eligible}}}_{=1 \text{ if eligible for loans}} + \underbrace{\gamma^x X_{ist-1}}_{\text{firms' controls}} + \underbrace{\gamma_{st}}_{\text{sector} \times \text{year FE}} + \underbrace{\gamma_i}_{\text{firm FE}} + \epsilon_{ist}$$



β^k : Difference in the spread gap between eligible and ineligible firms between year k and 2016 (with 90% confidence interval)

Credit spreads of eligible firms decreased more relative to credit spreads of ineligible firms.

Empirical Fact 2: Borrowing Costs Sensitivity to Debt Ratio

$$\text{Credit Spread}_{ist} = \beta_0 \text{Debt Ratio}_{ist-1} + \beta_1 D_{is}^{\text{eligible}} \text{Debt Ratio}_{ist-1} + \beta_2 \text{Debt Ratio}_{ist-1} D_t^{\text{After}} + \beta_3 D_{is}^{\text{eligible}} \text{Debt Ratio}_{ist-1} D_t^{\text{After}} + \gamma_{st} + \gamma_i + \epsilon_{ist}$$

	Spread (bp)
β_0 Debt Ratio	0.46*** (0.17)
β_1 Debt Ratio \times Eligible	-0.12 (0.18)
β_2 Debt Ratio \times After	-0.05 (0.10)
β_3 Debt Ratio \times Eligible \times After	-0.26*** (0.09)
Observations	57,625
R ²	0.05

Difference in credit spread sensitivity Before:

- No discernible difference between eligible and ineligible firms

Credit spread sensitivity After:

- Ineligible firms: no discernible change
- Eligible firms: decrease

How Does Improved Credit Access Affect Investment?

Empirical strategy

- Before (2014-16) & After (2017-19)
- Diff-in-Diff with 4 groups: $\underbrace{\{\text{eligible, ineligible}\}}_{\text{eligibility}} \times \underbrace{\{\text{before credit spread high, low}\}}_{\text{pre-policy borrowing costs}}$

We cannot precisely measure the level of financial constraint..BUT

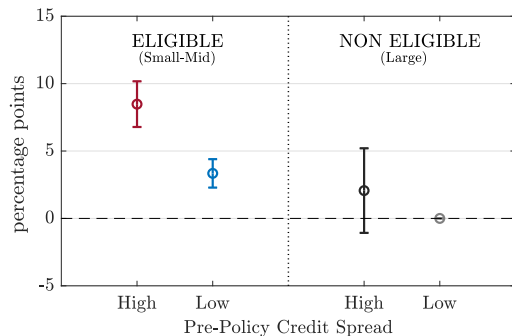
- Firms with *higher borrowing costs in Before* may have faced tighter financial constraint.

Firms with higher borrowing costs in Before are expected to increase investment more

Empirical Fact 3: Investment

$$\text{Investment}_{ist} = \beta_1 D_{is}^{\text{eligible}} D_{is}^H D_t^{\text{After}} + \beta_2 D_{is}^{\text{eligible}} (1 - D_{is}^H) D_t^{\text{After}} \\ + \beta_3 (1 - D_{is}^{\text{eligible}}) D_{is}^H D_t^{\text{After}} + \gamma^x X_{ist-1} + \gamma_{st} + \gamma_i + \epsilon_{ist}$$

$D_{is}^H = 1$ if a firm's pre-policy credit spread is in the upper 10th percentile



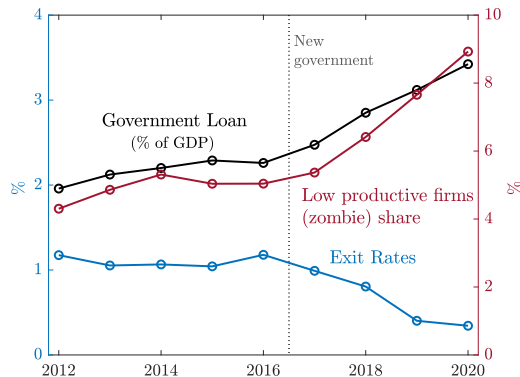
- Greater investment response by firms with higher pre-policy credit spread
- Ineligible: no significant effect

Variables

Continuous

Eventstudy

Exit Rates Decreased and the Share of Low-Productivity Firms Increased



Classify low-productivity firms based on definition of **zombie firms** [Detail](#) [Alternative](#)

1. debt service > operating profit for 3 years in a row
2. over 10-year old

Empirical strategy

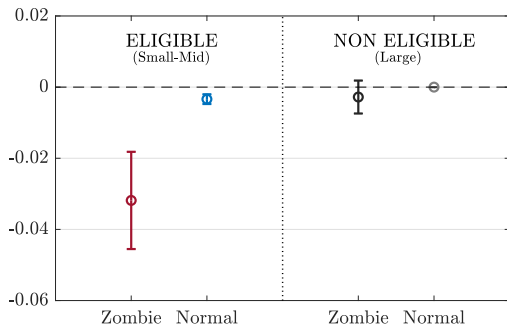
- Before (2014-16) & After (2017-19)
- Diff-in-diff with 4 groups:

$$\underbrace{\{\text{eligible}, \text{ineligible}\}}_{\text{eligibility}} \times \underbrace{\{\text{zombie}, \text{normal}\}}_{\text{one-year lagged indicator}}$$

Empirical Fact 4: Exit

$$\text{Exit}_{it} = \beta_1 D_i^{\text{eligible}} D_{it-1}^Z D_t^{\text{After}} + \beta_2 D_i^{\text{eligible}} (1 - D_{it-1}^Z) D_t^{\text{After}} + \beta_3 (1 - D_i^{\text{eligible}}) D_{it-1}^Z D_t^{\text{After}} + \gamma_x X_{it-1} + \gamma_t + \epsilon_{it}$$

$D_{ist-1}^z = 1$ if zombie in $t - 1$



- Higher survival probability of low-productivity firms
- Ineligible: no significant effect

Variables

Exit and Zombie

Before exit

Spread and Exit

After the policy: significant increase in Korean government loans after 2017

- Credit spread of eligible firms (SMEs) decreased more than ineligible firms (large).
- Investment increased more for eligible firms with high pre-policy credit spread.
- Exit rates decreased more for eligible low-productivity firms.

→ *Model to quantify the aggregate effect!*

Model

Model Summary: Heterogeneous Firms Dynamics Model

- Final good firms convert intermediate good (Y) into a final good (y_F)
- Intermediate good firms differ in cash-on-hand (x), capital (k), productivity (z)
 - Produce homogeneous good using capital (k). • Sell at price p Price
 - Repay & continue vs default & exit • Default risk \rightarrow endogenous borrowing constraint
- Risk-neutral private lenders
- Government loans are available to active firms (not potential entrants)
 - Loans at subsidized rate ($0 \sim$ risk free rate) and up to fixed limit ($\overline{b_g}$)
 - The loan program is financed with lump-sum tax from households.
- Representative household consumes profit from firms. (no labor)

Cash Shortage

- Cash on hand (x) depends on: capital (k), debt (b, b_g) and two idiosyncratic shocks:

- Persistent productivity (AR(1)): z Transitory productivity shock (i.i.d): ϕ

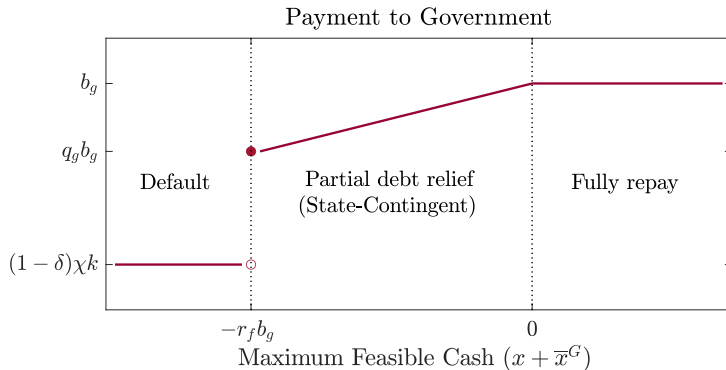
$$\underbrace{x(k, b + b_g, z, \phi)}_{\text{cash on hand}} = \underbrace{(1 - \tau)pz \exp(\phi) k^\alpha}_{\text{After-tax revenue}} - \underbrace{(f + f_k k)}_{\text{Operating cost}} - \underbrace{b}_{\text{Private loans}} - \underbrace{b_g}_{\text{Gov't loans}} + \underbrace{\tau(\delta k + r_f(b + b_g))}_{\text{Tax benefit}}$$

- Maximum fund a firm can raise:

$$\bar{x}^G(k, z) = \max_{k', b', b'_g} \underbrace{q(k', b', b'_g, z) b'}_{\text{private loan price}} + \underbrace{\overset{\beta}{\textcolor{brown}{q}_g} b'_g}_{\text{fixed rate}} - \underbrace{(k' - (1 - \delta)k)}_{\text{investment}} - \underbrace{\psi(k, k')}_{\text{adjustment cost}} \quad \text{s.t.} \quad \underbrace{b'_g \leq \overline{b}_g}_{\text{up to limit}}$$

- Maximum feasible cash = $x + \bar{x}^G(k, z) < 0 \rightarrow$ Cash shortage

Default Rule and Government Loans



Government loans decrease financing cost by

1. lending at risk free rate
2. debt relief if cannot pay interest

Debt price

Gov't loans

Value fcn

Continuing Firm's Problem

► Value of continuing firms:

$$V(x, k, z) = \max_{k', b', b'_g} d + \beta \sum_{z'} \pi(z' | z) \left[\int_{\phi' > \tilde{\phi}^G(k', b', b'_g, z')} V(x'(k', b', b'_g, z', \phi'), k', z') d\Phi(\phi') \right] \\ + \underbrace{\beta \sum_{z'} \pi(z' | z) \left[\left(\Phi(\tilde{\phi}^G(k', b', b'_g, z')) - \Phi(\hat{\phi}^G(k', B', z')) \right) V(x'(k', b', b'_g, z', \tilde{\phi}^G), k', z') \right]}_{\text{Value from government's debt relief}}$$

subject to

1. $d = x - \psi(k, k') + q(k', b', b'_g, z) b' + q_g b'_g \geq 0$
2. $x(k', b', b'_g, z', \phi') = (1 - \tau) p z' \exp(\phi') k'^\alpha - f_k k' - f - (b' + b'_g) + \tau(\delta k' + r_f(b' + b'_g))$
3. $\tilde{\phi}^G(k', b', b'_g, z') = \log \left(\frac{-\bar{x}^G(k', z') + f + f_k k' + b' + b'_g - \tau(\delta k' + r_f(b' + b'_g))}{(1 - \tau) p z' k'^\alpha} \right)$: Full-repayment cutoff
4. $\hat{\phi}^G(k', b', b'_g, z') = \log \left(\frac{-\bar{x}^G(k', z') + f + f_k k' + b' + b'_g - (1 - q_g) b'_g - \tau(\delta k' + r_f(b' + b'_g))}{(1 - \tau) p z' k'^\alpha} \right)$: Default cutoff

- ▶ A mass (M_e) of potential entrants receive a signal ν about productivity.

(Clementi and Palazzo, 2016)

- Productivity z distribution upon entry : $G(z \mid \nu)$

- ▶ Value of potential entrant with signal ν

$$V^e(\nu) = \max_{k', b'} \beta \sum_{z'} \int_{\phi' > \hat{\phi}} V(x'(k', b', z', \phi'), k', z') d\Phi(\phi') dG(z' \mid \nu)$$

$$\text{s.t.} \quad -\psi(k_e, k') + q^e(k', b', \nu)b' \geq 0$$

- ▶ To enter, firms must pay entry fee c_e .
- ▶ Firms with good signal for productivity $\nu \geq \hat{\nu}$ will enter: $V^e(\hat{\nu}) = c_e$

- Final good firms convert intermediate good (Y) into a final good (y_F),

$$\max_Y \underbrace{\bar{z}(Y)^{\alpha_y}}_{y_F} - pY$$

- Y : Sum of intermediate good firms' production.

$$Y(p^*) = \int y(x, k, z) d\mu(x, k, z) \quad p^*: \text{market-clearing price}, \quad \mu(x, k, z): \text{firm measure}$$

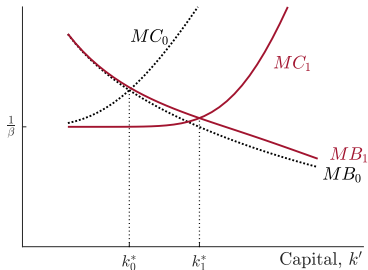
- \bar{z} : Intermediate good firms' average productivity.

$$\bar{z} = \int_{z_i} z_i m(z_i) d(z_i) \quad \text{where, } w(z_i) = \frac{\int \mathbf{1}(z=z_i) y(x, k, z) d\mu(x, k, z)}{\int y(x, k, z) d\mu(x, k, z)}$$

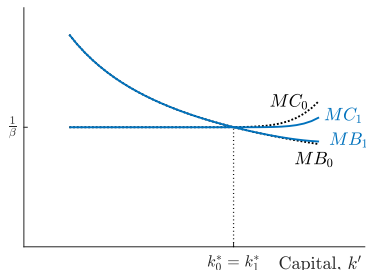
- Default risk and positive credit spread \rightarrow dispersion of capital by cash-on-hand
- Government loans: financing cost \downarrow \rightarrow more capital by constrained firms
 \rightarrow *Capital allocation is closer to the one in the economy without default risk*

Same current capital (k) and productivity (z)

(a) Low cash on hand: more borrowing



(b) High cash on hand: less borrowing



Quantification

Quantitative Exercises Overview

What I do: parameters target untarget1 untarget2 untarget3 policy fcn

1. Calibrate the model without government loans to match Korean firm data (2010-2016)
 - Main target moments: investments, spread, exit rates, sale-asset ratio at exit and entry
2. Introduce government loans:
 - Calibrated to capture changes in exit rates after the policy change
3. Transition path between two steady states with and without gov't loans

Main exercises:

1. Model validation: data vs simulated firms → micro effects
 - Simulated firms: 3-year after the introduction of gov't loans
2. Transition path and steady state comparison → macro effects

Model Validation: Normal vs Zombie Firms (Untargeted)

Zombie firms:

- Data: Operating profit < debt service for 3 consecutive years + over 10 years old
- Model: Negative cash-on-hand for 3 consecutive years + over 10 years old

Mean Difference: Zombie from Normal

		Data	Model
(%)	Log capital size	115.2	111.0
	Debt to Asset Ratio	9.7	10.1
(pp)	Profitability	-11.2	-15.5
	Investment	-12.2	-7.1

- Share of zombie firms before the policy
 - Data: 5.1% Model: 8.0%
- Change in the share of zombie firms
 - Data: 2.5pp Model: 4.0pp
- Zombie firms are relatively
 - Large, indebted, unprofitable, low-investment

Investment Increases More for Firms with High Pre-policy Spread

Data panel regression:

$$\text{Investment}_{ist} = \beta_1 D_{is}^{\text{eligible}} D_{is}^H D_t^{\text{After}} + \beta_2 D_{is}^{\text{eligible}} (1 - D_{is}^H) D_t^{\text{After}} + \beta_3 (1 - D_{is}^{\text{eligible}}) D_{is}^H D_t^{\text{After}} + \gamma^x X_{ist-1} + \gamma_{st} + \gamma_i + \epsilon_{ist}$$

Regression with simulated firms: $\text{Investment}_{it} = \alpha_1 D_i^H D_t^{\text{After}} + \gamma^x X_{it-1} + \gamma_t + \gamma^h D_i^H + \epsilon_{it}$

- Two groups by pre-policy credit spread ($D^H = 1$: High pre-policy credit spread)

Heterogeneity by pre-policy credit spread

Δ Investment (pp)	
Data ($\beta_1 - \beta_2$)	Model (α_1)
5.14	4.02
[3.41 6.86]	(0.28)

Exit Rate Decreases More for Less Productive Firms

Data panel regression:

$$\text{Exit}_{it} = \beta_1 D_i^{\text{eligible}} D_{it-1}^Z D_t^{\text{After}} + \beta_2 D_i^{\text{eligible}} (1 - D_{it-1}^Z) D_t^{\text{After}} + \beta_3 (1 - D_i^{\text{eligible}}) D_{it-1}^Z D_t^{\text{After}} + \gamma_x X_{it-1} + \gamma_t + \epsilon_{it}$$

Regression with simulated firms: $\text{Exit}_{it} = \alpha_1 D_{it-1}^Z D_t^{\text{After}} + \gamma^z D_{it-1}^Z + \gamma_t + \epsilon_{it}$

- Two groups by zombie indicator ($D^z = 1$: zombie)

Heterogeneity by zombie indicator

Δ Probability to exit	
Data ($\beta_1 - \beta_2$)	Model (α_1)
-0.028	-0.023
[-0.012 -0.045]	(0.009)

Exit ↓ + Investment by financially constrained firms ↑ → Eq. price ↓ Price

- Firm's average production and profitability decreases
- Discourages potential entrants from entering the market
 - Indirectly worsening composition of active firms' productivity
 - Fewer high-productive firms enter and more low-productivity firms survive
 - Potential entrants at entry margin are more productive than incumbent firms at exit margin.

Aggregate output and aggregate productivity

$$Y = \underbrace{M^{1-\alpha}}_{\text{Size effect}} \times \underbrace{\mathbf{E} \left[\tilde{z}^{\frac{1}{1-\alpha}} \right]^{1-\alpha}}_{\text{Composition}} \times \underbrace{\frac{Y}{Y^*}}_{\text{Capital allocation}} \times \underbrace{K^\alpha}_{\text{Capital qtys.}}$$

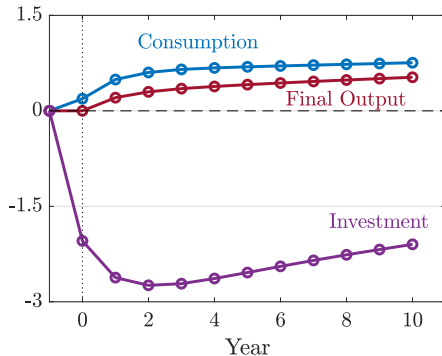
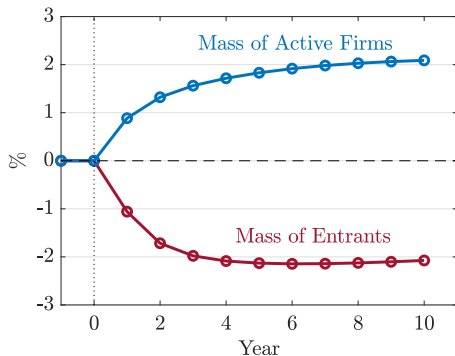
Aggregate Productivity

$$\tilde{z} = \sum_z z \pi(z | z_{-1}) \quad M = \int d\mu(x_{-1}, k_{-1}, z_{-1}) \quad K = \int k(x_{-1}, k_{-1}, z_{-1}) d\mu(x_{-1}, k_{-1}, z_{-1})$$

- Y^* : Maximum output given the mass of firms M and aggregate capital K
 - Capital is distributed for the marginal product of capital to be equalized across firms.

Macro: Transition Path

- Entrants \downarrow (GE) but incumbents exit rates \downarrow (GE + gov't loans) \rightarrow active firms \uparrow
- Investment decreases (GE + composition), but output increases due to more active firms.

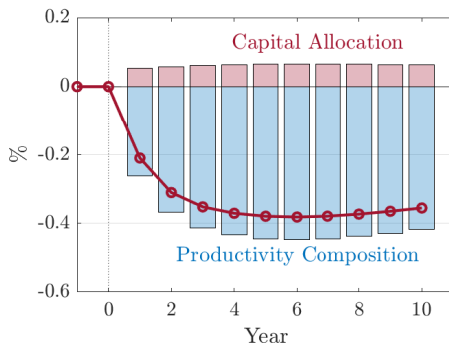
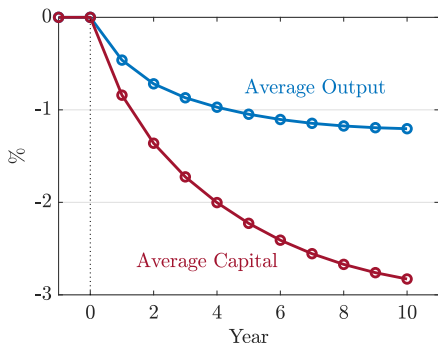


Average Firms' Size Decreases and Average Productivity Decreases

- Average output and capital decreases.
- Aggregate productivity decreases.
 - Capital allocation \uparrow : increased invt. by constrained firms and less capital dispersion
 - Productivity composition \downarrow : less exit by low-productivity firms + less entry

Decomposition

Bartik



Long Run Effects: Steady States Comparison

- Aggregate productivity effects mostly materialize in the first 10 years.
- Aggregate effects after 10 years mostly come from changes in the mass of active firms.

	Δ		Δ		Δ
Productivity	-0.3	Active Firms	+2.6	Capital	-0.4
(Capital allocation)	+0.1	Entrants	-2.2	Final output	+1.1
(Composition)	-0.3			Consumption	+1.3

Conclusion

- Effects of a significant increase in government loans for SMEs using Korea's case
 1. Credit spread of SMEs (eligible) decreased more than large firms (ineligible)
 2. Investment increased more for eligible firms with high pre-policy credit spread.
 3. Exit rates decreased more for eligible low-productivity firms.
- Heterogeneous response that captures trade-off: model \longleftrightarrow data
- Quantify the aggregate productivity effects of the government loans (over 10-year)
 - Productivity (-0.3%): improved capital allocation (+0.1%) but worsened composition (-0.4%)

THANK YOU!

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Appendix: Empirical

How to Define Zombie Firms

Zombie firms: continuously unable to cover debt costs from current profits

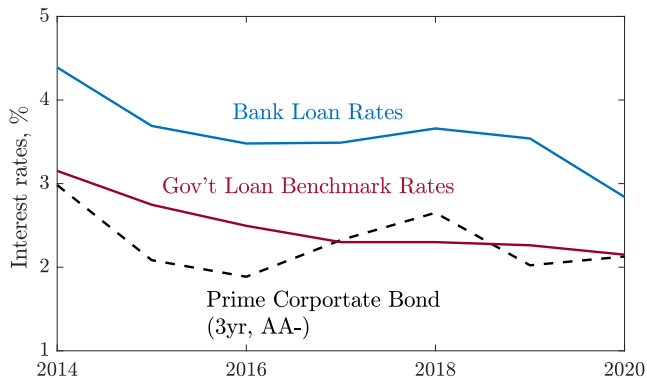
(Banerjee and Hofmann, 2018, McGowan, Andrews and Millot, 2017, Hong, Igan and Lee, 2021)

1. Meet one of the following conditions for 3 consecutive years.

- Interest coverage ratio (ICR) = $\frac{\text{Operating profit}}{\text{Interest expense}} < 1$
- Negative operating profit

2. Firm's age ≥ 10 years (For exclusion of start-ups)

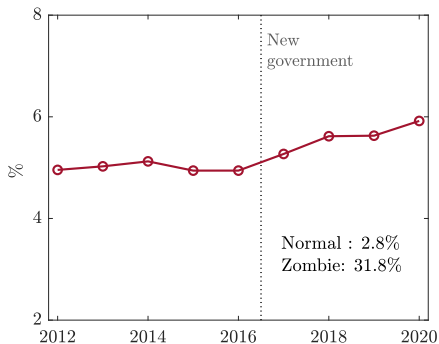
Government Loans Benchmark Rates

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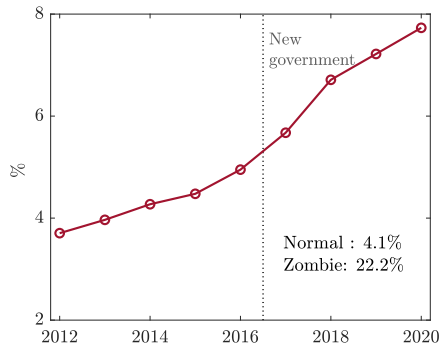
Share of Firms with Indicators Lower Than the Cutoff

Cutoff: the indicator's 5th percentile for each sector in the year 2016

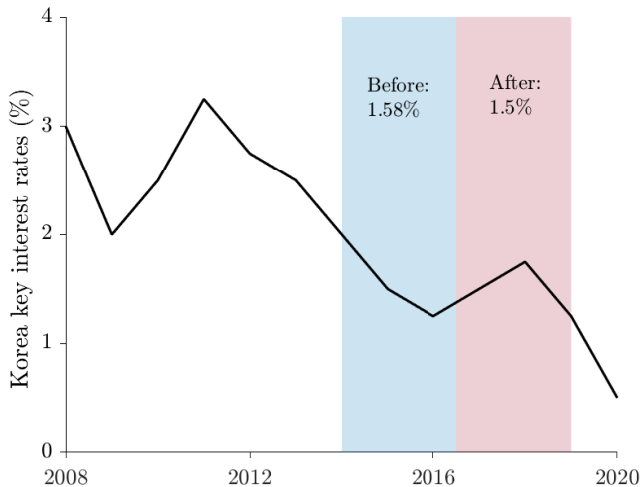
(a) Sales-Cost Ratio



(b) Sale-Asset Ratio



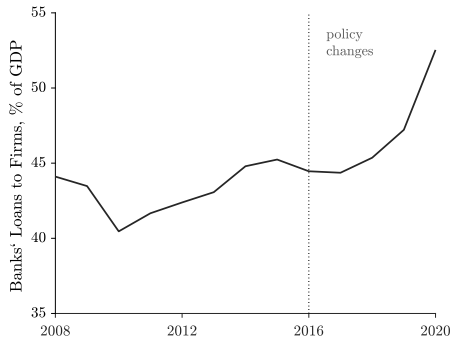
Bank of Korea Key Rates



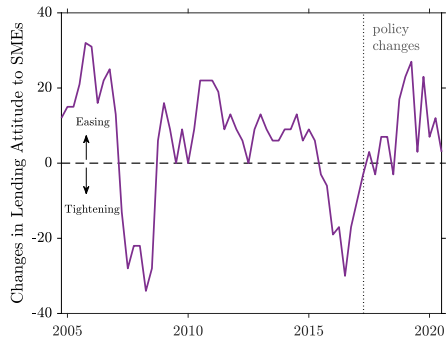
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The Private credit market has also become easing.

(a) Banks' loan to firms

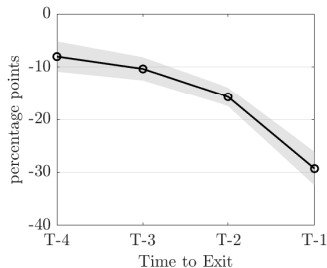


(b) Banks' lending attitude toward SMEs

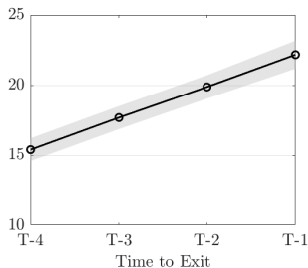


Cash shortage \uparrow , debt ratio \uparrow , and credit spreads $\uparrow \rightarrow$ exit

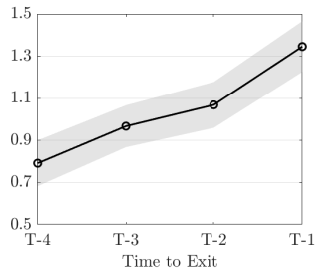
(a) Net-income ratio



(b) Debt ratio



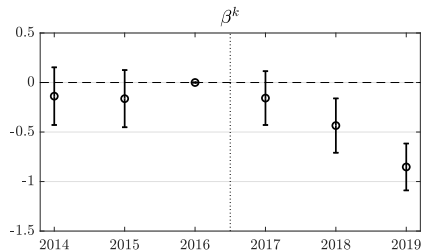
(c) Credit spread



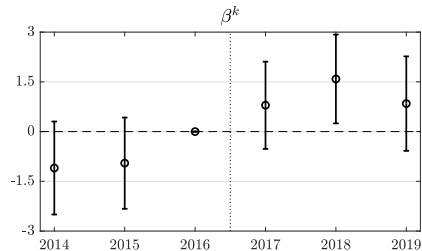
Notes: These plots show the relative financial state of firms with specific distance to exit. Specifically, those are series of coefficient of $y_i = \alpha + \sum_{k=1}^4 \beta_k D_i^{T-k} + \epsilon_i$, where D_i^{T-k} is an indicator whether a specific firm i closes down and exits after k periods. The shaded area indicates the 90% confidence interval.

$$Y_{it} = \sum_{k \neq 2016} \beta^k \text{Year}_k D_i^{sme} + \gamma_t + \epsilon_t$$

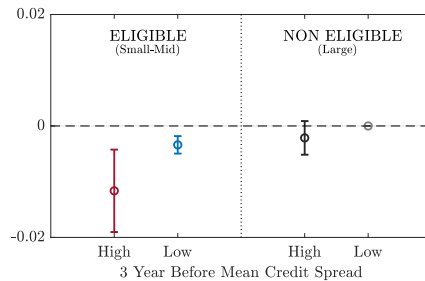
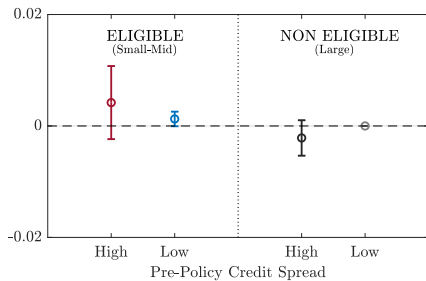
(a) Exit rates (pp)



(b) Zombie share (pp)



Credit Spread and Exit Rates

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Detailed Explanatory Variables

$$\text{Spread}_{ist} = \sum_{k \neq 2016} \beta^k \text{Year}_t D_{is}^{sme} + \gamma^x X_{ist-1} + \gamma_{st} + \gamma_i + \epsilon_{ist}$$

- D^{sme} : SMEs indicator X_{ist} : equity to asset, cash to asset, debt to asset
- γ_{st} : sector-year fixed effect γ_i : firm fixed effect

Spread

$$\text{Investment}_{ist} = \beta_1 D_{is}^{sme} D_{is}^H D_t^{\text{After}} + \beta_2 D_{is}^{sme} (1 - D_{is}^H) D_t^{\text{After}} + \beta_3 (1 - D_{is}^{sme}) D_{is}^H D_t^{\text{After}} + \gamma^x X_{ist-1} + \gamma_{st} + \gamma_i + \epsilon_{ist}$$

- D^{sme} : SMEs indicator γ_{st} : sector-year fixed effect γ_i : firm fixed effect
- D^H : an indicator of whether the pre-policy credit spread is in the upper 10th percentile
- X_{ist} : log of tangible asset, operating profit to asset

Investment

$$\text{Exit}_{it} = \beta_1 D_i^{sme} D_{it-1}^Z D_t^{\text{After}} + \beta_2 D_i^{sme} (1 - D_{it-1}^Z) D_t^{\text{After}} + \beta_3 (1 - D_i^{sme}) D_{it-1}^Z D_t^{\text{After}} + \gamma_x X_{it-1} + \gamma_t + \epsilon_{it}$$

- D^{sme} : indicator of SMEs D^Z : indicator of zombie firms γ_t : year fixed effect
- X_{it} : Interaction terms of indicator of SMEs and zombie firms

Exit

Sector (s) level regression using regional data (r)

- Given government loans in period t , sector s has a higher exposure to the policy:
 - Higher share of small-mid enterprises (SMEs) in region r of relatively higher output share

$$y_{st} = \beta \text{Exposure to Gov' Loan}_{st} + \gamma_t + \gamma_s + \epsilon_{st}$$

$$\text{Exposure to Gov' Loan}_{st} = \sum_{r=1}^{13} \underbrace{\frac{\text{number of SMEs}_{sr}}{\text{number of firms}_{sr}}}_{\text{SMEs share in } r \text{ region } s \text{ industry}} \times \overbrace{\frac{\text{total output}_r}{\text{total output}}}_{\text{Shock}} \times \text{Gov}_t$$

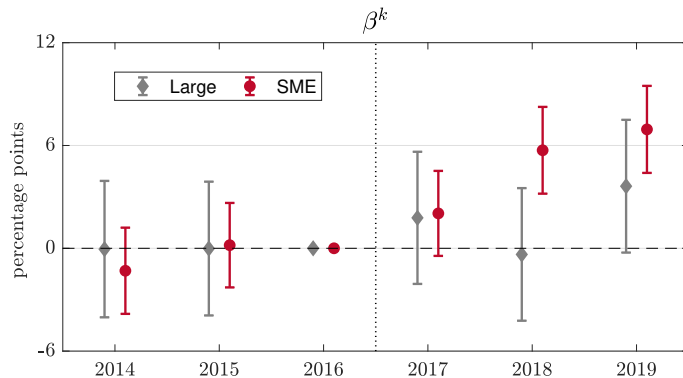
output share in region r

	Exit rates	Investment	Zombie share	Zombie K share	$\Delta \log \frac{\text{sales}}{\text{assets}}$
β	-0.009** (0.003)	-0.065*** (0.021)	0.027* (0.013)	-0.029 (0.051)	-0.002** (0.001)

$$\text{Investment}_{ist} = \beta_1 D_{is}^{sme} \text{Before CR}_{is} D_t^{\text{After}} + \beta_2 \text{Before CR}_{is} D_t^{\text{After}} + \gamma^x X_{ist-1} + \gamma_{st} + \gamma_i + \epsilon_{ist}$$

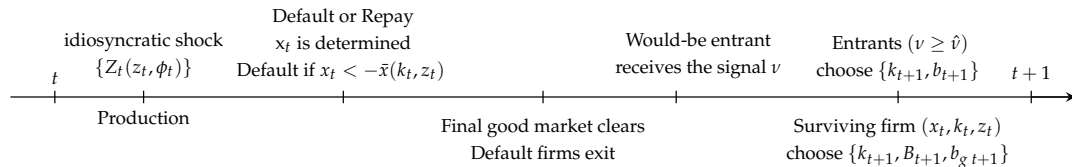
	Investment(pp)
Before CR \times SME \times After (β_1)	1.33*** (0.28)
Before CR \times After (β_2)	0.05 (0.26)

$$\text{Investment}_{ist} = \sum_{k \neq 2016} \beta^k \text{Year}_k D_i^{\text{High}} + \gamma^x X_{ist-1} + \gamma_{st} + \gamma_i + \epsilon_{ist}$$



Appendix: Quantitative

Timeline



z_t : firm's AR(1) idiosyncratic productivity ϕ_t : firm's i.i.d. idiosyncratic shock

x : cash on hand \bar{x} : maximum funds the firm can raise k : capital b : debt

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$$c(k_t, k_{t+1}) = \begin{cases} (k_{t+1} - (1 - \delta)k_t) + p_k^+ \frac{(k_{t+1} - (1 - \delta)k_t)^2}{2(1 - \delta)k_t} & \text{if } k_{t+1} - (1 - \delta)k_t \geq 0 \\ (k_{t+1} - (1 - \delta)k_t) + p_k^- \frac{(k_{t+1} - (1 - \delta)k_t)^2}{2(1 - \delta)k_t} & \text{if } k_{t+1} - (1 - \delta)k_t < 0 \end{cases}$$

where, $p_k^+ < p_k^-$

$$q(k', B', b'_g, z) = \beta \sum_{z'} \left[\left(1 - \Phi(\hat{\phi}^G) \right) + \Phi(\hat{\phi}^G) R^G(B', b'_g, k') \right] \pi(z' | z)$$

where,

$$\hat{\phi}^G(k', B', b'_g, z') = \log \left(\frac{-\bar{x}^G(k', z') + f + f_k k' + B' - (1 - q_g) b'_g - \tau(\delta k + r_f B')}{(1 - \tau) p z' k'^\alpha} \right)$$

$$R^G(B', b'_g, k') = \min \left(1, \max \left(0, \frac{\chi(1 - \delta)k' - b'_g - \eta}{B' - b'_g} \right) \right)$$

$$V^e(\nu) = \max_{k', b'} \beta \sum_{z'} \int_{\phi' > \hat{\phi}} V(x'(k', b', z', \phi'), k', z') d\Phi(\phi') dG(z' | \nu)$$

$$\text{s.t.} \quad -c(k_e, k') + q^e(k', b', \nu)b' \geq 0$$

$$x(k', b', z', \phi') = (1 - \tau)pz' \exp(\phi')k'^\alpha - f_k k' - f - b' + \tau(\delta k' + r_f b')$$

$$\hat{\phi}(k', b', z') = \log \left(\frac{-\bar{x}(k', z') + f + f_k k' + b' - \tau(\delta k' + r_f b')}{(1 - \tau)pz'k'^\alpha} \right)$$

$$q_e(k', b', \nu) = \beta \sum_{z'} \left[\left(1 - \Phi(\hat{\phi})\right) + \Phi(\hat{\phi}) R(b', k') \right] dG(z' | \nu)$$

$$\text{s.t.} \quad \hat{\phi}(k', b', z') = \log \left(\frac{-\bar{x}(k', z') + f + f_k k' + b' - \tau(\delta k' + r_f b')}{(1 - \tau)pz'k'^\alpha} \right)$$

$$R(b', k') = \min \left(1, \max \left(0, \chi \frac{(1 - \delta)k'}{b'} - \eta \right) \right)$$

Government loans: **fixed limit** (\bar{b}_g) and **contingent rates** ($0 \sim r_f = \text{risk free rate}$)

► Cash shortage: $x^{FR} + \bar{x}^G(k, z)$

$$\blacksquare x^{FR}(k, b + b_g, z, \phi) = \underbrace{(1 - \tau)pz \exp(\phi) k^\alpha}_{\text{After-tax revenue}} - \underbrace{(f + f_k k)}_{\text{Operating cost}} - b - b_g + \underbrace{\tau(\delta k + r_f(b + b_g))}_{\text{Tax benefit}}$$

1. No cash shortage: $x^{FR} + \bar{x}^G(k, z) \geq 0 \rightarrow \text{pay } b_g \text{ } (r_f)$

2. Cash shortage less than some limit: $-\overbrace{(1 - q_g) b_g}^{r_f b_g} \leq x^{FR} + \bar{x}^G(k, z) < 0$
 \rightarrow Partial debt relief: pay b_g + cash shortage ($0 \sim r_f$)

3. Cash shortage greater than some limit: $x^{FR} + \bar{x}^G(k, z) < -(1 - q_g) b_g \rightarrow \text{Default}$

Detailed equation

$$\frac{\beta \sum_{z'} \pi(z' | z) \left[\int_{\phi' > \tilde{\phi}_g} MPK(k', z', \phi') d\Phi(\phi') + \left(\Phi(\tilde{\phi}_g) - \Phi(\hat{\phi}_g) \right) MPK(k', z', \tilde{\phi}_g) + \left(-\frac{\partial \hat{\phi}_g}{\partial k'} \right) \phi(\hat{\phi}_g) \tilde{V} \right]}{\sum_{z'} \pi(z' | z) \left[\left(1 - \Phi(\hat{\phi}_g) \right) + \frac{\partial \hat{\phi}_g}{\partial B'} \phi(\hat{\phi}_g) \tilde{V} \right]}$$

$$= \frac{1 - \frac{\partial q}{\partial k'} (B'(x, k', z) - b_g)}{q(1 - \epsilon)}$$

where, $\epsilon = -\frac{\partial q}{\partial B'} \frac{(B' - b_g)}{q}$

$$MPK(k', z', \phi') = pz' \exp(\phi') \alpha k'^{\alpha-1} - f_k - \frac{\partial c(k', k''(x'(k', B'(x, k', z), z', \phi'), k', z'))}{\partial k'}$$

$$\tilde{V} = V\left(x'\left(k', B'(x, k', z), z', \tilde{\phi}^G\right), k', z'\right)$$

Solution Algorithm: Firm decision rules

1. Given the price p , construct $x_{\min}(k, z) = -\bar{x}(k, z)$ and bond price schedule $q(k', b', z)$
2. Solve for the cutoff $x_{\max}(k, z) = \hat{x}(k, z)$ which makes firms' decisions not dependent on the level of x .

$$\hat{x}(k, z) = c\left(k, \hat{k}'(k, z)\right) - q\left(\hat{k}'(k, z), \hat{b}'(k, z), z\right) \hat{b}'(k, z)$$

where \hat{k}', \hat{b}' is a solution to this problem

$$V_{nb}(k, z) = \max_{k', b'} -c(k, k') + q(k', b', z)b' + \beta \sum_{z'} \pi(z' | z) \int_{\phi' > \hat{\phi}} V(x'(k', b', z', \phi'), k', z') d\Phi(\phi')$$

3. Solve for decisions at the intermediate points between $x_{\min}(k, z)$ and $x_{\max}(k, z)$.

Solution Algorithm: Firm decision rules

4. Update value function using obtained policy functions with linear interpolations.

$$V^{n+1}(x, k, z) = x - c(k, k'(x, k, z)) + q(k'(x, k, z)b'(x, k, z), z)b'(x, k, z) \\ + \beta \underbrace{\sum_{z'} \int_{\phi' > \hat{\phi}} V^n(x'(k'(x, k, z), b'(x, k, z), z', \phi'), k'(x, k, z), z')}_{W(k'(x, k, z), b'(x, k, z), z)}$$

where

$$V^{n+1}(x', k', z') = x' + V_{nb}^n(k', z') \quad \text{if} \quad x' \geq \text{xmax}(k', z')$$

5. Iterate the process until $W(k, b, z)$ converges.

Solution Algorithm: Stationary Distribution and p

1. Given the policy function, update the distribution until it converges.

$$\begin{aligned}\mu'(x_i, k_j, z') = & \sum_{x, k, z} \int_{\phi' \geq \hat{\phi}(k', b', z')} \omega_x(x_i, x'(k'(x, k, z), b'(x, k, z), z', \phi')) \omega_k(k_j, k'(x, k, z)) dF(\phi') \pi(z' | z) \mu(x, k, z) \\ & + M \int_{\nu \geq \hat{\nu}} \int_{\phi' \geq \hat{\phi}(k', b', z')} \omega_x(x_i, x'(k'(\nu), b'(\nu), z', \phi')) \omega_k(k_j, k'(\nu)) dF(\phi') H(z' | \nu) dG(\nu)\end{aligned}$$

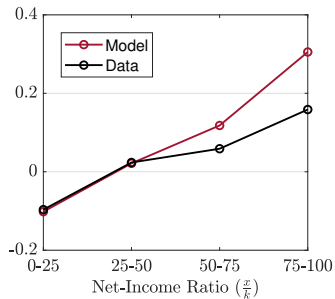
2. Determine the price with a bisection search.
3. Repeat the procedure until convergence.

<i>Description</i>	<i>Parameter</i>	<i>Source</i>
<i>Fixed parameters</i>		
Discount rate	$\beta = 0.97$	Annual interest rate 3%
Share of capital	$\alpha = 0.3$	Standard business cycle models
Depreciation	$\delta = 0.1$	Standard business cycle models
Tax rate	$\tau = 0.275$	Korea's corporate tax rate
Bond recovery rate	$\chi = 0.47$	Xiao (2020)
Persistence of z	$\rho_z = 0.9$	Foster, Haltiwanger and Syverson (2008)
Returns to scale	$\alpha_y = 0.85$	Atkeson and Kehoe (2005)
<i>Fitted parameters from moment matching</i>		
Volatility of z, ϕ	$\sigma_z = 0.1, \quad \sigma_\phi = 0.13$	} Internally calibrated
Invest & dis-invest adj cost	$p_k^+ = 1.8, \quad p_k^- = 2.8$	
Fixed & capital proportional cost	$f = 0.52, \quad f_k = 0.07$	
Default cost	$\eta = 0.2$	
Entry cost & initial capital	$c_e = 3.2 \quad k_e = 0.2$	
Pareto exponent	$\xi = 3.2$	
Government loans	$\bar{b}_g = 0.134$	

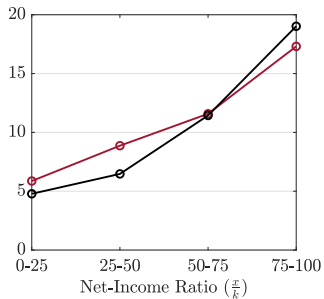
<i>Description</i>	<i>Data</i>	<i>Model</i>
<i>Incumbents</i>		
Mean investment	0.11	0.11
Mean investment ($\frac{x}{k} < \text{median}$)	0.06	0.07
Mean investment ($\frac{x}{k} \geq \text{median}$)	0.15	0.14
Mean spread (%p)	1.46	1.61
Exit rates (%)	1.10	1.12
<i>Entrants</i>		
Median relative size at enter	0.16	0.17
Mean relative sale-asset ratio at enter	1.81	1.55
Age 1 firms' mean investment	0.43	0.46
<i>Firms that exit</i>		
Mean net-income asset ratio at exit	-0.27	-0.30
Mean relative sale-asset ratio at exit	0.61	0.59

Untargeted Moments: Cross-Sectional Moments

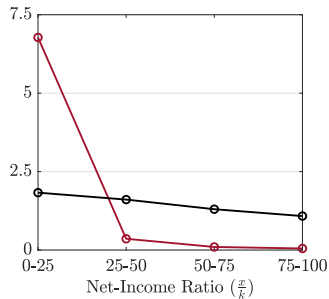
(a) Net-Income Ratio



(b) Investment

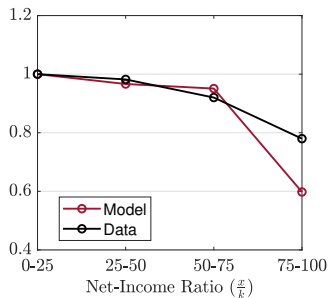


(c) Credit Spread

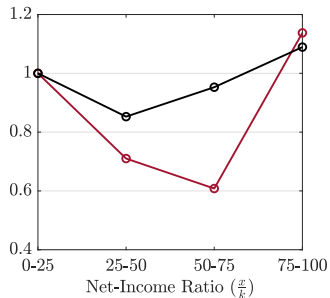


Untargeted Moments: Cross-Sectional Moments

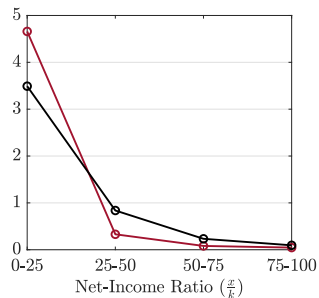
(a) Relative Size



(b) Relative Std. of Size



(c) Exit Rates

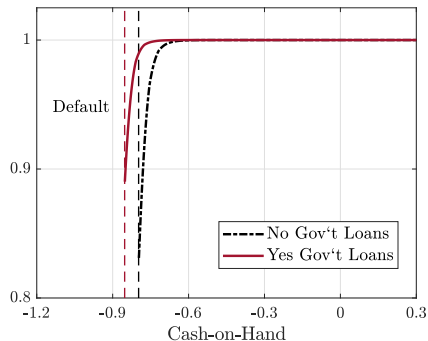


- Overall model captures well cross-sectional distribution except spreads.

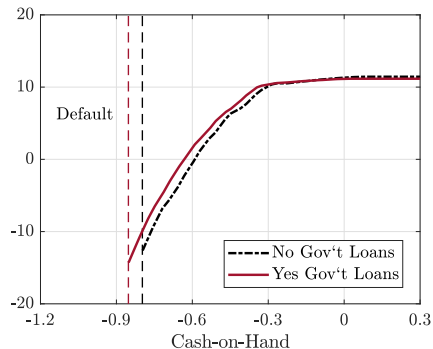
Moments	Net-income asset ratio ($\frac{x}{k}$)			
	[0,25]	[25,50]	[50,75]	[75,100]
<i>Data</i>				
Net-income asset ratio	-0.10	0.02	0.06	0.16
Investment	0.05	0.06	0.11	0.19
Spread	1.83	1.61	1.30	1.08
Exit rate (%)	3.49	0.84	0.23	0.09
Log size (Relative)	1.00	0.98	0.92	0.78
Std of log size (Relative)	1.00	0.85	0.95	1.09
<i>Model</i>				
Net-income asset ratio	-0.10	0.02	0.12	0.31
Investment	0.06	0.09	0.12	0.17
Spread	6.78	0.36	0.10	0.05
Exit rates (%)	4.66	0.33	0.08	0.05
Log size (Relative)	1.00	0.97	0.95	0.60
Std of log size (Relative)	1.00	0.71	0.61	1.14

Firms Decision Rule with Government Loans (p fixed)

(a) Survival Probability (%)

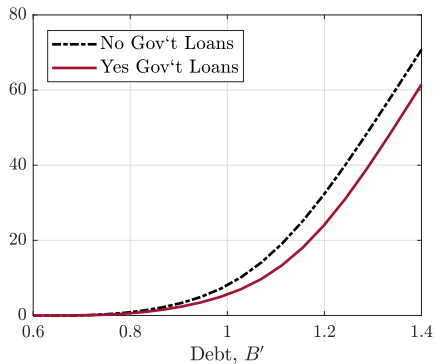


(b) Investment (%)

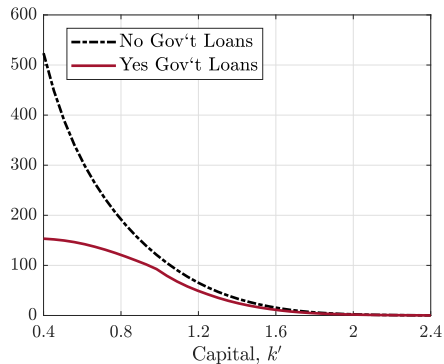


Credit Spread Schedules (p fixed)

(a) Credit spread against debt (%p)

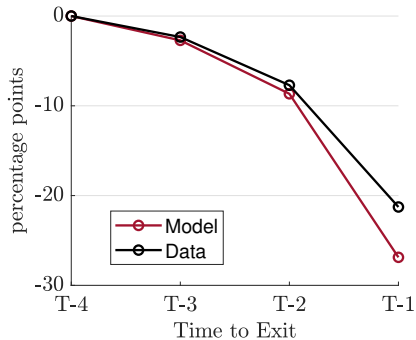


(b) Credit spread against capital (%p)

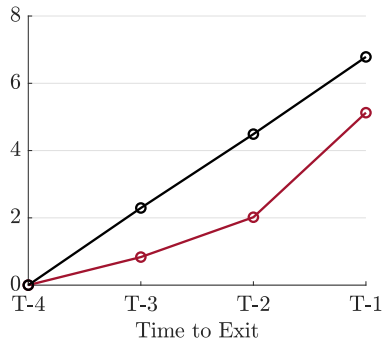


Model Validation: Financial States Before Firm Exits (Untargeted)

(a) Net-income ratio

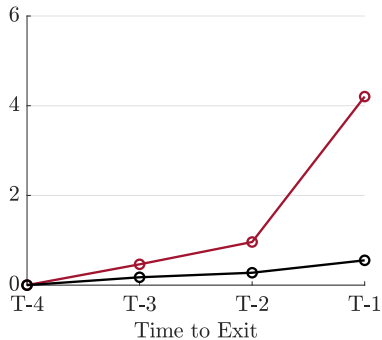


(b) Debt ratio

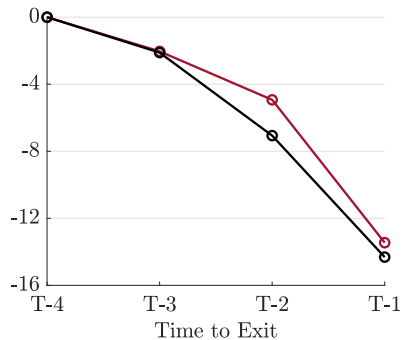


Model Validation: Financial States Before Firm Exits (Untargeted)

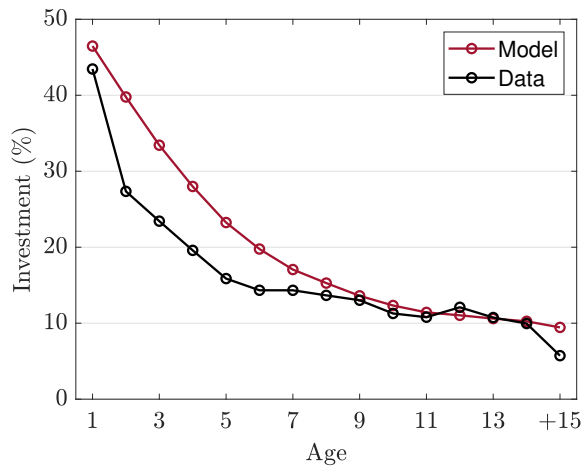
(a) Credit Spreads



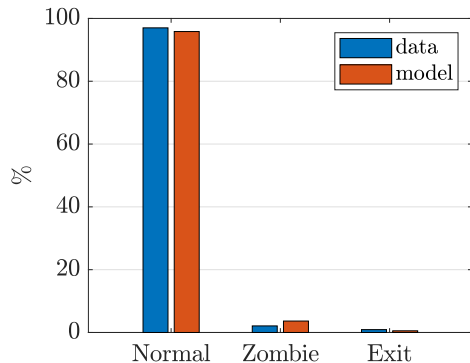
(b) Investment



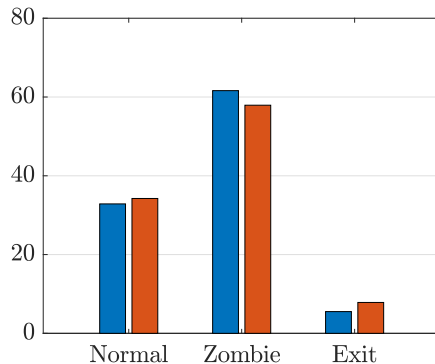
Model Validation: Investment by Age



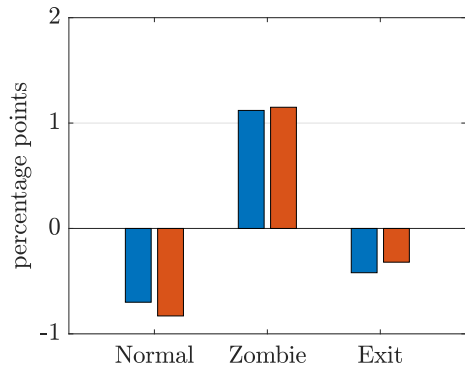
Normal firms



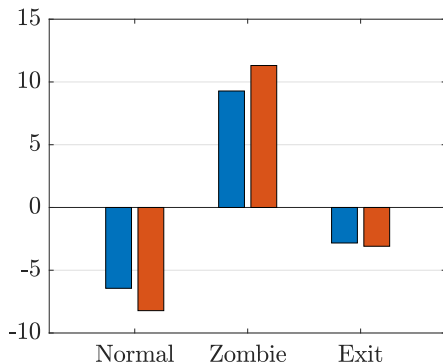
Zombie firms



Normal firms



Zombie firms



Data panel regression:

$$\text{Exit}_{it} = \beta_1 D_i^{sme} D_{it-1}^{High} D_t^{\text{After}} + \beta_2 D_i^{sme} (1 - D_{it-1}^{High}) D_t^{\text{After}} + \beta_3 (1 - D_i^{sme}) D_{it-1}^{High} D_t^{\text{After}} + \gamma_t + \epsilon_{it}$$

- D_{it-1}^{High} : Indicator 3-year average credit spread is in the upper 10th percentile

Regression with simulated firms: $\text{Exit}_{it} = \alpha_1 D_{it-1}^{High} D_t^{\text{After}} + \gamma^z D_{it-1}^{High} + \gamma_t + \epsilon_{it}$

- Two groups by lagged 3-year mean credit spread

Heterogeneity by lagged 3-year average credit spread

Data ($\beta_2 - \beta_1$)	Model (α_1)
-0.008	-0.013
[-0.017 0.001]	(0.007)