STAT401_HW3

김정현

Q1.

```
options(digits=4)
mu <- c(2, -1)
sigma <- matrix(c(2, 2, 2, 5), byrow = TRUE, nrow = 2)
eigen_result <- eigen(sigma)
```

(a)

```
a11 = round(eigen_result$vectors[1, 1],4)
a12 = round(eigen_result$vectors[2, 1],4)
a21 = round(eigen_result$vectors[1, 2],4)
a22 = round(eigen_result$vectors[2, 2],4)

cat(paste("PC 1:",a11, "x_1 + ", a12, "x_2"))

## PC 1: 0.4472 x_1 +  0.8944 x_2

cat("\n")

cat(paste("PC 2:", a21, "x_1 + ", a22, "x_2"))

## PC 2: -0.8944 x_1 +  0.4472 x_2
```

First PC: $Y_1 = 0.4472X_1 + 0.8944X_2$ Second PC: $Y_1 = -0.8944X_1 + 0.4472X_2$

(b)

```
round(eigen_result$values[1]/sum(eigen_result$values),4)

## [1] 0.8571
```

The proportion of total population variance explained by first PC is 0.8571.

(c)

```
i = 2
j = 1
cat("Correlation between X_2 and Y_1 : ",sqrt(eigen_result$values[j])*a12 /
sqrt(sigma[i,i]))

## Correlation between X_2 and Y_1 :  0.9798
```

Correlation between X_2 and Y_1 is 0.9798.

(d)

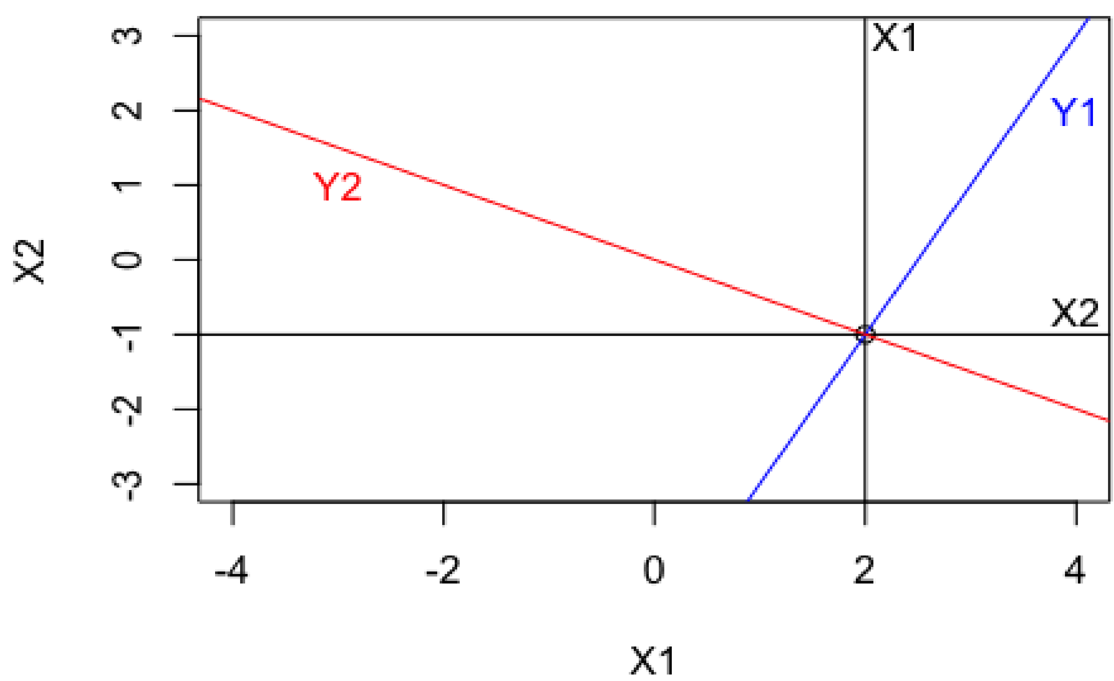
```r
obs <- matrix(c(3,1), byrow = T, nrow = 2)
a1 <- matrix(c(a11, a12), byrow = F, nrow = 1)
a1 %*% (obs-mu)

##       [,1]
## [1,] 2.236
```

The PC score for first principal component is 2.236 (same with $\sqrt{5}$).

(e)

```r
pc1 <- mu[2] - a12/ a11 * mu[1]
pc2 <- mu[2] - a22/ a21 * mu[1]
plot(mu[1],mu[2], xlim=c(-4,4), ylim=c(-3,3), xlab='X1', ylab='X2')
abline(h=-1); abline(v=2)
abline(pc1, a12 / a11, col='blue')
abline(pc2, a22 / a21, col='red')
text(mu[1]+0.3, 3, "X1") ;text(4, mu[2]+0.3, "X2")
text(4, 2, "Y1", col='blue'); text(-3, 1, "Y2", col='red')
```



Since λ_1 is the largest eigenvalue, the major axis lie on direction a_1 and the minor axis lie on direction a_2.

Q2.

```
# Mean vector
x_bar <- c(95.5, 164.4, 55.7, 93.4, 18.0, 31.1)

# Covariance matrix
S <- matrix(c(3266, 1344, 732, 1176, 163, 238,
              1344, 722, 324, 537, 80, 118,
              732, 324, 179, 281, 39, 57,
              1176, 537, 281, 475, 64, 95,
              163, 80, 39, 64, 10, 14,
              238, 118, 57, 95, 14, 21), nrow = 6, byrow = TRUE)
```

(a)

```
q2.pca.cov = princomp(covmat = S)
summary(q2.pca.cov)

## Importance of components:
##                          Comp.1   Comp.2   Comp.3   Comp.4    Comp.5
Comp.6
## Standard deviation     66.9242 12.33604 5.679053 2.813162 1.1730752
6.545e-01
## Proportion of Variance  0.9585  0.03257 0.006902 0.001694 0.0002945
9.166e-05
## Cumulative Proportion   0.9585  0.99102 0.997920 0.999614 0.9999083
1.000e+00
```

The data can be summarized by 1 dimension (with cumulative proportion approximate 95.8%), which is smaller than 6 dimensions.

(b)

```
std_devs <- sqrt(diag(S))

R <- matrix(nrow = 6, ncol = 6)

for (i in 1:6) {
  for (j in 1:6) {
    R[i, j] <- S[i, j] / (std_devs[i] * std_devs[j])
  }
}

rownames(R) <- colnames(R) <- c("X1", "X2", "X3", "X4", "X5", "X6")

print(R)

##        X1     X2     X3     X4     X5     X6
## X1 1.0000 0.8752 0.9574 0.9442 0.9019 0.9088
## X2 0.8752 1.0000 0.9013 0.9170 0.9415 0.9583
## X3 0.9574 0.9013 1.0000 0.9637 0.9218 0.9297
## X4 0.9442 0.9170 0.9637 1.0000 0.9286 0.9512
```

```
## X5 0.9019 0.9415 0.9218 0.9286 1.0000 0.9661
## X6 0.9088 0.9583 0.9297 0.9512 0.9661 1.0000
```

```r
summary(princomp(covmat = R))
```

```
## Importance of components:
##                           Comp.1  Comp.2   Comp.3   Comp.4   Comp.5   Comp.6
## Standard deviation     2.3782 0.42023 0.240086 0.224189 0.190853 0.152003
## Proportion of Variance 0.9427 0.02943 0.009607 0.008377 0.006071 0.003851
## Cumulative Proportion  0.9427 0.97209 0.981702 0.990078 0.996149 1.000000
```

The data can be summarized by 1 dimension (with cumulative proportion approximate 94.3%), which is smaller than 6 dimensions.

(c)

The proportion of total variance has similar value. However, the eigen values and eigen vector are different since scaling is made in correlation matrix.

Q3.

(a)

```r
radio <- read.table('radiotherapy.dat', header = TRUE, sep = "")[,-6]
sapply(radio, var)
```

```
##     X1     X2     X3     X4     X5
## 4.6548 0.6128 0.5714 0.1104 0.8622
```

Since the variance has difference in scale (especially in Symptoms), it is better to use correlation matrix R.

(b)

```r
q3.pca = prcomp(radio, scale = T)
round(q3.pca$rotation, 3)
```

```
##      PC1    PC2    PC3    PC4    PC5
## X1 0.445  0.231  0.608  0.603  0.127
## X2 0.432  0.572  0.117 -0.679  0.105
## X3 0.356 -0.779  0.333 -0.342  0.196
## X4 0.463 -0.039 -0.665  0.231  0.537
## X5 0.523 -0.105 -0.252  0.077 -0.804
```

(c)

```r
summary(q3.pca)
```

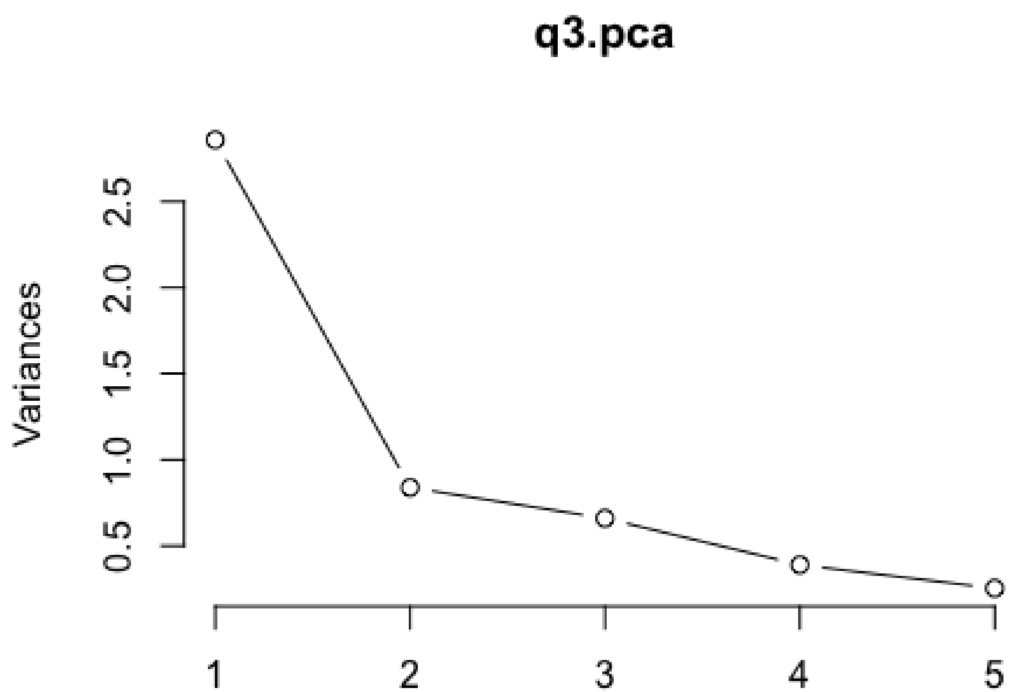
```
## Importance of components:
##                             PC1   PC2   PC3   PC4    PC5
## Standard deviation     1.691 0.916 0.812 0.624 0.5030
```

```
## Proportion of Variance 0.572 0.168 0.132 0.078 0.0506
## Cumulative Proportion  0.572 0.739 0.871 0.949 1.0000
```

1) By total proportion of variance: Since adding second proportion explains over 70% variance, choosing 2 principal components is appropriate.

2) By using scree plot: By drawing the scree plot, choosing 2 principal components is appropriate.

```
screeplot(q3.pca, type = "l")
```



3) By choosing variance larger than 1: Choosing 1 principal component is appropriate.

Therefore, using 2 principal component is appropriate.

(d)

1) First principal component explains the overall effect of 5 variables.

2) Second principal component: By considering the absolute value of coefficients larger than 0.2, exclude Food-consumed and appetite. Then, the second principal component can be interpreted as active reason (Symptom, Activity) against non-active reason (Sleep).

(e)

Since 2 PC explain 73.95% of total sample variance, the data is summarized with 2 PC given in this data.