STAT401_HW4

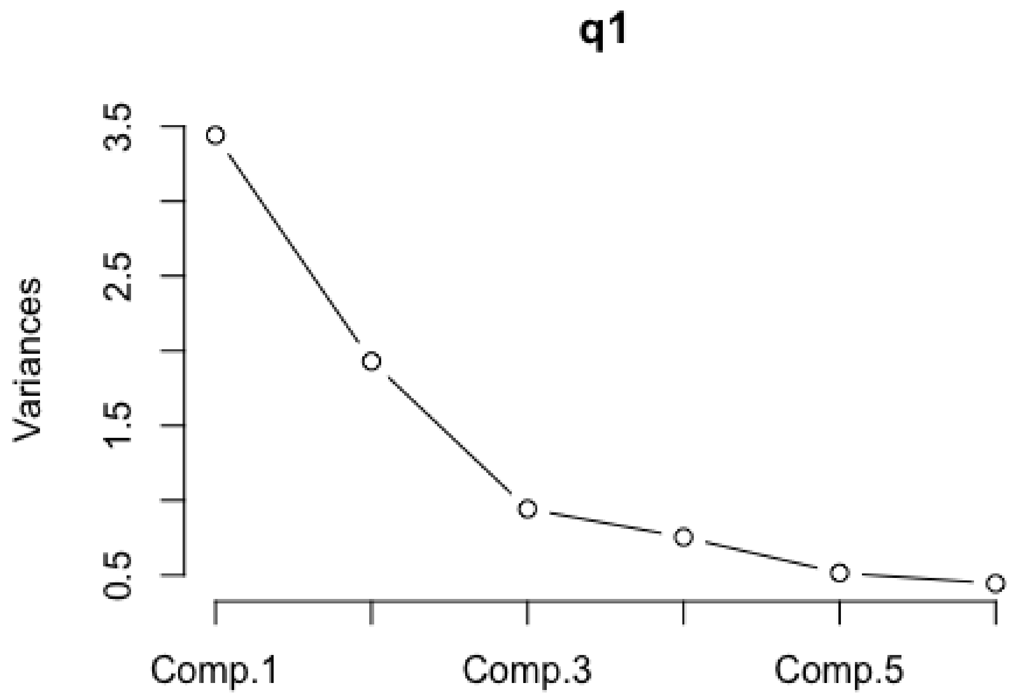
김정현

Q1.

```
options(digits=3)
library(psych)
rmat<-matrix(
    c(1,-0.04,0.61,0.45,0.03,-0.29,-0.3,0.45,0.3,
    -0.04,1,-0.07,-0.12,0.49,0.43,0.3,-0.31,-0.17,       0.61,-
0.07,1,0.59,0.03,-0.13,-0.24,0.59,0.32, 0.45,-0.12,0.59,1,-0.08,-0.21,-
0.19,0.63,0.37, 0.03,0.49,0.03,-0.08,1,0.47,0.41,-0.14,-0.24, -0.29,0.43,-
0.13,-0.21,0.47,1,0.63,-0.13,-0.15, -0.3,0.3,-0.24,-0.19,0.41,0.63,1,-0.26,-
0.29, 0.45,-0.31,0.59,0.63,-0.14,-0.13,-0.26,1,0.4, 0.3,-0.17,0.32,0.37,-
0.24,-0.15,-0.29,0.4,1),nrow = 9,ncol=9)
```

(a)

```
q1=princomp(covmat=rmat,cor=T)
summary(q1)

## Importance of components:
##                          Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7
Comp.8
## Standard deviation       1.855  1.389  0.971 0.8682 0.7174 0.6665 0.6498
0.5710
## Proportion of Variance  0.382  0.214  0.105 0.0838 0.0572 0.0494 0.0469
0.0362
## Cumulative Proportion   0.382  0.597  0.702 0.7853 0.8425 0.8918 0.9388
0.9750
##                          Comp.9
## Standard deviation       0.474
## Proportion of Variance  0.025
## Cumulative Proportion   1.000

q1$sdev^2

## Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8 Comp.9
##  3.442  1.930  0.942  0.754  0.515  0.444  0.422  0.326  0.225

screeplot(q1, npcs=6,type="l")
```



1) Choose the principal components at the total variance over 70%, then 3 PCs explain 70.2% of the total variation.
2) Choose the principal components where eigenvalues are larger than 1, then choose 2 PCs.
3) Choose the principal components based on the scree plot, then choose 3 PCs.

Therefore, we can choose 3 factors to estimate factor loadings.

(b)

```
f1 = fa(rmat, nfactors=3, fm="pa", rotate="none")
f1$communality

## [1] 0.593 0.434 0.667 0.543 0.565 0.781 0.521 0.756 0.256
```

(c)

```
diag(f1$uniq)

##        [,1]  [,2]  [,3]  [,4]  [,5]  [,6]  [,7]  [,8]  [,9]
##  [1,] 0.407 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
##  [2,] 0.000 0.566 0.000 0.000 0.000 0.000 0.000 0.000 0.000
##  [3,] 0.000 0.000 0.333 0.000 0.000 0.000 0.000 0.000 0.000
```

```
##  [4,] 0.000 0.000 0.000 0.457 0.000 0.000 0.000 0.000 0.000
##  [5,] 0.000 0.000 0.000 0.000 0.435 0.000 0.000 0.000 0.000
##  [6,] 0.000 0.000 0.000 0.000 0.000 0.219 0.000 0.000 0.000
##  [7,] 0.000 0.000 0.000 0.000 0.000 0.000 0.479 0.000 0.000
##  [8,] 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.244 0.000
##  [9,] 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.744
```

(d)

```
f1$loadings[7,1]

##    PA1
## -0.576
```

Correlation between 7th statement and the first factor is -0.576.

(e)

```
rmat-f1$loadings%*%t(f1$loadings)-diag(f1$uniq)

##             [,1]     [,2]     [,3]     [,4]     [,5]     [,6]     [,7]
[,8]
##  [1,]  0.000000 -0.00739  0.01826 -0.03166  0.00409 -0.00448  0.01044
0.000981
##  [2,] -0.007392  0.00000 -0.01603  0.03529  0.00625  0.03555 -0.04222 -
0.041507
##  [3,]  0.018262 -0.01603  0.00000  0.01016 -0.01017  0.02068 -0.00646 -
0.015184
##  [4,] -0.031659  0.03529  0.01016  0.00000 -0.00347 -0.05301  0.04567
0.014738
##  [5,]  0.004090  0.00625 -0.01017 -0.00347  0.00000 -0.02074  0.01138
0.040537
##  [6,] -0.004477  0.03555  0.02068 -0.05301 -0.02074  0.00000  0.00743
0.005962
##  [7,]  0.010440 -0.04222 -0.00646  0.04567  0.01138  0.00743  0.00000 -
0.018985
##  [8,]  0.000981 -0.04151 -0.01518  0.01474  0.04054  0.00596 -0.01899
0.000000
##  [9,]  0.019803  0.04423 -0.01877  0.02269 -0.05397  0.05945 -0.05392 -
0.018845
##           [,9]
##  [1,]  0.0198
##  [2,]  0.0442
##  [3,] -0.0188
##  [4,]  0.0227
##  [5,] -0.0540
##  [6,]  0.0594
##  [7,] -0.0539
##  [8,] -0.0188
##  [9,]  0.0000
```

We can check that our residual matrix is almost 0.

Repeat using Varimax

```
f2 = fa(rmat, nfactors=3, fm="pa", rotate="varimax")
f2$communality

## [1] 0.593 0.434 0.667 0.543 0.565 0.781 0.521 0.756 0.256

diag(f2$uniq)

##         [,1]  [,2]  [,3]  [,4]  [,5]  [,6]  [,7]  [,8]  [,9]
##  [1,] 0.407 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
##  [2,] 0.000 0.566 0.000 0.000 0.000 0.000 0.000 0.000 0.000
##  [3,] 0.000 0.000 0.333 0.000 0.000 0.000 0.000 0.000 0.000
##  [4,] 0.000 0.000 0.000 0.457 0.000 0.000 0.000 0.000 0.000
##  [5,] 0.000 0.000 0.000 0.000 0.435 0.000 0.000 0.000 0.000
##  [6,] 0.000 0.000 0.000 0.000 0.000 0.219 0.000 0.000 0.000
##  [7,] 0.000 0.000 0.000 0.000 0.000 0.000 0.479 0.000 0.000
##  [8,] 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.244 0.000
##  [9,] 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.744
```

Note that communalities and specific variances are same with rotate = “none” method.

```
f2$loadings[7,1]

##    PA1
## -0.218
```

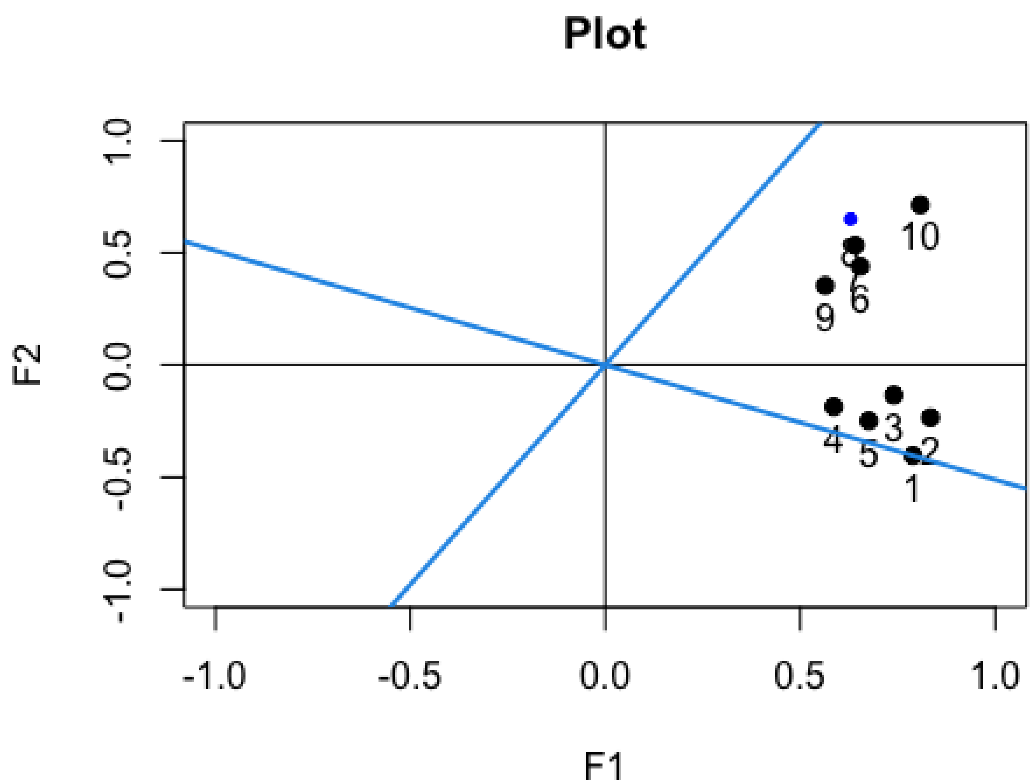
Correlation is -0.218, which is weaker than rotate = “none” method.

```
rmat-f2$loadings%*%t(f2$loadings)-diag(f2$uniq)

##             [,1]     [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
##  [1,] -3.33e-16 -0.00739  1.83e-02 -3.17e-02  4.09e-03 -4.48e-03  1.04e-02
##  [2,] -7.39e-03  0.00000 -1.60e-02  3.53e-02  6.25e-03  3.55e-02 -4.22e-02
##  [3,]  1.83e-02 -0.01603 -4.44e-16  1.02e-02 -1.02e-02  2.07e-02 -6.46e-03
##  [4,] -3.17e-02  0.03529  1.02e-02 -5.55e-16 -3.47e-03 -5.30e-02  4.57e-02
##  [5,]  4.09e-03  0.00625 -1.02e-02 -3.47e-03 -1.11e-16 -2.07e-02  1.14e-02
##  [6,] -4.48e-03  0.03555  2.07e-02 -5.30e-02 -2.07e-02 -3.33e-16  7.43e-03
##  [7,]  1.04e-02 -0.04222 -6.46e-03  4.57e-02  1.14e-02  7.43e-03 -2.22e-16
##  [8,]  9.81e-04 -0.04151 -1.52e-02  1.47e-02  4.05e-02  5.96e-03 -1.90e-02
##  [9,]  1.98e-02  0.04423 -1.88e-02  2.27e-02 -5.40e-02  5.94e-02 -5.39e-02
##             [,8]      [,9]
##  [1,]  9.81e-04  1.98e-02
##  [2,] -4.15e-02  4.42e-02
##  [3,] -1.52e-02 -1.88e-02
##  [4,]  1.47e-02  2.27e-02
##  [5,]  4.05e-02 -5.40e-02
##  [6,]  5.96e-03  5.94e-02
##  [7,] -1.90e-02 -5.39e-02
##  [8,] -5.55e-16 -1.88e-02
##  [9,] -1.88e-02 -2.22e-16
```

The residual matrix is close to zero compared to rotate = “none” method.

Q2.

```
l1<-matrix(c(0.789,0.834,0.74,0.586,0.676,0.654,0.641,0.629,0.564,0.808,
-0.403,-0.234,-0.134,-0.185,-0.248,0.44,0.534,0.651,0.354,0.714),ncol = 2,
nrow = 10)

fa.plot(l1,xlim=c(-1,1),ylim=c(-1,1))
abline(a=0,b=l1[1,2]/l1[1,1],col=4,lwd=2)
abline(a=0,b=-l1[1,1]/l1[1,2], col=4,lwd=2)
```




```
a1<-acos(l1[1,1]/sqrt(l1[1,1]^2+l1[1,2]^2))/(2*pi)*360
cat("Angle:", a1)

## Angle: 27.1
```

The angle needed is 27.1.

(b)

By this result, We can divide by (x6, x7, x8, x9, x10) and (x1, x2, x3, x4, x5) here.

Q3.

(a)

```
s.dat <- read.csv('sales.dat', header = T, sep = '')
s.f2 <- fa(s.dat, nfactors = 2, fm = "ml", rotate = "none")
print("Communality:")

## [1] "Communality:"

s.f2$communalities

##    X1    X2    X3    X4    X5    X6    X7
## 0.931 0.930 0.877 0.995 0.526 0.386 0.971

print("Specific Variance:")

## [1] "Specific Variance:"

diag(s.f2$uniq)

##        [,1]   [,2]  [,3]    [,4]  [,5]  [,6]   [,7]
## [1,] 0.0692 0.0000 0.000 0.00000 0.000 0.000 0.0000
## [2,] 0.0000 0.0704 0.000 0.00000 0.000 0.000 0.0000
## [3,] 0.0000 0.0000 0.123 0.00000 0.000 0.000 0.0000
## [4,] 0.0000 0.0000 0.000 0.00499 0.000 0.000 0.0000
## [5,] 0.0000 0.0000 0.000 0.00000 0.474 0.000 0.0000
## [6,] 0.0000 0.0000 0.000 0.00000 0.000 0.614 0.0000
## [7,] 0.0000 0.0000 0.000 0.00000 0.000 0.000 0.0288

print("Communality*Communality + Specific Variance:")

## [1] "Communality*Communality + Specific Variance:"

s.f2$loadings %*% t(s.f2$loadings) + diag(s.f2$uniq)

##       X1    X2    X3    X4    X5    X6    X7
## X1 1.000 0.930 0.883 0.572 0.664 0.554 0.931
## X2 0.930 1.000 0.875 0.541 0.655 0.562 0.937
## X3 0.883 0.875 1.000 0.700 0.675 0.480 0.845
## X4 0.572 0.541 0.700 1.000 0.592 0.150 0.413
## X5 0.664 0.655 0.675 0.592 1.000 0.341 0.619
## X6 0.554 0.562 0.480 0.150 0.341 1.000 0.602
## X7 0.931 0.937 0.845 0.413 0.619 0.602 1.000

s.f3 <- fa(s.dat, nfactors = 2, fm = "ml", rotate = "none")
print("Communality:")

## [1] "Communality:"

s.f3$communalities

##    X1    X2    X3    X4    X5    X6    X7
## 0.931 0.930 0.877 0.995 0.526 0.386 0.971
```

```r
print("Specific Variance:")

## [1] "Specific Variance:"

diag(s.f3$uniq)

##        [,1]   [,2]  [,3]    [,4]  [,5]  [,6]   [,7]
## [1,] 0.0692 0.0000 0.000 0.00000 0.000 0.000 0.0000
## [2,] 0.0000 0.0704 0.000 0.00000 0.000 0.000 0.0000
## [3,] 0.0000 0.0000 0.123 0.00000 0.000 0.000 0.0000
## [4,] 0.0000 0.0000 0.000 0.00499 0.000 0.000 0.0000
## [5,] 0.0000 0.0000 0.000 0.00000 0.474 0.000 0.0000
## [6,] 0.0000 0.0000 0.000 0.00000 0.000 0.614 0.0000
## [7,] 0.0000 0.0000 0.000 0.00000 0.000 0.000 0.0288

print("Communality*Communality + Specific Variance:")

## [1] "Communality*Communality + Specific Variance:"

s.f3$loadings %*% t(s.f3$loadings) + diag(s.f3$uniq)

##       X1    X2    X3    X4    X5    X6    X7
## X1 1.000 0.930 0.883 0.572 0.664 0.554 0.931
## X2 0.930 1.000 0.875 0.541 0.655 0.562 0.937
## X3 0.883 0.875 1.000 0.700 0.675 0.480 0.845
## X4 0.572 0.541 0.700 1.000 0.592 0.150 0.413
## X5 0.664 0.655 0.675 0.592 1.000 0.341 0.619
## X6 0.554 0.562 0.480 0.150 0.341 1.000 0.602
## X7 0.931 0.937 0.845 0.413 0.619 0.602 1.000
```

(c)

```r
s.f1 <- fa(s.dat, nfactors = 1, fm = "ml", rotate = "none")
s.f4 <- fa(s.dat, nfactors = 4, fm = "ml", rotate = "none")

chi1 <- function(model, nfactors) {
    chi.q <- model$STATISTIC
    prob <- model$PVAL
    cat("n =", nfactors, ": Chi Square =", chi.q, "with prob <", prob, "\n")
}

chi1(s.f1, 1)

## n = 1 : Chi Square = 163 with prob < 2.02e-27

chi1(s.f2, 2)

## n = 2 : Chi Square = 117 with prob < 1.25e-21

chi1(s.f3, 3)

## n = 3 : Chi Square = 117 with prob < 1.25e-21
```

Even though we use 3-factor model, our p-value is still smaller than 0.05, which is still significant. Also, 4-factor model is not appropriate since s < 0 in this case.

```
chi1(s.f4, 4)

## n = 4 : Chi Square = 18 with prob < NA
```

We check that 4 factor model does not work. Therefore, we can pick 3 factor model.

(d)

```
f.ml1 = fa(s.dat, nfactors=2, n.obs=length(s.dat), fm="ml", rotate="none")
apply(f.ml1$scores, 2, mean)

##       ML1       ML2
## -1.02e-15 -3.40e-15

apply(f.ml1$scores, 2, var)

##   ML1   ML2
## 0.996 0.978
```

Mean is close to 0 and Variance is close to 1.

(e)

```
f.ml1

## Factor Analysis using method =  ml
## Call: fa(r = s.dat, nfactors = 2, n.obs = length(s.dat), rotate = "none",
##     fm = "ml")
## Standardized loadings (pattern matrix) based upon correlation matrix
##     ML1   ML2   h2    u2 com
## X1 0.70  0.67 0.93 0.069 2.0
## X2 0.67  0.69 0.93 0.070 2.0
## X3 0.80  0.49 0.88 0.123 1.7
## X4 0.98 -0.17 1.00 0.005 1.1
## X5 0.65  0.31 0.53 0.474 1.4
## X6 0.25  0.57 0.39 0.614 1.4
## X7 0.56  0.81 0.97 0.029 1.8
##
##                        ML1  ML2
## SS loadings           3.33 2.28
## Proportion Var        0.48 0.33
## Cumulative Var        0.48 0.80
## Proportion Explained  0.59 0.41
## Cumulative Proportion 0.59 1.00
##
## Mean item complexity =  1.6
## Test of the hypothesis that 2 factors are sufficient.
##
## df null model =  21  with the objective function =  10.9 with Chi Square =
500
```

```
## df of  the model are 8  and the objective function was  2.63
##
## The root mean square of the residuals (RMSR) is  0.06
## The df corrected root mean square of the residuals is  0.09
##
## The harmonic n.obs is  50 with the empirical chi square  6.79  with prob <
0.56
## The total n.obs was  50  with Likelihood Chi Square =  117  with prob <
1.3e-21
##
## Tucker Lewis Index of factoring reliability =  0.382
## RMSEA index =  0.522  and the 90 % confidence intervals are  0.446 0.614
## BIC =  85.9
## Fit based upon off diagonal values = 0.99
## Measures of factor score adequacy
##                                                    ML1  ML2
## Correlation of (regression) scores with factors   1.00 0.99
## Multiple R square of scores with factors          1.00 0.98
## Minimum correlation of possible factor scores     0.99 0.96
```

x1, x3, x4, x5 is allocated in factor 1 and x2, x6, x7 is allocated in factor2.

```
simpsum <- cbind(round(as.matrix(s.dat[,c(1,3,4,5)]),1) %*% c(1,1,1,1),
    round(as.matrix(s.dat[,c(2,6,7)]),1) %*% c(1,1,1))
simpsum

##       [,1] [,2]
##  [1,]  212  125
##  [2,]  203  117
##  [3,]  214  135
##  [4,]  235  145
##  [5,]  230  152
##  [6,]  219  130
##  [7,]  216  134
##  [8,]  264  188
##  [9,]  234  152
## [10,]  241  170
## [11,]  236  154
## [12,]  228  151
## [13,]  244  158
## [14,]  220  150
## [15,]  226  149
## [16,]  192  114
## [17,]  227  148
## [18,]  232  157
## [19,]  217  147
## [20,]  240  143
## [21,]  206  117
## [22,]  220  168
## [23,]  201  116
```

```
## [24,]  238  163
## [25,]  251  172
## [26,]  216  132
## [27,]  244  169
## [28,]  238  180
## [29,]  210  121
## [30,]  237  176
## [31,]  250  172
## [32,]  209  111
## [33,]  218  141
## [34,]  215  124
## [35,]  252  174
## [36,]  234  176
## [37,]  230  126
## [38,]  230  142
## [39,]  251  175
## [40,]  248  168
## [41,]  232  154
## [42,]  220  132
## [43,]  247  156
## [44,]  186  111
## [45,]  218  137
## [46,]  245  161
## [47,]  209  121
## [48,]  195  107
## [49,]  237  158
## [50,]  239  168
```

One can check that factor scores in (d) and (e) are different.