MATH 243 -- Winter 2022
Mathematica Lab 0
Name -- Solutions

Textbook Sections: None
Topics: A review of Mathematica commands learned in MATH 242, as well as proper formatting of a Mathematica notebook.
New Mathematica Commands: None

Proper Formatting

Creating a text cell vs a coding cell.

There are two types of *cells* that you use in Mathematica: cells for typing text (such as this very cell I'm typing in right now), and cells for coding.
Mathematica automatically creates a coding cell whenever you start a new cell.
To create a cell for typing text, click **Format -> Style -> Text**. This will create a text cell for you to type in. Other styles such as Section and Subsection are for larger fonts that you might use to write a chapter/section title (e.g., how this very text cell is under the subsection called "Creating a text cell vs a coding cell.").

The basics.

1. Special functions and numbers (sin(x), cos(x), e^x, π, etc...) must be capitalized and use brackets when applicable. For example, if you want to use sin(x), then you must type **Sin[x]**. Similarly, if you want to use the number π, then you must type **Pi**.
2. It is best to *not* type every line of code in its own coding cell! After you evaluate a cell (using Shift+Enter or Shift+Return), Mathematica will automatically start a new cell. Be sure to click back into the coding cell and continue coding *inside* the same coding cell.
***Of course, you don't want to do *all* of you coding in a single cell. Use your best judgement for when to start a new coding cell. Just don't start a new cell for every new line!
3. If you want to delete a cell of any sort, simply click/highlight the vertical bar on the right (by the screen scroller) and click "Delete".
4. If you want to type text in a coding cell, then type it as follows: (* this is my text *).
You must put the text between (* *).
5. You can suppress outputs (i.e., Mathematica will still run, but it will not show the output) by ending a line with a semicolon ;

6. You can simplify an output in Mathematica by using the command **FullSimplify[ ]**.
For example, without FullSimplify you might get $2x + x - 1$. But with FullSimplify you will get $3x - 1$.
7. If you want to type a mathematical symbol (e.g., π, τ, θ, ϕ, etc...), then you can either: (1) click **Palettes -> Basic Math Assistant** or **Palettes -> Special Characters**; or (2) click Esc + "type the symbol out" + Esc. For example, the letter π can be made by clicking Esc, typing pi, then clicking Esc again.

Troubleshooting Mathematica.

Sometimes Mathematica simply stops working even though we've typed all the write stuff. One way to troubleshoot Mathematica is to essentially restart it. Do this by clicking **Evaluation -> Quit Kernel -> Local (and click "yes")**. Then, click **Evaluation -> Start Kernel -> Local**.

Previous Mathematica Commands

Defining functions.

You can define a function (e.g., $f(x) = x^2$) by typing in the function as so: **f[x_] := x^2**
You must use the brackets and underscore. The underscore tells Mathematica which variables denote independent variables/inputs.
***Whenever you define a new function, it is good habit to clear the name used for that function. This way, incase you use the same name later in a different setting, Mathematica will not get confused. For example, you can clear the function $f(x)$ by typing: **Clear[f]**
***NEVER use capital letters to define a function.

Plotting functions of a single variable.

Once you have defined a function, say $f(x)$, you might want to plot that function.
You can plot a function $f(x)$ by typing into the Mathematica: **Plot[f[x] , {x,a,b}]**
You must use brackets when using the function $f(x)$. Also, **a** and **b** are the left and right x –values that you want to plot $f(x)$ over.

Additional accessories you might find useful:
Axes -> True This will put axes on your plot.
AxesLabel -> {"x" , "y"} This will label your axes with x and y. You can label these axes whatever you want, and not necessarily x and y.
PlotRange -> {c,d} This will force the y –axis to show $c \leq y \leq d$.
PlotStyle -> Red This will make the graph red. There are other color options (yellow, blue, black, green, etc...).

If you want to plot two (or more) functions on the same plot, say $f(x)$ and $g(x)$, you type: **Plot[{ f[x],**

g[x] }, {x,a,b}]
Note that we put our two functions in curly braces and separate them with a comma.
If you want to label $f(x)$ with a blue curve and $g(x)$ with a red curve, then you type: **Plot[{ f[x],g[x] }, {x,a,b}, PlotStyle -> {Blue,Red}]**
You use PlotStyle just a normal, but put the two respective colors in curly braces.

There are other accessories that you can add onto your plots. Please see Wolfram's documentation online for more details.

Differentiation functions.

There are two ways to take a derivative of a function $f(x)$. Assume we've already defined the function, say we call it **f[x_]**.
1. You can type **f'[x]**.
2. You can use the command **D[f[x], x]**.

Similarly, you can take second derivatives (and higher) by either typing: **f''[x]** or **D[f[x], x,x]**.
***We will mostly use the second option in this course, so get in the habit of using it.

Integrating functions.

f a function $f(x)$ by typing: **Integrate[f[x], x]**
The x is the second position denotes "with respect to x" (which mathematically is like writing dx in integrals to denote "with respect to x").

You can compute the definite integral of a function $f(x)$ from $x = a$ to $x = b$ by typing: **Integrate[f[x], {x,a,b}]**

Equation solving.

You can solve an equation $p(x) = 0$ by using the commands: **Solve[p[x] == 0]** or **NSolve[p[x] == 0]**.
For our purposes, the first choice will usually suffice. Note that you must use double equal signs "==".

Computing limits.

In order to compute a limit $\lim_{x\to a} f(x)$, you can type the following command: **Limit[f[x], x -> a]**

Examples

Example 1.

Define the function $f(x) = \sin(2\,x) + e^x$.

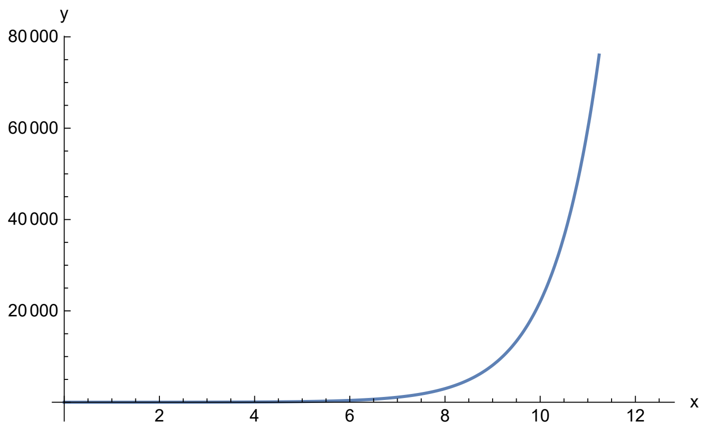
Compute the derivative of $f(x)$.

Plot the function $f(x)$ for $0 \leq x \leq 4\pi$. Be sure to include and label your axes.

In[176]:=
```
Clear[f]
f[x_] := Sin[2 * x] + Exp[x]
D[f[x], x]
Plot[f[x], {x, 0, 4 * Pi}, Axes → True, AxesLabel → {"x", "y"}]
```

Out[ ]= $e^x + 2\,\text{Cos}[2\,x]$

Out[ ]=



Example 2.

Define the function $g(x) = \frac{x-1}{x^2-1}$.

Solve for the roots of $g(x)$, that is, when does $g(x) = 1$?

Compute the first and second derivatives of $g(x)$. Use **FullSimplify** to simplify the output.

Compute the limit $\lim_{x \to 1} g(x)$.

Plot the function $g(x)$ for $-2 \leq x \leq 2$ and color the curve red. Be sure to include and label your axes.

In[226]:=
```
Clear[g]
g[x_] := (x - 1) / (x^2 - 1)
Solve[g[x] == 1]
FullSimplify[D[g[x], x]]
FullSimplify[D[g[x], x, x]]
Limit[g[x], {x → 1}]
Plot[g[x], {x, -2, 2}, Axes → True, AxesLabel → {"x", "y"}, PlotStyle → Red]
```

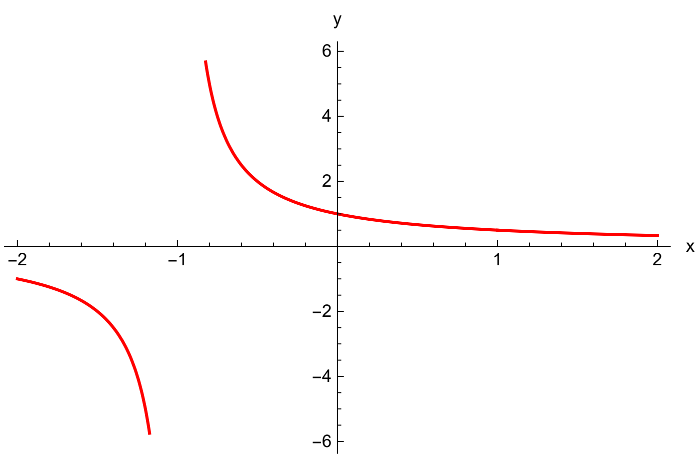
Out[ ]= {{x → 0}}

Out[ ]= $-\frac{1}{(1+x)^2}$

Out[ ]= $\frac{2}{(1+x)^3}$

Out[ ]= $\frac{1}{2}$

Out[ ]=



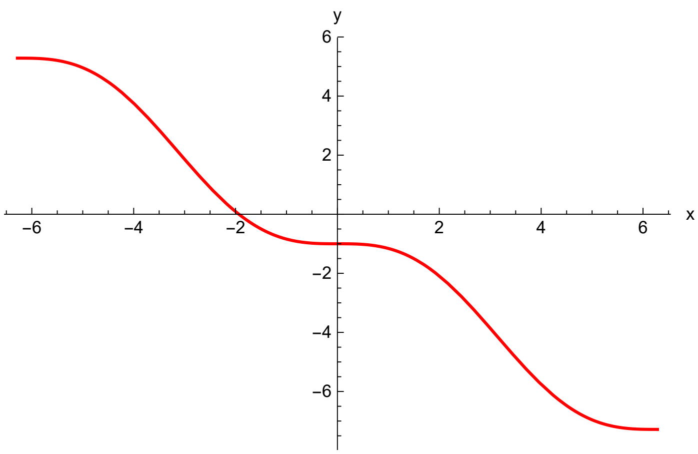
Exercises

Exercise 1. Define the function $f(x) = \sin(x) - x - 1$.
(a) Compute the derivative of $f(x)$.
(b) Compute the integral $\int_0^5 f(x)\,\mathrm{dx}$.
(c) Plot $f(x)$ for $-2\pi \leq x \leq 2\pi$ and color the curve red. Be sure to include and label your axes.

In[112]:=
```
Clear[f]
f[x_] := Sin[x] - x - 1

(* Part A *)
f'[x]

(* Part B *)
Integrate[f[x], {x, 0, 5}]

(* Part C *)
Plot[f[x], {x, -2 * Pi, 2 * Pi}, Axes → True, AxesLabel → {"x", "y"}, PlotStyle → Red]
```

Out[114]= $-1 + \mathrm{Cos}[x]$

Out[115]= $-\frac{33}{2} - \mathrm{Cos}[5]$

Out[116]=


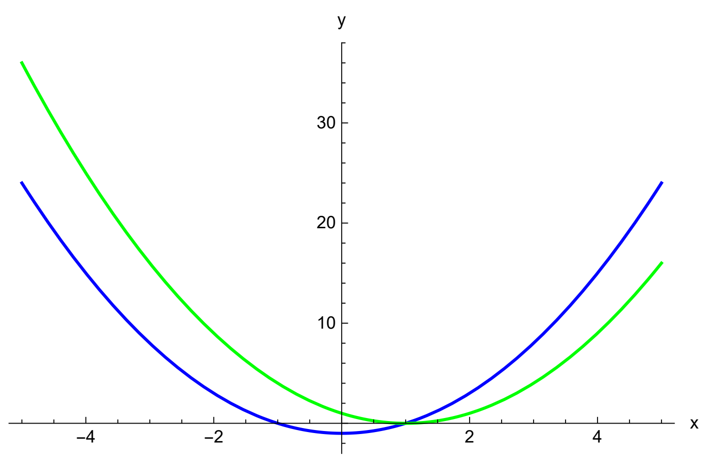
Exercise 2. Consider an arbitrary polynomial $p(x) = a\,x^2 + b\,x + c$.
(a) Create/define two functions called **quad1[a_,b_,c_]** and **quad2[a_,b_,c_]** that solves $a\,x^2 + b\,x + c = 0$ by using the quadratic formula. Note that **quad1** and **quad2** are for the two roots from the quadratic formula, i.e., the plus/minus. Then, use **quad1** and **quad2** to solve $3\,x^2 - 4\,x + 1 = 0$.

Essentially, I am asking you to code the quadratic formula.
(b) Now using **Solve[ ]**, solve $3x^2 - 4x + 1 = 0$. Your answers should match what you got for part (a).
(c) On the same set of axes (labelled, of course), plot $x^2 - 1$ and $x^2 - 2x + 1$ for $-5 \leq x \leq 5$. Make the first quadratic blue and the second quadratic green.

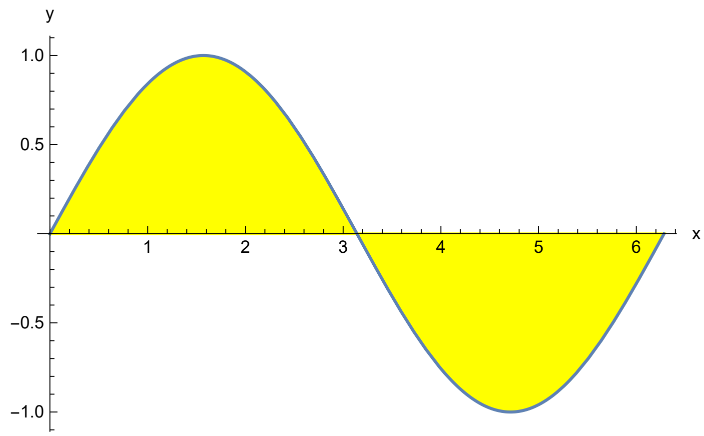
In[309]:=
```
Clear[quad1, quad2]
(* Part A *)
quad1[a_, b_, c_] := (-b + Sqrt[b^2 - 4 * a * c]) / (2 * a)
quad2[a_, b_, c_] := (-b - Sqrt[b^2 - 4 * a * c]) / (2 * a)
{quad1[3, -4, 1], quad2[3, -4, 1]}

(* Part B *)
Solve[3 * x^2 - 4 * x + 1 == 0]

(* Part C *)
Plot[{x^2 - 1, x^2 - 2 * x + 1}, {x, -5, 5},
 Axes → True, AxesLabel → {"x", "y"}, PlotStyle → {Blue, Green}]
```

Out[•]= $\left\{1, \frac{1}{3}\right\}$

Out[•]= $\left\{\left\{x \to \frac{1}{3}\right\}, \{x \to 1\}\right\}$

Out[•]=


Exercise 3. There are SO MANY other commands and applications that Mathematica can do! However, someone won’t always be there to teach you. Sometimes, you need to look up how to do things on your own.
In Mathematica, it is possible to shade “underneath” a curve (as shown below).



Your task in this exercise is to replicate the figure shown above. The plot shown is of the function $f(x) = \sin(x)$ for $0 \leq x \leq 2\pi$. The color of the shading/filling is yellow.

You will need to look up how to “fill underneath a curve in Mathematica.”

You can find the necessary information at

https://reference.wolfram.com/language/ref/Plot.html.

You can also search on Google if you wish.

Scroll to the bottom of the page and click **Options -> Filling** and **Options -> Filling Style**.

In[ ]:=
```
Plot[Sin[x], {x, 0, 2 * Pi}, Axes → True,
  AxesLabel → {"x", "y"}, Filling → Axis, FillingStyle → Yellow]
```

Out[ ]=