Multivariable Calculus Problem Set 1

Topics: dot and cross products, projections, lines and planes, space curves

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Exercise 1. Let $\mathbf{a} = \langle 3, -4, 5\rangle$ and $\mathbf{b} = \langle 2, -1, 1\rangle$.
(a) Compute $\mathbf{a} \cdot \mathbf{b}$. Are $\mathbf{a}$ and $\mathbf{b}$ orthogonal/perpendicular?
(b) Compute the angle between $\mathbf{a}$ and $\mathbf{b}$ using the dot product.
(c) Compute the area of the parallelogram formed by $\mathbf{a}$ and $\mathbf{b}$ *without* actually computing their cross product. **Hint: there might be a useful theorem you can use.*
(d) Compute the scalar and vector projections of $\mathbf{b}$ onto $\mathbf{a}$.

Exercise 2. On the same set of axes (in $\mathbb{R}^3$) draw $\mathbf{a} = \langle 3, 0, 0\rangle$ and $\mathbf{b} = \langle 1, 3, 0\rangle$. Without computing the cross product, does the vector $\mathbf{a} \times \mathbf{b}$ point upwards or downwards?
**Hint: use the righthand rule.*

Exercise 3. Compute $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{b} \times \mathbf{a})$ and $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b})$.

Exercise 4.
(a) Construct a nonzero vector $\mathbf{b}$ that is orthogonal/perpendicular to the vector $\mathbf{a} = \langle \alpha, -\alpha, 3\alpha\rangle$, where α is a constant.
(b) Using your answer from part (a), construct another vector that is orthogonal/perpendicular to *both* $\mathbf{a}$ and $\mathbf{b}$.

Exercise 5. Compute the parametric, symmetric, and vector equations of the line that passes through the points $P(1, 2, 3)$ and $Q(0, 3, -1)$.

Exercise 6. Find an equation for the plane that passes through the point $(6, -1, 3)$ and contains the line $\dfrac{x}{3} = y + 4 = \dfrac{z}{2}$.

Exercise 7. Find parametric, symmetric, and vector equations for the line that is perpendicular to the plane $x - y - z = 2$ and passes through the point $(1, 0, 1)$.

Exercise 8. Consider the following two lines:

$$L1: \; x = 2 + t, \; y = 3 - 2t, \; z = 1 - 3t$$
$$L2: \; x = 3 + s, \; y = -4 + s, \; z = 2 - 7s$$

Are these lines parallel, skew, or intersecting? If intersecting, what is the point of intersection?

Exercise 9.
(a) Find the domain of the curve $\mathbf{r}(t) = \left\langle e^t, \ln(t+1), \dfrac{1}{t-1} \right\rangle$.
(b) Compute the unit tangent vector at $t = 0$, that is, compute $\mathbf{T}(0)$.
(c) TRUE or FALSE: $\lim_{t \to -1^+} \mathbf{r}(t)$ exists.