PROBLEM SET #1 SOLUTIONS

#1 $\vec{a} = \langle 3,-4,5\rangle$, $\vec{b} = \langle 2,-1,1\rangle$

(a) $\vec{a}\cdot\vec{b} = 6+4+5 = 15 \neq 0$ NOT ORTHOGONAL!

(b) $\vec{a}\cdot\vec{b} = \|\vec{a}\|\|\vec{b}\|\cos(\theta)$

$$15 = \sqrt{9+16+25}\,\sqrt{4+1+1}\,\cos(\theta)$$

$$\theta = \cos^{-1}\left(\frac{15}{\sqrt{50}\sqrt{6}}\right) = \cos^{-1}\left(\frac{3}{\sqrt{12}}\right) = \cos^{-1}\left(\frac{3}{2\sqrt{3}}\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

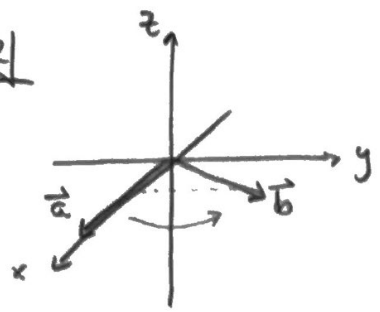
(c) $A = \|\vec{a}\times\vec{b}\| = \|\vec{a}\|\|\vec{b}\|\sin(\theta) = \sqrt{50}\sqrt{6}\sin\left(\frac{\pi}{6}\right)$

$$= 10\sqrt{3}\,\frac{1}{2} = 5\sqrt{3}$$

(d) $\text{comp}_{\vec{a}}\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|} = \dfrac{15}{\sqrt{50}} = \dfrac{3}{\sqrt{2}} = \dfrac{3\sqrt{2}}{2}$

$$\text{proj}_{\vec{a}}\vec{b} = \left(\text{comp}_{\vec{a}}\vec{b}\right)\frac{\vec{a}}{\|\vec{a}\|} = \frac{3\sqrt{2}}{2}\,\frac{1}{\sqrt{50}}\langle 3,-4,5\rangle = \left\langle \frac{9}{10}, -\frac{6}{5}, \frac{3}{2}\right\rangle$$

#2



$\vec{a}\times\vec{b}$ is facing/pointing upwards.

#3 $(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{a}) = -(\vec{a}\times\vec{b})\times(\vec{a}\times\vec{b}) = \vec{0}$

$$\vec{a}\cdot(\vec{a}\times\vec{b}) = 0$$

#4

(a) $\vec{b} = \langle b_1, b_2, b_3 \rangle$

$$0 = \vec{a} \cdot \vec{b} = \alpha b_1 - \alpha b_2 + 3\alpha b_3 = \alpha (b_1 - b_2 + 3b_3)$$

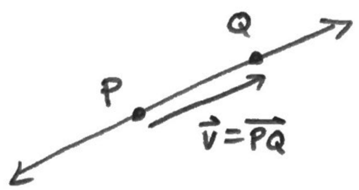
Let $\vec{b} = \langle 1, 1, 0 \rangle$.

(b) $$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & -\alpha & 3\alpha \\ 1 & 1 & 0 \end{vmatrix} = \langle -3\alpha, 3\alpha, 2\alpha \rangle$$

#5 NEED: ① Point ✓
② Vector ✓

$P(1,2,3)$, $Q(0,3,-1)$

$\overrightarrow{PQ} = \langle -1, 1, -4 \rangle$



Parametric Equations
$$\begin{cases} x(t) = 1 - t \\ y(t) = 2 + t \\ z(t) = 3 - 4t \end{cases}, \quad t \in \mathbb{R}$$

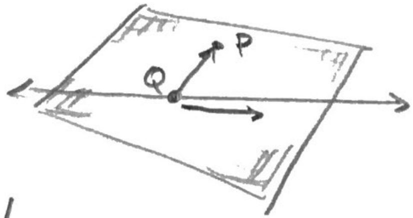
Symmetric Equation
$$\frac{x-1}{-1} = \frac{y-2}{1} = \frac{z-3}{-4}$$

Vector Equation
$$\vec{r}(t) = \langle 1-t, 2+t, 3-4t \rangle, \quad t \in \mathbb{R}$$

#6 $P(6,-1,3)$, $\dfrac{x}{3} = y+4 = \dfrac{z}{2}$

NEED: ① Point ✓

② Normal Vector ✓



↳ Need two vectors "in" the plane.

$\vec{v} = \langle 3, 1, 2 \rangle$

$\vec{u} = \overrightarrow{QP} = \langle 6, 3, 3 \rangle$

$Q(0,-4,0)$

↑

Set $z=0$.

$$\vec{n} = \vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 3 & 3 \\ 3 & 1 & 2 \end{vmatrix} = \langle 3, -3, -3 \rangle$$

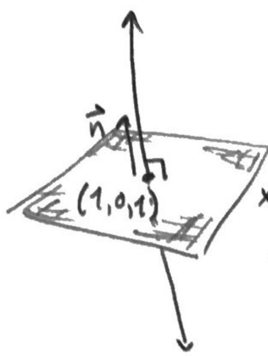
$$3(x-6) - 3(y+1) - 3(z-3) = 0$$

$$\Updownarrow$$

$$x - y - z = 4$$

#7 NEED: ① Point ✓

② Vector ✓



$x - y - z = 2$

$\vec{n} = \langle 1, -1, -1 \rangle$

Parametric Equations
$$\begin{cases} x(t) = 1 + t \\ y(t) = 0 - t \\ z(t) = 1 - t \end{cases}, \quad t \in \mathbb{R}$$

Symmetric Equation
$$\frac{x-1}{1} = \frac{y-0}{-1} = \frac{z-1}{-1}$$

Vector Equation
$$\vec{r}(t) = \langle 1+t, -t, 1-t \rangle, \quad t \in \mathbb{R}$$

#8 $\vec{v}_1 = \langle 1, -2, -3 \rangle$ $\quad\quad$ $\vec{v}_2 = \langle 1, 1, -7 \rangle$

Clearly $\vec{v}_1$ and $\vec{v}_2$ are <u>NOT PARALLEL!</u>

$$2 + t = 3 + s \qquad 3 - 2t = -4 + s \qquad 1 - 3t = 2 - 7s$$

$$-1 + t = s$$

$$3 - 2t = -4 - 1 + t$$
$$8 = 3t$$
$$\frac{8}{3} = t$$
$$\frac{5}{3} = s$$

$$1 - 3\left(\frac{8}{3}\right) \stackrel{?}{=} 2 - 7\left(\frac{5}{3}\right)$$
$$-7 \neq -\frac{29}{3}$$

$$\boxed{\text{SKEW!}}$$

#9 $\vec{r}(t) = \left\langle e^t, \ln(t+1), \frac{1}{t-1} \right\rangle$

(a) e^t: $(-\infty, +\infty)$

$\ln(t+1)$: $t + 1 > 0$, $t > -1$ $\quad (-1, +\infty)$

$\frac{1}{t-1}$: $(-\infty, 1) \cup (1, +\infty)$

$$\boxed{(-1, 1) \cup (1, +\infty)}$$

(b) $\vec{r}\,'(t) = \left\langle e^t, \frac{1}{t+1}, \frac{-1}{(t-1)^2} \right\rangle$

$\vec{r}\,'(0) = \langle 1, 1, -1 \rangle$

$\|\vec{r}\,'(0)\| = \sqrt{3}$

$\vec{T}(0) = \left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \right\rangle$

(c) $\boxed{\text{FALSE}}$ because $\lim_{t \to -1^+} \ln(t+1) = -\infty$ $\boxed{\text{DNE}}$