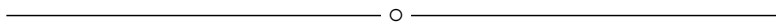


# MULTIVARIABLE CALCULUS PROBLEM SET 1

**Topics:** dot and cross products, projections, lines and planes, space curves



**Exercise 1.** Let  $\mathbf{a} = \langle 3, -4, 5 \rangle$  and  $\mathbf{b} = \langle 2, -1, 1 \rangle$ .

- (a) Compute  $\mathbf{a} \cdot \mathbf{b}$ . Are  $\mathbf{a}$  and  $\mathbf{b}$  orthogonal/perpendicular?
- (b) Compute the angle between  $\mathbf{a}$  and  $\mathbf{b}$  using the dot product.
- (c) Compute the area of the parallelogram formed by  $\mathbf{a}$  and  $\mathbf{b}$  *without* actually computing their cross product. *\*Hint: there might be a useful theorem you can use.*
- (d) Compute the scalar and vector projections of  $\mathbf{b}$  onto  $\mathbf{a}$ .

**Exercise 2.** On the same set of axes (in  $\mathbb{R}^3$ ) draw  $\mathbf{a} = \langle 3, 0, 0 \rangle$  and  $\mathbf{b} = \langle 1, 3, 0 \rangle$ . Without computing the cross product, does the vector  $\mathbf{a} \times \mathbf{b}$  point upwards or downwards?

*\*Hint: use the righthand rule.*

**Exercise 3.** Compute  $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{b} \times \mathbf{a})$  and  $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b})$ .

**Exercise 4.**

- (a) Construct a nonzero vector  $\mathbf{b}$  that is orthogonal/perpendicular to the vector  $\mathbf{a} = \langle \alpha, -\alpha, 3\alpha \rangle$ , where  $\alpha$  is a constant.
- (b) Using your answer from part (a), construct another vector that is orthogonal/perpendicular to *both*  $\mathbf{a}$  and  $\mathbf{b}$ .

**Exercise 5.** Compute the parametric, symmetric, and vector equations of the line that passes through the points  $P(1, 2, 3)$  and  $Q(0, 3, -1)$ .

**Exercise 6.** Find an equation for the plane that passes through the point  $(6, -1, 3)$  and contains the line  $\frac{x}{3} = y + 4 = \frac{z}{2}$ .

**Exercise 7.** Find parametric, symmetric, and vector equations for the line that is perpendicular to the plane  $x - y - z = 2$  and passes through the point  $(1, 0, 1)$ .

**Exercise 8.** Consider the following two lines:

$$L1 : x = 2 + t, y = 3 - 2t, z = 1 - 3t$$

$$L2 : x = 3 + s, y = -4 + s, z = 2 - 7s$$

Are these lines parallel, skew, or intersecting? If intersecting, what is the point of intersection?

**Exercise 9.**

- (a) Find the domain of the curve  $\mathbf{r}(t) = \left\langle e^t, \ln(t+1), \frac{1}{t-1} \right\rangle$ .
- (b) Compute the unit tangent vector at  $t = 0$ , that is, compute  $\mathbf{T}(0)$ .
- (c) TRUE or FALSE:  $\lim_{t \rightarrow -1^+} \mathbf{r}(t)$  exists.