Multivariable Calculus Problem Set 3

Topics: limits, continuity, partial derivatives, chain rule

Exercise 1. Sketch the domain of the function $h(x,y) = e^{\sqrt{y-x^2}} + \dfrac{1}{\sqrt{1-y}}$.

Exercise 2. For the following, compute the limit or show the limit does not exist.

(a) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{x^3y - x^3}{x^2y^2 - x^2}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2 + 3y^2}$

(c) $\displaystyle\lim_{(x,y)\to(1,0)} \frac{xy - y}{(x-1)^2 + y^2}$

Exercise 3. Use the squeeze theorem to compute the limit $\displaystyle\lim_{(x,y)\to(0,0)} xy\sin\left(\frac{1}{x^2+y^2}\right)$.

Exercise 4. Consider the function $f(x,y,z) = x^2y^3z^4 + \sin(x^2 + 5y)$.
Compute $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial f}{\partial y}$, $\dfrac{\partial f}{\partial z}$, and $\dfrac{\partial^2 f}{\partial z \partial x}$.

Exercise 5. Let $z = x^4 + x^2y$, $x = s + 2t - u$, and $y = stu^2$.
(a) Draw the tree diagram for this problem.
(b) Using your tree diagram, compute $\dfrac{\partial z}{\partial t}$ at the point when $(s,t,u) = (4,2,1)$.

Exercise 6. Consider the equation $\tan(x^2z^2) = \cosh(y^2x) + 2\ln z$. Compute $\dfrac{\partial z}{\partial x}$.

Exercise 7. The elevation at a point (x,y) on a mountain is given by $h(x,y)$. A person is on a hike such that their position after t minutes is given by $x = \sqrt{4t+9}$, $y = \sin(t\pi/8)$, where x and y are measured in meters. The elevation equation satisfies $h_x(5,1) = 0.5$ and $h_y(5,1) = 0.1$. How fast is the elevation increasing on the hiker's path after 4 minutes?