Equation of a plane

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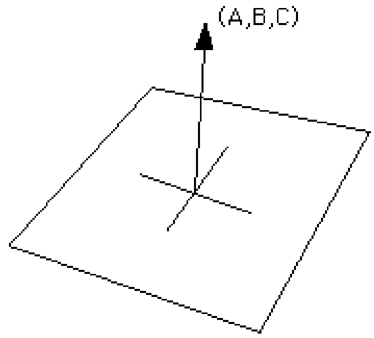

The standard equation of a plane in 3 space is

$$Ax + By + Cz + D = 0$$

The normal to the plane is the vector (A,B,C).

Given three points in space (x1,y1,z1), (x2,y2,z2), (x3,y3,z3) the equation of the plane through these points is given by the following determinants.

$$A = \begin{vmatrix} 1 & y1 & z1 \\ 1 & y2 & z2 \\ 1 & y3 & z3 \end{vmatrix} \quad B = \begin{vmatrix} x1 & 1 & z1 \\ x2 & 1 & z2 \\ x3 & 1 & z3 \end{vmatrix} \quad C = \begin{vmatrix} x1 & y1 & 1 \\ x2 & y2 & 1 \\ x3 & y3 & 1 \end{vmatrix} \quad D = - \begin{vmatrix} x1 & y1 & z1 \\ x2 & y2 & z2 \\ x3 & y3 & z3 \end{vmatrix}$$

Expanding the above gives
A = y1 (z2 - z3) + y2 (z3 - z1) + y3 (z1 - z2)
B = z1 (x2 - x3) + z2 (x3 - x1) + z3 (x1 - x2)
C = x1 (y2 - y3) + x2 (y3 - y1) + x3 (y1 - y2)
- D = x1 (y2 z3 - y3 z2) + x2 (y3 z1 - y1 z3) + x3 (y1 z2 - y2 z1)

Note that if the points are colinear then the normal (A,B,C) as calculated above will be (0,0,0).

The sign of s = Ax + By + Cz + D determines which side the point (x,y,z) lies with respect to the plane. If s > 0 then the point lies on the same side as the normal (A,B,C). If s < 0 then it lies on the opposite side, if s = 0 then the point (x,y,z) lies on the plane.

Alternatively

If vector **N** is the normal to the plane then all points **p** on the plane satisfy the following

N . **p** = k

where . is the dot product between the two vectors.

ie: **a** . **b** = $(a_x,a_y,a_z) . (b_x,b_y,b_z) = a_x b_x + a_y b_y + a_z b_z$

Given any point **a** on the plane

N . (**p** - **a**) = 0