

Abstract

[Place holder abdtract]

Contents

1	Introduction	2
2	Background	4
2.1	Network terminology and glossary	4
2.2	Network	5
2.2.1	Social network	6
2.3	Network properties	6
2.3.1	The small world effect	7
2.3.2	transitivity/clustering	7
2.3.3	Degree distribution	7
2.3.4	Degree correlations	8
2.3.5	Network resilience	8
2.3.6	community structure	9
2.4	Breadth First search	10
2.5	Data diffusion	10
2.6	Basic Diffusion Models	11
2.7	Linear threshold model	11
2.7.1	Independent cascade model	11
2.8	Matrix notations	12
2.8.1	Sparse Matrix	13
2.8.2	Breadth First Search as a matrix multiplication.	13
2.8.3	Semiring	14
2.9	BFS to data diffusion	14
2.10	Seed selection algorithm	14
2.10.1	the greedy algorithm	14
2.10.2	The degree algorithm	15
2.10.3	independent algorithm	15
2.10.4	random algorithm	16
2.11	Cache oblivious model	16
3	Related works	17

4	Method	18
4.1	PyC _x	18
4.2	R-mat	18
4.3	Adjacency matrices to graphs	20
4.4	The algorithm	20
5	Result	22
6	Discussion	26
7	Future work	27
8	Conclusion	28

List of Figures

2.1	Simple network	5
2.2	Citation network(Information network)	6
2.3	Examples of triandles in networks	8
2.4	A example of a typical degree distribution	9
2.5	Linear Threshold mode	12
2.6	Independent cascade model	12
2.7	Sparse matrix to graph	13
4.1	The R-mat model	19
4.2	How the adjacency matrix is flipped on the diagonal	20
5.1	The degree distribution of the graph that was used for the simulation	22
5.2	The result from the small graph	23
5.3	The result from the medium graph	24
5.4	The result from the large graph	25

List of Tables

5.1	Result from small graph	23
5.2	Result from medium graph	24
5.3	result from large graph	25

Chapter 1

Introduction

Using graphs as representation of real world network and graph traversal algorithms to iterate over graphs are seen in different scientific domains[1]. Graph simulation have been used to simulate how a disease would spread through a social network[2], predict how efficient viral marketing is and how a new trend would spread through a community[3]. Seeing how some data propagates through a network is known as information diffusion. We can compare the diffusion as a company trying to promote a new product to the public, the public would be the network and each node can either adopt the new trend, or ignore it. One important aspect of information diffusion is the starter set, or the seed nodes. The seed node is the initial starter nodes that are activated, much like a person receiving a free sample from a company trying to promote a new product. By giving someone such a free product it can help promote the product, but choosing which person to give the sample to is important. Choosing someone with few friends and few connection would be a waste of product, giving too much would hurt the company's profit and bit giving would result in not enough exposure.

To select the best seed nodes is an NP-hard problem[3]. One of the solutions is the greedy algorithm[4] proposed by Kempe et al. The greedy algorithm goes through all the nodes and computes the effect on the network that nodes had. The greedy algorithm takes the k top most influential nodes as the starter node. When a new seed node is to be added to the starter set S , the algorithm iterates over all the nodes in the network that are not in S and compute the spread.

This project is a collaboration with the Arctic Green Computing Group. The task assigned was: "Information diffusion is a field of network research where a message, starting at a set of seed nodes, is propagated through the edges in a graph according to a simple model. Simulations are used to measure the coverage and speed of the diffusion and are useful in modelling a variety of phenomena such as the spread of disease, memes on the Internet, viral marketing and emergency messages in disaster scenarios.

The effectiveness of a given spreading model is dependent on the initially infected nodes, or seeds. Seed selection for an optimal spread is an NP hard

problem and is normally approximated by selecting high-degree nodes or using heuristic methods such as discount-degree or choosing nodes at different levels of the k-core.

This project will explore the feasibility of hardware accelerated seed selection in large graphs as a variation of breadth-first search (BFS) where the decision to visit and infect a child node relies on the outcome of a coin flip. The student is expected to conduct a thorough review of the literature on seed selection algorithms, diffusion models, with emphasis on parallelisation and hardware acceleration and cache models to reduce memory bottlenecks. Hardware design and simulations are possible if time permits.”

I have chosen to interpret the task as to understand the fundamental concepts of graph theory, the core concept of information diffusion and seed selection. I have chosen to focus on one specific model of information diffusion which is the independent cascade model, which will be further discussed in section [BACKGROUND]. I have been looking at different proposed solutions that might be able to improve the independent cascade model and the seed selection.

I have in this report performed a similar experiment as the experiment done in [?] with the greedy algorithm, the high degree algorithm, the random algorithm and a modified version of the greedy algorithm we are calling independent greedy algorithm in [METHODE]. We will go through some of the notation commonly used in graph theory in [Background] and we will look at how we can perform standard graph algorithms such as BFS as matrix-vector multiplication. We will propose some idea that could potentially improve the seed selection algorithm, the independent cascade model or the diffusion of information.

Chapter 2

Background

In this chapter, we will look at the fundamental concepts and background information for the different diffusion model, the seed selection problem and performing breadth-first search using matrix multiplication. This chapter will contain notations that we would use throughout the report. One aspect we will focus on, is how we can perform graph algorithm such as breadth first search as matrix multiplication. The motivation for transforming breadth first search as matrix-vector multiplication is that displaying the graph algorithm as a matrix multiplication can display the data access pattern for the algorithm and can be readily optimized[5]. We will look at the independent cascade model, which is a special case of breadth first search[1]. By looking at how to improve BFS, we can apply such optimization to ICM and the seed selection algorithm.

2.1 Network terminology and glossary

The fundamental unit in a network is *Vertex*(*pl.vertices*), sometimes called a node. For this report, both vertex and node will be used. The "bridge" or the line connecting two vertices is called a *Edge*, which serves as a connection between vertices as shown in Figure 2.1. Different network have different types of edge, some is *Directed*, while others is *undirected*. Directed edge is an edge that runs in only one direction (such as a one-way road between two points), and undirected runs in both directions. Directed edges can be thought of as sporting arrows indicating their orientation, while undirected edges have no such orientation. A graph is directed if all of its edges are directed.

Each node have a value that is called *Degree*. Degree for a vertex v_1 is the amount of edges connected to v_1 . Note that the degree is not necessarily equal to the number of vertices adjacent to a vertex, since there may be more than one edge between any two vertices. A directed graph has both an in-degree and an out-degree for each vertex, which are the numbers of in-coming and out-going edges respectively. The *Component* to which a vertex belongs is that set of vertices that can be reached from it by paths running along edges of the graph.

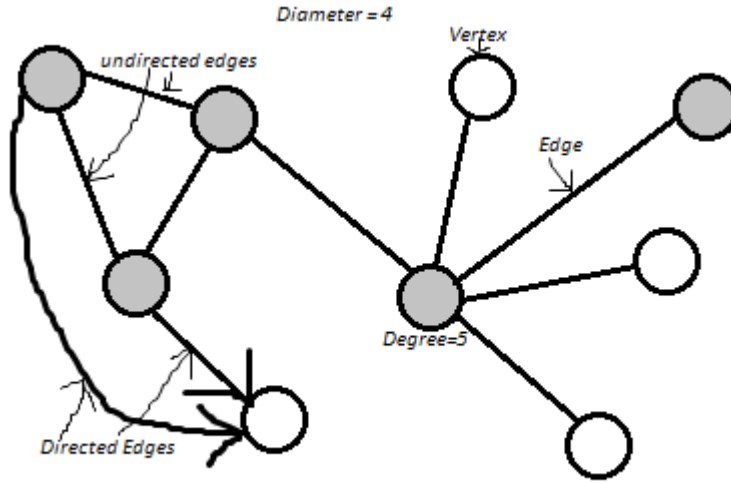


Figure 2.1: Simple network

In a directed graph a vertex has both an in-component and an out-component, which are the sets of vertices from which the vertex can be reached and which can be reached from it.

A *Geodesic path* is the shortest path through the network from one vertex to another. Note that there may be and often is more than one geodesic path between two vertices. The *Diameter* of a network, however, is the length (in number of edges) of the longest geodesic path between any two vertices. A few authors have also used this term to mean the average geodesic distance in a graph, although strictly the two quantities are quite distinct

2.2 Network

A *network* is a collection of vertices and edges[6]. The edges serves as a connection, or a "bridge" between the nodes, while the nodes can represent something, or containing information. In the real world, multiple systems takes the form of networks around the world, examples are the internet, the World Wide Web, social media like Facebook, twitter etc. There are different types of network. These include from *social network*, *information networks*, *technological networks*, and *biological network*. Different network have different properties, we will look at those most relevant to social networks.

2.2.1 Social network

A social network is a set of individuals connected to each other via some form of contact or interactions[6]. The nodes are people while the edges are the connections between peoples. The social network display information regarding connection, interaction or location of a set of people. It forms patterns regarding friendships, business interactions between companies and families history. The social network is often used in social science[6]. Some notable experiments are [?], which we will discuss more in ??

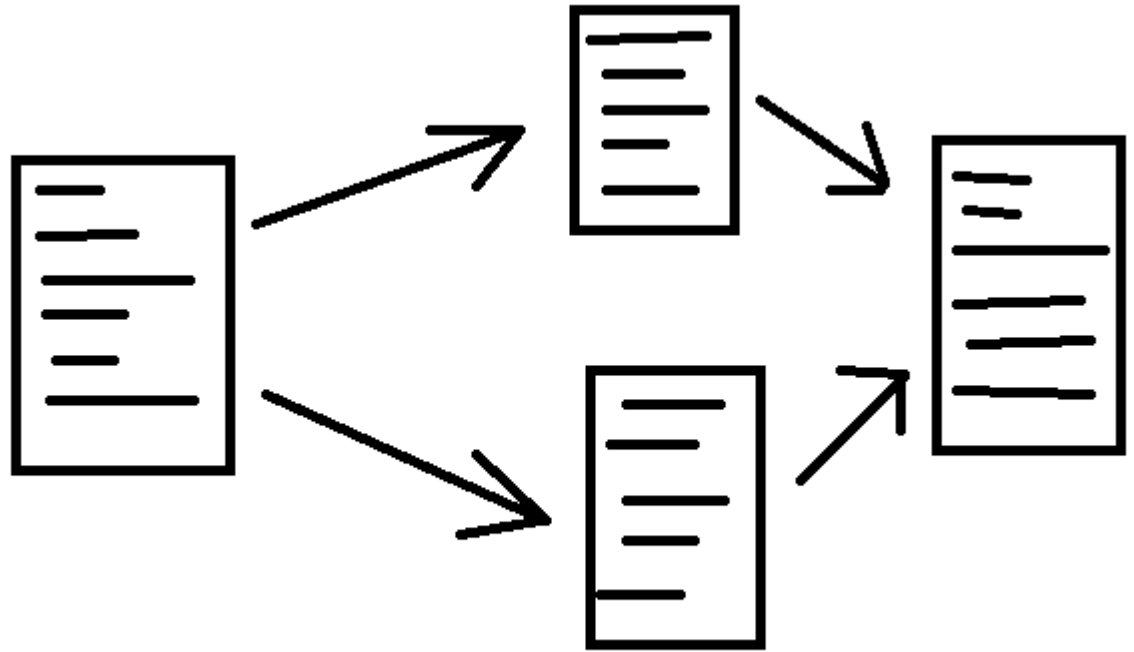


Figure 2.2: Citation network(Information network)

2.3 Network properties

Network properties are different characteristic properties that network displays. We will focus on those that are more relevant to social network and how it is relevant to the ICM, data diffusion and seed selection.

2.3.1 The small world effect

The small world effect was first demonstrated by Stanley Milgram in the 1960s during his famous letter passing experiment[7]. The experiment was about passing letters from person to person to reach a designated target with only small steps. For the published case, the chain was around six [8], meaning there were only six passes necessary for the letter to reach its destination. This shows us that for most pair of vertices in a network can reach each other with a short path. A more precise wording is that "Networks are said to show the small world effect if the value of l scales logarithmic or slower with the network size for a fixed mean degree." [6]. We have defined l to be the mean geodesic distance between vertex pairs in a network.

The small world problem can be summarized as: "what is the probability that any two people, selected arbitrarily from a large population, such as that of the United States, will know each other?". [9]. This in itself is not that interesting, the [?] asked even tho person a and z does not know each other, do they have a set of individuals $\{b_1, b_2, \dots, b_n\}$ who are mutual friend or even a "chain" of such individual($a - b - c - \dots - y - z$).

For the data diffusion problem, this kind of effect would result in that the diffusion through a network would need around 6 steps to have traveled through the entire network. Meaning that most node can reach each other through a relatively small step.

2.3.2 transitivity/clustering

In graphs and networks, there would often have a special connection pattern called *triangles*. Triangles is where three vertices: v_a, v_b, v_c are all connected to each other as shown in Figure:2.3. We can look at such a connection as person A is friends with B and C. There are a chance that B and C is friends with each other too. Transitivity is used to determine how many such components are present in the graph.

For data diffusion and seed selection, this would mean that picking nodes that are neighbor to each other, would have a smaller spread than picking two not neighboring vertices. By picking two nodes that are connected to each other, they would likely share common neighbors and thus having a smaller reach.

2.3.3 Degree distribution

As mentioned earlier, each node has a degree. The degree distribution shows how the different degrees in the network are distributed. From the degree distribution we can see how many nodes have specific degree. Different networks have different shaped degree distribution.

For random graphs, the distribution would most likely be a poisson distribution or binomial. For social graph, the degree distribution is often in the shape of Figure 2.4. For the information diffusion, this distribution shows us how many high degree nodes there are in the network, those nodes would likely

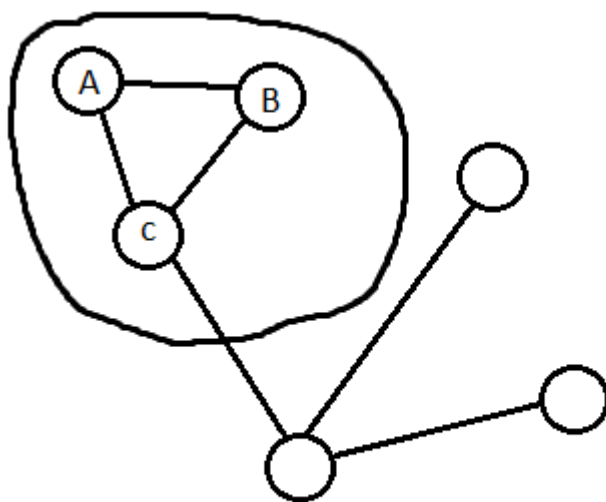


Figure 2.3: Examples of triandles in networks

be high prioritized node and have a large impact. One of the algorithm uses the distribution to select the seed nodes. We will discuss this in later sections.

2.3.4 Degree correlations

As 5.1 show, the network have a degree distribution with few high degree nodes and many low degree nodes. One interesting properties is the degree correlations. Degree correlations is how the high degree and low degree nodes connects to each other. One question is if high degree nodes tends to connect to other high degree nodes, or do they prefer to connect to low degree nodes. It turns out that both incidents is found in networks[?]. Most social networks are assortative, meaning vertices have a selective linking, where high degree vertex tends to connects to other high degree vertex, while technological network and biological network are most likely disassortative[?].

For data diffusion, this kind of behavior would result in wasting a seed by picking majority of high degree node for starting seed, since most of them would be connected to each other. One solution is by mixing the selection, choose some percentage to be high degree, and some with lower degree.

2.3.5 Network resilience

Most network model, there is the need to remove nodes from the network. Removal of a node can have no effect on the network, or it can be devastating.



Figure 2.4: A example of a typical degree distribution

Network resilience looks at how the network can resist to such a removal. There are two different removal schemes, the random removal where nodes are randomly picked and removed, or the targeted removal where specific nodes are removed depending on the criteria.

The experiment mentioned in [?] by Albert *et.al* showed that for a subset of network representing the internet and the world wide web, targeted removal had a larger impact than random removal. The targeted removal removed the highest degree nodes from the network, and the random removal removed nodes randomly. The random removal had a minimal effect on the network, while the targeted removal had a much larger impact, the mean vertex-vertex distance increased. They proposed that the internet was highly resilient to random removal, while much more vulnerable to targeted removal.

There are other studies that proposed a different interpretation about the data found.

In an example of information diffusion, a targeted removal would result in massive change to the diffusion path. By removing high degree nodes would result in removing influential nodes and limit the spread of the data. Removing important node connections two different communities would result in isolation and no path towards other communities.

2.3.6 community structure

One property that is often observed in a social network, is the community structure. The community structure is where a group of vertices having high

density of edges with each other, while having low density of edges to other "community". We can see an example of the community structure clearly displayed from [?]. Where we can see the playground was divided into different groups.

This type of community structure would have a large impact on how the algorithm would select a seed for information diffusion. If all the seed would be selected in one community, the probability of spreading over to other community would be smaller, then having a seed be in the other community.

2.4 Breadth First search

Breadth first search is an tree traversal algorithm. BFS start at the root node v_r . The algorithm then stores all v_r 's children node in an *queue*. The algorithm then takes the first node from the queue, v_1 and stores all the children node to v_1 in the back of the queue, this process continuous until the queue is empty and all the nodes have been iterated over.

Breadth first search is common graph iteration algorithm. The breadth first search is often limited by the irregular memory access where the algorithm have too find the data stored in different space in the memory. As mentioned earlier, Independent cascade model is one special case of the Breadth first search.

Algorithm 1 Breadth First Search

```

1:  $dist[\forall v \in V] = -1; currentQ, nextQ = \emptyset$ 
2:  $step = 0; dist[root] = step$ 
3: ENQUEUE( $nextQ, root$ )
4: while  $nextQ \neq \emptyset$  do
5:    $currentQ = nextQ; nextQ = \emptyset$ 
6:    $step = step + 1$ 
7:   while  $currentQ \neq \emptyset$  do
8:      $u = DEQUEUE(currentQ)$ 
9:     for  $v \in Adj[u]$  do
10:      if  $dist[v] == -1$  then
11:         $dist[v] = step$ 
12:        ENQUEUE( $nextQ, v$ )
return  $dist$ 

```

2.5 Data diffusion

Data diffusion is looking at how information is propagated through a network or a graph. An example would be how a new Internet meme, a new product or how a new disease is spread through a community. The process consist of a set of starter nodes, which we will call seed nodes, that are "infected". During each time-step, there are a percentage p_g that the "infected" nodes

would "infect" its neighbors. We would need to first pick out a set of initial starter node. These k seed nodes is a set of k nodes that in the initial time-step is infected. They will pass on the information/infection during each time-step and the information/infection will propagate through the network.

2.6 Basic Diffusion Models

When we talk about data diffusion, we can look at how disease or technological innovations is spread through a social network. We can simulate those kind of behavior with different diffusion models. There are two basic diffusion models used to simulate the propagation of information through a network[3], the *linear threshold model*(LTM) and the *independent cascade model*(ICM)[3].

This process is similar to how a new product is promoted via facebook. Each node is a person, that person can buy the new product(activated), or ignore the new product(inactive). Each person will see their friend promote the new product and potentially buy the product. There are several different criteria for each person to buy the product(activates). They can have a percentage chance to be affected by the advertisement(ICM), or they will only be interested if a percentage of his or her friends have promoted it(LTM). Some people might have a bigger friend circle than other(high degree), while other have bigger impact on a person(large p_x). Some might be harder to promote too(weighted edges), while some user have no friend(singletons). The different model we will focus on, is the independent cascade model and the linear threshold model.

2.7 Linear threshold model

The linear threshold model uses a threshold θ_v between the interval $[0,1]$, which represent the fraction of v 's neighbors that need to be active to activate node v . The Linear Threshold Model activates the current node v when the weight $b_{v,w} > \theta$ from its neighbor w outweighs the θ_v . This is more in line with the situation where each person has a chance to adopt to a new trend if exposed to the trend enough time by his close friends. An example would be a product promoted on social network like twitter and Facebook. The user will adopt the new trend if he is exposed to it from enough friends or idols. As shown in Figure 2.5a, the threshold for vertex v to be activated is θ_v , a percentage of the total neighbour, in this instance, three.

We can look at the linear threshold model as the

2.7.1 Independent cascade model

The independent cascade model changes states with a probability $p_{v,w}$, where v is current node, and w is its neighbor. During each propagation, the node v has a $p_{v,w}$ chance to change state if the neighbor w changed state. If during time-step t a node v changed state, its neighbor w would have a $p_{v,w}$ chance to change state in the next time step. An example here would be spread of a

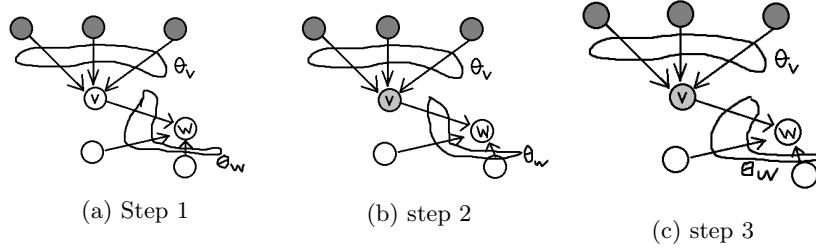


Figure 2.5: Linear Threshold mode

disease. The current node v have a chance (θ_v) to activate its neighbor. As shown in Figure 2.6a, there is an probability of p_v to infect the neighbors to vertex v , in this figure, the probability is the same, bit that is not necessary true. the probability can be independent, each vertex can have different probability to activate neighbor, or we can have a global probability.

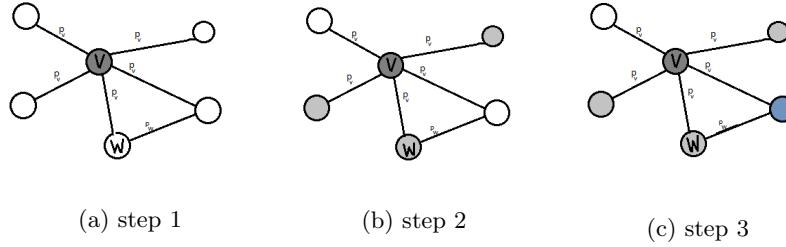


Figure 2.6: Independent cascade model

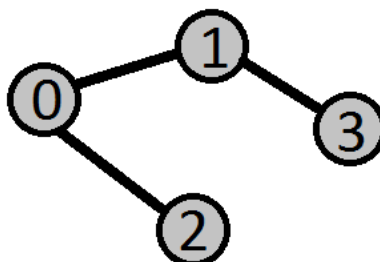
We can see the similarity between the breadth first search and information diffusion. Breath first search, as mentioned before, starts at the root node and add the root nodes children in a queue, then takes the first child node in the queue and adds all its children to the queue. Each new child node is added to the back of the queue, while finished node is placed in a *finished_queue*. The sequences of nodes tested is the same as during an information diffusion, where the diffusion neighbor is placed in the *boarder_queue* and having a probability to be infected. If the node is infected, then that nodes neighbor would be added to the *boarder_queue*, while if the node is not infected, the neighbor to the node would be safe. The information diffusion can therefore be looked at as an breadth first search, with an probability to add the child to the queue.

2.8 Matrix notations

By using a linear algebraic approach to solve graph algorithms often gives us a variety of benefit include easier implementation, higher performance and syntactic simplicity[?]. There are multiple reasons to do graph algorithms as linear



(a) The adjacency matrix



(b) The graph corresponding to the adjacency matrix

Figure 2.7: Sparse matrix to graph

algebra. This can show potentially improvement, potentially optimization and be more visually clear how the algorithm is. Another reason is that matrix-vector multiplication is a clear illustration and unlike pointer chasing, easy to optimize.

2.8.1 Sparse Matrix

One way to represent a graph in matrix format, is by adjacency matrix, We can represent a social graph with few connection in the form of a sparse adjacency matrix. A sparse matrix is a matrix with only few non-zero element. In an adjacency matrix, a cell containing 1 means a connection is presence, while a 0 is lack no connection. So, as we can see in the Figure 2.7a, as we can see at coordinate $[1,3] = 1$. This tells us that there is a edge between vertex 1 and vertex 3. The adjacency matrix is commonly used to represent an graph such as Figure 2.7a and Figure 2.7b.

2.8.2 Breadth First Search as a matrix multiplication.

By looking at the adjacency matrix, we can see that the matrix is a graph represented in a matrix format. The BFS can be achieved by looking at the BFS operations as a matrix multiplication [?]. If we look at the graph as a connectivity matrix, we can apply matrix multiplication to generate the breadth first search.

We can also look at how an breadth first search can be executed by applying sparse matrix multiplication. We can see that the adjacency matrix is an sparse matrix. We can use sparse matrix multiplication to do graph algorithm, one example is the BFS.

2.8.3 Semiring

A *semiring* is a set of elements with two binary operations. The two operations are often known as "addition" and "multiplication". By this definition, [SYMBOL FOR BOOLEAN] and [SYMBOL FOR INTEGER]. are both semirings. One way to perform graph algorithm, is apply matrix multiplication over such semirings. One of the common notation in writing the semiring, is $a+b$ and $a*b$. This can be confusion, so often there will be some special semiring operator.

2.9 BFS to data diffusion

We can draw an distinctive line between the breadth first search and data diffusion. The breadth first search adds all the child node from the parent node to a queue, then change the current node to the first node in the queue and repeats until the entire graph is iterated over, or the queue is empty. This is in theory the same as data diffusion, where information is passed to the connected vertex. The new "infected node" can then pass on the information along to the other nodes that are connected to the new node and so on. This is in practice, the same as breadth first search, minus the searching part.

We can in then draw the conclusion that Independent cascade model, is a modified version of the breadth first search, where each child note have a specific percent to be infected. The iteration is the same as a breadth first approach, but the result of the addition of node is dependent on a random "coin-toss".

2.10 Seed selection algorithm

The seed selection algorithm, is the algorithm used to select the initial k seed nodes to be choosen at the start. We can compare it to a new technological product or a cosmetic company trying to promote a new product. By selecting a few influential persons to give a free sample. the new trend would most likely spread and catch on. The seed selection algorithm would be the algorithm to select the few influentnial individuals. There are multiple different scheme to choose from, in this section, we will focus on four different, greedy algorithm, degree algorithm, random algorithm and the indipendent greedy algorithm.

2.10.1 the greedy algorithm

The greedy algorithm was proposed by Kempe [4]. The algorithm is a greedy algorithm where it finds the starter set of nodes S by iterating through all the vertex in V and calculate the total amount of spread. The spread is saved and the k most influential nodes would be choose.

Algorithm 2 Greedy Algorithm

- 1: Start with $A = \emptyset$
 - 2: **while** $|A| \leq l$ **do**
 - 3: For each node x , use repeated sampling to approximate $\sigma(A \cup x)$ to within $(1 \pm \varepsilon)$ with probability 1δ
 - 4: Add the node with largest estimate for $\sigma(A \cup x)$ to A .
 - 5: Output the set A of nodes.
-

2.10.2 The degree algorithm

Another popular algorithm is the degree algorithm[4]. Unlike the greedy algorithm, the degree sort all the node according to their degree distribution. The algorithm picks the top k nodes according to the degree distribution. This approach does not take the degree correlation into account, by picking only high degree node, there would be multiple overlapping activated node.

Algorithm 3 Degree Algorithm

- 1: Start with $A = \emptyset$
 - 2: **while** $|A| \leq l$ **do**
 - 3: For each node x , use repeated sampling to compute $\text{DegreeMax}(x)$.
 - 4: Add the node with largest degree to A .
 - 5: Output the set A of nodes.
-

2.10.3 independent algorithm

Another algorithm is the independent greedy algorithm. The algorithm iterates through the network, computing the spread of each node. The algorithm then chooses the vertex with the largest coverage independent of the other previous chosen nodes. This algorithm is a special case of the greedy algorithm mentioned above.

Algorithm 4 Independent Algorithm

- 1: Start with $A = \emptyset$
 - 2: **while** $|A| \leq l$ **do**
 - 3: For each node x , use repeated sampling to approximate $\sigma(A \cup x)$ to within $(1 \pm \varepsilon)$ with probability 1δ
 - 4: Add the node with largest estimate for $\sigma(x)$ to A .
 - 5: Output the set A of nodes.
-

2.10.4 random algorithm

The last one is the random algorithm. The random algorithm just pick a random seed node. This approach is the simplest to implement and easiest. The downside is that this is random and there are no strategical choosing of seed node.

2.11 Cache oblivious model

A cache oblivious model ignores the cache size and line and designs the algorithm to be cross platform and optimized. An algorithm is *cacheaware* if it contains parameter that can be tuned to optimize the cache complexity for the cache size[?]. Such algorithm have the disadvantage where a adaptation is needed for a new architecture[?]. The cache oblivious model is designed for two level of memory, and assumes that such an optimization is optimized for multiple levels as well. There are multiple algorithm designed that shows that such a model actually improves the algorithm. One of such is an cache oblivious sparse matrix multiplication. Sparse matrix multiplication is notorious to be inefficient use of the cache system, it forces the user to jump through data in main memory[?]. The sparse matrix approach proposed [?] focus on reordering the matrix and row in an cache oblivious manner. There are results that shows that a cache oblivious approach have given improvement to computation time for some matrices during sparse matrix multiplication, but for cache-friendly structure, this is shown to be a minor improvement and even small loss.

Chapter 3

Related works

related work, there are multiple people working on this kind of problem. One of the more well known is Kemnp, where he tries to solve how to maximize the spread.

Other solution include the hybrid FPGA-CPU architecture, which reduces the computation time for a BFS over a small world graph by XX%[1]

As we can see, there are many different research around this subjects, even tho there aren't many that looks at how to accelerate seed selection and the Independent cascade model in hardware, which we will look more at in the end of this report.

—————am looking into the different related works from the hybrid BFS paper from Yaman.

Chapter 4

Method

For this report, we created a python simulation to simulate the data diffusion and the seed selection. We utilized the Pycx libraries to create the GUI, and applied the R-mat generator to create the different network. Our diffusion simulation finds k seed node, and for each k , runs the simulation 50 times. Each run runs until there are no more vertex to be activated. Since each node can only try to activate their neighbors once, regardless of the result, that node can't try to activate the same neighbors again. For our simulation, when no more nodes can be tested, the simulation stops.

4.1 PyCx

PyCx an libraries that help python to generate an GUI[?]. The pycx have a clear structure, initialize, observe and update. The initialize part, the graph is generated, the starter seed is found and the position to the graph is generated. The observe part is where python generate graphic for our simulation. for each step, the observe is called to generate a new frame. the update section i called every step, for our program, the diffusion is calculated as each step we can see how the data is diffused. The simulation allow the information and data to be displayed

4.2 R-mat

One problem during graph analyzation and calculation is finding suitable graphs to analyses. Generate graphs with desired properties is not easy to do. One solution proposed by Chakrabartiy *etal* is to use the "recursive matrix" or R-mat model. The R-mat model generates graph with only a few parameters, the generated graph will naturally have the small world properties and follows the laws of normal graphs, and have a quick generation speed[10]. The R-mat models goal is to generate graphs that matches the degree distribution,

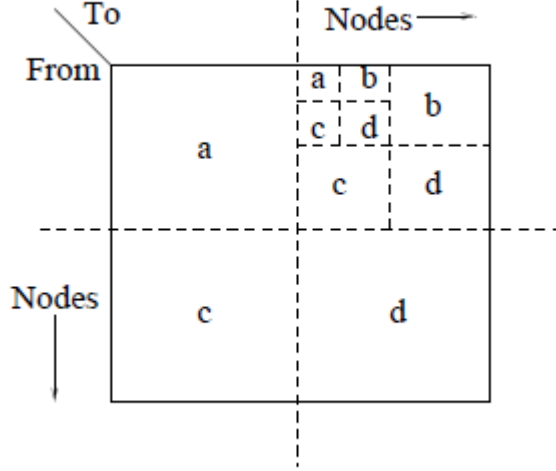


Figure 4.1: The R-mat model

exhibits a "community" structure and have a small diameter and matches other criteria.[10].

The R-mat generator generates social network with the community structure. The different probability for the four partition is : $A = 0.57, B = 0.19, C = 0.19, D = 1 - A - B - C = 0.05$. These different probability was used by GRAPH500[CITATION NEEDED]. The R-mat generated three different adjacency matrices of different size.

The algorithm to generate such a recursive matrix is as follow: The idea is to partition the adjacency matrix into four equally sized part branded A,B,C,D, like shown in Figure4.2. The adjacency matrix starts by having all element set to 0. Each new edge is "dropped" onto the adjacency matrix. Which section the edge would be placed in, is chosen randomly. Each section have a probability of a, b, c, d , and $a + b + c + d = 1$. After a section is chosen, the partition that was chosen is partitioned again. This continuous until the chosen section is a 1x1 square and the edge is dropped there.

From the algorithm, we can see that the R-mat generator is capable to generate graphs with total numbers of node $V = 2^x$. Since the algorithm partitioned the matrix into four part. This is approach would only generate a directed graph. To generate undirected graph, $b = c$ and the adjacency matrix must make a "copy flip" on the diagonal elements, like Figure 4.2.

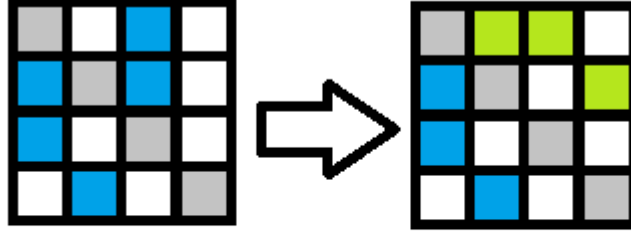


Figure 4.2: How the adjacency matrix is flipped on the diagonal

4.3 Adjacency matrices to graphs

For this report, three different sized adjacency matrix was created. One of the restriction to the R-mat generator is that the size of the adjacency matrix have to be 2^n . This resulted in that our adjacency matrix was originally of the size, 128×128 , 512×512 and 1024×1024 . The resulted graphs had multiple singletons and unconnected nodes, for the sake of the simulation, those nodes were discarded and resulted in a graph with 75 nodes and 307 edges, 287 nodes with 2415 edges and 617 nodes and 8374 edges. This was not surprising consider that for a larger graph, those singletons would most likely result in the outskirts and small community.

4.4 The algorithm

The simulation creates an social network by reading from the adjacency matrix. We implemented 4 different algorithm. The greedy algorithm, the degree algorithm, random algorithm, and a independent greedy algorithm. The algorithm implemented, finds the k vertices to be the starter nodes. k is $[1, 2, \dots, 20]$. This is to be able to see how the size of the starting nodes affect the coverage. For each k , the simulation applies the diffusion for 50 times. This is to find the mean coverage to remove the randomness

The greedy algorithms finds the most influential nodes in relation with the other previous picked seeds. The algorithm starts by finding the most influential nodes s_1 from the entire graph G , in this instance, $k=1$. The most influential node is found by just choosing a node and see how much spread it will result in, the value is stored with the node, and after the entire G is iterated over, the node with the highest value is chosen. Then the algorithm stores the node in S and applies data diffusion and stores the effect this runs had. The next run, $k=2$

and the greedy algorithm finds the most influential node s_2 where $s_2 \neq s_1$ and $s_2 + s_1 = \text{maxCoverage}$. This is repeated until $k=20$. The new run, the seed selection will keep the previous selected seed and during the finding maximum coverage phase, the previous chosen seed will have impact on the run.

The degree algorithm chooses the vertex with the highest degree. Unlike the greedy algorithm. The degree algorithm just finds the vertices with the highest degree. The algorithm chooses s_1, s_2, \dots, s_k from G that have the highest degree. One of the problem would be that higher degree nodes would often be connected to each other, the community structure that was mentioned in previous section. The degree histogram shows us that there are very few high degree node, while having more low degree nodes.

The random algorithm picks random vertices as the starter nodes. This is the simplest algorithm, where each runs, a new random node is added to the set S . Then the diffusion is applied.

One experimental algorithm we tested was the independent greedy algorithm, where the algorithm choose the largest impact node as a seed node. This algorithm is like the greedy algorithm, with one major difference, the algorithm chooses a new seed node for each run. The algorithm picks the k seed nodes each independently to the previous picked node. This results in that the chosen nodes would often result in

Chapter 5

Result

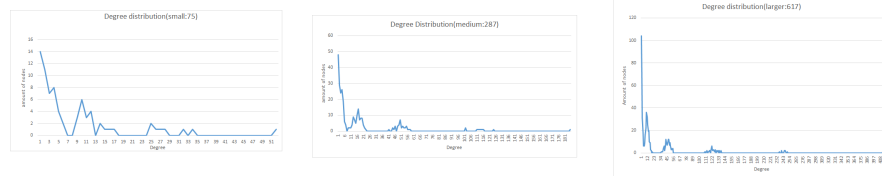
The result from the multiple runs we have gotten is as shown in the different figures. Figure 5.2 shows the spread and coverage from the simulation of the small graph. As we can see, the four different algorithm had little difference. The y-akses shows us the average coverage after 50 runs, and the x-akses gives us how many seed node was selected. We can see from figure 5.2, figure 5.3 and figure ?? that the resulted coverage was around 0.3-0.45. The smaller graph would result in a smaller spread, while larger graph results in large spread. Figure 5.2 have a more linear increase as the seed nodes increased. Figure 5.3 and Figure ?? on the other hand, had an massive increase in coverage after the addition of the second seed node, then the increase slowed down drastically.

The network that we ran our simulation on, resulted in a degree distribution as Figure 5.1a, Figure 5.1b, and Figure 5.1c. The degree distribution is similar to the 2.4. We can see that there are one vertex with high degree, while the majority of the nodes have a small degree.

We can see that the greedy algorithm performed better then the other algorithm for the large and medium graph, except the random algorithm for the large graph, where it was best.

For each of the

Our result as a table:



(a) Degree distribution of the small graph.

(b) Degree distribution of the medium graph.

(c) Degree distribution of the large graph.

Figure 5.1: The degree distribution of the graph that was used for the simulation

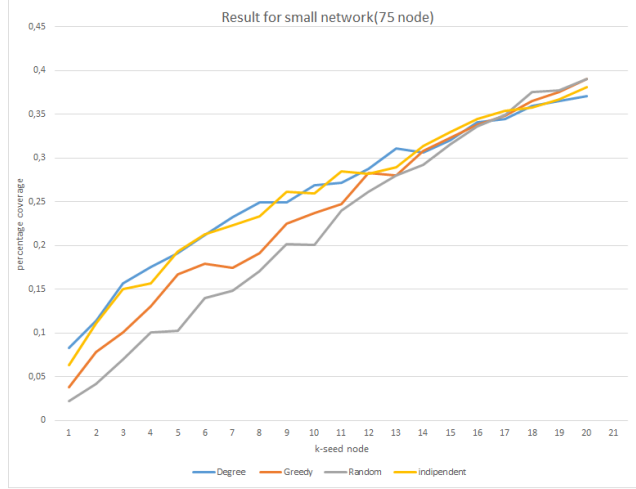


Figure 5.2: The result from the small graph

k	Degree	Greedy	Random	Indipendent
1	0.0832	0.0381333333333	0.0224	0.634666666667
2	0.114133333333	0.0778666666667	0.0413333333333	0.110933333333
3	0.157066666667	0.100533333333	0.0696	0.149866666667
4	0.175733333333	0.130666666667	0.100266666667	0.156266666667
5	0.190933333333	0.166666666667	0.102933333333	0.193066666667
6	0.212	0.178933333333	0.139466666667	0.212266666667
7	0.232533333333	0.174666666667	0.148533333333	0.2232
8	0.2488	0.1912	0.1704	0.233333333333
9	0.249066666667	0.225066666667	0.201333333333	0.261066666667
10	0.2688	0.237333333333	0.200266666667	0.2592
11	0.271466666667	0.246933333333	0.239466666667	0.285066666667
12	0.287733333333	0.2832	0.261066666667	0.281866666667
13	0.310933333333	0.280266666667	0.279733333333	0.2896
14	0.306133333333	0.308	0.292533333333	0.313866666667
15	0.320266666667	0.322666666667	0.3152	0.329066666667
16	0.3408	0.338133333333	0.336	0.3448
17	0.344533333333	0.348266666667	0.3496	0.354133333333
18	0.359733333333	0.348266666667	0.375466666667	0.3576
19	0.3648	0.375733333333	0.377333333333	0.366933333333
20	0.370933333333	0.389866666667	0.3904	0.381333333333

Table 5.1: Result from small graph

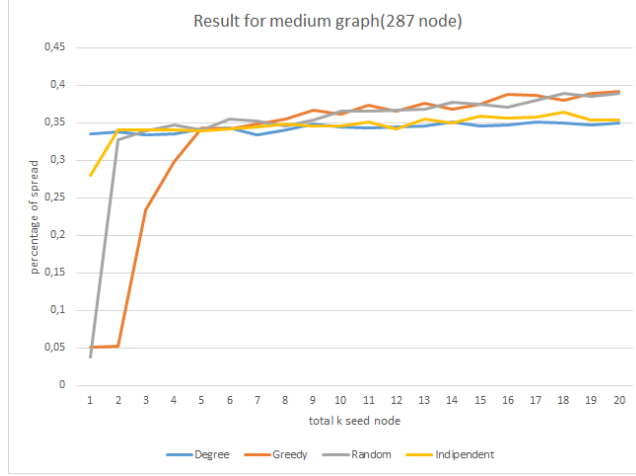


Figure 5.3: The result from the medium graph

k	Degree	Greedy	Random	Indipendent
1	0.334773519164	0.0506620209059	0.0384668989547	0.279790940767
2	0.337421602787	0.0529616724739	0.327456445993	0.340069686411
3	0.334216027875	0.23393728223	0.339442508711	0.340975609756
4	0.335400696864	0.298048780488	0.34668989547	0.341324041812
5	0.341742160279	0.342926829268	0.340557491289	0.339930313589
6	0.34362369338	0.341672473868	0.354773519164	0.341463414634
7	0.334076655052	0.348571428571	0.352055749129	0.345156794425
8	0.341324041812	0.355191637631	0.346550522648	0.348780487805
9	0.347944250871	0.366968641115	0.353867595819	0.345644599303
10	0.344390243902	0.362369337979	0.365156794425	0.345365853659
11	0.343554006969	0.373449477352	0.365783972125	0.350871080139
12	0.34425087108	0.366271777003	0.36668989547	0.341881533101
13	0.346132404181	0.375609756098	0.368432055749	0.354912891986
14	0.351358885017	0.368710801394	0.377979094077	0.349337979094
15	0.345714285714	0.374285714286	0.37456445993	0.359442508711
16	0.347804878049	0.38850174216	0.371567944251	0.355818815331
17	0.351010452962	0.386620209059	0.379721254355	0.357909407666
18	0.350383275261	0.380766550523	0.389128919861	0.364320557491
19	0.347386759582	0.388989547038	0.385714285714	0.354285714286
20	0.349268292683	0.391986062718	0.389477351916	0.353658536585

Table 5.2: Result from medium graph

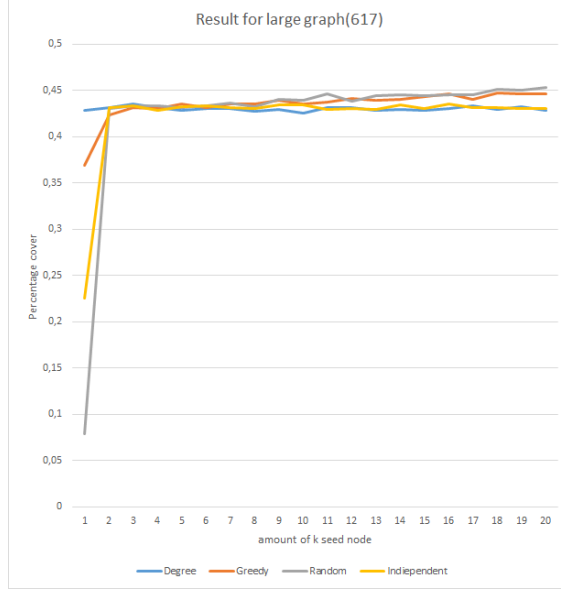


Figure 5.4: The result from the large graph

k	Degree	Greedy	Random	Indipendent
1	0.428363047002	0.368881685575	0.0788330632091	0.225705024311
2	0.431993517018	0.423857374392	0.430696920583	0.431928687196
3	0.435364667747	0.431831442464	0.433873581848	0.433387358185
4	0.431150729335	0.430470016207	0.433614262561	0.428849270665
5	0.42829821718	0.435818476499	0.431863857374	0.432544570502
6	0.430275526742	0.432025931929	0.433614262561	0.433776337115
7	0.430210696921	0.435559157212	0.436369529984	0.431377633712
8	0.427487844408	0.435170178282	0.432414910859	0.430923824959
9	0.429951377634	0.439643435981	0.440713128039	0.434424635332
10	0.425380875203	0.435170178282	0.439189627229	0.435072933549
11	0.431280388979	0.437471636953	0.446612641815	0.429692058347
12	0.431993517018	0.441847649919	0.438995137763	0.431118314425
13	0.428200972447	0.439384116694	0.444278768233	0.429627228525
14	0.429529983793	0.440842787682	0.445316045381	0.434586709887
15	0.428719611021	0.443824959481	0.444829821718	0.431118314425
16	0.431118314425	0.446126418152	0.4456726094	0.43510534846
17	0.433938411669	0.440097244733	0.445510534846	0.43209076175
18	0.42982171799	0.447066450567	0.451280388979	0.431345218801
19	0.432350081037	0.446677471637	0.450437601297	0.431021069692
20	0.428784440843	0.446839546191	0.453452188006	0.430632090762

Table 5.3: result from large graph

Chapter 6

Discussion

The result was somewhat surprising, the greedy algorithm that I thought would be the best algorithm in this simulation was not a huge improvement over the other algorithm. The random algorithm performed better than I expected and the degree algorithm was in some stages better than the greedy algorithm. The independent coverage algorithm performed better or equally good as most of the other algorithm. Toward the end where we choose twenty seed nodes, most of the algorithm performed somewhat the same.

These results can be explained by looking at our simulated social networks. The networks we ran the simulation over was a small graph compared to real world graphs, by choosing at most 20 seed nodes, a large percentage of the graph would be covered. Since the graphs were all connected in a network, the small world effect would result in that every vertex would be subjected to activation. Our global percentage was 0.05.

One paper showed how implementing a parallelized BFS algorithm on a distributed system reduced the communication times by a factor of 3.5, compared to a common vertex based approach[11]. In scientific application, graph based computation is pervasive and often data-intensive, this especially for distributed systems.

Our simulation runs until there are no more nodes to test. This can explain some of the result we got. Since the simulation runs until no more nodes can be activated, the coverage after 50 runs would most likely activate most of the node. This can result in no distinctive better algorithm.

Chapter 7

Future work

One interesting aspect that does not have huge previous work on is a hardware approach to speed up the seed selection and the information diffusion. We have seen different algorithms to select the most appropriate seed, the problem is that for larger graph, the seed selection algorithm is very taxing for the system. For this report, the graphs generated is still small, yet the largest of the used graph proved to be quite slow for the system. It would be interesting to see how an CPU-FPGA heterogeneous platform like [1] would improve the information diffusion.

Another interesting branch to explore would be to see how a paralyzed independent cascade model would measure against the non-parallel version. There are studies exploring the paralyzed BFS[11]

For this report, we created the simulation by storing arrays and "pointer chasing", as mentioned in ??, by applying sparse matrix-vector multiplication, we could have explored other potentially beneficial optimization.

The cache oblivious sparse matrix-vector multiplication is one aspect we have yet to touch upon. It would be interesting to further investigate how much of an improvement this would potentially give compare to a standard sparse matrix multiplication.

—————still lacking a lot of information, would add more detail of how each point mentioned can be expanded.

Chapter 8

Conclusion

As we have shown in this report, there are multiple different algorithms to select the seed. Information diffusion can simulate different aspect of real life. The problem is very taxing on a normal home computer. BFS can be optimized, ICM is a special case of the BFS. There are different optimization, matrix multiplication, maybe something with hardware.

The result we got from the simulation was somewhat supprising, the result from a similar simulation[?]. From [?] we see that the greedy algorithm is better then the other algorithm and the random algorithm performed the worst. For the simulation where the simulation runs until there are no more nodes to activate, the greedy algorithm performed the same as the other. This can be resulted by the stop criteria.

For the future work, it would be interesting to see if the matrix-vector multiplication can improve the seed selection algorithm or the information diffion. One interesting aspect is to see if there are possible to improve it via some form of hardware solution(maybee look at yamans paper.)

Bibliography

- [1] Magnus Jahre Yaman Umuroglu, Donn Morrison. Hybrid breadth-first search on a single-chip fpga-cpuheterogeneous platform. 2015.
- [2] V. S. Anil Kumar Madhav V. Marath Aravind Srinivasan Zoltn Toroczka Nan Wang⁵ Stephen Eubank, Hasan Guclu. Modelling disease outbreaks in realistic urban social networks. 2003.
- [3] Éva Tardos David Kampe, Jon Klein. Maximizing the spread of influence through a social network. 2003.
- [4] Éva Tardos David Kampe, Jon Klein. Maximizing the spread of influence through a social network. 2003.
- [5] John Gilbert Jeremy Kepner. Graph algorithms in the language of linear algebra. 2011.
- [6] M. E. J. Newman. *The structure and function of complex networks*. 2003.
- [7] Stanley Milgram. The small-world problem. 1967.
- [8] Stanley Milgram Fjeffrey Travers. An experimental study of the small world problem. 1969.
- [9] Stanley Milgram Jeffrey Travers. An experimental study of the small world problem. 1969.
- [10] Christos Faloutsos Deepayan Chakrabarti, Yiping Zhanz. R-mat: A recursive model for graph mining. 2004.
- [11] Kamesh Madduri Aydib Buluc. Parallel breadth-first search on distributed memory systems. 2011.