

TDT4900 Computer Science, Master's Thesis

Optimization of Seed Selection for Information Diffusion with High Level Synthesis

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Abstract

Information Diffusion is often used for different simulations in network research because it simulates how information propagates thorough a network. Measuring spread and speed, we can find influential targets in the network. Such targets are optimal targets to pass message during disaster scenario, vaccinate to prevent spreading of a disease, or even targets for viral marketing.

High Level Synthesis (HLS) have in recent years matured greatly. With HLS, designing custom architectures is no longer a dream.

Assignment

Information diffusion is a field of network research where a message, starting at a set of seed nodes, is propagated through the edges in a graph according to a simple model. Simulations are used to measure the coverage and speed of the diffusion and are useful in modelling a variety of phenomena such as the spread of disease, memes on the Internet, viral marketing and emergency messages in disaster scenarios.

The effectiveness of a given spreading model is dependent on the initially infected nodes, or seeds. Seed selection for an optimal spread is an NP hard problem and is normally approximated by selecting high-degree nodes or using heuristic methods such as discount-degree or choosing nodes at different levels of the k-core.

High-level synthesis (HLS) is becoming an important tool in the optimization/acceleration of algorithms in hardware. Starting with an algorithm written in a high-level language such as C or C++, HLS aids with hardware design by providing a methodology and tools that guide the developer through the design process.

This project should employ HLS as a design methodology for hardware accelerated seed selection in large graphs. The student will study seed selection for a given diffusion model, write a high-level model, and use HLS to implement a hardware design that exploits parallelism in the seed selection algorithm in order to improve performance over a GPCPU implementation.

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Chapter 1

Introduction

1.1 Motivation

Information Diffusion is a field in network research where a message, or data, is propagated through a *network* or a *graph*. The message originates from a chosen set of nodes, known as *seed nodes*. These seed nodes pass the message to its neighbours through the edges and thus propagate the message over the entire network. The effectiveness of the Diffusion is measured through the spread and the speed of propagation and is dependent on the chosen seed nodes. By finding the most optimal set of seed node, we can potentially stop an epidemic by vaccinating influential nodes, we can find important target for viral marketing by giving free sample, and use this information to quickly spread message during disaster scenarios[2] [3].

There are multiple studies done regarding information diffusion, [4], [2], [5], [6]. But as far as we know, there are none that focus on optimizing the seed selection in hardware. The current seed selection algorithm is a greedy solution[7], where every set of nodes is tested and the set with best coverage and time is chosen. This is a time consuming process and highly parallelizable, which makes it a good candidate for *Field-programmable gate arrays*(FPGAs).

High Level Synthesis (HLS) transform high level behaviour and constraints to lower level design.[8]. It makes it possible to implement an algorithm in high level language, C or C++, and generate an optimal design in *verilog* or *VHDL*. Verilog and VHDL are hardware descriptive languages designed to describe digital systems [9].

Unlike traditional hardware design, HLS allows programmers with limited knowledge of hardware design to create an optimal custom *Intellectual property core*(IP-core). In HLS, programmers can test out different optimization schemes

in a short period of time, thus reducing development overhead.

In this thesis, we have implemented a simple IP-core that performs information diffusion using the *Independent Cascade model* (ICM) as *Breadth-First Search* (BFS) over boolean semirings. This is done by using HLS as the development tool.

1.2 Assignment Interpretation

From the assignment text, these task were chosen as the main focus of this thesis:

Task 1 (*mandatory*) Implement Information Diffusion as Sparse matrix vector multiplication, with high level language C.

Task 2 (*mandatory*) Tailor the implementation of Information Diffusion for synthesise with Vivado HLS.

Task 3 (*optional*) Implement said design on a Zynq FPGA board.

Task 4 (*optional*) Extend the system to be able to handle graph in the size of toy graphs(containing 2^{26} nodes)

1.3 Report Structure

We have here the basic outline for this report and a short overview of the remainder of this report:

Chapter 2: Background contains the theory regarding network, information diffusion, matrix vector multiplication and high level synthesis.

Chapter 3: Related Work gives a short introduction of the state of the art of HLS implementations, information diffusion research and different optimization of BFS.

Chapter 4: Design and Implementation present our implementation of our IP-core and give a brief introduction regarding HLS implementation and optimization.

Chapter 5: Result and Discussion will compare the result our core gener-

ated compared to a C-simulation. We will also discuss some of the design choices regarding the IP-core.

Chapter 6: Future Work present how our design can be further improved.

Chapter 7: Conclusion provides a concluding remarks regarding this paper and a summary of the identified tasks.

Chapter 2

Background

In this chapter, we will look at the fundamental concepts and theory for the different diffusion models, seed selection algorithms and performing BFS over boolean semiring. We will also have a look at HLS and specifications of the Zedboard. This chapter will contain notations that we will use throughout the report.

We will look at the independent cascade model, which is a special case of breadth first search [10]. By looking at how to improve BFS, we can apply such optimization to ICM and the seed selection algorithm.

2.1 Information Diffusion

Information diffusion is looking at how information is propagated through a network. A vertex can be either activated(infected) or inactivated(healthy/noninfected), each node can spread the contagion(activation,infection) to their neighbour. Some examples would be how a meme, a new trend or a new disease is spread through a community. The process consist of a set of starter nodes, which we will call seed nodes, that are "infected" at initial timestep. During each time-step, there are a percentage p_g where the "infected" nodes would "infect" its neighbours. Seed nodes is a set of k nodes that in the initial time-step are infected. They will pass on the information/infection during each time-step and the information/infection will propagate through the network.

2.2 Basic Diffusion Models

When we talk about data diffusion, we can look at how diseases or technological innovations would spread through a social network. We can simulate those kind of behaviors with different diffusion models. There are two basic diffusion models, the *linear threshold model*(LTM) and the *independent cascade model*(ICM) [11].

To illustrate how these two models work, consider them example as a new product that is promoted via social media. Each node is a person that can either buy the new product (activated), or ignore it (inactive). Each person will then see their friends promote the new product and potentially buy the promoted product. There are several different criteria for each person to buy the product (activates). They can either have a percentage chance to be affected by the advertisement (ICM), or they will only be interested if a percentage of his or hers friends have promoted it (LTM).

Some people might have a larger circle of friends then others(high degree), while others have larger impact on a person(large p_x). Some might be harder to promote too(weighted edges), while some users have no friends(singletons).

2.3 Linear Threshold Model

The linear threshold model uses a threshold θ_v between the interval $[0,1]$, which represent the fraction of vs neighbours that needs to be active to activate node v . The actual number of the activated neighbours is known as the weight of v , b_v . For example, consider Figure ?? and let's assume that the weight of all the nodes is 0.5, meaning over half of its neighbours must be activated for the node to be activated. As we can see in Figure 2.1a, current node v will get activated when $b_v \geq \theta_v$. In figure 2.1b we can see that v is now activated. The next time-step, Figure 2.1c, node w is checking if it will too, be activated. We can see here that $b_w < \theta_w$, so node w will not be activated.

We can look at the linear threshold model as a cosmetic company trying to promote their new product via social media. Each users of the product would display the new cosmetic product through social media. Each users would then be exposed to the product through their friends update. Each user would adopt the new product after seeing a percentage of their friends using the product.

2.3.1 Independent Cascade Model

The independent cascade model (ICM) have a local or global probability for activation. In figure 2.2, each node have their own local probability. In 2.2a, the node v is activated, during the next time-step, all of node v 's neighbours use their internal probability to determine if they are activated or not. The result

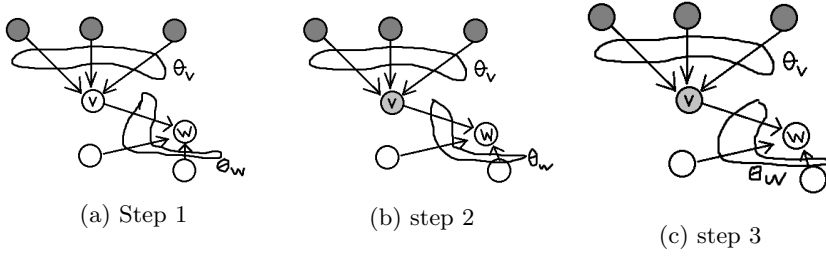


Figure 2.1: Linear Threshold mode

was three activation out of five. In the next time-step 2.2c node w was able to activate the blue node. For a ICM with global probability, each node would then have the same probability to activate its neighbor. As we can see from Figure 2.2, each neighbor to v is activated individually. As in 2.2b, only three nodes were activated. In ICM, each node can only try to activate the neighbor once. In figure 2.2b, the node that was not activated by v , got activated in 2.2c by the node w .

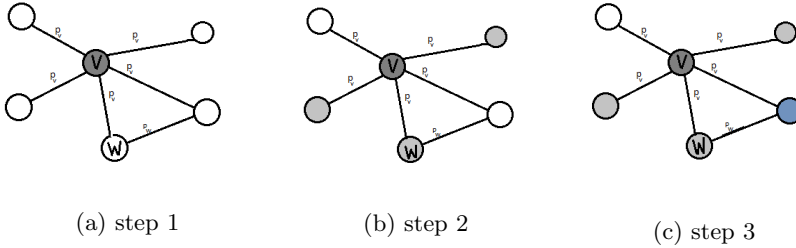


Figure 2.2: Independent cascade model

We can compare the ICM with the same example we have been using, the cosmetic company promoting a product. Each new user have a chance(pp_v) to promote the product to a new user. The new user would then continue promoting the new product to his/hers friends.

As mentioned earlier, the ICM is a special case of the breadth first search. If we set the transmissison probability to 1.0, meaning there are 100% chance to activate the neighbors, the ICM is equal to BFS. BFS iterates through the graph by appending the child node not examined into the queue. Then examines a new node from the queue and repeating the process until all nodes have been examined. The main difference between the ICM and BFS, is that in ICM, there

is a chance that the child node is not added into the queue. During each step, a random "coin toss" is tested to determine if the child node would be added into the queue. Other than that, both algorithms iterate through the network via edges.

2.4 Breadth First Search

BFS is a tree traversal algorithm. BFS starts at the root node v_r . The algorithm then stores all v_r 's children nodes in a *queue*. The algorithm then takes the first node from the queue, v_1 and stores all the children nodes to v_1 in the back of the queue. This process continues until the queue is empty and all the nodes have been iterated over.

BFS is a common graph iteration algorithm, but is often limited by the irregular memory access where the algorithm has to find the data stored in different spaces in memory.

Algorithm 1 Breadth First Search

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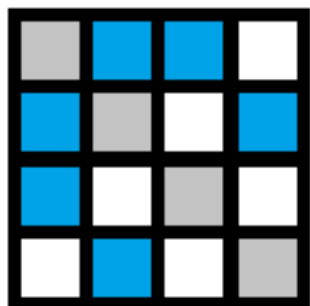
1:  $dist[\forall v \in V] = -1; currentQ, nextQ = \emptyset$ 
2:  $step = 0; dist[root] = step$ 
3: ENQUEUE( $nextQ, root$ )
4: while  $nextQ \neq \emptyset$  do
5:    $currentQ = nextQ; nextQ = \emptyset$ 
6:    $step = step + 1$ 
7:   while  $currentQ \neq \emptyset$  do
8:      $u = DEQUEUE(currentQ)$ 
9:     for  $v \in Adj[u]$  do
10:      if  $dist[v] == -1$  then
11:         $dist[v] = step$ 
12:      ENQUEUE( $nextQ, v$ )
return  $dist$ 

```

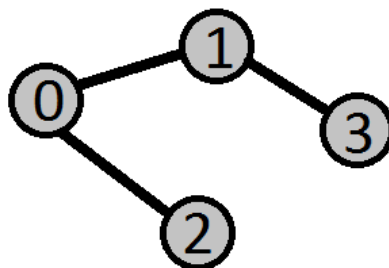
2.4.1 BFS to Data Diffusion

The motivation for transforming breadth first search as matrix-vector multiplication is that displaying the graph algorithm as a matrix multiplication can display the data access pattern for the algorithm and can be readily optimized [12].

As mentioned before, ICM is a special case of the breadth first search. By modifying the algorithm proposed earlier, we can in theory perform ICM with matrix-vector multiplication.



(a) The adjacency matrix



(b) The graph corresponding to the adjacency matrix

Figure 2.3: Sparse matrix to graph

2.5 Matrix Notations

Nodes and edges are not the only way to present a graph, graphs and networks can be represented as sparse adjacency matrices [12]. We can see that such an idea have been proposed in earlier literature [13]. By representing graphs as a sparse matrix, we can often discover different ways to optimize the algorithm, we can have a different structure to store data. The adjacency matrix in particular, is a interesting way to represent the graph. A graph $G = (V, E)$, G have N vertices and M edges, this correspond to a $N \times N$ adjacency matrix called A . If $A(i,j)=1$, then there is an edge from v_i to v_j . Otherwise its 0. In Figure:??, we can see how a undirected graph can be represented as an adjacency matrix. To generate a undirected graph as a adjacency matrix, the matrix must be mirrored diagonally, meaning if $A(i,j)=1$, then $A(j,i)=1$, if this is not true, then the matrix would be representing a directed graph.

2.5.1 Sparse Matrix

A sparse matrix is a matrix containing few nonzero. Social graph with few edges would often be represented as a sparse matrix. Since sparse matrices only have few non-zero elements, by storing only the non-zero elements, we can have savings in memory.

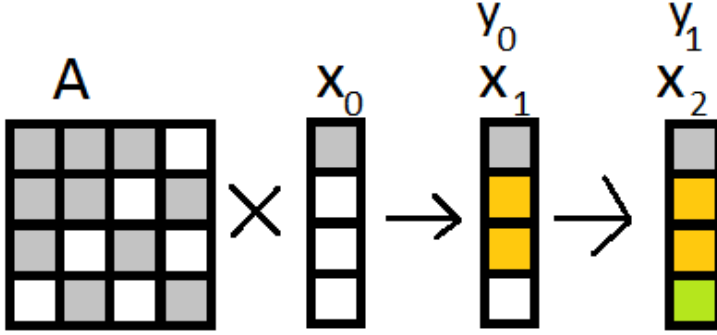


Figure 2.4: BFS on Boolean semiring

2.5.2 Breadth First Search as Matrix Multiplication.

From [12], we can see that BFS can be recast as algebraic operations. BFS can be performed by applying matrix-vector multiplication over Boolean semirings [10]. The graph is represented as an adjacency matrix A , then for the root node, a vector $x(\text{root})=1$ is multiplied with the matrix A . $A \times x_0 = y_0$. y_0 is the result after the first matrix-vector multiplication and in the next iteration, $x_1 = y_0$. We can see from the Figure 2.4

2.5.3 Semiring

A *semiring* is a set of elements with two binary operations. The two operations are often known as "addition" (+) and "multiplication" (\times). As we shown in previous section, the algorithm perform matrix multiplication uses the two operations, multiplications and addition. In [10], the AND and OR operator was chosen instead of the normal addition and multiplication.

2.6 Seed Selection Algorithm

The seed selection algorithm, is the algorithm used to select the initial k seed nodes to be chosen at the start of the information diffusion. Each selected nodes

is in the initial timestep activated. During each timestep, the seed nodes will propagate the activation along the network depend on what diffusion model is used. We can compare it to a new gadget or a cosmetic company trying to promote a new product. By selecting a few influential persons to give a free sample, the new trend would most likely spread through *viral marketing* [14]. The seed selection algorithm would be the algorithm to select the few influential individuals to receive this free sample. There are multiple different scheme to choose from, in this section, we will focus on four different algorithms, greedy algorithm, degree algorithm, random algorithm and the independent greedy algorithm.

2.6.1 The Greedy Algorithm

The greedy algorithm [7] proposed by Kempe et al, is known to be the best algorithm according to the result from [7]. The greedy algorithm starts by iterate over the entire network and finds one node that have the largest coverage and stores that node in the set A . The algorithm then activates that node from A , and computes the coverage of each node in the network with the previous chosen node. After the algorithm finds the second node, that node is stored in A with the previous node and we have now chosen two seed node. The algorithm continues until we have choosen k seed nodes, where each have been tested to have maximum coverage in relation with the previous chosen nodes. To compute the maximum coverage, the algorithm have to test every combination of nodes together, this results in heavy computation and the algorithm would therefore not scale well.

We can look at the greedy algorithm as a special case of BFS with a transistion probability. Each node in the network will be chosen as the seed node for the

Algorithm 2 Greedy Algorithm

- 1: Start with $A = \emptyset$
 - 2: **while** $|A| \leq l$ **do**
 - 3: For each node x , use repeated sampling to approximate $\sigma(A \cup x)$ to within $(1 \pm \varepsilon)$ with probability 1δ
 - 4: Add the node with largest estimate for $\sigma(A \cup x)$ to A .
 - 5: Output the set A of nodes.
-

2.6.2 The Degree Algorithm

Another popular algorithm is the degree algorithm [7]. Unlike the greedy algorithm, does not compute the coverage of node, the algorithm picks the top k nodes according to the degree distribution instead. The node chooses the top k

nodes with the highest degree and stores them as the seed nodes. This approach benefits over the greedy algorithm by not having as much computation time as the greedy algorithm since only one iteration is needed to compute the degree to node. The disadvantage is that this algorithm does not take the degree correlation into account. As mentioned in section ??, high degree nodes would often have common node as neighbor. This would result in multiple overlapping activated node choosen.

Algorithm 3 Degree Algorithm

- 1: Start with $A = \emptyset$
 - 2: **while** $|A| \leq l$ **do**
 - 3: For each node x , use repeated sampling to compute $\text{DegreeMax}(x)$.
 - 4: Add the node with largest degree to A .
 - 5: Output the set A of nodes.
-

2.6.3 Independent Algorithm

Another algorithm is the independent greedy algorithm. The algorithm iterates through the network, computing the spread of each node. The algorithm then chooses the vertex with the largest coverage independent of the other previous chosen nodes. This algorithm is a special case of the greedy algorithm mentioned above.

Algorithm 4 Independent Algorithm

- 1: Start with $A = \emptyset$
 - 2: **while** $|A| \leq l$ **do**
 - 3: For each node x , use repeated sampling to approximate $\sigma(A \cup x)$ to within $(1 \pm \varepsilon)$ with probability 1δ
 - 4: Add the node with largest estimate for $\sigma(x)$ to A .
 - 5: Output the set A of nodes.
-

2.6.4 Random Algorithm

The last one is the random algorithm. The random algorithm just picks a random seed node. This approach is the simplest to implement and easiest. The downside is that this is random and there are no strategic choosing of seed node.

2.7 High Level Synthesis

High Level Synthesis convert algorithms implemented on higher level down to *Register Transfer Level* (RTL)[15]. RTL is models of digital circuits displaying flow of data between register, logical operations and such. It is commonly used to describe low level digital systems. HLS is known to be able to reduce development effort and cost of creating specialized hardware compared to traditional hand-drawn RTL designs[16][17][15]. By taking high level language such as C, C++ and SystemC implementations and generate the optimal architecture. HLS allows the user to generate custom *Intellectual Property*(IP) core. An IP-core is a custom created data core that have an output and an input port.

2.8 ZedBoard

The Zedboard that we used for this project, is *Xilinx Zynq-7000 All programmable System-on-chip(SoC)* Z-7020. Consist of a dual core *ARM cortex-A9 MPCore* based processing system(PS) and an *Artix-7 XC7Z020* FPGA. The FPGA is the *programmable logic*(PL). The Zedboard have 512 MB DDR3 RAM, 256MB Quad-SPI Flash and 4GB SD card.[18]. The system offers the flexibility and scalability of an FPGA[19].

The FPGA use *Advance eXtensible Interface*(AXI4)bus protocol. There are three types if AXI4 interfaces:

- **AXI4 Lite**- Simple, memory mapped communication. Useful for small single read.
- **AXI4-Stream** - for continues streaming of data.
- **AXI4** - For memory mapped applications.

A component with PL implemented would be able to connect to the PS through a AXI4 bus port. The close coupling between t

2.9 RMat

One problem during graph analyzation and calculation is finding suitable graphs to analyses. Generate graphs with desired properties is not easy to do. One solution proposed by Chakrabartiy et al is to use the "recursive matrix" or R-mat model. The R-mat model generates graph with only a few parameters, the generated graph will naturally have the small world properties and follows the laws of normal graphs, and have a quick generation speed [1]. The R-mat models goal is to generate graphs that matches the degree distribution, exhibits a "

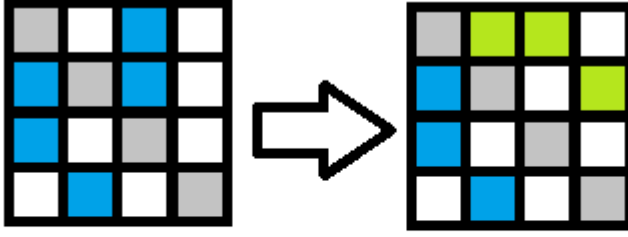


Figure 2.5: How the adjacency matrix is flipped on the diagonal

community " structure and have a small diameter and matches other criteria. [1]. The algorithm to generate such a recursive matrix is as follows: The idea is to partition the adjacency matrix into four equally sized part branded A,B,C,D, as shown in Figure2.5. The adjacency matrix starts by having all element set to 0. Each new edge is "dropped" onto the adjacency matrix. Which section the edge would be placed in, is chosen randomly. Each section have a probability of a , b , c , d , and $a + b + c + d = 1$. After a section is chosen, the partition that was chosen is partitioned again. This continues until the chosen section is a 1×1 square and the edge is dropped there. From the algorithm, we can see that the R-mat generator are capable to generate graphs with total numbers of node $V = 2^x$. Since the algorithm partitioned the matrix into four part. This is approach would only generate a directed graph. To generate undirected graph, $b = c$ and the adjacency matrix must make a "copy flip" on the diagonal elements, like Figure 2.5.

Chapter 3

Related Work

In this chapter, we will look at the state of research regarding High Level Synthesis, information diffusion, and optimization of independent cascade model and breadth-first search.

3.1 Information Diffusion

There are multiple studies done regarding information diffusion. One study shows how information diffusion can be applied during an disease outbreak[2], viral marketing[14], coordinat during crisis situation[20].

Different models of information diffusion have been done on blogs[21][22], and twitter[23]. We can see that in an age of social media, the studies of information diffusion is more relevant then ever.

While [6] have argued that the emerging of social networks and media, have changed the traditional model. The activation is no longer only relying on neighbour nodes, but also an external influence. They found that large amount of information volume in Twitter is the result of network diffusion, while a small amount is due to external events and factors outside the network[6]. Another studie shows that during the 2011 Egyptian Uprising, how a large amount of rebel movement were "tweeted"[20] during the uprising.

As we mentioned in Chapter 2, we mainly focus on two common information diffusion models, ICM and LTM. But there are different models too. [5] proposed several different problems with traditional models where each node is either *activated*(infected, influenced, '1') or *inactive*(healthy, not reached, '0'), and passes the *contagion*(information, data, infection, influence) to neighbouring nodes through the edges. The report mentioned different assumptions that such models make. Among them is that a complete graph is provided, the spread of

contagion is from a known source, and that the structure in the network is sufficient to explain the the behaviour[5]. The report propose an alternative model, *Linear Influence Model*(LIM), where the focus is on the global influence that an infected node has on the rate of diffusion through the implicit network. This model makes the assumption that newly activated nodes are dependent on previous activated nodes. The LIM does not need explicit knowledge of the entire network, instead the model takes the newly activated nodes and model them as a *influence function*, which is used to find the global influence.

3.2 High Level Synthesis

High Level Synthesis as a concept have been around since the mid-1980s and early-1990s[15][24]. Carnegie-Mellon University design automation (CMU-DA)[25][26] was a pioneering early version of HLS tools. The tool gathers quickly considerable interest. A number of HLS tools were built in later years mostly for prototyping and research[27][28][29]. Some of these was able to produce real chips, but the reason for lack of further development and adaptation, was that RTL synthesis was not a widely accepted and a immature field. This often lead to suboptimal solutions.

Around the year 2000, new HLS tools was developed in academia and in the industry. These tools, used hihg level language, C and C++. Vivado HLS, designed by Xilinx [30], is one such HLS tool. The Vivado HLS became free during their 2015.4 update[31]. This resulted in an revived interest in HLS. The community around HLS is also evolving, on the Xilinx-forum, there are multiple answers and active members. We can see that the solution designed by HLS tools is close to traditional hand-crafted designs[32].

[15] goes into the history of HLS throughout the ages. The paper discuss different problems and reason for failed commercialization of early HLS tools, Cong et al, concluded that the problem with the early HLS tools can be summarized by the following reasons: *Lack of comprehensive design language support, Lack of reusable and portable design specification, Narrow focus on datapath synthesis, lack satisfactory QoR, Lack of a compelling Reason/event to adopt new design methodology*. Early version of HLS tools was not a C-to-RTL transformation. Most of them needed a custom *Hardware Descriptive Language*(HDL). Lacking of reusable and portable design specification resulted in that HLS tools required user to include detailed information regarding timing and interface information into source code. This resulted in a target dependent solution and can't easily port to other devices. The narrow focus on datapath synthesis resulted in a lack of focus on interface to other hardware modules and platform integration. Those aspect was left to the users to solve system integration problem. The lack of foundation to accurately measure HLS result and often failed ot meet timing and

power requirement with early HLS tools were another limiting factor. The last reason, was that there where no real driving force to turn developers over to such a young and early development format. HLS tools shows interesting capabilities, but most developers did not want to move from the safe and tested RTL design methodology. The paper concludes with that current(2011)HLS tools showed huge potentials in becoming standard in selected deployment.

3.2.1 Applications using HLS

In [33], HLS was used to design an accelerator for database analytic and SQL operation. The design was implemented on a Virtex-7 xc7vx690t-g1761-2 FPGA with focus on accelerating operations as join, data filter, sort, merge and string matching. The accelerator was implemented in C++ in Vivado HLS and optimized with UNROLL directive, PIPELINE directive and ARRAY_PARTITION. The UNROLL directive unroll all of the specified loops, while the PIPELINE directive allows multiple accelerator to process data at each clock cycle. The ARRAY_PARTITION directive partition data into registers. The accelerator showed promises, giving a 15-140 \times speedup compared to Postgres software DBMS running selected TPCB queries.

[34] explored the advantage and disadvantages of HLS implementation of image processing. They argued that custom algorithms on FPGA platforms will most likely result in an improvement, but the algorithms must be tailored to the platforms. The author conducted different case studies to show both the strengths and the weaknesses of HLS. The report goes through image filtering, connected component analysis and two dimensional fast-Fourier transformation(FFT). One example that the author brings up, is during image filtering where HLS was not able to identify the standard accessing pattern during special cases. This resulted in that the HLS built additional hardware to counter such a exception. The report concluded that while HLS can significantly reduce development time and improve utilization of the design space, it is still important to focus on careful design. The report concludes that HLS can offer many benefits, and is an improvement over conventional RTL-designs, but is not an replacement for hardware designers or clever designs.

[35] implemented an fast Fourier transform (FFT) algorithm for different digital system processing application in HLS. There the authors used Simulink for verification of design, and implemented it in HLS.

[17] discuss improvements to the current HLS tools with polyhedral transformation. Here they discuss a problem with HLS, which is that unless the code is inherently compatible, HLS can not apply most of the optimizations. Zuo et al. proposed the polyhedral model, the model takes data dependent multi-block program as input and performs three steps: Classification of array access pat-

terns, performance Metric, and implementation. During the classification of array access patterns, a set of data access pattern is defined and classified. Then the appropriate loop transformation is applied. The next step, the performance of each loop transformation with data-dependency is estimated, and the best improvement is chosen. In the final step, the chosen solution, loop transformation and inserting HLS directive is applied. Then a interface block for the data-dependent blocks is generated. The generated communication block is then optimized depending on how it behaves. The paper concludes with that the polyhedral model can find important loop transformation, thus enable optimization such as pipeline and parallelization.

[32] is a case-study where HLS is used to implement two compute heavy machine learning techniques with different computational properties. The two algorithms that was tested was *Lloyd's Algorithm* and a *Filtering Algorithm*. The result was that for the first case, a similar performance between the HLS solution and a hand-written solution, while the second algorithm was severely worse with HLS, if the developer did not customized for HLS.

3.3 Different optimization scheme

There are a large amount of different optimization research on graph traversal, especially on Breadth First Search.

[10] proposed a hybrid FPGA-CPU heterogeneous platform for BFS. The idea is to run the first couple of steps on the CPU core, then switch over to the FPGA accelerator to explore the rest of the graph. The CPU is better suited for calculating while there is a smaller frontier, while the FPGA core is better suited for a larger frontier. By exploiting the characteristics of small-world networks where the frontier is much larger after two-three iterations, one can significantly improve computation time. The report proposed an alternative method of performing breadth-first search. By performing it as a sparse matrix vector multiplication over a boolean semiring, more parallelization option was discovered. The result was a speedup of 7.8 compared to a pure software implementation, and 2 \times better compared to an accelerator only implementation.

[36] propose a hybrid solution, combining a conventional top-down algorithm and a novel bottom-up algorithm. The optimization in this paper focuses on examining fewer edges, thus reduce computation time and trying to circumvent one major drawback with BFS; memory-bound on shared memory. The top down approach is the traditional algorithm, where a frontier expands and visits all nodes on that level, before each node checks its neighbour for unvisited vertices. Unvisited vertices is placed in the frontier and marked as visited. The bottom up algorithm, contrary to the top down algorithm, is where each children vertex tries to find a potential parent. A neighbour vertex can be parent if the neighbour is

also in the frontier. This results in that after a vertex finds its parent, there is no need to traverse the rest of the frontier. By using different approach during different time, the report was able to achieve a speedup of $3.3 - 7.8\times$ on synthetic graphs and $2.4 - 4.6\times$ on real social network graphs.

[37]

Chapter 4

Design and Implementation

In this chapter, we will present our implementation of the SpMV over boolean semirings, how we implemented with HLS and how we connected the component in the FPGA.

4.1 IP-core

For this project, we have implemented an IP-core with Vivado HLS. The IP-core applies a modified version of SpMV over a adjacency matrix and a set of vertices known as seed nodes. We will have a brief introduction to HLS programming and how to customize the algorithm for HLS.

4.1.1 IP-core

Our implementation of the SpMV is done in HLS. Our algorithm takes in an adjacency matrix and a set of seed nodes as input, and outputs a result vector showing which node was activated after either all nodes are activated, or no more node can be. Unlike normal SpMV, where each iteration will activate all their neighbours, an ICM is dependent on a RNG and global or local probability. For this project, a global probability of 5% was used. Each node have a 5% chance to be activated, e.g. if v_1 was not activated by V_r on the first iteration, V_r can not reactivate v_1 on the next iteration.

To prevent a reactivation as mentioned above. For each iteration, a frontier vector will be send in instead of the result from previous run as mentioned in Seciton 2.5.2. A vector is a list of vertices. The frontier vector is generated by comparing the result from this run with a list of activated vertices. The vertices

that was not on the list of activated node would be new activated vertices and would be the frontier.

Our algorithm applies SpMV over the matrix and the frontier. For each iteration, each element in a row of the matrix is applied an AND (&&) operations with the corresponding element in the frontier vector. The resulting bit will be applied a OR (||) operation between the other :

```
(matrix_row[0] && frontier[0]) || (matrix_row[1] && frontier[1]) || ...
2: (matrix_row[n] && frontier[n])=result[row]
```

Unlike the breadth first search on boolean semiring mentioned in earlier chapter, each node will have a chance to not be activated (set as '1'), even if `matrix_row[x]` and `frontier[x] = 1`. This resulted that for each &&- operation, we need to && another *coin toss*, which determined if the activation takes place. The coin-toss is determined by the RNG and the global probability.

The algorithm will continue until either all of the nodes are activated, or no more node can be activated. This is solved with the `dist_gen()` function as we mention [INSERT PSEUDOCODE]. The function stops the algorithm when either frontier is empty (all the nodes are 0), or `y (result)` is full (all nodes = 1).

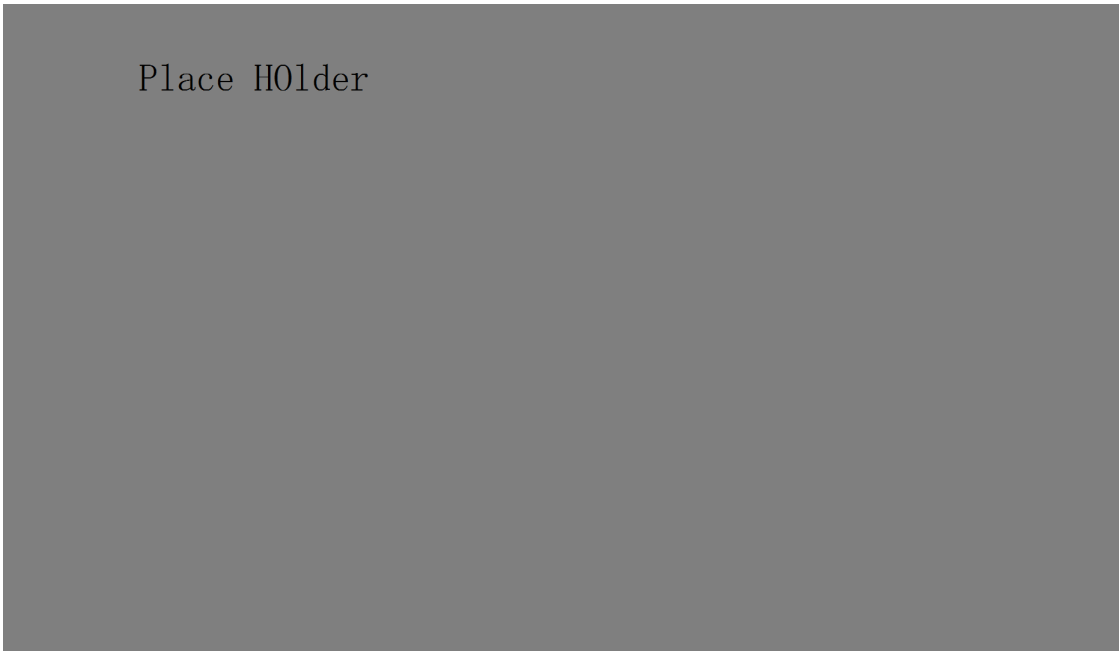
In HLS the implementation of the ICM-SpMV was done in two level. The top level was the `TopLevelWrapper`. The `TopLevelWrapper` was set as the main function in our HLS implementation. The function takes in the address to the location to the matrix, the address to the result, and the address frontier. Since we are working with ICM, a global probability, a random seed as the initial state for the LFSR is also set as input. `TopLevelWrapper` stores the useful data over on local buffer, where it is sent to the datapath-function. The datapath-function is a sparse matrix vector multiplication.

The address to matrix, result, frontier, and the random seed and global probability is all mapped as AXI4-Lite while the matrix, frontier and the result are memory mapped. This allows us to send in the address of the memory location where the variables are stored and apply our algorithm.

Since ICM is dependent on a random function, we ran each sets of seed nodes 50 times to find the average runtime and coverage to find the most optimal set of nodes.

4.2 Linear-Feedback Shift Register

The LFSR is a commonly used pseudo random number generator(RNG)[38]. Different sized LFRS is able to generate a wider range of pseudo random number. LFSR generates a pseudorandom number based on the previous number. The



Place Holder

Figure 4.1: 16-bit Linear-Feedback Shift Register

LFSR implemented for this project, is a 16 bit LFSR, the shift register is able to generate a pseudorandom number in the range from $0-2^{16}$ (65536). We take the bit from position: 16,14,13 and 11, and XOR all the bits. resulting in $((16 \text{ XOR } 14) \text{ XOR } 13) \text{ XOR } 11 = \text{bit}$. The bit is pushed into bit-position 1 and the entire registers shifts towards right 1 bit. This allows the IP-core to generate a pseudo random number based on a initial seed input. From Figure: 4.1 we can see that to continuously generate pseudo random numbers, the output from the LFSR is used as the next iterations seed.

4.3 Design flow of HLS

For this project, Xilinx Vivado HLS 2016.1 was used. The usual workflow in designing with Vivado HLS is as follow.

1. Define your function/algorithm.
2. Simulate as compile code.

3. Synthesise.
4. Co-simulate.

The first step in designing with Vivado HLS is to define the algorithm that will be synthesise, for this report, its the matrix vector multiplication. After identifying different requirement and dependency, the algorithm is implemented in C with Vivado HLS. Vivado HLS have some limitation regarding the implementations:

- No dynamic memory (Need to be static), Vivado HLS does not support malloc, free, new or delete.
- NO STD, FILE-IO, etc, (no system calls).
- avoid recursive functions.

The next step is to *run C simulation*. This will verify that the C implementation is correct, the test in the testbench are created by the designer. After verifying that the implementation is correct, next step is to synthesise the implementation. Vivado HLS will then generate the appropriate Verilog or VHDL, depending on what the designer require. The finished generated solution is then reviewed. Vivado displays the "Synthesis Report", containing crucial information regarding the generated solution. There we can find the performance estimates of the generated core, the utilization estimates, and the interface to the generated core. After all this is done, Vivado offers a *run C/RTL Cosimulation*. Vivado will then run both the C simulation and testbench, and the same testbench on a simulated version of the implemented core. This function allows the designer to verify that the generated core have the same behaviour as the simulated C implementation.

After Cosimulation is done, the IP-core is ready for export, *Export RTL* generates the necessary RTL files and export the IP core. The exported IP-core can be found in the project folder and ready to be implement to the FPGA.

There are some steps that Vivado HLS requires before the project can start. In the begining, Vivado HLS would require the designer to specify which function is the top level of the implementation. That top function would determine which port the IP core would have and what type of AXI4 protocol to implement. Vivado HLS also enables the user to specify to which platform this implementation is for.

The input and output port of the IP core is determined by the variables that functions require. Inputs that is read from, will automatically set as input, while variable that is only written to, will be set as output. Another thing about output is that for the core to understand that this is something that changes, it is often set as a pointer (For C code).VIVADO HLS generates control signal automatically.

4.4 Network and graph generator

For this project, we choose to implement a R-mat generator as mentioned in Chapter: 2.9 and [1]. The generator is implemented in *Python* [39] with *numpy*. The generator generates adjacency matrix with 2^k nodes, where k is known as the scale of the sparse matrix. The total amount of edges the graph contains, is set as $totalamountofedge = k \times edgefactor$. The edge factor is the ratio of the graph's edge count to its vertex count[40]. Our adjacency matrix was stored as a .txt format and was read as input for our IP core. We used the following variables to generate our adjacency matrix:

- **A** = 0.57
- **B** = 0.19
- **C** = 0.19
- **D** = 1-0.57-0.19-0.19 = 0.05
- **Edge factor** = 16
- **k (Scale)** = 10

By using the random-function provided by Python, we placed '1' in it's correct place. After the matrix is generated, we further applied a diagonal copying as shown in Figure:2.5. This is obligatory to generate an undirected graph.

For this project, a graph with size 1024 was used. This is a microscopic graph compared to graphs used in network analysis.

4.5 How is it connected

To connect the IP core, we will use Xilinx Vivado (2016.1). In *Block design*, we connected our xutom IP-Core to a *ZYNQ7 processing system*[18]. The Zynq7 Processing system act as our PS, while the custom IP core act as our PL. The custop IP-core was connected to one of the High performance slave interface. For our implementation, the RNG was included with the core. This allows that

For our core, we used the architecture of memory mapped. We pass the address in the form of AXI4-lite. The core includes an dma-function, which the HLS would initiate for us. This allows us to utilize the High performance slave interface.

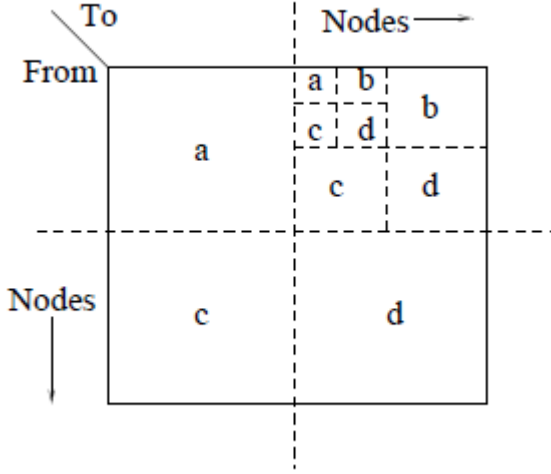


Figure 4.2: The R-mat model [1]

4.6 Xilinx SDK

Vivado offers a *Eclipse* based software develop kit (SDK). There we read the pregenerated graph from a SD card on the ZedBoard. We initiated the the custom IP core by setting all the required input signals.

4.7 Optimization

The function `matrix_vector_multiplication()` performs a single matrix vector multiplication. From the pseudocode, we can see that there is a room for parallization of the SpMV. The outer for loop from the pseudocode, can be paralized since the for loop is not dependent on the variable from the inner for-loop.

Another paralization is during the simulation, after the `spmv`, the frontier needs to be calculated. And a `converged()` function is called in the end to determine if the simulation is finished. The frontier calculation and `converged` can pe run in parallel.

include the different directives, PIPELINE

The IP-core (currently) is only using around 2% of the resource available on the FPGA(Zedboard). This gives us a lot of room for parallization of different

core. Implemented 2 bus, one for input stream, one for output stream. The output stream consist of the result_vector

4.8 Potential improvement

: For the second option, where the RNG core is not implemented into the IP CORE, we would have to have teh random number set as AXI4 stream from the buffer, since we would need to call a stream of random number from the core.

4.9 global vs local probability

For this project, a global activation chance was implemented, giving each node the same probability to be activated. A interesting model would be there are a personal probability of activation. Each node might have unique activation chance. Our implementation.

Chapter 5

Result and Discussion

as we can see, the algorithm was able to finish a

5.1 Performance

As we can see, the hardware implementation is better then the C-simulated implementation, even at this low scale, we can see that the

5.2 Discussion

Analysis of the performance

!

problem that was encountered

One problem that was encountered during this project was that the output signal from the synthesiser was not the correct direction. The output signal was often set as input signal. The HLS would automatically set the values as output signal or input signal. The return value from a function would be set as the output signal, while the variable that the function takes, would be set as the input signal. Another way to specify that something is the output signal would be to explicitly set them as pointer arguments. This will in set the signal to be output signal.

Another problem that I often encountered, was that the Vivado HLS often stop working. The problem was fixed by creating a new project and include the previous files.

Another problem, was that some times the Vivado HLS was not able to Cosimulate the implementation the first time. I was not able to find a solution to this problem except resynthesis the project.

There were incidence where the implementation on the Zedboard did not behave as the implementation. This was solved by reprogramming the device and sometimes, restarting the Zedboard.

Chapter 6

Future work

Information diffusion and seed selection in general computes gigantic graphs, and thus is very time consuming. There is therefore several performance related improvements that have yet to be explored.

1. **Different architecture** for this implementation, we used a core including the LFSR, it would be interesting to explore different architecture. One solution that we did not have the chance to explore, is implement a large buffer connected to a single LFSR that continuously generates random number. The implemented cores would then each pop one random number for each cointoss. This solution requires a large buffer and would potentially generate large overhead with reading from the buffer. A large enough buffer would also be required since there are in worst case scenario for a single SpMV run, we would need n^2 cointoss. The potentially benefits of such a design would be better space utilization. A smaller core would use less resource of the Zedboard. This can result in more parallelization.
2. **Use larger graph** In graph theory, graph used is often at scale(26-42). The smallest mentioned graph from Graph500, is 2^{26} , a toy graph. Our graph is not even close to such a large graph.
3. **Customize algorithm** As mentioned in Chapter 3, HLS can generate a close to hand written design if the algorithm is customized for HLS. A interesting potential improvement would be to analyse algorithm and explore different solutions and implementation.
4. **Compare different solutions** In this report, we have only showed the result from one architecture, it would be interesting to compare different shcems

5. **Memory Optimization.** For this algorithm, we store the entire adjacency matrix. This is inefficient for a sparse matrix. A potentially improvement would be to explore a different storage format for the adjacency matrix.

6. Try different seed selection algorithm

The community of the HLS is very active and frequently responds to forum post seeking help. Not many work that uses HLS[CITATION NEEDED]. Recently was free, used to cost money.

Learn more about HLS so it can be better utilized. different scheme to further work: - implement parallel - different scheme, rng on the outside - Use other data structure. - larger graph - compare this solution to other solution, - implement more efficient memory storage - use other storage method since its sparse. - would be interesting to gather information on energy consumption. - Implement a more general architecture to handle more total nodes. - If there would be enough memory on the board, would be interesting to run 50 times on the board and just return the average time and coverage

Chapter 7

Conclusion

For placeholder, need to have some text here.

This paper works as a proof of concept, where we can see that with minimal knowledge regarding HLS, I was able to generate a simple Sparse Matrix vector multiplication IP-core. We can see from the result from co-simulation, that custom core ran much better then the other. HLS was easy to use and provided with a quick development time. There are still some problem with the HLS tools, but compared to earlier version, this is much better.

- HLS was great tool
- The custom core appears to be good.
- result is promising, shows potentiall
- HLS is great, allows rapid development and optimization.
- There is room for improvement, but still a promising field.
- There are still some random bugs in HLS.
- Not many clear tutorials for HLS

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