HW 3

Q7 - Q11

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Question 7

Exercise 3.1.1

$$A = \{x \in \mathbb{Z}; x \text{ is an integer multiple of } 3\}$$
$$B = \{x \in \mathbb{Z}; x \text{ is a perfect square}\}$$
$$C = \{4, 5, 9, 10\}$$
$$D = \{2, 4, 11, 14\}$$
$$F = \{4, 6, 16\}$$

(a) $27 \in A$
True. 27 is an integer multiple of 3. Therefore, 27 is an element of set A.

(b) $27 \in B$
False. 27 is not a perfect square of an integer. Therefore, 27 is not an element of set B.

(c) $100 \in B$
True. 100 is a perfect square of 10. Therefore, 100 is an element of set B.

(d) $E \subseteq C$ **or** $C \subseteq E$
False. $6 \notin C$ but $6 \in E$. Therefore, $E \nsubseteq C$. Also, $10 \notin E$ but $10 \in C$. Therefore, $C \nsubseteq E$.

(e) $E \subseteq A$
True. 3, 6, and 9 are integer multiples of 3. Therefore, $3 \in A$, $6 \in A$, and $9 \in A$.

(f) $A \subset E$
False. $12 \in A$ but $12 \notin A$. Therefore, $A \not\subset E$

(g) $E \in A$
False. E is a set, not an element.

Exercise 3.1.2

$$A = \{x \in \mathbb{Z}; x \text{ is an integer multiple of } 3\}$$
$$B = \{x \in \mathbb{Z}; x \text{ is a perfect square}\}$$
$$C = \{4, 5, 9, 10\}$$
$$D = \{2, 4, 11, 14\}$$
$$F = \{4, 6, 16\}$$

(a) $15 \subset A$
False. 15 is not a set, but an element.

(b) $\{15\} \subset A$
True. 15 is a integer multiple of 3. $15 \in A$, so $\{15\} \subseteq A$. Also, $3 \notin \{15\}$ but $3 \in A$. Therefore, $\{15\}$ is a proper subset of A.

(c) $\emptyset \subset C$

True. $\emptyset$, an empty set, is a subset of C. $4 \in C$ but $4 \notin \emptyset$. Therefore, $\emptyset$ is a proper subset of C.

(d) $D \subseteq D$

True. $D = D$. Therefore, $D \subseteq D$.

(e) $\emptyset \in B$

False. $\emptyset$ is a set, not an element.

Exercise 3.1.5

(b) {3, 6, 9, 12, ...}

$A = \{x \in \mathbb{N}^+; x \text{ is an integer multiple of } 3\}$
Set A is infinite.

(d) {0, 10, 20, 30, ..., 1000}

$B = \{x \in \mathbb{N}; x \text{ is a integer multiple of 10 where } x \leq 1000\}$
Set B is finite. Its cardinality is 101.

Exercise 3.2.1

(a) $2 \in X$

True. 2 is an element of the set X.

(b) $\{2\} \subseteq X$

True. 2 is an element of the set X, so the subset, {2} is one of the subsets of X.

(c) $\{2\} \in X$

False. the subset {2} is not an element of the set X.

(d) $3 \in X$

False. 3 is not an element of the set X.

(e) $\{1, 2\} \in X$

True. $\{1, 2\}$ is an element of the set X.

(f) $\{1, 2\} \subseteq X$

True. Since $1 \in X$ and $2 \in X$, $\{1, 2\}$ is a subset of the set X.

(g) $\{2, 4\} \subseteq X$

True. Since $2 \in X$ and $4 \in X$, $\{2, 4\}$ is a subset of the set X.

(h) $\{2, 4\} \in X$

False. $\{2, 4\}$ is not an element of the set X.

(i) $\{2, 3\} \subseteq X$

False. Although 2 is an element of the set X, 3 is not. Therefore, $\{2, 3\}$ is not a subset of the set X.

(j) $\{2, 3\} \in X$

False. Although 2 and 3 are elements of the set X, $\{2, 3\}$ is not an element of the set, but a subset. Therefore, $\{2, 3\} \in X$ is not true.

(k) $|X| = 7$

False. Since there are 6 elements in the set X, $1, \{1\}, \{1, 2\}, 2, \{3\}$, and 4, the cardinality of the set X is 6.

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Exercise 3.2.4

(b) Let $A = \{1, 2, 3\}$. What is $\{X \in P(A) : 2 \in X\}$?

$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$
the elements of P(A) that contains 2 are $\{2\}, \{1, 2\}, \{2, 3\}$, and $\{1, 2, 3\}$.

$$X = \{\{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$$

Question 9

Exercise 3.3.1

$$A = \{-3, 0, 1, 417\}$$
$$B = \{-12, -5, 1, 4, 6\}$$
$$C = \{x \in \mathbb{Z} : x \text{ is odd}\}$$
$$D = \{x \in \mathbb{Z} : x \text{ is positive}\}$$

(c) $A \cap C = \{-3, 1, 17\}$

(d) $A \cup (B \cap C) = \{-5, -3, 0, 1, 4, 17\}$

(e) $A \cap B \cap C = \{1\}$

Exercise 3.3.3

$$A_i = \{i^0, i^1, i^2\}$$
$$B_i = \left\{x \in \mathbb{R} : -i \leq x \leq \tfrac{1}{i}\right\}$$
$$C_i = \left\{x \in \mathbb{R} : -\tfrac{1}{i} \leq x \leq \tfrac{1}{i}\right\}$$

(a) $\bigcap_{i=2}^{5} A_i$

$A_2 = \{1, 2, 4\} \qquad A_3 = \{1, 3, 9\} \qquad A_4 = \{1, 4, 16\} \qquad A_5 = \{1, 5, 25\}$

$$\begin{aligned}\bigcap_{i=2}^{5} A_i &= \{1, 2, 4\} \cap \{1, 3, 9\} \cap \{1, 4, 16\} \cap \{1, 5, 25\} \\ &= \{1\}\end{aligned}$$

(b) $\bigcup_{i=2}^{5} A_i$

$A_2 = \{1, 2, 4\} \qquad A_3 = \{1, 3, 9\} \qquad A_4 = \{1, 4, 16\} \qquad A_5 = \{1, 5, 25\}$

$$\begin{aligned}\bigcup_{i=2}^{5} A_i &= \{1, 2, 4\} \cup \{1, 3, 9\} \cup \{1, 4, 16\} \cup \{1, 5, 25\} \\ &= \{1, 2, 3, 4, 5, 9, 16, 25\}\end{aligned}$$

(e) $\bigcap_{i=1}^{100} C_i$

when $i = 1, C_1 = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$

when $i = 100, C_{100} = \left\{x \in \mathbb{R} : -\frac{1}{100} \leq x \leq \frac{1}{100}\right\}$

$$\begin{aligned}\bigcap_{i=1}^{100} C_i &= C_1 \cap C_2 \cap C_3 \cap ... \cap C_{100} \\ &= C_{100} \\ &= \left\{x \in \mathbb{R} : -\tfrac{1}{100} \leq x \leq \tfrac{1}{100}\right\}\end{aligned}$$

(f) $\bigcup_{i=1}^{100} C_i$

when $i = 1, C_1 = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$

when $i = 100, C_{100} = \left\{x \in \mathbb{R} : -\frac{1}{100} \leq x \leq \frac{1}{100}\right\}$

$$\begin{aligned}\bigcup_{i=1}^{100} C_i &= C_1 \cup C_2 \cup C_3 \cup ... \cup C_{100} \\ &= C_1 \\ &= \{x \in \mathbb{R} : -1 \leq x \leq 1\}\end{aligned}$$

Exercise 3.3.4

$$A = \{a, b\}$$
$$B = \{b, c\}$$

(b) $P(A \cup B)$

$A \cup B = \{a, b, c\}$
$P(A \cup B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$

(d) $P(A) \cup P(B)$

$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
$P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$
$P(A) \cup P(B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$

Question 10

Exercise 3.5.1

(b) $B \times A \times C$

$B \times A \times C =$ (foam, tall, non-fat)

(c) $B \times C$

	foam	no-foam
non-fat	(foam, non-fat)	(no-foam, non-fat)
whole	(foam, whole)	(no-foam, whole)

$B \times C =$ {(foam, non-fat), (no-foam, non-fat), (foam, whole), (no-foam, whole)}

Exercise 3.5.3

(b) $\mathbb{Z}^2 \subseteq \mathbb{R}^2$

True. Since $\mathbb{Z} \subseteq \mathbb{R}$, if $x \in Z$ and $y \in Z$, then $x \in R$ and $y \in R$. So, if $(x, y) \in \mathbb{Z}^2$, then $(x, y) \in \mathbb{R}^2$

(c) $\mathbb{Z}^2 \cap \mathbb{Z}^3 = \emptyset$

True. $\mathbb{Z}^2$ is a set of ordered pairs of integers and $\mathbb{Z}^3$ is a set of ordered triple of integers. So, the two sets do not have elements in common.

(e) for any three sets, A, B, and C, if $A \subseteq B$, then $A \times C \subseteq B \times C$

True. If $(a, b) \in A$, then $(a, b) \in B$. Therefore, if $(x, y) \in A \times C$, then $(x, y) \in B \times C$.

Exercise 3.5.6

(d) $\{xy$: **where** $x \in \{0\} \cup \{0\}^2$ **and** $y \in \{1\} \cup \{1\}^2\}$

$\{0\} \cup \{0\}^2 = \{0, 00\}$

$\{1\} \cup \{1\}^2 = \{1, 11\}$

Let $A = \{xy$: where $x \in \{0\} \cup \{0\}^2$ and $y \in \{1\} \cup \{1\}^2\}$

$A = \{01, 011, 001, 0011\}$

(e) $\{xy : x \in \{aa, ab\}$ **and** $y \in \{a\} \cup \{a\}^2\}$

$\{a\} \cup \{a\}^2 = \{a, aa\}$

let $B = \{xy : x \in \{aa, ab\}$ and $y \in \{a\} \cup \{a\}^2\}$

$B = \{aaa, aba, aaaa, abaa\}$

Exercise 3.5.7

$$A = \{a\}$$
$$B = \{b, c\}$$
$$C = \{a, b, d\}$$

(c) $(A \times B) \cup (A \times C)$

$A \times B = \{ab, ac\}$
$A \times C = \{aa, ab, ad\}$
$(A \times B) \cup (A \times C) = \{aa, ab, ac, ad\}$

(f) $P(A \times B)$

$A \times B = \{ab, ac\}$
$P(A \times B) = \{\emptyset, \{ab\}, \{ac\}, \{ab, ac\}\}$

(g) $P(A) \times P(B)$

$P(A) = \{\emptyset, \{a\}\}$
$P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$

	$\emptyset$	$\{a\}$
$\emptyset$	$(\emptyset, \emptyset)$	$(\{a\}, \emptyset)$
$\{b\}$	$(\emptyset, \{b\})$	$(\{a\}, \{b\})$
$\{c\}$	$(\emptyset, \{c\})$	$(\{a\}, \{c\})$
$\{b, c\}$	$(\emptyset, \{b, c\})$	$(\{a\}, \{b, c\})$

$$P(A) \times P(B) = \{(\emptyset, \emptyset), (\{a\}, \emptyset), (\emptyset, \{b\}), (\{a\}, \{b\}), (\emptyset, \{c\}), (\{a\}, \{b\}), (\emptyset, \{c\}), (\{a\}, \{c\}), (\emptyset, \{b, c\}), (\{a\}, \{b, c\})\}$$

Question 11

Exercise 3.6.2

(b) $(B \cup A) \cap \left(\overline{B} \cup A\right) = A$

1	$(B \cup A) \cap \left(\overline{B} \cup A\right)$	
2	$A \cup \left(B \cap \overline{B}\right)$	Distributive law, 1
3	$A \cup \emptyset$	Complement law, 2
4	A	Identity Law, 3

(c) $\overline{A \cap \overline{B}} = \overline{A} \cup B$

1	$\overline{A \cap \overline{B}} = \overline{A} \cup B$	
2	$\overline{A} \cup \overline{\overline{B}}$	De Morgan's law, 1
3	$\overline{A} \cup B$	Double complement law, 2

Exercise 3.6.3

(b) $A - (B \cap A) = A$
Let $A = \{1, 2, 3\}$ and $B = \{1, 2\}$.
$A \cap B = \{1, 2\}$
$A - (A \cap B) = \{3\} \neq A$

(d) $(B - A) \cup A = A$
Let $A = \{2, 3\}$ and $B = \{1, 2\}$.
$B - A = \{1\}$
$(B - A) \cup A = \{1, 2, 3\} \neq A$

Exercise 3.6.4

(b) $A \cap (B - A) = \emptyset$

1	$A \cap (B - A)$	
2	$A \cap \left(B \cap \overline{A}\right)$	Set Subtraction law, 1
3	$\left(A \cap \overline{A} \cap B\right)$	Associative law, 2
4	$\emptyset \cap B$	Complement Law, 3
4	$\emptyset$	Domination law, 4

(c) $A \cup (B - A) = A \cup B$

1	$A \cup (B - A) = A \cup B$	
2	$A \cup \left(B \cap \overline{A}\right)$	Set subtraction law, 1
3	$(A \cup B) \cap \left(A \cup \overline{A}\right)$	Distributive law, 2
4	$(A \cup B) \cap U$	Complement law, 3
5	$A \cup B$	Identity law, 4