HW 6

Q5

Joo Hyun Lee
jhl504@nyu.edu
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Question 5

(a) $5n^3 + 2n^2 + 3n = \Theta(n^3)$

Let $f(n) = 5n^3 + 2n^2 + 3n$ and $g(n) = n^3$. We will prove that $c_1 g(n) \leq f(n) \leq c_2 g(n)$.

For any $n \geq 1$, $5n^3 \leq 5n^3 + 2n^2 + 3n$. So, $c_1 = 5$.
Therefore, $5g(n) \leq f(n)$ and $f = \Omega(g)$

For any $n \geq 1$, $5n^3 + 2n^2 + 3n \leq 5^3 + 2n^3 + 3n^3 = 10n^3$. So, $c_2 = 10$.
Therefore, $f(n) \leq 10g(n)$ and $f = O(g)$.

If we take $c_1 = 5, c_2 = 10$, and $n_0 = 1$, for all $n \geq n_0$, $5n^3 + 2n^2 + 3n = \Theta(n^3)$. ■

(b) $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

Let $f(n) = \sqrt{7n^2 + 2n - 8}$ and $g(n) = n$. We will prove that $c_1 g(n) \leq f(n) \leq c_2 g(n)$.

For any positive $n, \sqrt{7n^2 + 2n - 8} \leq \sqrt{7n^2 + 2n^2} = \sqrt{9n^2} = 3n$. So, $c_2 = 3$.
Therefore, $f(n) \leq 3g(n)$ and $f = O(g)$.

For $2n - 8 \geq 0, \sqrt{7n^2} \leq \sqrt{7n^2 + 2n - 8}$. So, $c_1 = \sqrt{7} \approx 2$.
Therefore, $2g(n) \leq f(n)$ and $f = \Omega(g)$.

Since $2n - 8 \geq 0$, $n_0 = \sqrt{4} = 2$.

If we take $c_1 = 2, c_2 3$, and $n_0 = 2$, for all $n \geq 2, \sqrt{7n^2 + 2n - 8} = \Theta(n)$. ■