HW 1

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Question 1

A.1) $(\mathbf{10011011})_\mathbf{2}$

$= 1 \cdot 2^0 + 1 \cdot 2^1 + 0 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + 0 \cdot 2^5 + 0 \cdot 2^6 + 1 \cdot 2^7$

$= (155)_{10}$

A.2) $(\mathbf{456})_\mathbf{7}$

$= 6 \cdot 7^0 + 5 \cdot 7^1 + 4 \cdot 7^2$

$= (237)_{10}$

A.3) $(\mathbf{38A})_\mathbf{16}$

$= 10 \cdot 16^0 + 8 \cdot 16^1 + 3 \cdot 16^2$

$= (906)_{10}$

A.4) $(\mathbf{2214})_\mathbf{5}$

$= 4 \cdot 5^0 + 1 \cdot 5^1 + 2 \cdot 5^2 + 2 \cdot 5^3$

$= (309)_{10}$

B.1) $(\mathbf{69})_\mathbf{10}$

$$\begin{array}{cccccccc} \underline{0} & \underline{1} & \underline{0} & \underline{0} & \underline{0} & \underline{1} & \underline{0} & \underline{1} \\ 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$2^6 < 69 < 2^7 \rightarrow$ "1" in the 2^6 digit $\rightarrow 69 - 2^6 = 5$

$2^2 < 5 < 2^3 \rightarrow$ "1" in the 2^2 digit $\rightarrow 5 - 2^2 = 1$

$1 = 2^0 \rightarrow$ "1" in the 2^0 digit $\rightarrow 1 - 1 = 0$

$$\therefore (69)_{10} = (1000101)_2$$

B.2) $(485)_{10}$

$$\begin{array}{cccccccccc} \underline{0} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{0} & \underline{0} & \underline{1} & \underline{0} & \underline{1} \\ 2^9 & 2^8 & 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$2^8 < 485 < 2^9 \rightarrow$ "1" in the 2^8 digit $\rightarrow 485 - 256 = 229$
$2^7 < 229 < 2^8 \rightarrow$ "1" in the 2^7 digit $\rightarrow 229 - 128 = 101$
$2^6 < 101 < 2^7 \rightarrow$ "1" in the 2^6 digit $\rightarrow 101 - 64 = 37$
$2^5 < 37 < 2^6 \rightarrow$ "1" in the 2^5 digit $\rightarrow 37 - 32 = 5$
$2^2 < 5 < 2^8 \rightarrow$ "1" in the 2^2 digit $\rightarrow 5 - 4 = 1$
$2^0 = 1 \rightarrow$ "1" in the 2^0 digit $\rightarrow 1 - 1 = 0$

$$\therefore (485)_{10} = (111100101)_2$$

B.3) $(6D1A)_{16}$

$6 = 0 \cdot 2^0 + 1 \cdot 2^1 + 1 \cdot 2^2 + 0 \cdot 2^3 = 0110$
$\text{D} = 13 = 1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 = 1101$
$1 = 1 \cdot 2^0 + 0 \cdot 2^1 + 0 \cdot 2^2 + 0 \cdot 2^3 = 0001$
$\text{A} = 10 = 1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 + 0 \cdot 2^3 = 1010$

$$\therefore (6\text{D}1\text{A})_{16} = (110110100011010)_2$$

C.1) $(1101011)_2$

$= 1 \cdot 2^0 + 1 \cdot 2^1 + 0 \cdot 2^2 + 1 \cdot 2^3 + 0 \cdot 2^4 + 1 \cdot 2^5 + 1 \cdot 2^6$
$= 2^0(1 \cdot 2^0 + 1 \cdot 2^1 + 0 \cdot 2^2 + 1 \cdot 2^3) + 2^4(0 \cdot 2^0 + 1 \cdot 2^1 + 1 \cdot 2^2)$
$= 2^0(11) + 2^4(6)$
$= (6\text{B})_{16}$

C.2) $(895)_{10}$

$895 \div 16 = 55\text{R}15 \rightarrow$ "F"
$55 \div 16 = 3\text{R}7 \rightarrow$ "7"
$3 \div 16 = 0\text{R}3 \rightarrow$ "3"

$$\therefore (895)_{10} = (37\text{F})_{16}$$

Question 2

1) $(7566)_8 + (4515)_8$

$$
\begin{array}{r}
111 \\
7566_8 \\
+\underline{\;4515}_8 \\
14303_8
\end{array}
$$

2) $(10110011)_2 + (1101)_2$

$$
\begin{array}{r}
111111 \\
10110011_2 \\
+\underline{\;00001101}_2 \\
11000000_2
\end{array}
$$

3) $(7A66)_{16} + (45C5)_{16}$

$$
\begin{array}{r}
11 \\
7A66_{16} \\
+\underline{\;45C5}_{16} \\
C02B_{16}
\end{array}
$$

4) $(3022)_5$ - $(2433)_5$

$$
\begin{array}{r}
46 \\
2\cancel{5}\cancel{1}7 \\
\cancel{3}\cancel{0}\cancel{2}\cancel{2}_5 \\
-\underline{\;2433}_5 \\
34_5
\end{array}
$$

Question 3

A.1) $(124)_{10}$

$$\begin{array}{cccccccc} \underline{0} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{0} & \underline{0} \\ 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$2^6 < 124 < 2^7 \rightarrow$ "1" in the 2^6 digit $\rightarrow 124 - 2^6 = 60$
$2^5 < 60 < 2^6 \rightarrow$ "1" in the 2^5 digit $\rightarrow 60 - 2^5 = 28$
$2^4 < 28 < 2^5 \rightarrow$ "1" in the 2^4 digit $\rightarrow 28 - 2^4 = 12$
$2^3 < 12 < 2^4 \rightarrow$ "1" in the 2^3 digit $\rightarrow 12 - 2^3 = 4$
$4 = 2^2 \rightarrow$ "1" in the 2^2 digit $\rightarrow 4 - 4 = 0$

$$\therefore (124)_{10} = (01111100)_{\text{8-bit 2's comp}}$$

A.2) $(-124)_{10}$

$$(124)_{10} = (01111100)_{\text{8-bit 2's comp}}$$

$$\begin{array}{r} 111112 \\ \cancel{1}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}00 \\ - \underline{\ 01111100}_2 \\ 10000100_2 \end{array}$$

$$\therefore (-124)_{10} = (10000100)_{\text{8-bit 2's comp}}$$

A.3) $(109)_{10}$

$$\begin{array}{cccccccc} \underline{0} & \underline{1} & \underline{1} & \underline{0} & \underline{1} & \underline{1} & \underline{0} & \underline{1} \\ 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$2^6 < 109 < 2^7 \rightarrow$ "1" in the 2^6 digit $\rightarrow 109 - 2^6 = 45$
$2^5 < 45 < 2^6 \rightarrow$ "1" in the 2^5 digit $\rightarrow 45 - 2^5 = 13$
$2^3 < 13 < 2^4 \rightarrow$ "1" in the 2^3 digit $\rightarrow 13 - 2^3 = 5$
$2^2 < 5 < 2^3 \rightarrow$ "1" in the 2^2 digit $\rightarrow 5 - 2^2 = 1$
$1 = 2^0 \rightarrow$ "1" in the 2^0 digit $\rightarrow 1 - 1 = 0$

$$\therefore (109)_{10} = (01101101)_{\text{8-bit 2's comp}}$$

A.4) (-79)₁₀

$$\begin{array}{cccccccc} \underline{0} & \underline{1} & \underline{0} & \underline{0} & \underline{1} & \underline{1} & \underline{1} & \underline{1} \\ 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$2^6 < 79 < 2^7 \rightarrow$ "1" in the 2^6 digit $\rightarrow 79 - 2^6 = 15$
$2^3 < 15 < 2^4 \rightarrow$ "1" in the 2^3 digit $\rightarrow 15 - 2^3 = 7$
$2^2 < 7 < 2^3 \rightarrow$ "1" in the 2^2 digit $\rightarrow 7 - 2^2 = 3$
$2^1 < 3 < 2^2 \rightarrow$ "1" in the 2^1 digit $\rightarrow 3 - 2^1 = 1$
$1 = 2^0 \rightarrow$ "1" in the 2^0 digit $\rightarrow 1 - 1 = 0$

$$(109)_{10} = (01001111)_{\text{8-bit 2's comp}}$$

$$\begin{array}{r} 11111112 \\ \cancel{1}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}_2 \\ -\ \underline{01001111_2} \\ 10110001_2 \end{array}$$

$$\therefore (-79)_{10} = (10110001)_{\text{8-bit 2's comp}}$$

B.1) $(00011110)_{\text{8-bit 2's comp}}$

$= 0 \cdot 2^0 + 1 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + 0 \cdot 2^5 + 0 \cdot 2^6 + 0 \cdot 2^7$
$= 2 + 4 + 8 + 16$
$= (30)_{10}$

B.2) $(11100110)_{\text{8 bit 2's comp}}$

$= 0 \cdot 2^0 + 1 \cdot 2^1 + 1 \cdot 2^2 + 0 \cdot 2^3 + 0 \cdot 2^4 + 1 \cdot 2^5 + 1 \cdot 2^6 - 1 \cdot 2^7$
$= 2 + 4 + 32 + 64 - 128$
$= (-26)_{10}$

B.3) $(00101101)_{\text{8 bit 2's comp}}$

$= 1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 + 0 \cdot 2^4 + 1 \cdot 2^5 + 0 \cdot 2^6 + 0 \cdot 2^7$
$= 1 + 4 + 8 + 32$
$= (45)_{10}$

B.4) $(10011110)_{\text{8 bit 2's comp}}$

$$
\begin{aligned}
&= 0 \cdot 2^0 + 1 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + 0 \cdot 2^5 + 0 \cdot 2^6 - 1 \cdot 2^7 \\
&= 2 + 4 + 8 + 16 - 128 \\
&= (-98)_{10}
\end{aligned}
$$

Question 4

Exercise 1.2.4

(b) $\neg(p \vee q)$

p	q	$(p \vee q)$	$\neg(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

(c) $r \vee (p \wedge \neg q)$

p	q	r	$\neg q$	$(p \wedge \neg q)$	$r \vee (p \wedge \neg q)$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	F	F

Exercise 1.3.4

(b) $(p \to q) \to (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(d) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

Question 5

Exercise 1.2.7

B: Applicant presents a birth certificate.
D: Applicant presents a driver's license.
M: Applicant presents a marriage license.

(b) The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

$$(B \land D) \lor (B \land M) \lor (D \land M)$$

(c) Applicant must present either a birth certificate or both a driver's license and a marriage license.

$$B \lor (D \land M)$$

Exercise 1.3.7

s: a person is a senior
y: a person is at least 17 years of age
p: a person is allowed to park in the school parking lot

(b) A person can park in the school parking lot if they are a senior or at least seventeen years of age.

$$(s \lor y) \to p$$

(c) Being 17 years of age is a necessary condition for being able to park in the school parking lot.

$$p \to y$$

(d) A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

$$p \leftrightarrow (s \land y)$$

(e) Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

$$p \to (s \vee y)$$

Exercise 1.3.9

y: the applicant is at least eighteen years old
p: the applicant has parental permission
c: the applicant can enroll in the course

(c) The applicant can enroll in the course only if the applicant has parental permission.

$$c \to p$$

(d) Having parental permission is a necessary condition for enrolling in the course.

$$c \to p$$

Question 6

Exercise 1.3.6

(b) Maintaining a B average is necessary for Joe to be eligible for the honors program.

If Joe wants to be eligible for the honors program, then he needs to maintain a B average

(c) Rajiv can go on the roller coaster only if he is at least four feet tall.

If Rajiv can goes on the roller coaster, then he is at least four feet tall.

(d) Rajiv can go on the roller coaster if he is at least four feet tall.

If Rajiv is at least four feet tall, he can go on the roller coaster.

Exercise 1.3.10

p is true
q is false
r is unknown

(c) $(p \vee r) \leftrightarrow (q \wedge r)$

False.
If r is true, then the expression is false.
if r is false, then the expression is false.

(d) $(p \wedge r) \leftrightarrow (q \wedge r)$

Unknown.
If r is true, then the expression is false.
If r is false, then the expression is true.

(e) $p \to (r \vee q)$

Unknown.
If r is true, then the expression is true.
if r is false, then the expression is false.

(f) $(p \wedge q) \to r$

True.
If r is true, then the expression is true.
if r is false, then the expression is true.

Question 7

Exercise 1.4.5

j: Sally got the job.
l: Sally was late for her interview
r: Sally updated her resume.

(b) If Sally did not get the job, then she was late for her interview or did not update her resume. If Sally updated her resume and was not late for her interview, then she got the job.

$$\neg j \to (l \vee \neg r)$$
$$(r \wedge \neg l) \to j$$

j	l	r	$\neg j \to (l \vee \neg r)$	$(r \wedge \neg l) \to j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

$\therefore$ the statements are logically equivalent.

(c) If Sally got the job then she was not late for her interview. If Sally did not get the job, then she was late for her interview.

$$j \to \neg l$$
$$\neg j \to l$$

j	l	$j \to \neg l$	$\neg j \to l$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

$\therefore$ the statements are not logically equivalent.

(d) If Sally updated her resume or she was not late for her interview, then she got the job. If Sally got the job, then she updated her resume and was not late for her interview.

$$(r \vee \neg l) \rightarrow j$$
$$j \rightarrow (r \wedge \neg l)$$

j	l	r	$(r \vee \neg l) \rightarrow j$	$j \rightarrow (r \wedge \neg l)$
T	T	T	T	F
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

$\therefore$ the statements are not logically equivalent.

Question 8

Exercise 1.5.2

(c) $(p \to q) \land (p \to r) \equiv p \to (q \land r)$

$(p \to q) \land (p \to r)$	
$(\neg p \lor q) \land (\neg p \lor r)$	Conditional Identity
$\neg p \lor (q \land r)$	Distributive Property
$p \to (q \land r)$	Conditional Identity

(f) $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

$\neg(p \lor (\neg p \land q))$	
$\neg((p \lor \neg p) \land (p \lor q))$	Distributive Property
$\neg(T \land (p \lor q))$	Complement Law
$\neg(p \lor q)$	Identity Law
$\neg p \land \neg q$	De Morgan's Law

(i) $(p \land q) \to r \equiv (p \land \neg r) \to \neg q$

$(p \land q) \to r$	
$\neg(p \land q) \lor r$	Conditional Identity
$\neg p \lor \neg q \lor r$	De Morgan's Law
$\neg p \lor r \lor \neg q$	Commutative Law
$\neg(p \land \neg r) \lor \neg q$	De Morgan's Law
$(p \land \neg r) \to \neg q$	Conditional Identity

Exercise 1.5.3

(c) $\neg r \lor (\neg r \to p)$

$\neg r \lor (\neg r \to p)$	
$\neg r \lor (\neg\neg r \lor p)$	Conditional Identity
$\neg r \lor (r \lor p)$	Double Negation Law
$(\neg r \lor r) \lor p$	Associative Law
$T \lor p$	Complement Law
T	Domination Law

,

(d) $\neg(p \to q) \to \neg q$

$\neg(p \to q) \to \neg q$	
$\neg(\neg p \vee q) \to \neg q$	Conditional Identity
$(p \wedge \neg q) \to \neg q$	De Morgan's law
$\neg(p \wedge \neg q) \vee \neg q$	De Morgan's Law
$\neg((p \wedge \neg q) \wedge q)$	De Morgan's Law
$\neg(p \wedge (\neg q \wedge q))$	Associative Law
$\neg(p \wedge F)$	Complement Law
$\neg F$	Domination Law
T	Complement Law

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Question 9

Exercise 1.6.3

(c) There is a number that is equal to its square.

$$\exists x(x = x^2)$$

(d) Every number is less than or equal to its square plus 1.

$$\forall x(x \leq x^2 + 1)$$

Exercise 1.7.4

S(x): x was sick yesterday
W(x): x went to work yesterday
V(x): x was on vacation yesterday

(b) Everyone was well and went to work yesterday.

$$\forall x(\neg S(x) \land W(x))$$

(c) Everyone who was sick yesterday did not go to work.

$$\forall x(S(x) \rightarrow W(x))$$

(d) Yesterday someone was sick and went to work.

$$\exists x(S(x) \land W(x))$$

Question 10

Exercise 1.7.9

	$P(x)$	$Q(x)$	$R(x)$
a	T	T	F
b	T	F	F
c	F	T	F
c	T	T	F
d	T	T	T

(c) $\exists x((x = c) \to P(x))$

True. Example a

(d) $\exists x(Q(x) \land R(x))$

True. Example: e

(e) $Q(a) \land P(d)$

True. $Q(a) = P(d) = T$

(f) $\forall x((x \neq b) \to Q(x))$

True. Example: a, c, d, e

(g) $\forall x(P(x) \lor R(x))$

False. Counterexample: c

(h) $\forall x(R(x) \to P(x))$

True.

(i) $\exists x(Q(x) \lor R(x))$

True. Example: a, c, d, e

Exercise 1.9.2

P	1	2	3
1	T	F	T
2	T	F	T
3	T	T	F

Q	1	2	3
1	F	F	F
2	T	T	T
3	T	F	F

S	1	2	3
1	F	F	F
2	F	F	F
3	F	F	F

(b) $\exists x \forall y Q(x, y)$

True. If $x = 2$, then $Q(2, y)$ is true for all values of y.

(c) $\exists y \forall x P(x, y)$

True. If $y = 1$, then $P(x, 1)$ is true for all values of x.

(d) $\exists x \exists y S(x, y)$

False. there is no pair that makes $S(x, y)$ true.

(e) $\forall x \exists y Q(x, y)$

False. If $x = 1$, then $Q(1, y)$ is false for all values of y.

(f) $\forall x \exists y P(x, y)$

True. $P(1, 1)$, $P(2, 1)$, and $P(3, 1)$ are true.

(g) $\forall x \forall y P(x, y)$

False. The expression is true when the entire table is filled with T's.

(h) $\exists x \exists y Q(x, y)$

True. The value of at least one row in at least one column is true

(i) $\forall x \forall y \neg S(x, y)$

True. For all combinations of x and y, the expression is true.

Question 11

Exercise 1.10.4

(c) There are two numbers whose sum is equal to their product.

$$\exists x \exists y(x + y = x \times y)$$

(d) The ratio of every two positive numbers is also positive.

$$\forall x \forall y \Big[(x > 0) \wedge (y > 0) \rightarrow \left(\tfrac{x}{y} > 0\right)\Big]$$

(e) The reciprocal of every positive number less than one is greater than one.

$$\forall x((x > 0) \wedge (x < 1)) \rightarrow \left(\tfrac{1}{x} > 1\right)$$

(f) There is no smallest number.

$$\neg \exists x \forall y(x < y)$$

(g) Every number other than 0 has a multiplicative inverse.

$$\forall x \exists y((x \neq 0) \rightarrow (x \cdot y = 1))$$

Exercise 1.10.7

P(x, y): x knows y's phone number. (A person may or may not know their own phone number.)
D(x): x missed the deadline.
N(x): x is a new employee.

(c) There is at least one new employee who missed the deadline.

$$\exists x(N(x) \wedge D(x))$$

(d) Sam knows the phone number of everyone who missed the deadline.

$$\forall y(D(y) \rightarrow P(\text{Sam}, y))$$

(e) There is a new employee who knows everyone's phone number.

$$\exists x \forall y((N(x) \wedge P(x, y)))$$

(f) Exactly one new employee missed the deadline.

$$\exists x \forall y((N(x) \land D(x)) \land (((x \neq y) \to N(y)) \to \neg D(y)))$$

Exercise 1.10.10

x: students at a university
y: math classes offered at the university
T(x,y) = student x has taken class y.

Sam is a student at the university and Math 101 is one of the courses offered at the university. Give a logical expression for each sentence.

(c) Every student has taken at least one class other than Math 101.

$$\forall x \exists y(T(x, y) \land (y \neq \text{Math 101}))$$

(d) There is a student who has taken every math class other than Math 101.

$$\exists x \forall y((y \neq \text{Math 101}) \to T(x, y))$$

(e) Everyone other than Sam has taken at least two different math classes.

$$\forall x \exists y \exists z((x \neq \text{Sam}) \to ((y \neq z) \land T(x, y) \land T(x, z))$$

(f) Sam has taken exactly two math classes.

$$\exists y \exists z \forall x((y \neq z) \land T(\text{Sam}, y) \land T(\text{Sam}, z)) \land (((w \neq z) \land (w \neq y) \to \neg T(\text{Sam}, w)))$$

Question 12

Exercise 1.8.2

P(x): x was given the placebo
D(x): x was given the medication
M(x): x had migraines

(b) Every patient was given the medication or the placebo or both.

logical expression: $\forall x(D(x) \lor P(x))$
negation: $\neg\forall x(D(x) \lor P(x))$
De Morgan's law: $\exists x\neg((D(x) \lor P(x))) \equiv \exists x(\neg D(x) \land \neg P(x))$

there exists a patient who was not given the placebo and the medication.

(c) There is a patient who took the medication and had migraines.

logical expression: $\exists x(D(x) \land M(x))$
negation: $\neg\exists x(D(x) \land M(x))$
De Morgan's law: $\forall x(\neg(D(x) \land M(x))) \equiv \forall x(\neg D(x) \lor \neg M(x))$

all patients were not given the medications or did not have migraines or both.

(d) Every patient who took the placebo had migraines.

logical expression: $\forall x(P(x) \rightarrow M(x))$
negation: $\neg\forall x(P(x) \rightarrow M(x))$
De Morgan's law: $\exists x(\neg(P(x) \rightarrow M(x))) \equiv \exists x(\neg(\neg P(x) \lor M(x))) \equiv \exists x(P(x) \land \neg M(x))$

there is a patient who was given the placebo and did not have migraines

(e) There is a patient who had migraines and was given the placebo.

logical expression: $\exists x(M(x) \land P(x))$
negatiion: $\neg\exists x(M(x) \land P(x))$
De Morgan's law: $\forall x(\neg(M(x) \land P(x))) \equiv \forall x(\neg M(x) \lor \neg P(x))$

all patients did not have migraines or was not given the placebo or both.

Exercise 1.9.4

(c) $\exists x \forall y (P(x, y) \to Q(x, y))$

$$\begin{aligned} \neg \exists x \forall y (P(x, y) \to Q(x, y)) &\equiv \forall x \exists y \neg(\neg P(x, y) \vee Q(x, y)) \\ &\equiv \forall x \exists y (P(x, y) \wedge \neg Q(x, y)) \end{aligned}$$

(d) $\exists x \forall y (P(x, y) \leftrightarrow P(y, x))$

$$\begin{aligned} \neg \exists x \forall y (P(x, y) \leftrightarrow P(y, x)) &\equiv \forall x \exists y [\neg(P(x, y) \leftrightarrow P(y, x))] \\ &\equiv \forall x \exists y [\neg\{(P(x, y) \to P(y, x)) \wedge (P(y, x) \to P(x, y))\}] \\ &\equiv \forall x \exists y [\neg\{(\neg P(x, y) \vee P(y, x)) \wedge (\neg P(y, x) \vee P(x, y))\}] \\ &\equiv \forall x \exists y [\{\neg(\neg P(x, y) \vee P(y, x))\} \vee \{\neg(\neg P(y, x) \vee P(x, y))\}] \\ &\equiv \forall x \exists y [(P(x, y) \wedge \neg P(y, x)) \vee (P(y, x) \wedge P(x, y))] \end{aligned}$$

(e) $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$

$$\neg \exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y) \equiv \forall x \forall y \neg P(x, y) \vee \exists x \exists y \neg Q(x, y)$$