

MT105A Study Guide

Jack Morton (jhm@jemscout.com)

Last updated: May 1, 2018

What will be on the exam?

The exam will be split into two parts, Section A and Section B, for 60 and 20 marks, respectively. The exam is always eight questions long - six questions in the first part and two in the second. ~~I would suggest doing Section B first.~~ In hindsight, do Section A before Section B, because the second section will definitely have a question designed to stump most students, and time is so limited that there isn't a moment to ruminate on it. Leave a question like that until the end.

Section B will present two 'wordy', multi-part questions, that will definitely cover the topics of optimization and constrained optimization, and may possibly cover compound interest or something else.

The optimization problems in Section B will almost definitely be economics problems related to cost, utility and/or profit. The questions may be related to an individual or a firm and the firm may be producing goods as a monopoly or not. Every exam I have looked at has a constrained optimization problem in which the student has to use the method of Lagrange multipliers. Many of the problems will ask the student to solve a general equation of optimization, with variables in place of real numbers, and often the final solution is provided and the instructions are to demonstrate the steps to arrive at that solution.

The key to the exam is to know all of the variations of the optimization problems and to ace this part of the test.

Section A will have six questions that will be much more brief, and cover topics such as:

- integration
- solving variables using a matrix method
- compound interest
- the Hessian test for critical points
- curve sketching
- anything else covered in the course material

If you can successfully answer questions relating to the first five topics then you will have no problem with Section A.

Standard derivatives to memorize:

$f(x)$	$f'(x)$
x^k	kx^{k-1}
e^x	e^x
$\ln(x)$	$\frac{1}{x}$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$

1. The sum rule:

If $h(x) = f(x) + g(x)$ then $h'(x) = f'(x) + g'(x)$

2. The product rule:

If $h(x) = f(x)g(x)$ then $h'(x) = f'(x)g(x) + f(x)g'(x)$

3. The quotient rule:

If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

4. The chain rule:

If $h(x) = s(r(x))$ then $h'(x) = s'(r(x))r'(x)$

Standard integrals to memorize:

$f(x)$	$\int f(x)dx$
$x^n, n \neq -1$	$\frac{x^{n+1}}{n+1} + c$
$\frac{1}{x}$	$\ln x + c$
$\ln x $	$x \ln(x) - x + c$
e^x	$e^x + c$
$e^{\alpha x}$	$\frac{1}{\alpha} e^{\alpha x} + c$
$\sin(x)$	$-\cos(x) + c$
$\cos(x)$	$\sin(x) + c$

1. The addition rule:

$$\int f(x) + g(x)dx = \int f(x)dx + \int g(x)dx$$

2. The constant rule:

$$\int k \times f(x)dx = k \times \int f(x)dx$$

3. Integration by parts:

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

The formula replaces one integral (that on the left) with another (that on the right); the intention is that the one on the right is a simpler integral to evaluate.

4. Integration by substitution:

When we change the variable by putting $x = x(u)$ in the integral of $f(x)$ then we must replace dx by $(\frac{dx}{du})du$

To determine $\int f(x)dx$ we can solve $\int f(x(u))x'(u)du$ and then substitute x back for $x(u)$

For definite integrals, $\int_{x=a}^{x=b} f(x)dx = \int_{u=\alpha}^{u=\beta} f(x(u))x'(u)du$ where α and β are the values of u that correspond to $x = a$ and $x = b$

Partial fractions:

An example:

$$\frac{x}{x^2+5x+4} = \frac{x}{(x+1)(x+4)} = \frac{A_1}{(x+1)} + \frac{A_2}{(x+4)}$$

Cross multiply to obtain:

$$A_1(x+4) + A_2(x+1) = x$$

$$\text{let } x = -4 \Rightarrow A_2(-4+1) = -4 \Rightarrow A_2 = \frac{4}{3}$$

$$\text{let } x = -1 \Rightarrow A_1(-1+4) = -1 \Rightarrow A_1 = -\frac{1}{3}$$

Therefore, the partial fraction is:

$$\frac{4}{3(x+4)} - \frac{1}{3(x+1)}$$

The second partial derivative test:

The second partial derivative test is a method used in multivariable calculus to determine if a critical point of a function is a local minimum, maximum or saddle point.

First find the critical points, where the partial derivatives for each variable is zero.

The Hessian function is: $H(x, y) = f_{xx}f_{yy} - (f_{xy})^2$

If, at a critical point, we have:

$H < 0$, then that point is a *saddle point*.

$H > 0$ and $f_{xx} > 0$ then that point is a *local minimum*.

$H > 0$ and $f_{xx} < 0$ then that point is a *local maximum*.

If $H = 0$ then the test fails!

Note that for single variable functions, the second part of the Hessian test defines the nature of the critical points, (ie. $f_{xx} > 0$ is the minimum, $f_{xx} < 0$ is the maximum).

Economic questions:

Total profit:

$$\pi(q) = TR(q) - TC(q)$$

Revenue is always quantity \times price. This may be in the form of a revenue equation, expressed in quantity or price (usually price), which must be multiplied by the second term.

For example, for two products, X and Y in quantities x and y , the total revenue will be:

$$TR(x, y) = xp_x + yp_y$$

Total cost:

$$TC(q) = VC(q) + FC(q)$$

Total cost equals variable cost plus fixed cost. If you solve total cost through integration, then the constant will be the fixed cost.

Average cost:

$$AC(q) = \frac{TC(q)}{q}$$

Average variable cost:

$$AVC(q) = \frac{VC(q)}{q}$$

Marginal revenue:

The revenue derived from the production of one more unit:

$$MR(q) = TR'(q)$$

Marginal cost:

The cost incurred from the production of one more unit:

$$MC(q) = TC'(q)$$

A common question is to maximize profit using a Lagrange multiplier...