MT105A Study Guide

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What will be on the exam?

The exam will be split into two parts, Section A and Section B, for 60 and 20 marks, respectively. The exam is always eight questions long - six questions in the first part and two in the second. I would suggest doing Section B first.

Section B will present two 'wordy', multi-part questions, that will definitely cover the topics of optimization and constrained optimization, and may possibly cover compound interest or something else.

The optimization problems in Section B will almost definitely be economics problems related to cost, utility and/or profit. The questions may be related to an individual or a firm and the firm may be producing goods as a monopoly or not. Every exam I have looked at has a constrained optimization problem in which the student has to use the method of Lagrange multipliers. Many of the problems will ask the student to solve a general equation of optimization, with variables in place of real numbers, and often the final solution is provided and the instructions are to demonstrate the steps to arrive at that solution.

The key to the exam is to know all of the variations of the optimization problems and to ace this part of the test.

Section A will have six questions that will be much more brief, and cover topics such as:

- integration
- solving variables using a matrix method
- compound interest
- the Hession test for critical points
- curve sketching
- anything else covered in the course material

If you can successfuly answer questions relating to the first five topics then you will have no problem with Section A.

Standard derivatives to memorize:

$f(x)$	$f'(x)$
x^k	kx^{k-1}
e^x	e^x
$\ln(x)$	$\frac{1}{x}$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$

1. The sum rule:

If $h(x) = f(x) + g(x)$ then $h'(x) = f'(x) + g'(x)$

2. The product rule:

If $h(x) = f(x)g(x)$ then $h'(x) = f'(x)g(x) + f(x)g'(x)$

3. The quotient rule:

If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x)-f(x)g'(x)}{g(x)^2}$

4. The chain rule:

If $h(x) = s(r(x))$ then $h'(x) = s'(r(x))r'(x)$

Standard integrals to memorize:

$f(x)$	$\int f(x)dx$
$x^n,\ n \neq -1$	$\frac{x^{n+1}}{n+1} + c$
$\frac{1}{x}$	$\ln\lvert x\rvert + c$
$\ln\lvert x\rvert$	$x\ln(x) - x + c$
e^x	$e^x + c$
$e^{\alpha x}$	$\frac{1}{\alpha}e^{\alpha x} + c$
$\sin(x)$	$-\cos(x) + c$
$\cos(x)$	$\sin(x) + c$

1. The addition rule:

$\int f(x) + g(x)dx = \int f(x)dx + \int g(x)dx$

2. The constant rule:

$\int k \times f(x)dx = k \times \int f(x)dx$

3. Integration by parts:

$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$

The formula replaces one integral (that on the left) with another (that on the right); the intention is that the one on the right is a simpler integral to evaluate.

4. Integration by substituion:

When we change the variable by putting $x = x(u)$ in the integral of $f(x)$ then we must replace dx by $(\frac{dx}{du})du$

To determine $\int f(x)dx$ we can solve $\int f(x(u))x'(u)du$ and then substitute x back for $x(u)$

For definite integrals, $\int_{x=a}^{x=b} f(x)dx = \int_{u=\alpha}^{u=\beta} f(x(u))x'(u)du$ where α and β are the values of u that correspond to $x = a$ and $x = b$

Partial fractions:

An example:

$$\frac{x}{x^2+5x+4} = \frac{x}{(x+1)(x+4)} = \frac{A_1}{(x+1)} + \frac{A_2}{(x+4)}$$

Cross multiply to obtain:

$A_1(x+4) + A_2(x+1) = x$

let $x = -4 \implies A_2(-4+1) = -4 \implies A_2 = \frac{4}{3}$

let $x = -1 \implies A_1(-1+4) = -1 \implies A_1 = -\frac{1}{3}$

Therefore, the partial fracion is:

$$\frac{4}{3(x+4)} - \frac{1}{3(x+1)}$$

The second partial derivative test

The second partial derivative test is a method used in multivariable calculus to determine if a critical point of a function is a local minimum, maxium or saddle point.

First find the critical points, where the partial derivatives for each variable is zero.

The Hessian function is: $H(x,y) = f_{xx}f_{yy} - (f_{xy})^2$

If, at a critical point, we have:

$H < 0$, then that point is a *saddle point.*

$H > 0$ and $f_{xx} > 0$ then that point is a *local minimum.*

$H > 0$ and $f_{xx} < 0$ then that point is a *local maximum.*

If $H = 0$ then the test fails!