CENTER FOR APPLIED MATHEMATICS SEMINAR

Techniques for the Fitting and Verification of Linear/Non-Linear Models using DATAPLOT

James J. Filliben*
National Bureau of Standards
Washington, D. C. 20234

Green Auditorium, Administration Building
10:30--12:00, Monday, November 20, 1978

ABSTRACT

An important practical problem which arises in a wide variety of NBS experimental activities is that of model-building and verification. Heretofore, linear models have always played (due to their simplicity and mathematical tractability) an important role in describing physical phenomena and in gaining insight into underlying mechanisms. Such insight has frequently resulted, however, in a growing awareness of the fundamental limitations of linear models and a begrudging acceptance of the fact that many phenomena are intrinsically non-linear.

The main purpose of this talk is to convey the fact that recent advances in statistical software have been made with the net effect that the non-linear modeling problem is now "solved" in the sense that fitting such models is now no longer a major programming effort for the analyst--rather, it has become a one line/one command operation. The use of the DATAPLOT FIT command frees the analyst from typical programming details, and allows the analyst to concentrate on the physical modeling problem at hand. Whereas before, the several iterations typical for exploratory non-linear model-building would take weeks and months, it now takes minutes. Enormous savings of time--the analyst's time--thus result.

This talk will demonstrate by several examples the ease with which the analyst may now carry out non-linear (or linear) modeling at NBS. In addition to providing details regarding the DATAPLOT FIT command (by which such non-linear modeling is done), this talk will also cover:

1) general principles for non-linear model construction;
2) guidelines for which models to choose (and avoid);
3) techniques for model verification;
4) considerations for choosing between competing models;
5) statistical and graphical techniques for assessing goodness of fit.

An indefinitely large number of non-linear models can be handled with the DATAPLOT FIT command; among the more popular and frequently-encountered models that will be explicitly included in the discussion and examples are the following:

1) linear and polynomial models;
2) exponential models;
3) models involving powers to be estimated;
4) square root models;
5) exponential over polynomial models;
6) Lorentzian models;
7) Gaussian models;
8) Bessel function models;
9) Chebychev function models;
10) rational function models.

The importance of this last family (rational functions) as a general and flexible modeling family will be discussed and demonstrated. The handout from this talk will illustrate the application of the DATAPLOT FIT command to over 25 different NBS linear/non-linear modeling problems; DATAPLOT instructions and corresponding output will be provided for each example.

*Stat. Eng. Lab., Admin. A-337, 301-921-3651

EXAMPLES

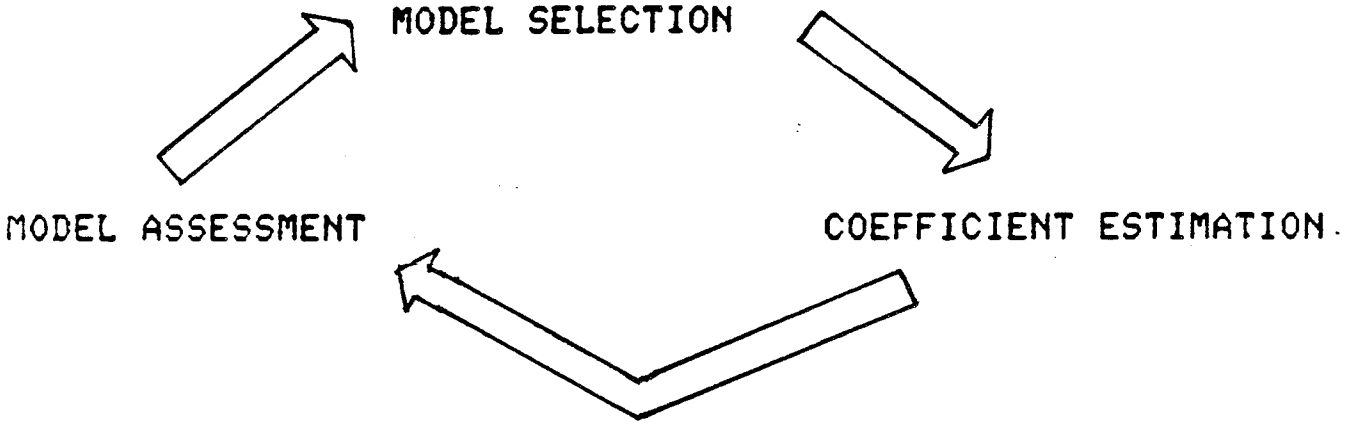
1.	ALASKA PIPELINE RADIOGRAPHIC DEFECT BIAS CURVE	LINEAR
2.	CONCRETE TENSILE STRENGTH STUDY	LINEAR
3.	FIRE RESEARCH IGNITION STUDY	LINEAR
4.	OIL CONSUMPTION STUDY	LINEAR
5.	NUCLEAR SAFEGUARDS TANK CALIBRATION CURVE	LINEAR
6.	SILICON 288.1 NM. CALIBRATION CURVE	QUADRATIC
7.	LOAD CELL CALIBRATION CURVE	LINEAR & QUADRATIC
8.	COMPUTER UTILIZATION STUDY	EXPONENTIAL
9.	SAFETY EQUIPMENT STUDY	EXPONENTIAL
10.	ERYTHEMA BIOMEDICAL STUDY	2 EXPONENTIALS
11.	QUEUING THEORY SERVER FUNCTION CURVE	POWER
12.	CONCRETE STRENGTH STUDY	POWER IN DENOMINATOR
13.	DENTAL RESEARCH MONOMOLECULAR ADSORBTION STUDY	EXP., QUAD., . ,RECIPROCAL
14.	QUANTUM DEFECTS FOR SULFUR I ATOM STUDY	ARCTANGENT
15.	CIRCULAR INTERFERENCE FILTER STUDY	LORENTZIAN & GAUSSIAN
16.	ULTRASONOVISION CALIBRATION CURVE	BESSEL
17.	ULTRASONIC REFERENCE BLOCK ANALYSIS	EXPONENTIAL/LINEAR
18.	CONCRETE PULL-OUT BOND STRENGTH STUDY	LINEAR/LINEAR
19.	SCANNING ELECTRON MICROSCOPE LINE SPACING STANDARDS	LINEAR/LINEAR & QUADRATIC/QUADRATIC
20.	SEMICONDUCTOR BORON DIFFUSION STUDY	QUADRATIC/QUADRATIC
21.	COPPER THERMAL EXPANSION CURVE	QUADRATIC/QUADRATIC & CUBIC/CUBIC
22.	SMOKE OBSCURATION FIRE RESEARCH STUDY	LINEAR/QUADRATIC & LINEAR/QUARTIC
23.	DOPPLER SPECTROMETER PARTICLE SIZE DIST. STUDY	LINEAR/QUADRATIC
24.	RESIDENTIAL TIME-TEMPERATURE CURVE	CUBIC/CUBIC
25.	PRACTICAL TEMPERATURE SCALE REFERENCE CURVE	CUBIC/CUBIC
26.	PHOSPHORUS-DOPED SILICON SEMICONDUCTOR STUDY	CUBIC/CUBIC

EOF AT LINE 35

35*>

MODEL BUILDING IS AN ITERATIVE PROCESS

PLOTTING IS A NECESSARY COMPONENT FOR
MODEL SELECTION AND VERIFICATION



MODEL ASSESSMENT IS PRIMARILY DONE BY PREDICTED VALUE AND
RESIDUAL ANALYSIS

1) RESIDUAL STANDARD DEVIATION
2) RESIDUAL PLOTS
3) PREDICTED VALUE SUPERIMPOSED ON RAW DATA PLOT

@DATAPLOT
⟨ANALYST PROGRAM OR INTERACTIVE SESSION (OR BOTH)⟩
END (OR STOP OR HALT OR EXIT)

ELEMENTS OF THE DATAPLOT LANGUAGE

AND
ANOVA
AMPLITUDE PLOT
ARGAND PLOT
AUTOCORRELATION PLOT

BATCH
BELL

CALCOMP
CHARACTERS
COHERENCY SPECTRUM
COMMENT
COMPLEX DEMODUALTION ... PLOT
CONTINUOUS TERMINAL
... CONTROL CHART
CO-SPECTRUM
CROSS-CORRELATION PLOT
CROSS-SPECTRUM
CUMULATIVE FREQUENCY PLOT
CUMULATIVE HISTOGRAM

DELETE
DEGREE
DISCRETE TERMINAL

ECHO
END
ERASE
EXACT ... RATIONAL FIT
EXIT

FIT
FOR
FRAME
FREQUENCY PLOT

GAIN PLOT
GRID

HALT
HARDCOPY
HELP
HISTOGRAM

LAG-1 AUTOCORRELATION PLOT
LET
LOGLOG
LOGX
LOGY

OFF
ON

PACK
PERCENT POINT PLOT
PHASE PLOT
PIE CHART
PLOT
PRINT
PRINTER
... PROBABILITY PLOT

QUADRATURE SPECTRUM

RADIAN
... RANDOM NUMBERS
READ
RESET
RUN-SEQUENCE PLOT

SEQUENCE
SIZE
SKIP
SMOOTH
SORT
SPECTRUM
STATUS
SUBSET

TEKTRONIX
TITLE

UNFRAME
UNGRID
UNLABEL
UNLOG
UNSORT
UNTITLE

VERSATEC
VERSUS

WRITE

XLABEL
X1LABEL
X2LABEL
X3LABEL
XLIM
XMAX
XMIN

YLABEL
YLIM
YMAX
YMIN

REFERENCES FOR FITTING CURVES TO DATA (FUNCTIONAL CONSIDERATIONS)

HASTINGS, C. (1955). APPROXIMATIONS FOR DIGITAL CBMPUTERS. PRINCETON UNIVERSOTY PRESS.

HOERL, ARTHUR E. CHAPTER 20 IN THE CHEMICAL BUSINESS HANDBOOK.

READ DATA

PLOT DATA

FIT DATA

SUPERIMPOSE DATA & PREDICTED VALUES

PLOT RESIDUALS

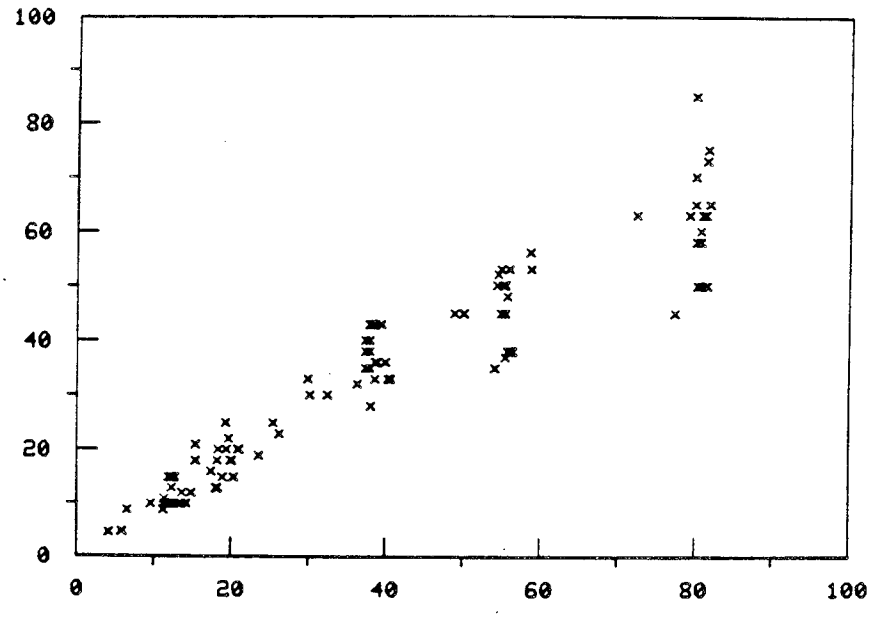
EXAMPLE 1-A

```
READ JJF6*DATA.BERGER1 TRUE MEAS
FIT MEAS = A0 + A1*TRUE
```

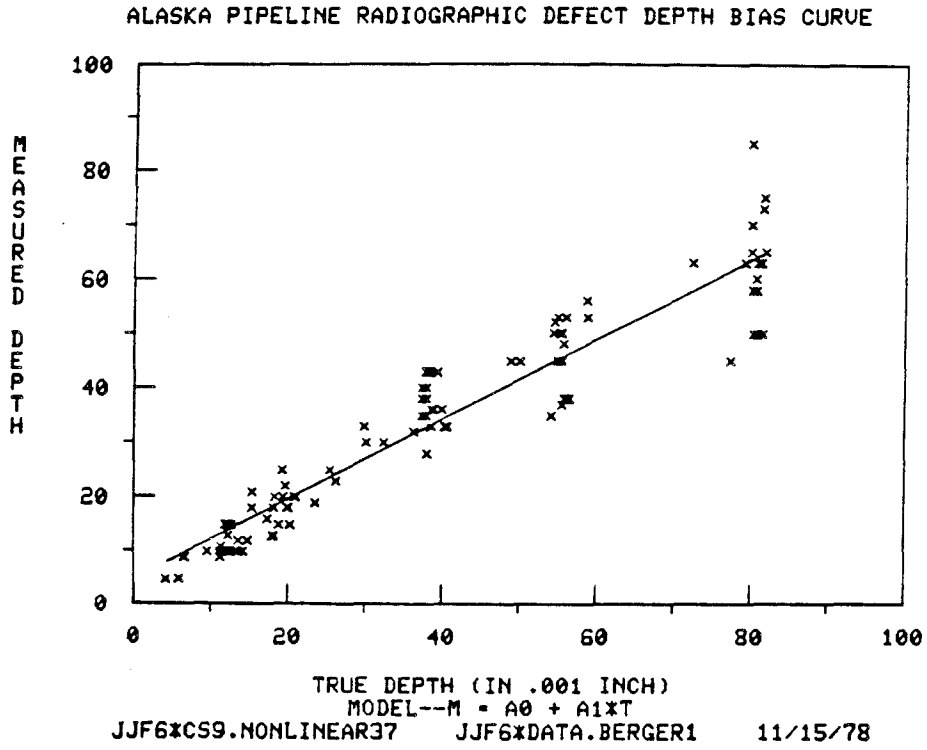
```
LEAST SQUARES NON-LINEAR FIT
      SAMPLE SIZE N =       107
      MODEL-- MEAS = A0 + A1*TRUE
      REPLICATION CASE
      REPLICATION STANDARD DEVIATION =        .6112686932+01
      REPLICATION DEGREES OF FREEDOM =           29
      NUMBER OF DISTINCT SUBSETS     =           78

ITERATION  CONVERGENCE  RESIDUAL  *  PARAMETER
 NUMBER      MEASURE    STANDARD  *  ESTIMATES
                        DEVIATION *
-----------------------------------*-----------
  1--       .10000-01    .11069+02 *   .10000+01    .10000+01
  2--       .50000-02    .60809+01 *   .49883+01    .73124+00

      FINAL PARAMETER ESTIMATES            (APPROX. ST. DEV.)
      1  A0                 4.99369        (  1.126    )
      2  A1                 .731111        (  .2455-01)

      RESIDUAL    STANDARD DEVIATION =          6.0809237957
      RESIDUAL    DEGREES OF FREEDOM =          105
      REPLICATION STANDARD DEVIATION =          6.1126869321
      REPLICATION DEGREES OF FREEDOM =            29
      LACK OF FIT F RATIO =      .9857 = THE  46.3056% POINT OF THE
      F DISTRIBUTION WITH     76 AND     29 DEGREES OF F EEDOM
```

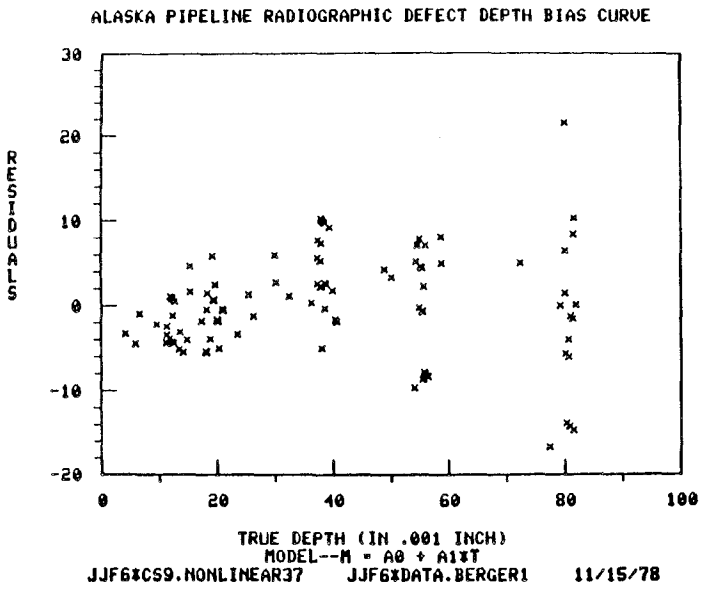
EXAMPLE 1-B

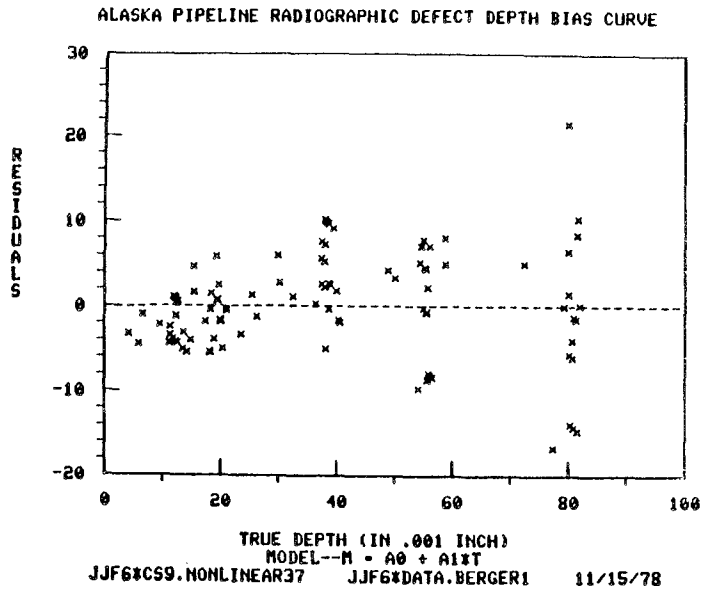
```
COMMENT EXAMPLE--ALASKA PIPELINE RADIOGRAPHIC DEFECT BIAS CURVE
COMMENT MODEL  --LINEAR
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6%DATA.BERGER1 TRUE MEAS
.
CHARACTERS X
LINES
PLOT MEAS TRUE
.
FIT MEAS = A0 + A1%TRUE
.
TITLE ALASKA PIPELINE RADIOGRAPHIC DEFECT DEPTH BIAS CURVE
YLABEL MEASURED DEPTH
XLABEL TRUE DEPTH (IN .001 INCH)
X2LABEL MODEL--M = A0 + A1%T
X3LABEL JJF6%CS9.NONLINEAR37    JJF6%DATA.BERGER1    11/15/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT MEAS PRED VS TRUE
.
YLABEL RESIDUALS
PLOT RES TRUE
.
LINES BLANK DASHED
PLOT RES TRUE AND
PLOT Y = 0 FOR X = 0 10 100
.
XLABEL
NORMAL PROBABILITY PLOT RES
```

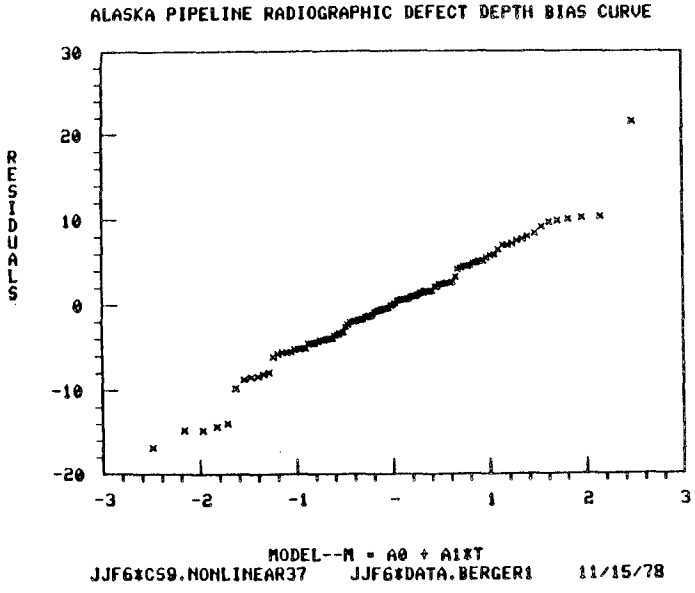


```
LEAST SQUARES NON-LINEAR FIT
      SAMPLE SIZE N =       107
      MODEL-- MEAS = A0 + A1*TRUE
      REPLICATION CASE
      REPLICATION STANDARD DEVIATION =        .6112686932+01
      REPLICATION DEGREES OF FREEDOM =          29
      NUMBER OF DISTINCT SUBSETS     =          78

ITERATION  CONVERGENCE  RESIDUAL  *  PARAMETER
 NUMBER      MEASURE    STANDARD  *  ESTIMATES
                        DEVIATION *
-----------------------------------*-----------
  1--       .10000-01    .11069+02 *   .10000+01    .10000+01
  2--       .50000-02    .60809+01 *   .49883+01    .73124+00

     FINAL PARAMETER ESTIMATES            (APPROX. ST. DEV.)
     1  A0                 4.99369        (  1.126    )
     2  A1                 .731111        (  .2455-01)

     RESIDUAL    STANDARD DEVIATION =          6.0809237957
     RESIDUAL    DEGREES OF FREEDOM =          105
     REPLICATION STANDARD DEVIATION =          6.1126869321
     REPLICATION DEGREES OF FREEDOM =           29
     LACK OF FIT F RATIO =      .9857 = THE   6.3056% POINT OF THE
     F DISTRIBUTION WITH     76 AND     29 DEGREES OF FREEDOM
```



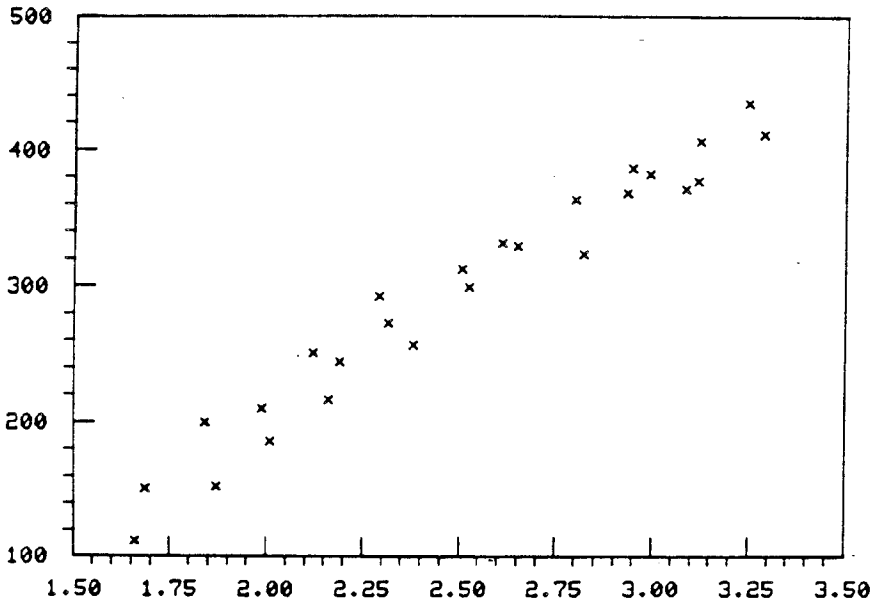

ALASKA PIPELINE RADIOGRAPHIC DEFECT DEPTH BIAS CURVE
RESIDUALS
30
20
10
0
-10
-20
0
20
40
60
80
100
TRUE DEPTH (IN .001 INCH)
MODEL--M = A0 + A1*T
JJF6*CS9.NONLINEAR37 JJF6*DATA.BERGER1 11/15/78


ALASKA PIPELINE RADIOGRAPHIC DEFECT DEPTH BIAS CURVE
RESIDUALS
30
20
10
0
-10
-20
0
20
40
60
80
100
TRUE DEPTH (IN .001 INCH)
MODEL--M = A0 + A1*T
JJF6*CS9.NONLINEAR37 JJF6*DATA.BERGER1 11/15/78


ALASKA PIPELINE RADIOGRAPHIC DEFECT DEPTH BIAS CURVE
RESIDUALS
30
20
10
0
-10
-20
-3
-2
-1
-
1
2
3
MODEL--M = A0 + A1*T
JJF6*CS9.NONLINEAR37 JJF6*DATA.BERGER1 11/15/78

EXAMPLE 2

```
COMMENT EXAMPLE--H. S. LEW CONCRETE TENSILE STRENGTH
COMMENT MODEL  --LINEAR
.
ECHO ON
HARDCOPY ON
BELL ON
.
SKIP 10
READ JJF6*DATA.LEW3 STREN MAT
.
CHARACTERS X
LINES
PLOT STREN MAT
.
FIT STREN = C0 + C1*MAT
.
TITLE SPLITTING TENSILE STRENGTH OF CONCRETE
YLABEL TENSILE STRENGTH
XLABEL MATURITY (= [TEMPERATURE(F) - 10] * ELAPSED DAYS)
X2LABEL MODEL--STREN = C0 + C1*MAT
X3LABEL JJF6*CS9.NONLINEAR38    JJF6*DATA.LEW3    1/76
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT STREN PRED VS MAT
.
YLABEL RESIDUALS
PLOT RES MAT
.
XLABEL
NORMAL PROBABILITY PLOT RES
```

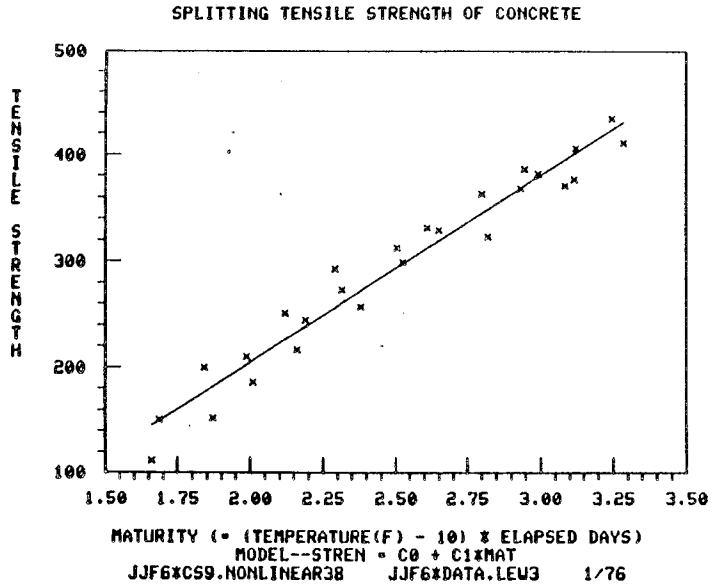


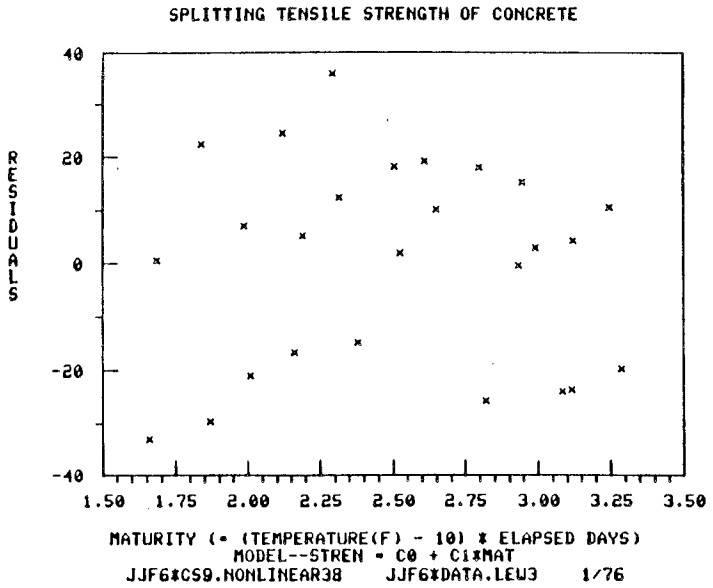
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 26
MODEL-- STREN = C0 + C1*MAT
NO REPLICATION CASE

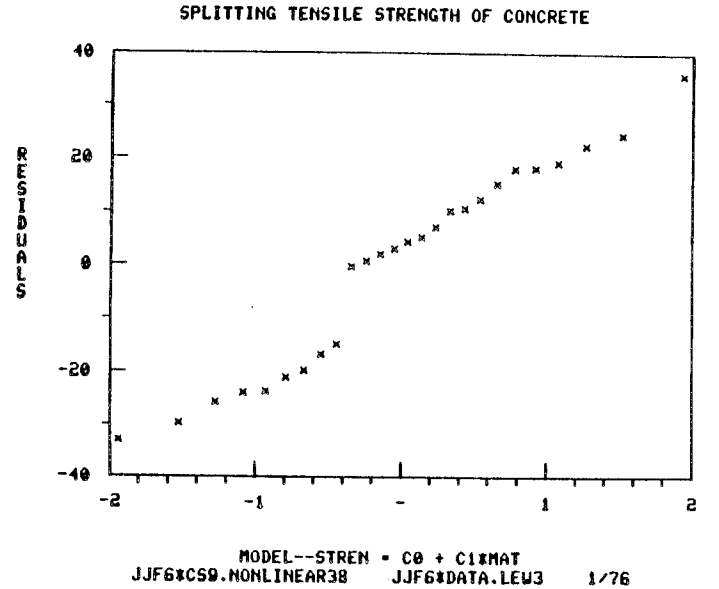
ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	*	PARAMETER ESTIMATES	
1--	.10000-01	.31561+03	*	.10000+01	.10000+01
2--	.50000-02	.19545+02	*	-.14376+03	.17456+03
3--	.25000-02	.19542+02	*	-.14532+03	.17518+03

FINAL PARAMETER ESTIMATES		(APPROX. ST. DEV.)
1 C0	-145.324	(19.93)
2 C1	175.178	(7.804)

RESIDUAL STANDARD DEVIATION = 19.5424284935
RESIDUAL DEGREES OF FREEDOM = 24

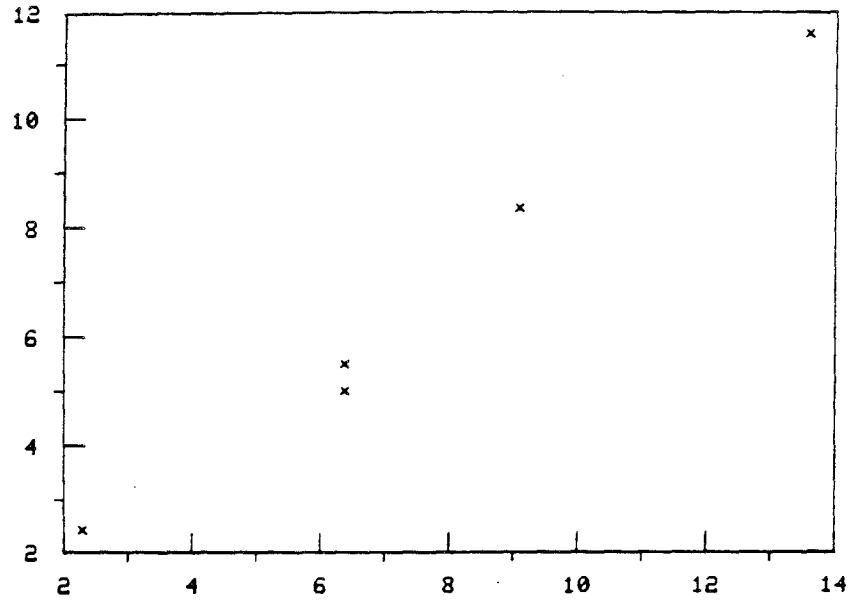




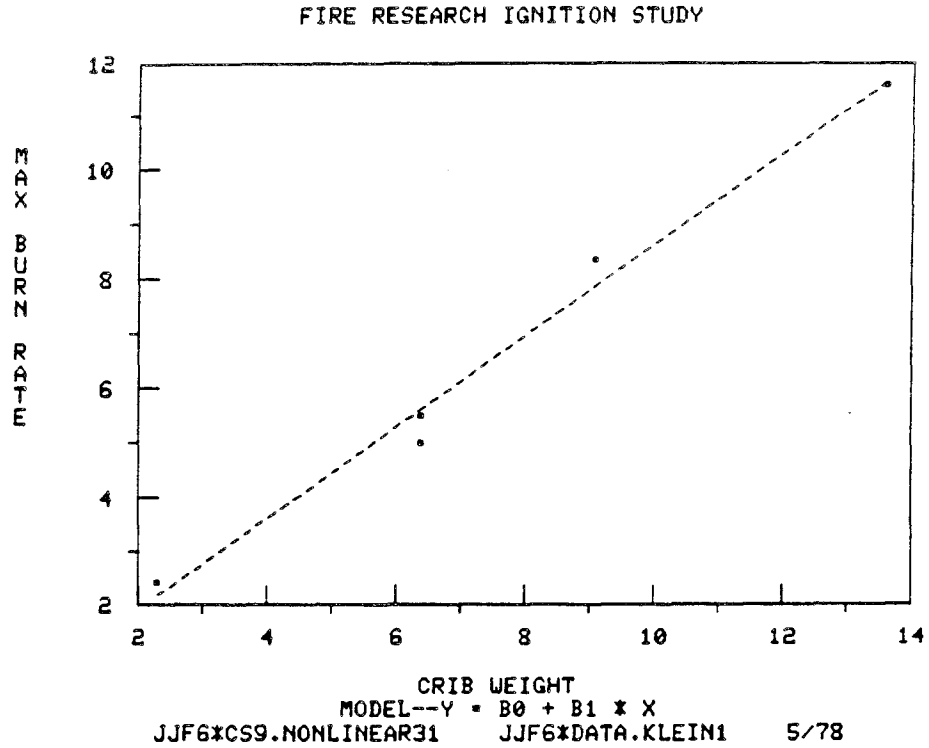


EXAMPLE 3

```
COMMENT EXAMPLE--DAVE KLEIN FIRE RESEARCH STUDY
COMMENT MODEL   --LINEAR
COMMENT NOTE    --SQUARE PLOT CHARACTERS AND DOTTED LINES
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6*DATA.KLEIN1 Y X
.
CHARACTERS X
LINES
PLOT Y X
.
FIT Y = B0+B1*X
.
TITLE FIRE RESEARCH IGNITION STUDY
YLABEL MAX BURN RATE
XLABEL CRIB WEIGHT
X2LABEL MODEL--Y = B0 + B1 * X
X3LABEL JJF6*CS9.NONLINEAR31    JJF6*DATA.KLEIN1    5/78
CHARACTERS SQUARE BLANK
LINES BLANK DASHED
PLOT Y PRED VS X
```

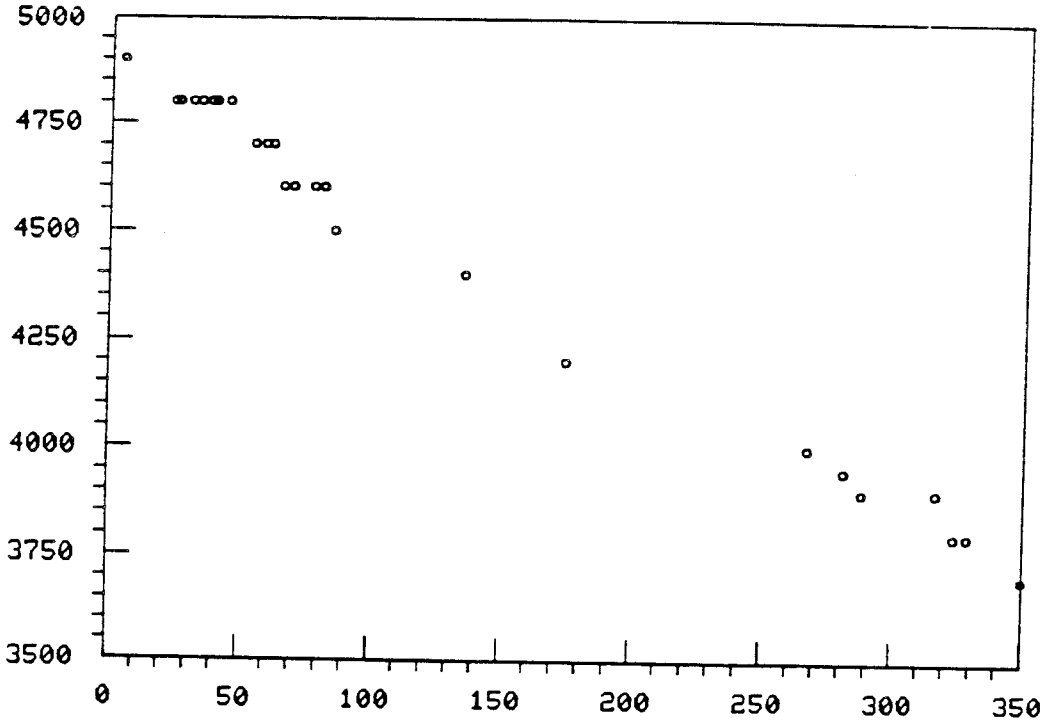


```
LEAST SQUARES NON-LINEAR FIT
      SAMPLE SIZE N =          5
      MODEL-- Y = B0+B1*X
      REPLICATION CASE
      REPLICATION STANDARD DEVIATION =          .3535533920+00
      REPLICATION DEGREES OF FREEDOM =                1
      NUMBER OF DISTINCT SUBSETS     =                4

ITERATION  CONVERGENCE  RESIDUAL  *  PARAMETER
 NUMBER      MEASURE    STANDARD  *  ESTIMATES
                        DEVIATION *
-----------------------------------*----------
  1--       .10000-01    .27175+01 *   .10000+01    .10000+01
  2--       .50000-02    .47950+00 *   .26015+00    .83597+00

      FINAL PARAMETER ESTIMATES            (APPROX. ST. DEV.)
      1  B0                  .260415       (  .4856    )
      2  B1                  .835924       (  .5764-01 )

      RESIDUAL    STANDARD DEVIATION =            .4794989079
      RESIDUAL    DEGREES OF FREEDOM =            3
      REPLICATION STANDARD DEVIATION =            .3535533920
      REPLICATION DEGREES OF FREEDOM =            1
      LACK OF FIT F RATIO =     2.2590 = THE  57.4297% POINT OF THE
      F DISTRIBUTION WITH       2 AND       1 DEGREES OF FREEDOM
```



EXAMPLE 4

```
COMMENT EXAMPLE--BOB MAY OIL CONSUMPTION STUDY
COMMENT MODEL   --LINEAR
COMMENT NOTE    --CIRCLES FOR PLOT CHARACTERS
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6*DATA.MAY1 DAY BH1 BH2 OIL
.
LET H = BH1 + BH2
.
CHARACTERS CIRCLE
LINES
PLOT OIL H
.
FIT OIL = A + B*H
.
TITLE OIL TANK LEVEL VS OIL BURNER HOURS
YLABEL TANK LEVEL /GAL/
XLABEL BURNER HOURS
X2LABEL MODEL--Y = A + B*H
X3LABEL JJF6*CS9.NONLINEAR20    JJF6*DATA.MAY1    11/7/78
CHARACTER CIRCLE BLANK
LINES BLANK SOLID
PLOT OIL PRED VS H
.
YLABEL RESIDUALS
PLOT RES H
```



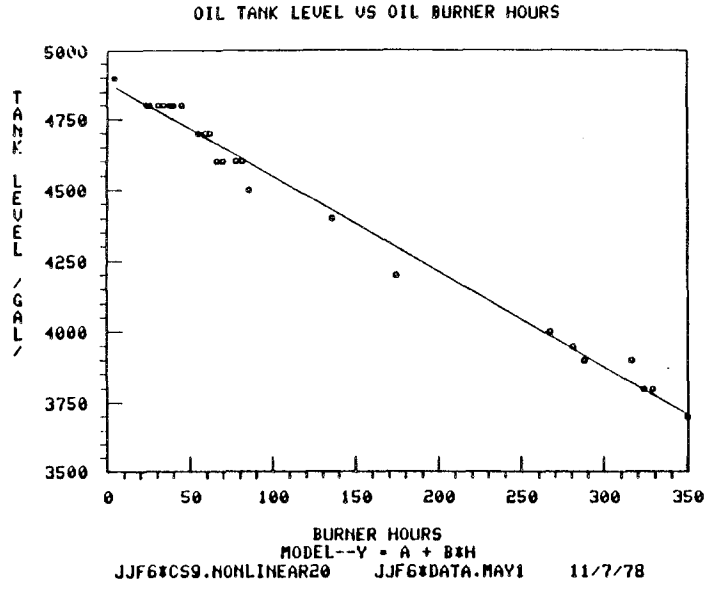
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 27
MODEL-- OIL = A + B*H
REPLICATION CASE
REPLICATION STANDARD DEVIATION = .0000000000
REPLICATION DEGREES OF FREEDOM = 2
NUMBER OF DISTINCT SUBSETS = 25

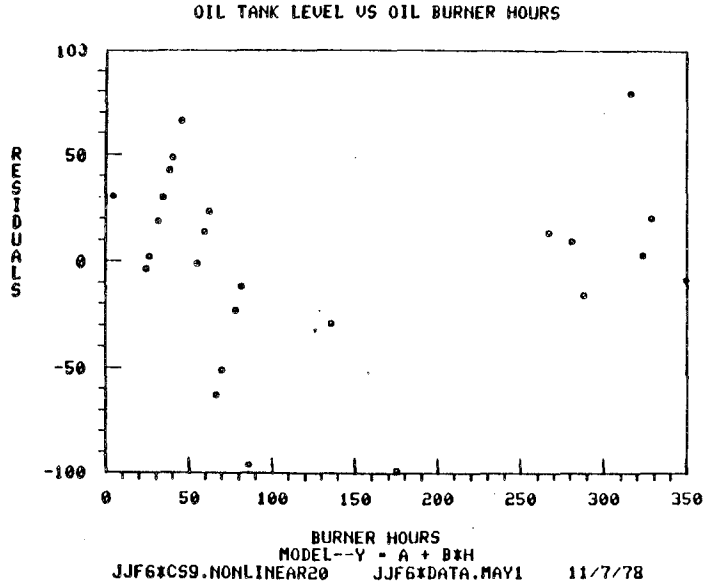
ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	*	PARAMETER ESTIMATES	
1--	.10000-01	.44979+04	*	.10000+01	.10000+01
2--	.50000-02	.42803+02	*	.48854+04	-.33596+01
3--	.25000-02	.42791+02	*	.48865+04	-.33677+01

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)

1	A	4886.54	(12.33)
2	B	-3.36768	(.6793-01)

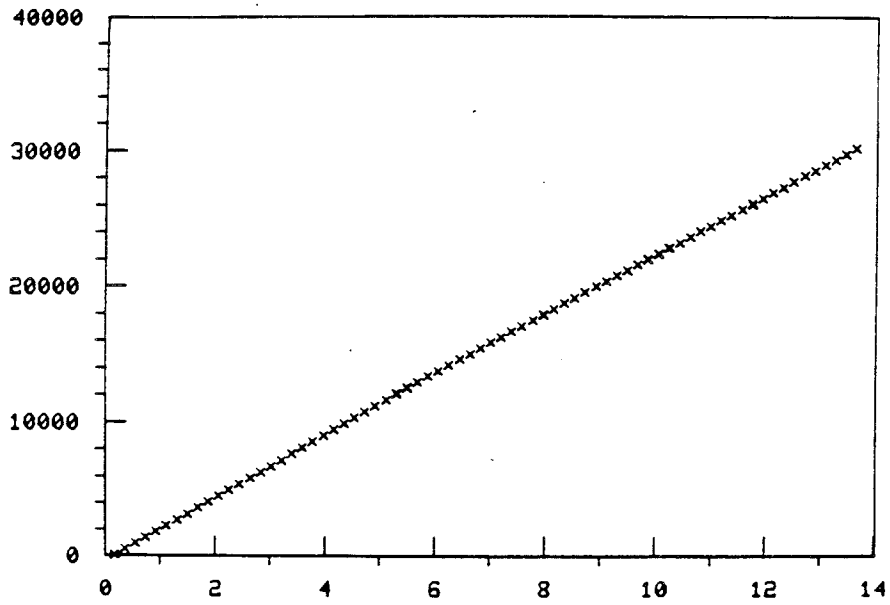
RESIDUAL STANDARD DEVIATION = 42.7909731865
RESIDUAL DEGREES OF FREEDOM = 25
REPLICATION STANDARD DEVIATION = .0000000000
REPLICATION DEGREES OF FREEDOM = 2
LACK OF FIT F RATIO = ********** = THE 100.0000% POINT OF THE
F DISTRIBUTION WITH 23 AND 2 DEGREES OF FREEDOM



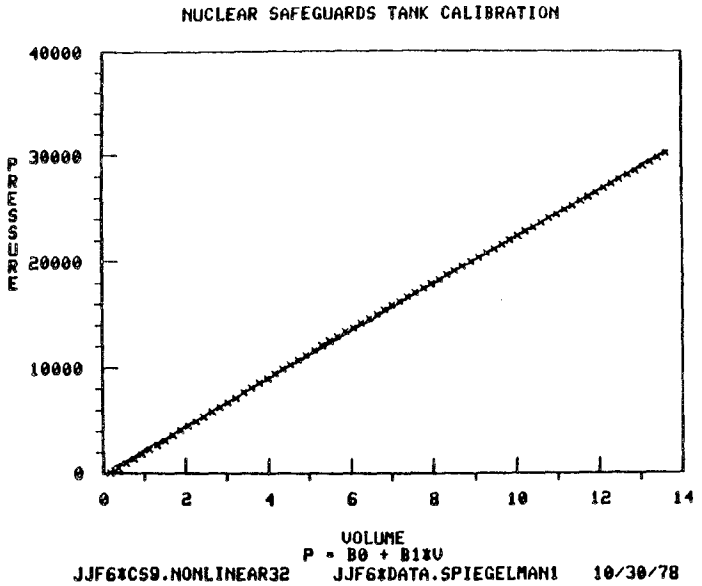


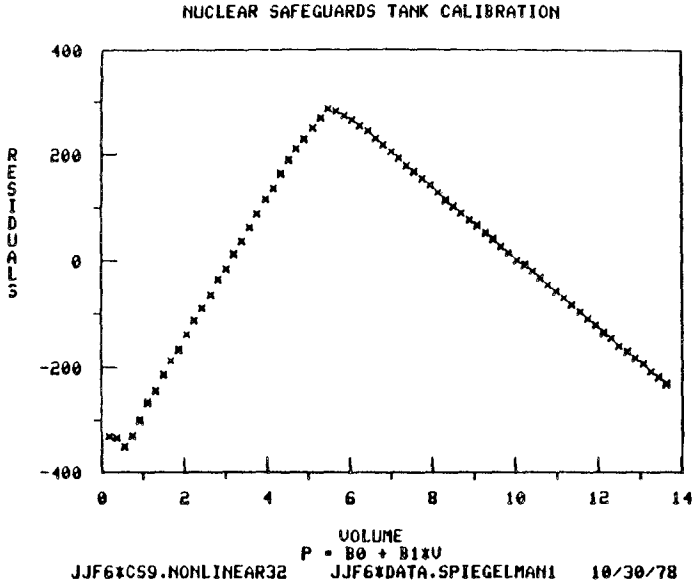
EXAMPLE 5

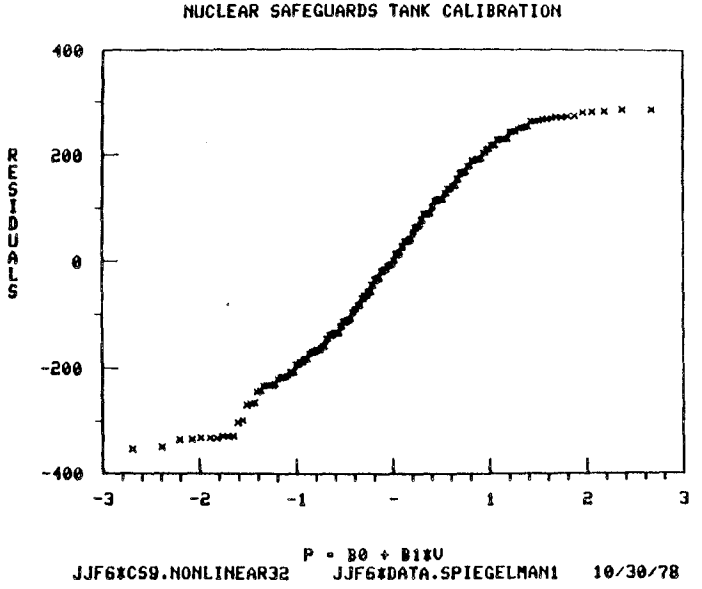
```
COMMENT EXAMPLE--CLIFF SPIEGELMAN NUCLEAR SAFEGUARDS CALIBRATION PROBLEM
COMMENT MODEL   --LINEAR
COMMENT NOTE    --VALUE OF RESIDUAL ANALYSIS
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6%DATA.SPIEGELMAN1 V P
.
CHARACTERS X
LINES
PLOT P V
.
FIT P = B0+B1%V
.
TITLE NUCLEAR SAFEGUARDS TANK CALIBRATION
YLABEL PRESSURE
XLABEL VOLUME
X2LABEL P = B0 + B1%V
X3LABEL JJF6%CS9.NONLINEAR32    JJF6%DATA.SPIEGELMAN1    10/30/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT P PRED VS V
.
YLABEL RESIDUALS
PLOT RES V
.
XLABEL
NORMAL PROBABILITY PLOT RES
```



```
LEAST SQUARES NON-LINEAR FIT
      SAMPLE SIZE N =      192
      MODEL-- P = B0+B1*V
      REPLICATION CASE
      REPLICATION STANDARD DEVIATION =        .1874172986+01
      REPLICATION DEGREES OF FREEDOM =              2
      NUMBER OF DISTINCT SUBSETS     =            190

ITERATION  CONVERGENCE  RESIDUAL  *  PARAMETER
 NUMBER      MEASURE    STANDARD  *  ESTIMATES
                        DEVIATION *
-------------------------------------*----------
  1--       .10000-01    .18087+05 *   .10000+01    .10000+01
  2--       .50000-02    .17841+03 *   .12856+03    .22147+04
  3--       .25000-02    .17837+03 *   .12214+03    .22156+04

      FINAL PARAMETER ESTIMATES            (APPROX. ST. DEV.)
      1  B0                 122.142        (  26.24   )
      2  B1                2215.65         (  3.249   )

      RESIDUAL    STANDARD DEVIATION =        178.3704223633
      RESIDUAL    DEGREES OF FREEDOM =          190
      REPLICATION STANDARD DEVIATION =          1.8741729856
      REPLICATION DEGREES OF FREEDOM =              2
      LACK OF FIT F RATIO =  9154.2249 = THE  99.9891% POINT OF THE
      F DISTRIBUTION WITH    188 AND      2 DEGREES OF FREEDOM
```

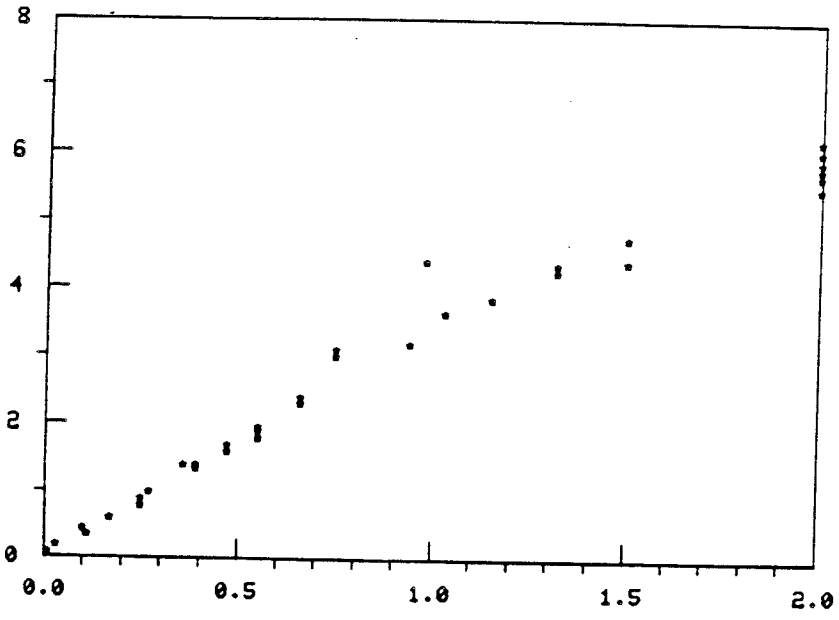




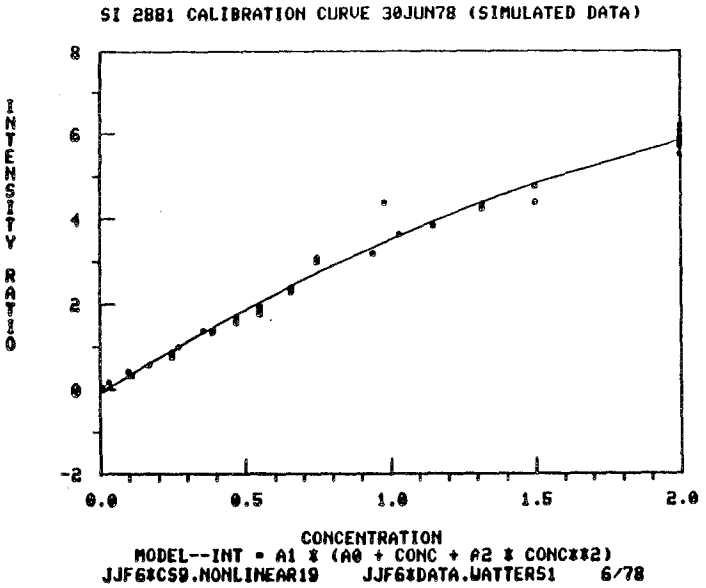


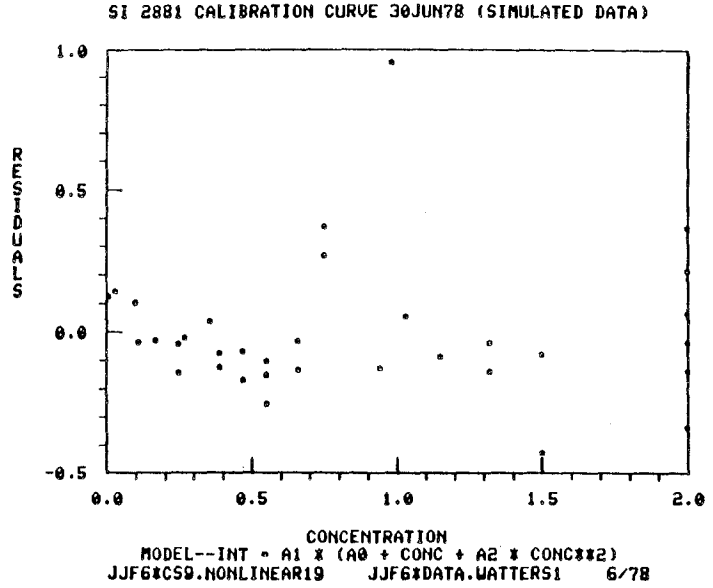
EXAMPLE 6

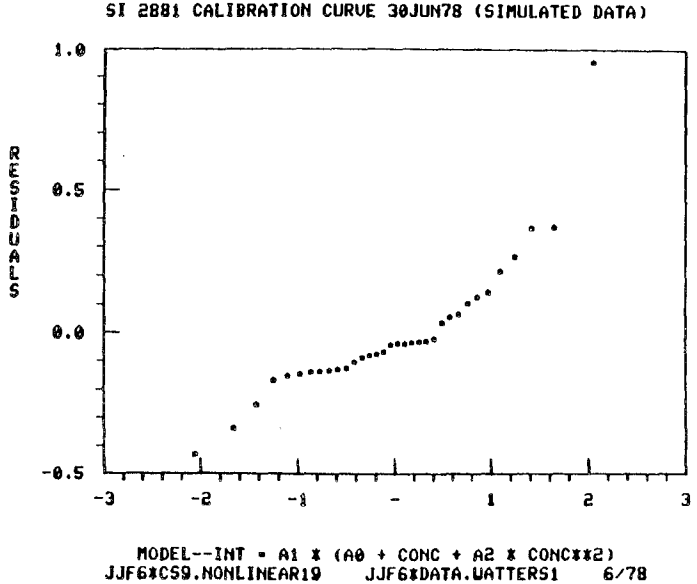
```
COMMENT EXAMPLE--BOB WATTERS CALIBRATION CURVE
COMMENT           (SIMULATED DATA)
COMMENT MODEL  --QUADRATIC
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6*DATA.WATTERS1 CONC INT
.
CHARACTERS STAR
LINES
PLOT INT CONC
.
FIT INT = A1*(A0+CONC+A2*CONC**2)
.
TITLE SI 2881 CALIBRATION CURVE 30JUN78 (SIMULATED DATA)
YLABEL INTENSITY RATIO
XLABEL CONCENTRATION
X2LABEL MODEL--INT = A1 * (A0 + CONC + A2 * CONC**2)
X3LABEL JJF6*CS9.NONLINEAR19    JJF6*DATA.WATTERS1    6/78
CHARACTERS STAR BLANK
LINES BLANK SOLID
PLOT INT PRED VS CONC
.
YLABEL RESIDUALS
PLOT RES CONC
.
XLABEL
NORMAL PROBABILITY PLOT RES
```



```
LEAST SQUARES NON-LINEAR FIT
      SAMPLE SIZE N =       34
      MODEL-- INT = A1*(A0+CONC+A2*CONC**2)
      REPLICATION CASE
      REPLICATION STANDARD DEVIATION =        .1713913567+00
      REPLICATION DEGREES OF FREEDOM =          14
      NUMBER OF DISTINCT SUBSETS     =          20

 ITERATION   CONVERGENCE   RESIDUAL  *  PARAMETER
  NUMBER       MEASURE     STANDARD  *  ESTIMATES
                           DEVIATION *
-----------------------------------*-----------
   1--       .10000-01     .71561+00 *   .10000+01    .10000+01    .10000+01
   2--       .11391+00     .62338+00 *   .12246+01    .72164+00    .47504+00
   3--       .17086+00     .53122+00 *   .15001+01    .54107+00    .35996+00
   4--       .8543A-A1     .48951+A0 *   .17991+01    .39529+00    .22344+00
   5--       .19222+00     .44010+00 *   .21029+01    .29492+00    .13369+00
   6--       .96108-01     .39478+00 *   .24360+01    .20816+00    .55575-01
   7--       .14416+00     .35440+00 *   .29745+01    .97933-01   -.44134-01
   8--       .72081-01     .28463+00 *   .34915+01    .41112-01   -.95845-01
   9--       .36041-01     .26024+00 *   .40468+01   -.11655-01   -.14390+00
  10--       .18020-01     .24820+00 *   .41930+01   -.16401-01   -.14829+00
  11--       .90102-02     .24819+00 *   .41991+01   -.16706-01   -.14856+00

      FINAL PARAMETER ESTIMATES              (APPROX. ST. DEV.)
      1  A1                 4.19913          (   .2616     )
      2  A0              -.167100-01         (   .2425-01)
      3  A2                 -.148567         (   .1957-01)

      RESIDUAL    STANDARD DEVIATION =          .2481906656
      RESIDUAL    DEGREES OF FREEDOM =          31
      REPLICATION STANDARD DEVIATION =          .1713913567
      REPLICATION DEGREES OF FREEDOM =          14
      LACK OF FIT F RATIO =     3.0004 = THE  97.8272% POINT OF THE
      F DISTRIBUTION WITH     17 AND     14 DEGREES OF FREEDOM
```

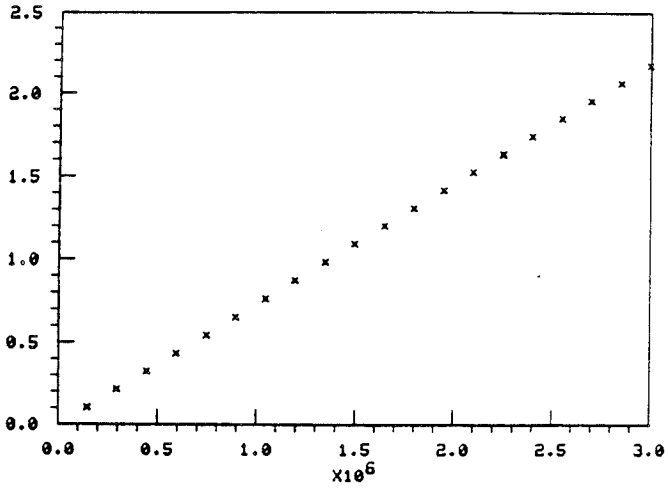




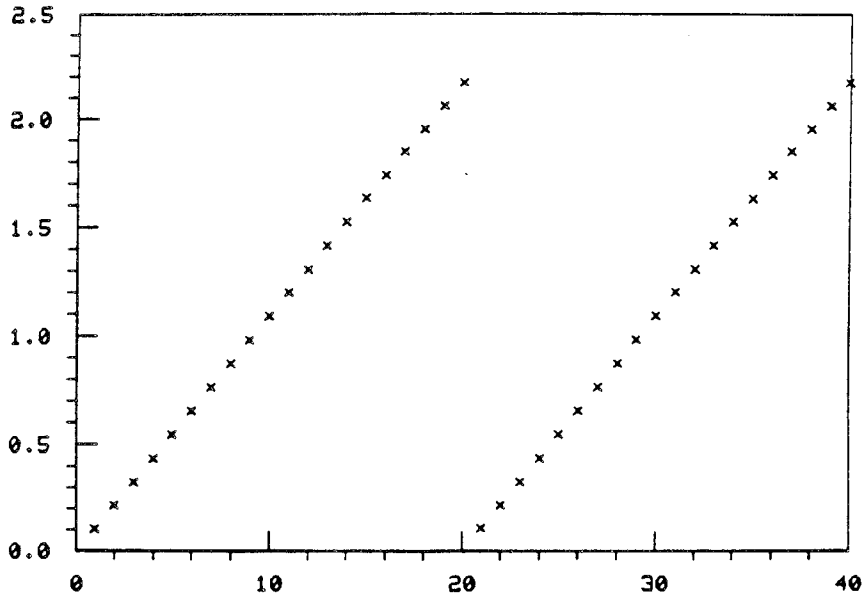


EXAMPLE 7

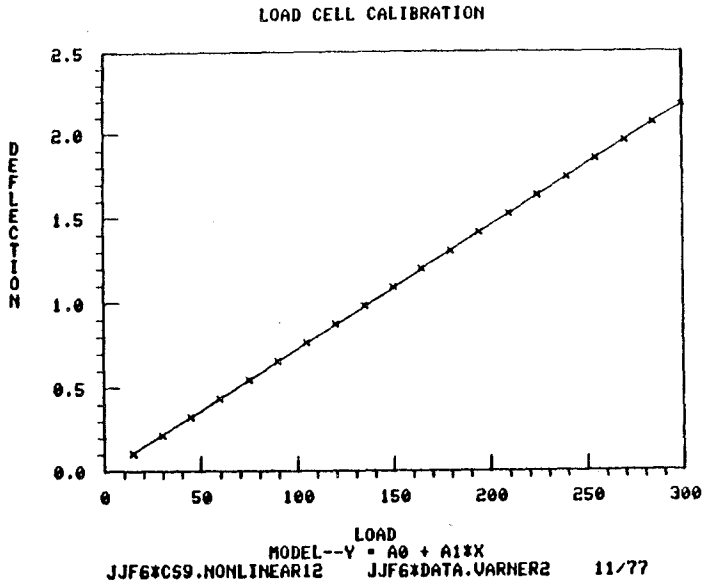
```
COMMENT EXAMPLE--PAUL PONTIUS LOAD CELL CALIBRATION
COMMENT MODEL   --LINEAR AND QUADRATIC
COMMENT NOTE    --VALUE OF RESIDUAL ANALYSIS
COMMENT NOTE    --FITTING, HISTOGRAMS, AND PROB. PLOTS FOR PARTIAL DATA
COMMENT NOTE    --STORING AND PRINTING COEF. AND RESIDUAL STANDARD DEV.
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6*DATA.VARNER2 Y X
.
CHARACTERS X
LINES
PLOT Y X
PRINT Y X
LET X=X/10000
PLOT Y
.
FIT Y = A0+A1*X
.
TITLE LOAD CELL CALIBRATION
YLABEL DEFLECTION
XLABEL LOAD
X2LABEL MODEL--Y = A0 + A1*X
X3LABEL JJF6*CS9.MONLINEAR12    JJF6*DATA.VARNER2    11/77
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT Y PRED VS X
.
YLABEL RESIDUALS
PLOT RES X
XLABEL
NORMAL PROBABILITY PLOT RES
.
FIT Y = A0+A1*X+A2*X**2
.
YLABEL DEFLECTION
XLABEL LOAD
X2LABEL MODEL--Y = A0 + A1*X + A2*X**2
PLOT Y PRED VS X
.
YLABEL RESIDUALS
PLOT RES X
XLABEL
NORMAL PROBABILITY PLOT RES
LET SBOTH=RESSD
.
FIT Y = B0+B1*X+B2*X*X FOR I = 1 1 20
.
YLABEL DEFLECTION
XLABEL LOAD
X2LABEL MODEL Y = B0 + B1*X + B2*X*X  (FIRST 20 OBSERVATIONS ONLY)
PLOT Y PRED VS X FOR I = 1 1 20
.
YLABEL RESIDUALS
PLOT RES X FOR I = 1 1 20
XLABEL
NORMAL PROBABILITY PLOT RES FOR I = 1 1 20
LET S1=RESSD
.
FIT Y = C0+C1*X+C2*X*X FOR I = 21 1 40
.
YLABEL DEFLECTION
XLABEL LOAD
X2LABEL MODEL Y = C0 + C1*X + C2*X*X  (LAST 20 OBSERVATIONS ONLY)
PLOT Y PRED VS X FOR I = 21 1 40
YLABEL RESIDUALS
PLOT RES X FOR I = 21 1 40
XLABEL
NORMAL PROBABILITY PLOT RES FOR I = 21 1 40
LET S2=RESSD
.
PRINT A0 B0 C0 A1 B1 C1 A2 B2 C2 SBOTH S1 S2
```

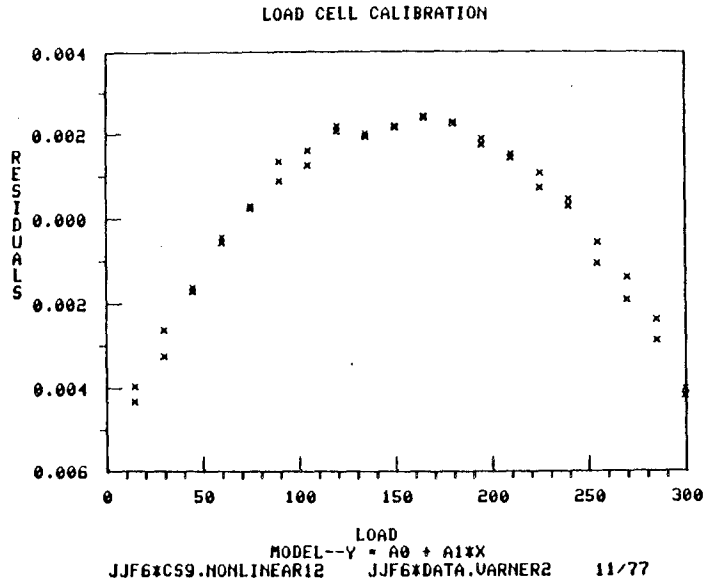


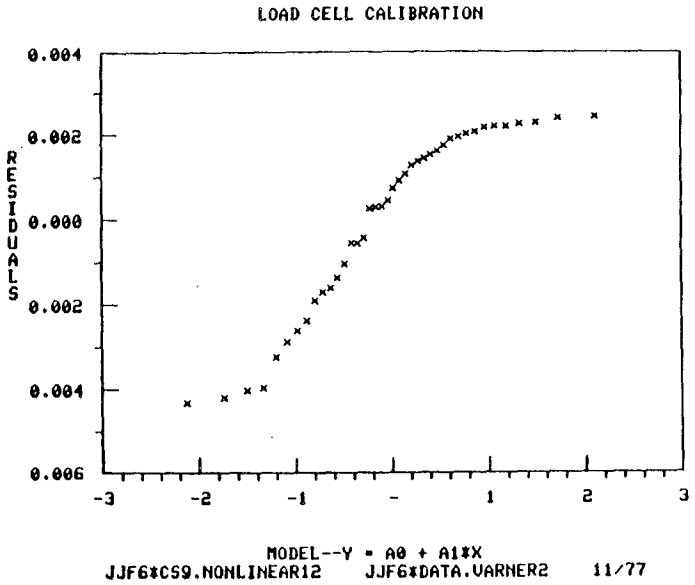
VARIABLES--	Y	X
1--	.1101900+00	.1500000+06
2--	.2195600+00	.3000000+06
3--	.3294900+00	.4500000+06
4--	.4389900+00	.6000000+06
5--	.5480300+00	.7500000+06
6--	.6569400+00	.9000000+06
7--	.7656200+00	.1050000+07
8--	.8748700+00	.1200000+07
9--	.9829200+00	.1350000+07
10--	.1091460+01	.1500000+07
11--	.1200010+01	.1650000+07
12--	.1308220+01	.1800000+07
13--	.1415990+01	.1950000+07
14--	.1523990+01	.2100000+07
15--	.1631940+01	.2250000+07
16--	.1739470+01	.2400000+07
17--	.1846460+01	.2550000+07
18--	.1953920+01	.2700000+07
19--	.2061280+01	.2850000+07
20--	.2168440+01	.3000000+07
21--	.1105200+00	.1500000+06
22--	.2201800+00	.3000000+06
23--	.3293900+00	.4500000+06
24--	.4388600+00	.6000000+06
25--	.5479800+00	.7500000+06
26--	.6573900+00	.9000000+06
27--	.7659600+00	.1050000+07
28--	.8747400+00	.1200000+07
29--	.9830000+00	.1350000+07
30--	.1091500+01	.1500000+07
31--	.1200040+01	.1650000+07
32--	.1308180+01	.1800000+07
33--	.1416130+01	.1950000+07
34--	.1524080+01	.2100000+07
35--	.1631590+01	.2250000+07
36--	.1739650+01	.2400000+07
37--	.1846960+01	.2550000+07
38--	.1954450+01	.2700000+07
39--	.2061770+01	.2850000+07
40--	.2168290+01	.3000000+07



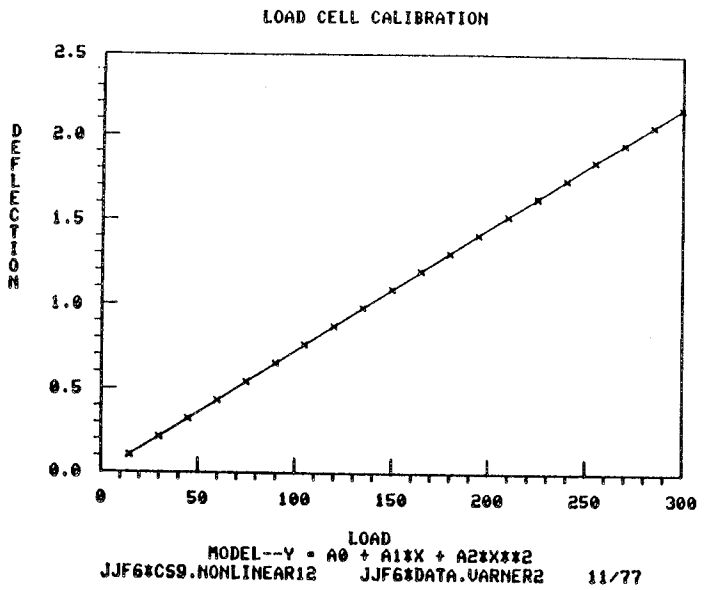
```
LEAST SQUARES NON-LINEAR FIT
      SAMPLE SIZE N =          40
      MODEL-- Y = A0+A1*X
      REPLICATION CASE
      REPLICATION STANDARD DEVIATION =        .2147274445-03
      REPLICATION DEGREES OF FREEDOM =           20
      NUMBER OF DISTINCT SUBSETS     =           20

ITERATION  CONVERGENCE  RESIDUAL  *  PARAMETER
 NUMBER      MEASURE    STANDARD  *  ESTIMATES
                        DEVIATION *
-----------------------------------*-----------
  1--       .10000-01    .18392+03 *   .10000+01    .10000+01
  2--       .50000-02    .37669-01 *  -.60833-01    .76448-02
  3--       .25000-02    .21713-02 *   .61355-02    .72211-02
  4--       .12500-02    .21713-02 *   .61498-02    .72210-02

      FINAL PARAMETER ESTIMATES              (APPROX. ST. DEV.)
      1  A0                  .614928-02   (  .7132-03)
      2  A1                  .722103-02   (  .3969-05)

      RESIDUAL    STANDARD DEVIATION =          .0021712666
      RESIDUAL    DEGREES OF FREEDOM =          38
      REPLICATION STANDARD DEVIATION =          .0002147274
      REPLICATION DEGREES OF FREEDOM =          20
      LACK OF FIT F RATIO =   214.7440 = THE 100.0000% POINT OF THE
      F DISTRIBUTION WITH     18 AND     20 DEGREES OF FREEDOM
```

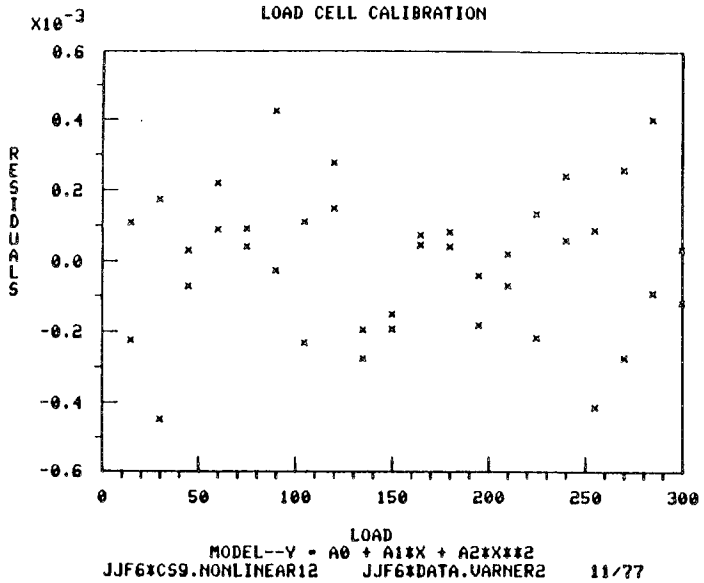


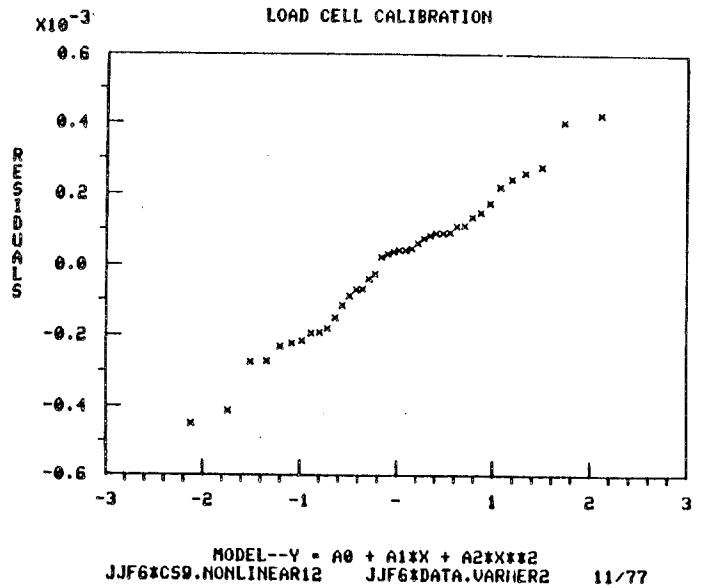




```
LEAST SQUARES NON-LINEAR FIT
      SAMPLE SIZE N =          40
      MODEL-- Y = A0+A1*X+A2*X**2
      REPLICATION CASE
      REPLICATION STANDARD DEVIATION =        .2147274445-03
      REPLICATION DEGREES OF FREEDOM =           20
      NUMBER OF DISTINCT SUBSETS     =           20

ITERATION  CONVERGENCE  RESIDUAL  *  PARAMETER
 NUMBER      MEASURE    STANDARD  *  ESTIMATES
                        DEVIATION *
-----------------------------------*------------
  1--       .10000-01    .44470+05 *   .61493-02   .72210-02   .10000+01
  2--       .50000-02    .24182+02 *   .14646+02  -.60841+00   .25395-02
  3--       .25000-02    .43889-01 *   .12114+00   .52749-02   .59413-05
  4--       .12500-02    .20802-03 *   .76938-03   .73190-02  -.31122-06
  5--       .62500-03    .20518-03 *   .67368-03   .73206-02  -.31607-06

      FINAL PARAMETER ESTIMATES           (APPROX. ST. DEV.)
      1  A0                .673531-03   (  .1079-03)
      2  A1                .732059-02   (  .1578-05)
      3  A2               -.316083-06   (  .4867-08)

      RESIDUAL    STANDARD DEVIATION =          .0002051765
      RESIDUAL    DEGREES OF FREEDOM =          37
      REPLICATION STANDARD DEVIATION =          .0002147274
      REPLICATION DEGREES OF FREEDOM =          20
      LACK OF FIT F RATIO =      .8107 = THE  33.3796% POINT OF THE
      F DISTRIBUTION WITH     17 AND     20 DEGREES OF FREEDOM
```





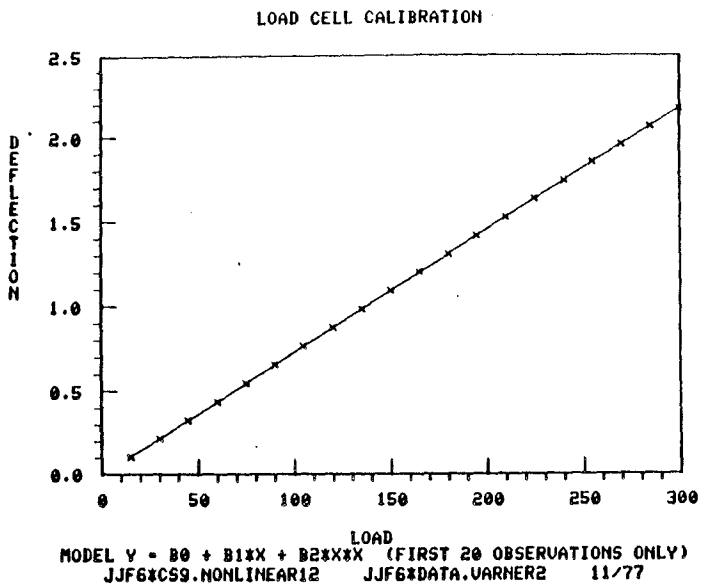


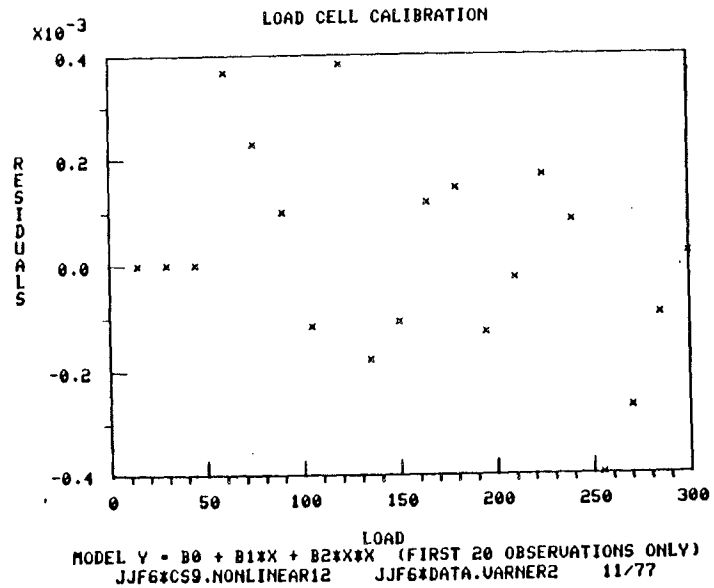
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 20
MODEL-- Y = B0+B1*X+B2*X*X
NO REPLICATION CASE

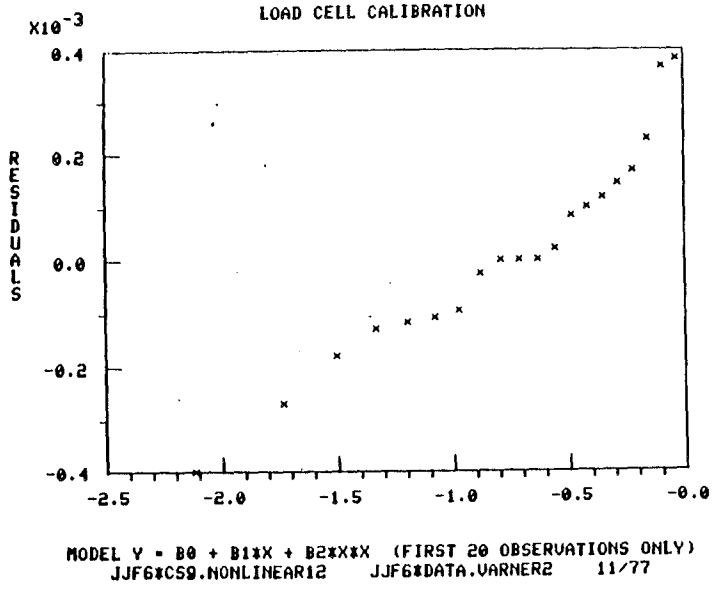
ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	*	PARAMETER ESTIMATES		
1--	.10000-01	.46578+05	*	.10000+01	.10000+01	.10000+01
2--	.50000-02	.28731+02	*	.68315+02	-.12348+01	.39696-02
3--	.25000-02	.86942-01	*	.23359+00	.34298-02	.11494-04
4--	.12500-02	.21712 03	*	.67197-03	.73196-02	-.31354-06
5--	.62500-03	.20640-03	*	.49040-03	.73227-02	-.32271-06
6--	.12014-01	.20640-03	*	.49100-03	.73226-02	-.32268-06

	FINAL PARAMETER ESTIMATES		(APPROX. ST. DEV.)
1	B0	.490636-03	(.1537-03)
2	B1	.732265-02	(.2247-05)
3	B2	-.322695-06	(.6926-08)

RESIDUAL STANDARD DEVIATION = .0002064030
RESIDUAL DEGREES OF FREEDOM = 17







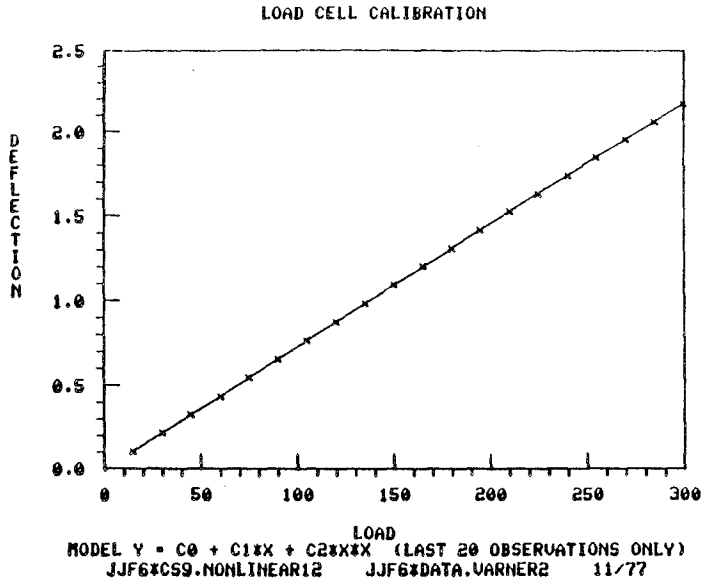
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 20
MODEL-- Y = C0+C1*X+C2*X*X
NO REPLICATION CASE

ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	*	PARAMETER ESTIMATES		
1--	.10000-01	.46578+05	*	.10000+01	.10000+01	.10000+01
2--	.50000-02	.28731+02	*	.68315+02	-.12348+01	.39696-02
3--	.25000-02	.86963-01	*	.23412+00	.34251-02	.11511-04
4--	.12500-02	.19213-03	*	.10386-02	.73155-02	-.30028-06
5--	.62500-03	.17981-03	*	.85651-03	.73185-02	-.30946-06
6--	.35596-02	.17980-03	*	.85639-03	.73185-02	-.30947-06

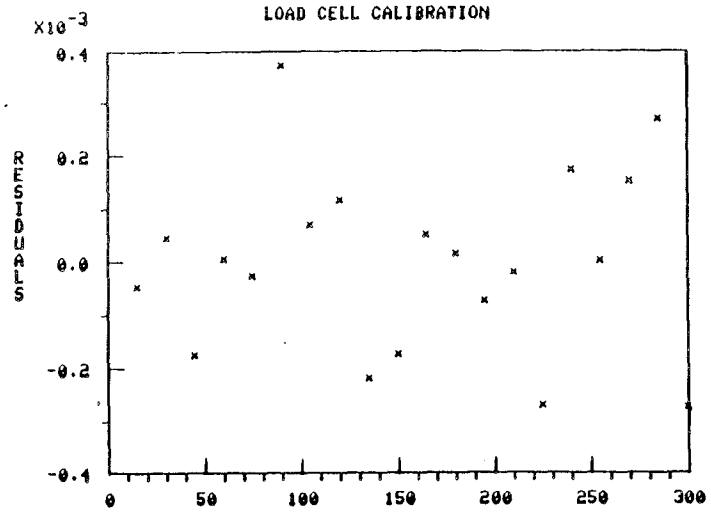
FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)

1	C0	.856317-03	(.1337-03)
2	C1	.731853-02	(.1955-05)
3	C2	-.309474-06	(.6025-08)

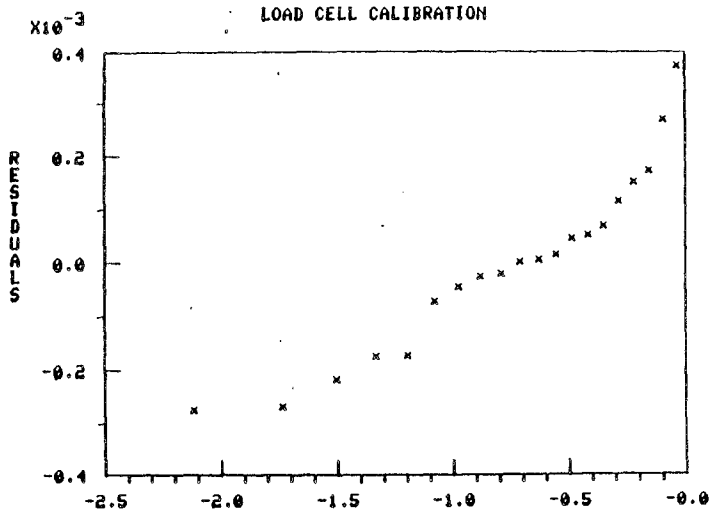
RESIDUAL STANDARD DEVIATION = .0001798039
RESIDUAL DEGREES OF FREEDOM = 17



MODEL Y = C0 + C1*X + C2*X*X (LAST 20 OBSERVATIONS ONLY)
JJF6*CS9.NONLINEAR12 JJF6*DATA.VARNER2 11/77



MODEL Y = C0 + C1*X + C2*X*X (LAST 20 OBSERVATIONS ONLY)
JJF6*CS9.NONLINEAR12 JJF6*DATA.VARNER2 11/77



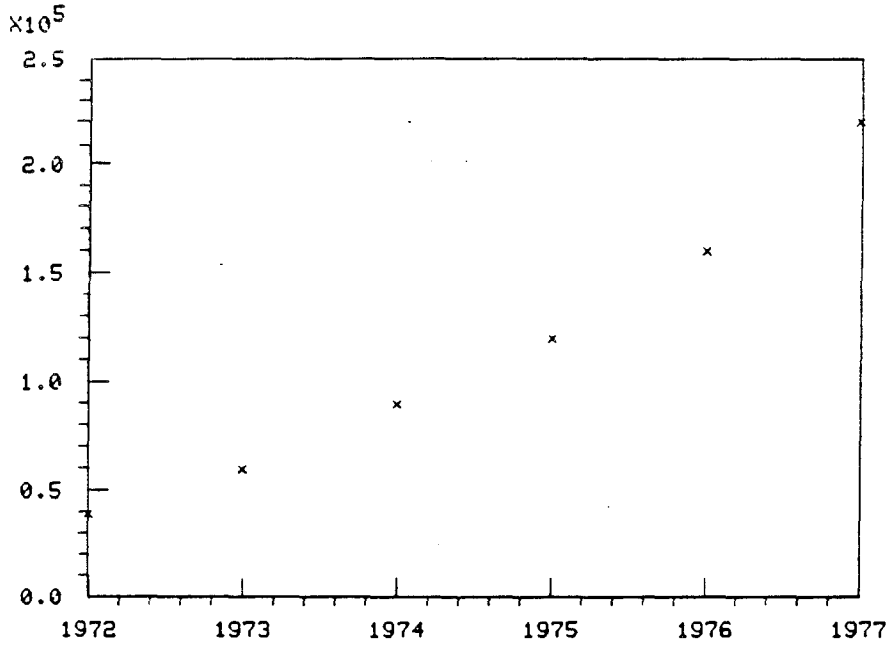
MODEL Y = C0 + C1*X + C2*X*X (LAST 20 OBSERVATIONS ONLY)
JJF6*CS9.NONLINEAR12 JJF6*DATA.VARNER2 11/77

PARAMETERS AND CONSTANTS--

A0	--	.6735314-03
B0	--	.4910012-03
C0	--	.8563168-03
A1	--	.7320592-02
B1	--	.7322649-02
C1	--	.7318532-02
A2	--	-[illegible]0828-06
B2	--	[illegible]827-06
C2	--	[illegible]736-06

EXAMPLE 8

```
COMMENT EXAMPLE--JEAN YANCEY COMPUTER UTILIZATION STUDY
COMMENT MODEL  --AN EXPONENTIAL
COMMENT NOTE   --EXTRAPOLATION
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6*DATA.YANCEY3 YEAR NUMCPU
.
CHARACTERS X
LINES
PLOT NUMCPU YEAR
.
LET MY=NUMCPU(6)
LET MX=YEAR(6)
LET A0=0
LET A1=1
LET A2=ALOG(MY)/(MX-1950)
FIT NUMCPU = A0+A1*EXP(A2*(YEAR-1950))
.
TITLE MINI-COMPUTER UTILIZATION STUDY (UNITED STATES)
YLABEL NUMBER OF CPU'S
XLABEL YEAR
X2LABEL MODEL--Y = A0 + A1 * EXP(A2 * (YEAR-1950))
X3LABEL JJF6*CS9.NONLINEAR24    JJF6*DATA.YANCEY3    11/1/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT NUMCPU PRED VS YEAR
.
PLOT NUMCPU YEAR AND
PLOT Y = A0+A1*EXP(A2*(X-1950)) FOR X = 1972 1 1985
.
CHARACTERS X BLANK BLANK
LINES BLANK SOLID DOTTED
PLOT NUMCPU YEAR AND
PLOT Y = A0+A1*EXP(A2*(X-1950)) FOR X = 1972 1 1977 AND
PLOT Y = A0+A1*EXP(A2*(X-1950)) FOR X = 1977 1 1985
```

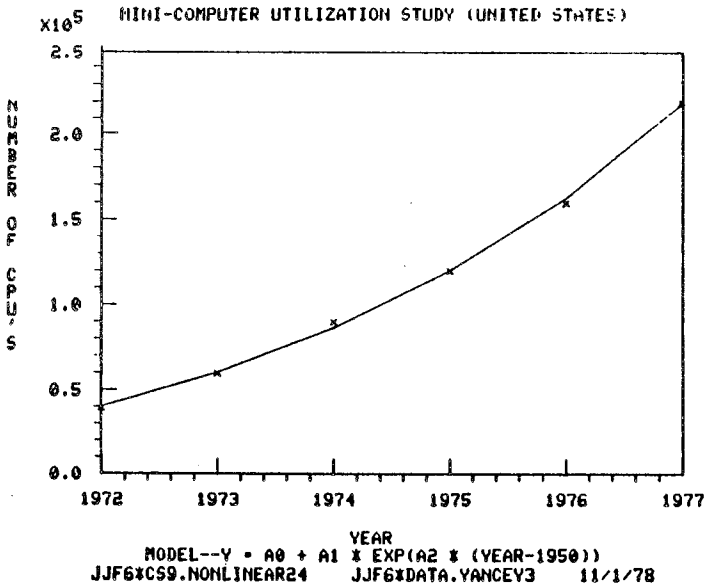


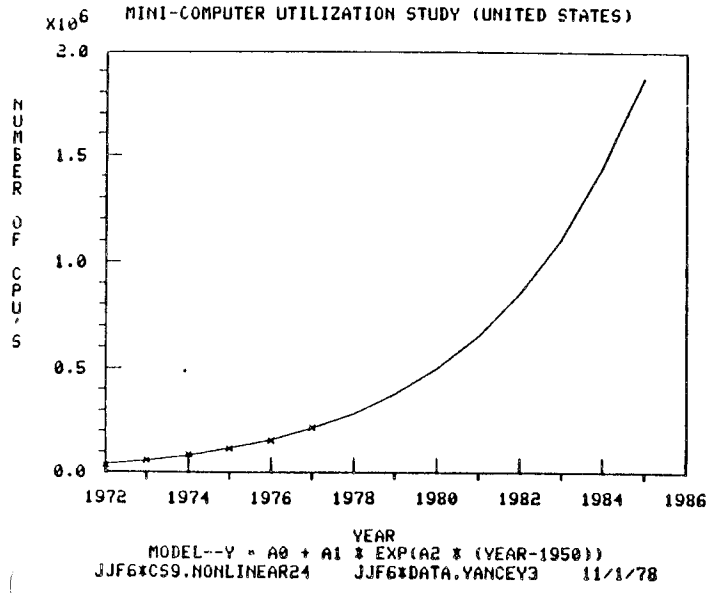
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 6
MODEL-- NUMCPU = A0+A1*EXP(A2*(YEAR-1950))
NO REPLICATION CASE

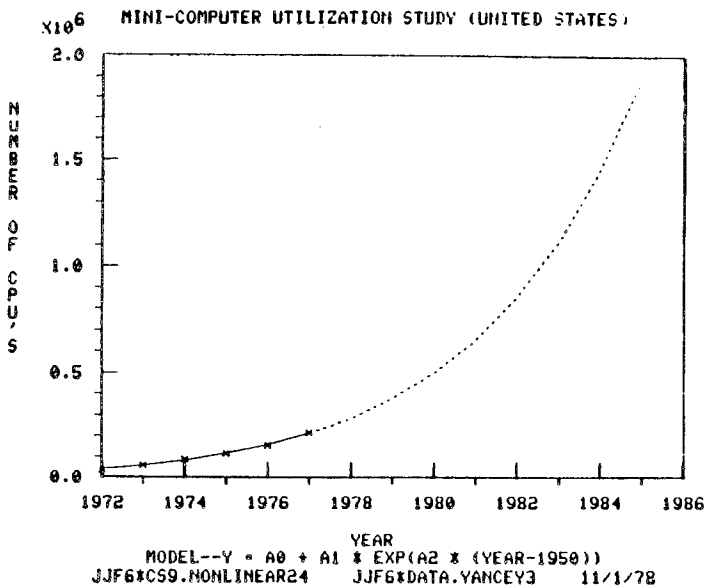
ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	*	PARAMETER ESTIMATES		
1--	.10000-01	.34002+05	*	.00000	.10000+01	.45561+00
2--	.22500-01	.23972+05	*	.20056+05	.19590+01	.42349+00
3--	.33750-01	.11854+05	*	.24147+05	.28056+01	.41226+00
4--	.16875-01	.10152+05	*	.21888+05	.38039+01	.40191+00
5--	.37969-01	.86752+04	*	.20203+05	.47758+01	.39440+00
6--	.18984-01	.81540+04	*	.18542+05	.58566+01	.38726+00
7--	.42715-01	.77000+04	*	.17296+05	.68398+01	.38195+00
8--	.21357-01	.74243+04	*	.16052+05	.79001+01	.37684+00
9--	.48054-01	.72035+04	*	.15109+05	.88236+01	.37299+00
10--	.24027-01	.70155+04	*	.14164+05	.97990+01	.36926+00
11--	.36041-01	.68083+04	*	.12521+05	.11591+02	.36307+00
12--	.18020-01	.65141+04	*	.11005+05	.13608+02	.35737+00
13--	.40546-01	.62314+04	*	.98528+04	.15380+02	.35315+00
14--	.20273-01	.60267+04	*	.87057+04	.17243+02	.34911+00
15--	.30409-01	.59102+04	*	.67471+04	.20623+02	.34251+00
16--	.15205-01	.55877+04	*	.49693+04	.24405+02	.33658+00
17--	.34210-01	.52415+04	*	.36320+04	.27701+02	.33225+00
18--	.17105-01	.50358+04	*	.23177+04	.31123+02	.32816+00
19--	.25658-01	.50172+04	*	.12488+03	.37184+02	.32163+00
20--	.12829-01	.46832+04	*	-.18203+04	.43803+02	.31591+00
21--	.28865-01	.43251+04	*	-.32678+04	.49446+02	.31179+00
22--	.14433-01	.41447+04	*	-.46629+04	.55172+02	.30797+00
23--	.32473-01	.40108+04	*	-.56967+04	.59871+02	.30515+00
24--	.16237-01	.39038+04	*	-.67031+04	.64593+02	.30250+00
25--	.24355-01	.37810+04	*	-.83735+04	.72693+02	.29926+00
26--	.12177-01	.36272+04	*	-.98578+04	.80963+02	.29451+00
27--	.27399-01	.35091+04	*	-.10947+05	.87633+02	.29179+00
28--	.13700-01	.34239+04	*	-.11990+05	.94202+02	.28927+00
29--	.20549-01	.33341+04	*	-.13668+05	.10509+03	.28538+00
30--	.10275-01	.32196+04	*	-.15118+05	.11580+03	.28202+00
31--	.23118-01	.31412+04	*	-.16159+05	.12416+03	.27963+00
32--	.11559-01	.30876+04	*	-.17133+05	.13214+03	.27747+00
33--	.17339-01	.30263+04	*	-.18638+05	.14478+03	.27425+00
34--	.86693-02	.29655+04	*	-.19893+05	.15654+03	.27156+00
35--	.13004-01	.29362+04	*	-.21655+05	.17349+03	.26794+00
36--	.65020-02	.28730+04	*	-.22988+05	.18799+03	.26520+00
37--	.97530-02	.28529+04	*	-.24599+05	.20590+03	.26203+00
38--	.48765-02	.28243+04	*	-.25655+05	.21904+03	.25993+00
39--	.24382-02	.28184+04	*	-.27103+05	.23691+03	.25720+00
40--	.12191-02	.28138+04	*	-.27783+05	.24653+03	.25585+00
41--	.60956-03	.28127+04	*	-.27899+05	.24844+03	.25557+00
42--	.30478-03	.28123+04	*	-.27909+05	.24863+03	.25554+00

	FINAL PARAMETER ESTIMATES		(APPROX. ST. DEV.)
1	A0	-27910.2	(.1482+05)
2	A1	248.650	(190.3)
3	A2	.255533	(.2548-01)

RESIDUAL STANDARD DEVIATION = 2812.2999267578
RESIDUAL DEGREES OF FREEDOM = 3



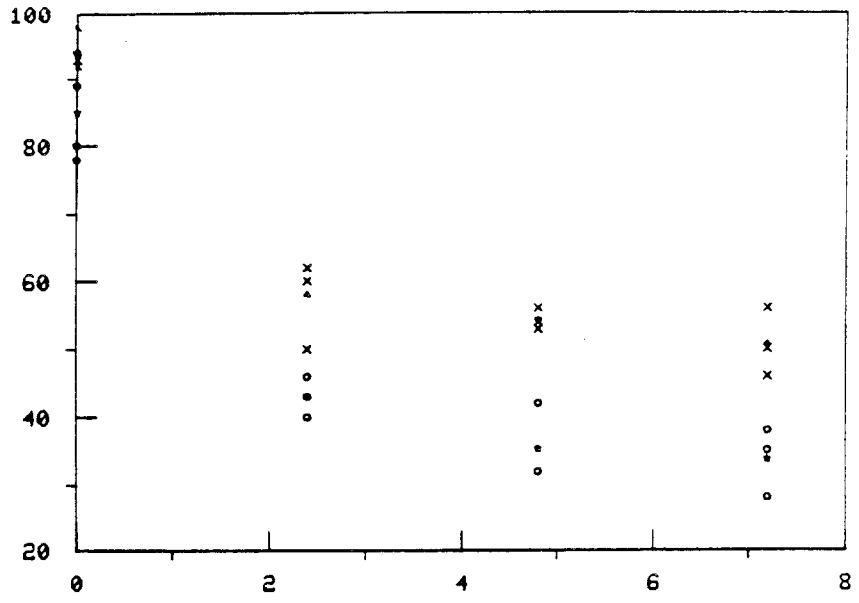




EXAMPLE 9

```
COMMENT EXAMPLE--BILL BEINE SAFETY EQUIPMENT STUDY
COMMENT MODEL   --AN EXPONENTIAL
COMMENT NOTE    --FIT SUBSETS AND SUPERIMPOSE FITTED CURVES
.
ECHO ON
HARDCOPY ON
BELL ON
VERSATEC ON
.
READ JJF6XDATA.BEINE6 Y X ID
PRINT 1 2 3
.
CHARACTERS CIRCLE X STAR TRIANGLE
LINES BLANK ALL
PLOT Y X ID
.
LET A0=30
LET A1=50
LET A2=1
FIT Y=A0+A1*EXP(-A2*X) SUBSET ID 3
.
LET B0=50
LET B1=40
LET B2=1
FIT Y=B0+B1*EXP(-B2*X) SUBSET ID 4
.
TITLE SAFETY EQUIPMENT STUDY
YLABEL PEAK ACCELERATION
XLABEL MOISTURE CONTENT (KG)
X2LABEL MODEL--Y = C0 + C1 * EXP(-C2 * X)
X3LABEL JJF6XCS9.NONLINEAR14    JJF6XDATA.BEINE6    11/7/78
LINES BLANK BLANK DOTTED DASHED
CHARACTERS CIRCLE X BLANK BLANK
.
PLOT Y X SUBSET ID 1 AND
PLOT Y X SUBSET ID 2 AND
PLOT Y = A0+A1*EXP(-A2*X) FOR X = 0 .1 8 AND
PLOT Y = B0+B1*EXP(-B2*X) FOR X = 0 .1 8
.
XLIMITS -2 10
.
PLOT Y X SUBSET ID 1 AND
PLOT Y X SUBSET ID 2 AND
PLOT Y = A0+A1*EXP(-A2*X) FOR X = 0 .1 8 AND
PLOT Y = B0+B1*EXP(-B2*X) FOR X = 0 .1 8
```

VARIABLES--	1	2	3
1--	.7800000+02	.0000000	.1000000+01
2--	.8000000+02	.0000000	.1000000+01
3--	.9400000+02	.0000000	.1000000+01
4--	.8900000+02	.0000000	.1000000+01
5--	.8530000+02	.0000000	.3000000+01
6--	.4000000+02	.2400000+01	.1000000+01
7--	.4600000+02	.2400000+01	.1000000+01
8--	.4300000+02	.2400000+01	.1000000+01
9--	.4300000+02	.2400000+01	.3000000+01
10--	.3200000+02	.4800000+01	.1000000+01
11--	.4200000+02	.4800000+01	.1000000+01
12--	.3200000+02	.4800000+01	.1000000+01
13--	.3530000+02	.4800000+01	.3000000+01
14--	.3800000+02	.7200000+01	.1000000+01
15--	.2800000+02	.7200000+01	.1000000+01
16--	.3500000+02	.7200000+01	.1000000+01
17--	.3370000+02	.7200000+01	.3000000+01
18--	.9800000+02	.0000000	.2000000+01
19--	.9200000+02	.0000000	.2000000+01
20--	.9300000+02	.0000000	.2000000+01
21--	.9430000+02	.0000000	.4000000+01
22--	.5000000+02	.2400000+01	.2000000+01
23--	.6000000+02	.2400000+01	.2000000+01
24--	.6200000+02	.2400000+01	.2000000+01
25--	.5800000+02	.2400000+01	.4000000+01
26--	.5300000+02	.4800000+01	.2000000+01
27--	.5600000+02	.4800000+01	.2000000+01
28--	.5400000+02	.4800000+01	.2000000+01
29--	.5430000+02	.4800000+01	.4000000+01
30--	.5600000+02	.7200000+01	.2000000+01
31--	.4600000+02	.7200000+01	.2000000+01
32--	.5000000+02	.7200000+01	.2000000+01
33--	.5070000+02	.7200000+01	.4000000+01



LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 4
MODEL-- Y=A0+A1*EXP(-A2*X)
NO REPLICATION CASE

ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	PARAMETER ESTIMATES		
1--	.10000-01	.11707+02	.30000+02	.50000+02	.10000+01
2--	.50000-02	.42795+01	.33781+02	.51517+02	.57344+00
3--	.25000-02	.64436+00	.33667+02	.51626+02	.69085+00
4--	.12500-02	.11395+00	.33438+02	.51860+02	.70327+00
5--	.62500-03	.11394+00	.33438+02	.51860+02	.70333+00

FINAL PARAMETER ESTIMATES		(APPROX. ST. DEV.)
1 A0	33.4377	(.1051)
2 A1	51.8601	(.1529)
3 A2	.703326	(.6990-02)

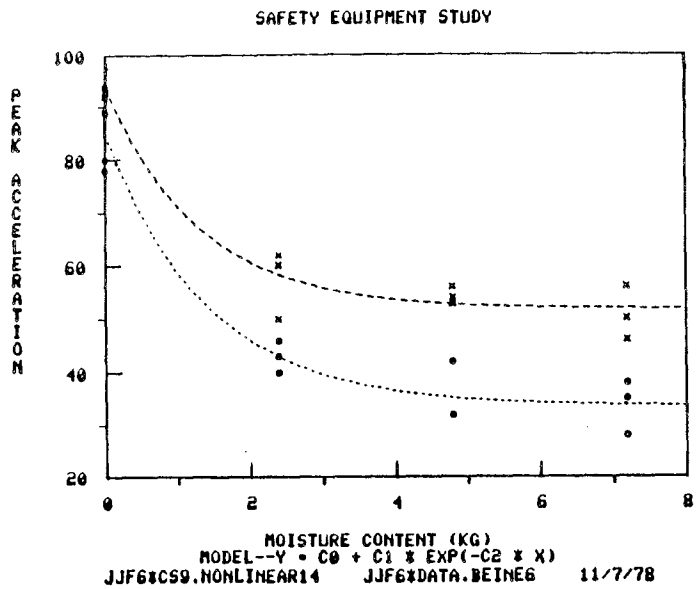
RESIDUAL STANDARD DEVIATION = .1139446376
RESIDUAL DEGREES OF FREEDOM = 1

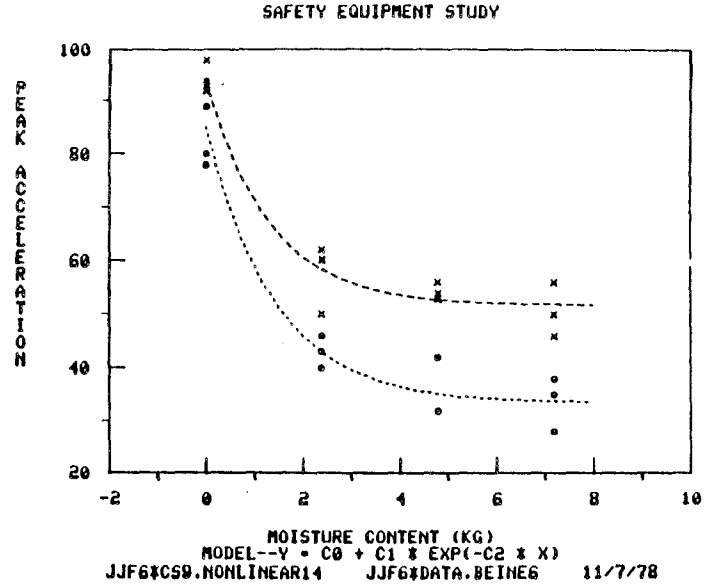
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 4
MODEL-- Y=B0+B1*EXP(-B2*X)
NO REPLICATION CASE

ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	PARAMETER ESTIMATES		
1--	.10000-01	.73358+01	.50000+02	.40000+02	.10000+01
2--	.50000-02	.23856+01	.51940+02	.42350+02	.71323+00
3--	.25000-02	.19792+01	.51686+02	.42578+02	.76409+00
4--	.12500-02	.19771+01	.51705+02	.42565+02	.77071+00

FINAL PARAMETER ESTIMATES		(APPROX. ST. DEV.)
1 B0	51.7084	(1.731)
2 B1	42.5631	(2.602)
3 B2	.771186	(.1700)

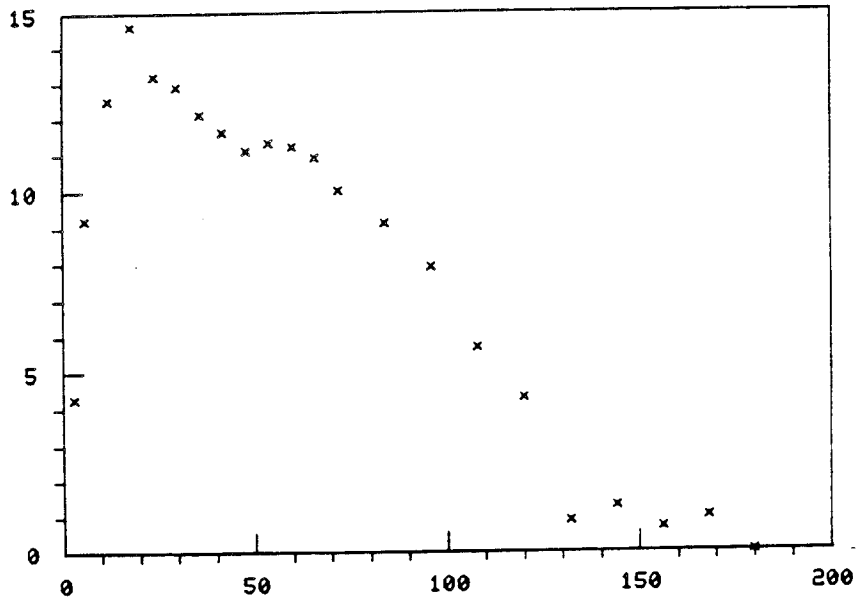
RESIDUAL STANDARD DEVIATION = 1.9770912677
RESIDUAL DEGREES OF FREEDOM = 1





EXAMPLE 10

```
COMMENT EXAMPLE--DOUG SHIER ERYTHEMA STUDY
COMMENT MODEL  --SUM OF 2 EXPONENTIALS
.
ECHO ON
HARDCOPY ON
BELL ON
.
SKIP 25
READ JJF6*DATA.SHIER1 REDNESS TIME
.
CHARACTERS X
LINES BLANK
PLOT REDNESS TIME
.
LET A=20
LET B=.1
FIT REDNESS = A * (EXP(-B*TIME/10) - EXP(-TIME/10))
.
TITLE ERYTHEMA (REDNESS OF A SKIN SORE) BIOMEDICAL STUDY
YLABEL REDNESS
XLABEL TIME (IN HOURS)
X2LABEL MODEL--Y = A * ((EXP(-B*X/10) - EXP(-X/10))
X3LABEL JJF6*CS9.NONLINEAR3    JJF6*DATA.SHIER1    11/76
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT REDNESS PRED VS TIME
.
YLABEL RESIDUALS
PLOT RES TIME
```

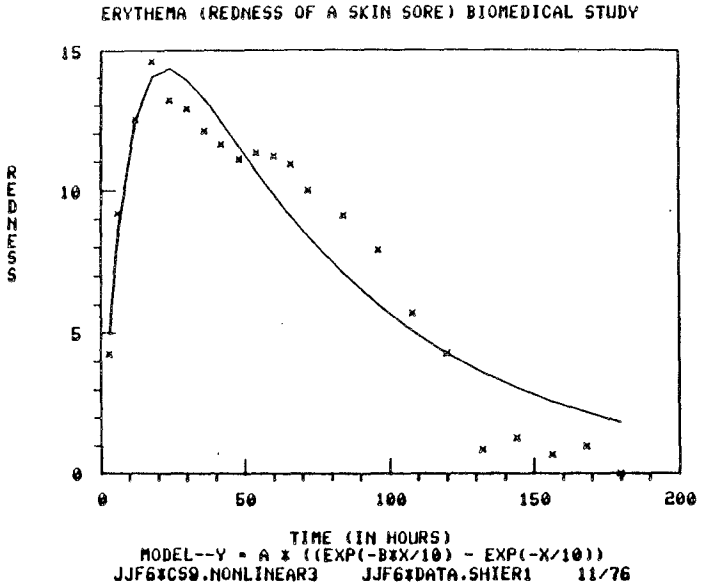


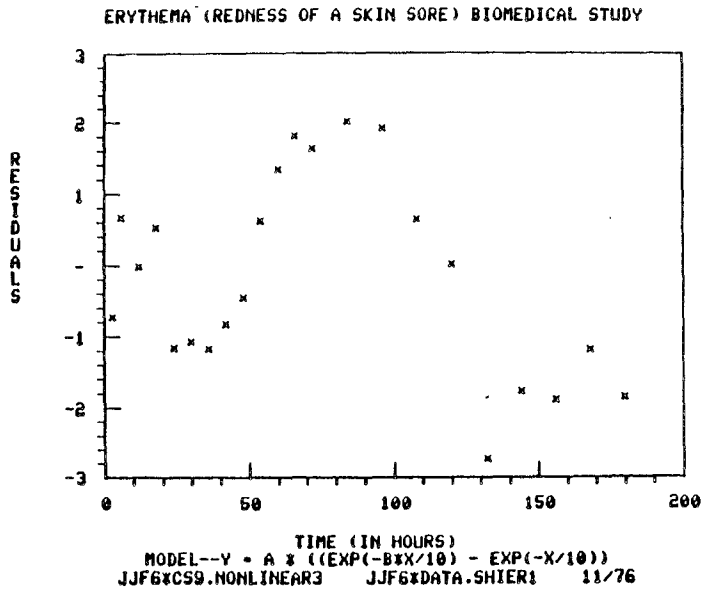
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 22
MODEL-- REDNESS = A * (EXP(-B*TIME/10) - EXP(-TIME/10))
NO REPLICATION CASE

ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	*	PARAMETER ESTIMATES	
1--	.10000-01	.19622+01	*	.20000+02	.10000+00
2--	.50000-02	.14315+01	*	.22889+02	.13957+00
3--	.25000-02	.14312+01	*	.23006+02	.14016+00

FINAL PARAMETER ESTIMATES		(APPROX. ST. DEV.)
1 A	23.0060	(1.489)
2 B	.140162	(.1265-01)

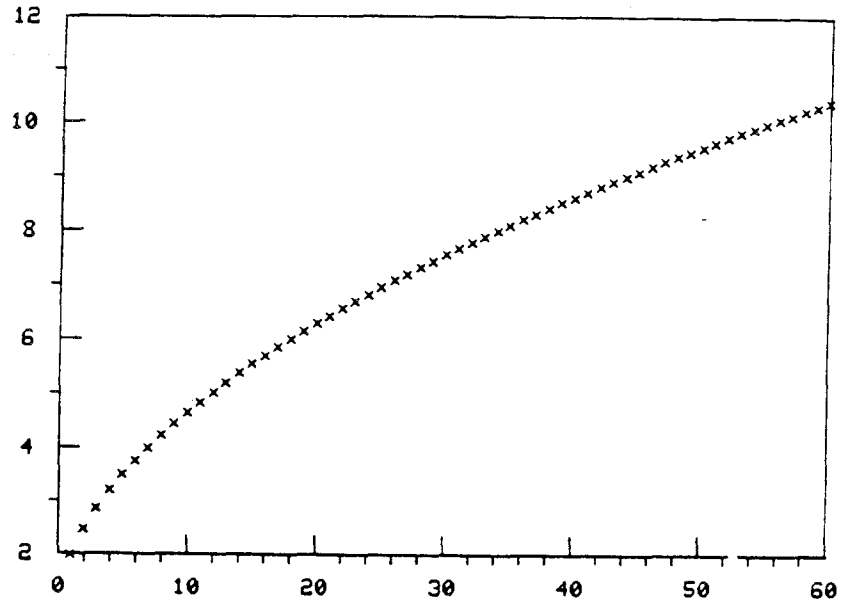
RESIDUAL STANDARD DEVIATION = 1.4311647564
RESIDUAL DEGREES OF FREEDOM = 20





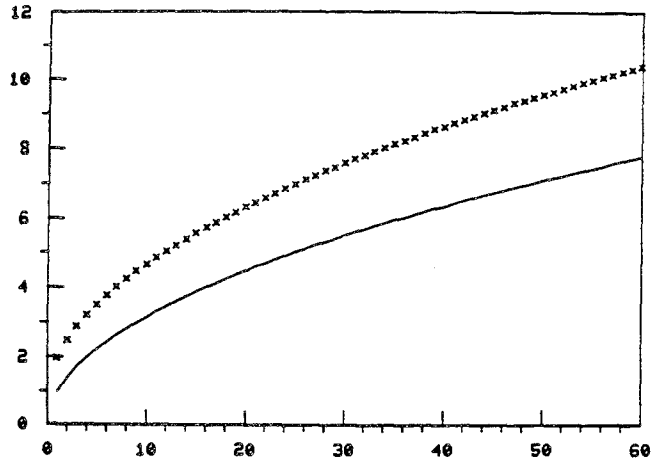
EXAMPLE 11

```
COMMENT EXAMPLE--DOUG SHIER QUEUEING THEORY SERVER FUNCTION PHI(X)
COMMENT MODEL   --POWER FUNCTION
COMMENT NOTE    --FULLY-SPECIFIED MODEL FIT CAPABILITY
.
ECHO ON
HARDCOPY ON
BELL ON
.
SKIP 25
READ JJF6*DATA.SHIER2 X Y
.
CHARACTERS X
LINES
PLOT Y X
.
CHARACTERS X BLANK
LINES BLANK SOLID
FIT Y = X**.5
PLOT Y PRED VS X
FIT Y = 1+X**.5
PLOT Y PRED VS X
FIT Y = 1 + X**.7
PLOT Y PRED VS X
FIT Y = 1 + X**.55
PLOT Y PRED VS X
.
LET B2=.55
FIT Y = B0 + B1*X**B2
.
TITLE QUEUEING THEORY SERVER FUNCTION PHI(X)
YLABEL Y
XLABEL X
X2LABEL MODEL--Y = B0 + B1 * X**B2
X3LABEL JJF6*CS9.NONLINEAR6    JJF6*DATA.SHIER2    11/76
PLOT Y PRED VS X
.
YLABEL RESIDUALS
PLOT RES X
```



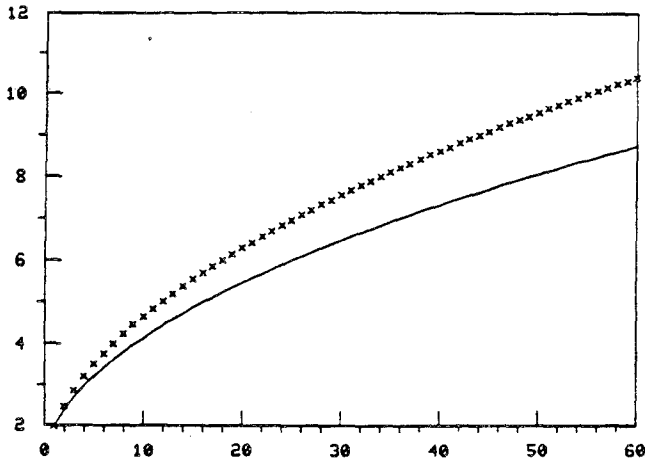
FULLY-SPECIFIED MODEL
SAMPLE SIZE N = 60
MODEL-- Y = X**.5
NO REPLICATION CASE

RESIDUAL STANDARD DEVIATION = 2.0607069731
RESIDUAL DEGREES OF FREEDOM = 60



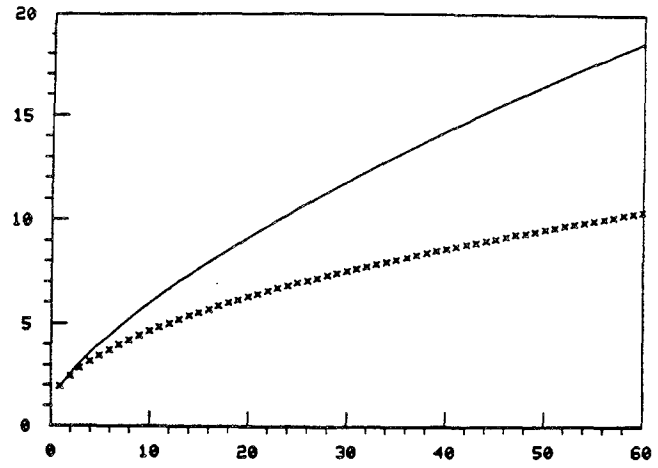
FULLY-SPECIFIED MODEL
SAMPLE SIZE N = 60
MODEL-- Y = 1+X**.5
NO REPLICATION CASE

RESIDUAL STANDARD DEVIATION = 1.1050422937
RESIDUAL DEGREES OF FREEDOM = 60



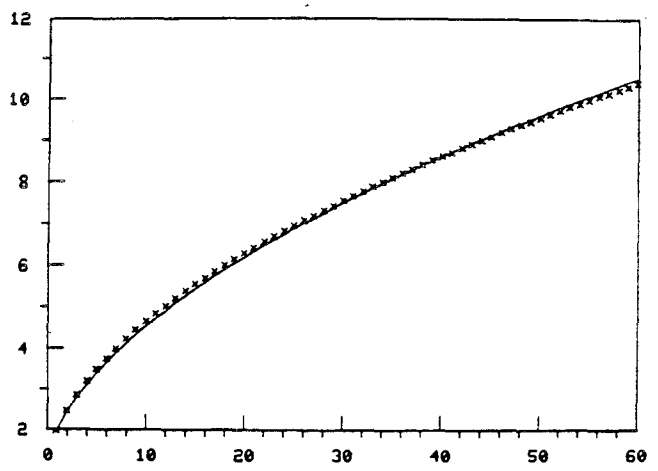
FULLY-SPECIFIED MODEL
SAMPLE SIZE N = 60
MODEL-- Y = 1+X**.7
NO REPLICATION CASE

RESIDUAL STANDARD DEVIATION = 4.8758707643
RESIDUAL DEGREES OF FREEDOM = 60



FULLY-SPECIFIED MODEL
SAMPLE SIZE N = 60
MODEL-- Y = 1 + X**.55
NO REPLICATION CASE

RESIDUAL STANDARD DEVIATION = .0772325629
RESIDUAL DEGREES OF FREEDOM = 60



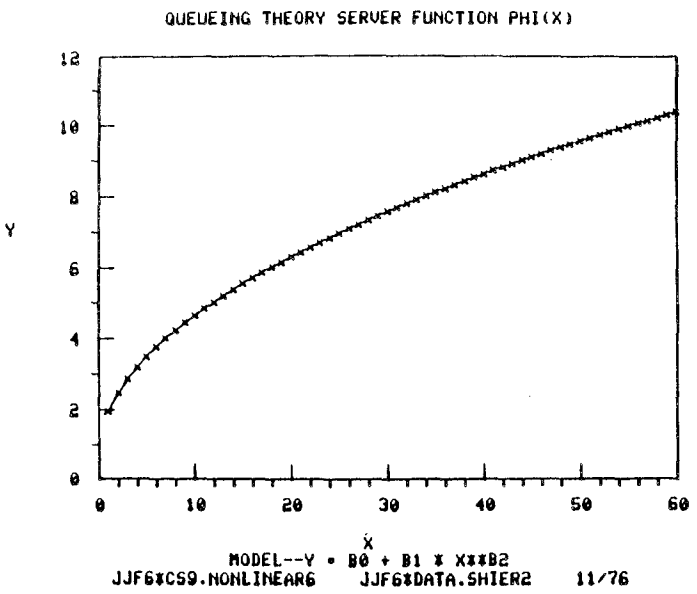
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 60
MODEL-- Y = B0 + B1*X**B2
NO REPLICATION CASE

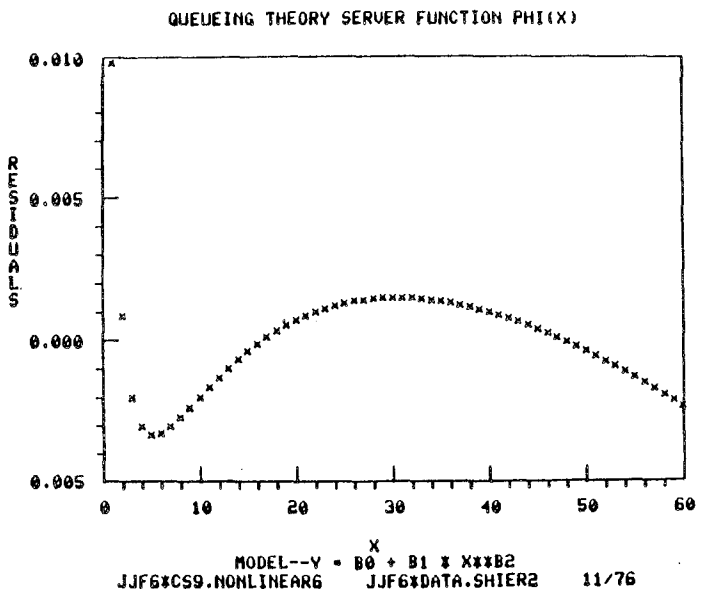
ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	PARAMETER ESTIMATES		
1--	.10000-01	.79239-01	.10000+01	.10000+01	.55000+00
2--	.50000-02	.65638-01	.85918+00	.11570+01	.51289+00
3--	.25000-02	.42829-02	.78295+00	.12089+01	.50614+00
4--	.12500-02	.19770-02	.77794+00	.12123+01	.50571+00
5--	.62500-03	.19752-02	.77785+00	.12123+01	.50569+00

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)

1	B0	.777848	(.2679-02)
2	B1	1.21235	(.1556-02)
3	B2	.505693	(.2539-03)

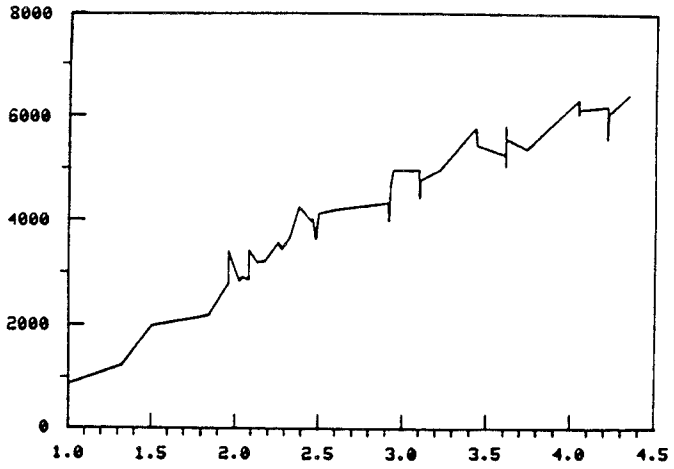
RESIDUAL STANDARD DEVIATION = .0019752173
RESIDUAL DEGREES OF FREEDOM = 57

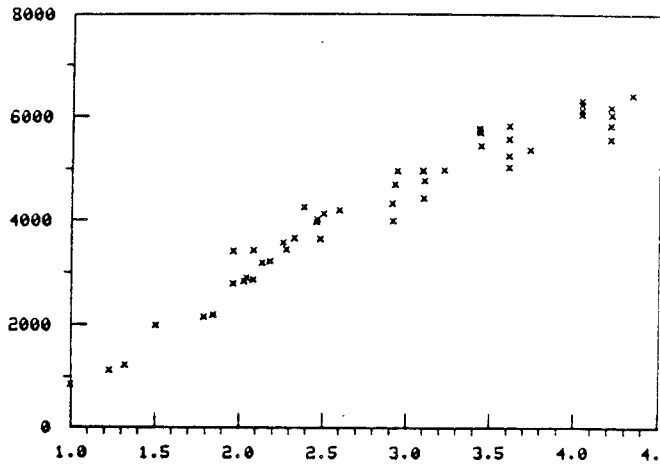




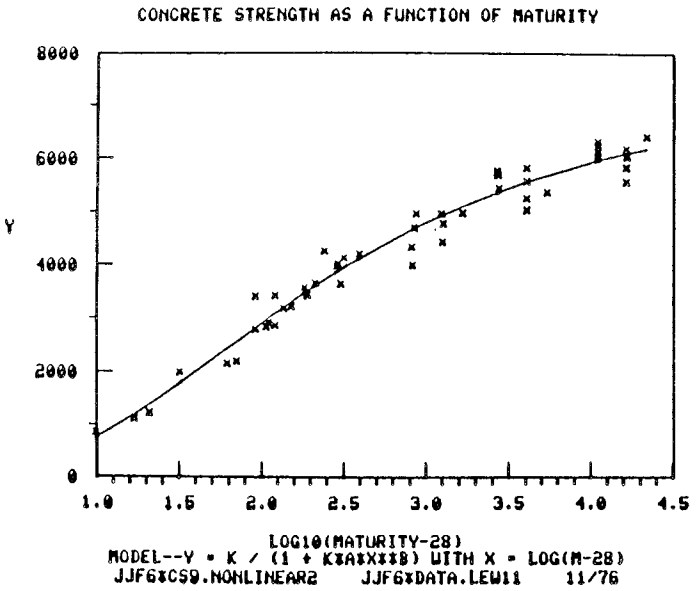
EXAMPLE 12

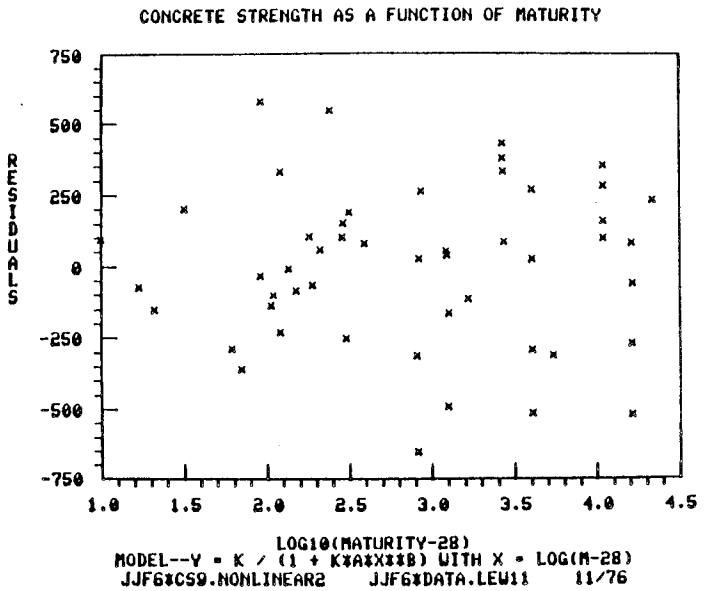
```
COMMENT EXAMPLE--LEW/WAMPLER CONCRETE STRENGTH
COMMENT MODEL  --POWER IN DENOMINATOR
.
ECHO ON
HARDCOPY ON
BELL ON
.
SKIP 25
READ JJF6*DATA.LEW11 M Y
LET X=ALOG10(M-28)
.
PLOT Y X
.
CHARACTERS X
LINES
PLOT Y X
.
LET K=9000
LET A=.001
LET B=-2
FIT Y=K/(1+K*A*X**B)
.
TITLE CONCRETE STRENGTH AS A FUNCTION OF MATURITY
YLABEL Y
XLABEL LOG10(MATURITY-28)
X2LABEL MODEL--Y = K / (1 + K*A*X**B) WITH X = LOG(M-28)
X3LABEL JJF6*CS9.NONLINEAR2    JJF6*DATA.LEW11    11/76
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT Y PRED VS X
.
YLABEL RESIDUALS
PLOT RES X
```





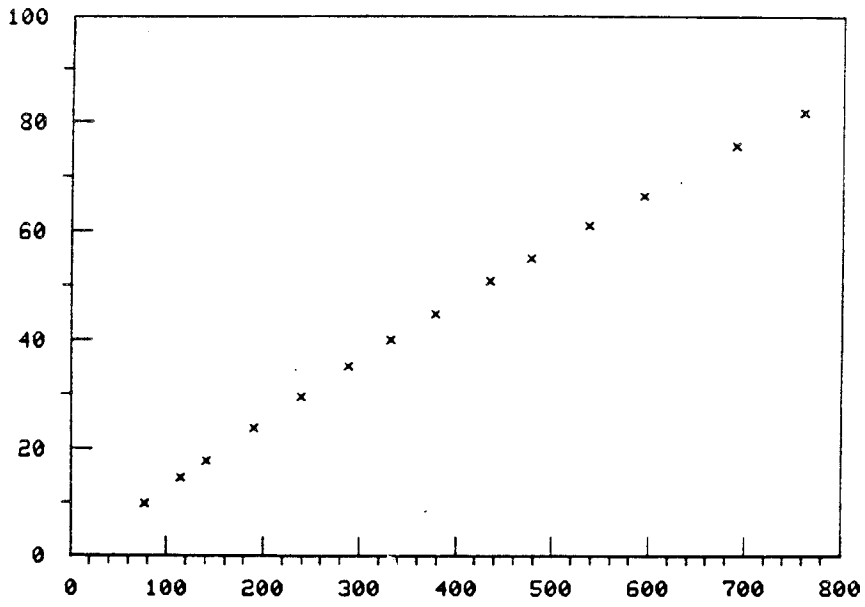
```
LEAST SQUARES NON-LINEAR FIT
      SAMPLE SIZE N =       50
      MODEL-- Y=K/(1+K*A*X**B)
      REPLICATION CASE
      REPLICATION STANDARD DEVIATION =        .4140350227+03
      REPLICATION DEGREES OF FREEDOM =           2
      NUMBER OF DISTINCT SUBSETS     =          48

ITERATION  CONVERGENCE  RESIDUAL  *  PARAMETER
 NUMBER      MEASURE    STANDARD  *  ESTIMATES
                        DEVIATION *
-----------------------------------*-----------
  1--       .10000-01    .35234+03 *   .90000+04   .10000-02  -.20000+01
  2--       .22500-01    .34590+03 *   .73315+04   .10952-02  -.23641+01
  3--       .11250-01    .28200+03 *   .77591+04   .11499-02  -.24228+01
  4--       .56250-02    .28200+03 *   .77506+04   .11494-02  -.24236+01

      FINAL PARAMETER ESTIMATES           (APPROX. ST. DEV.)
      1  K                  7750.42       (  544.0    )
      2  A                   .114950-02   (  .1741-03)
      3  B                 -2.42372       (  .2471    )

      RESIDUAL    STANDARD DEVIATION =        281.9972190857
      RESIDUAL    DEGREES OF FREEDOM =           47
      REPLICATION STANDARD DEVIATION =        414.0350227356
      REPLICATION DEGREES OF FREEDOM =           2
      LACK OF FIT F RATIO =      .4401 = THE  11.4769% POINT OF THE
      F DISTRIBUTION WITH     45 AND      2 DEGREES OF FREEDOM
```





EXAMPLE 13

```
COMMENT EXAMPLE--DUARIKA MISRA DENTAL RESEARCH STUDY
COMMENT MODEL   --EXPONENTIAL, QUADRATIC, SQUARE ROOT, RECIPROCAL
COMMENT NOTE    --COMPARING 4 NON-LINEAR MODLES
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6*DATA.MISRA1 V P
.
CHARACTERS X
LINES
PLOT V P
.
LET VM=400
LET C=.0003
FIT V = VM * (1 - EXP(-C*P))
.
TITLE DENTAL RESEARCH MONOMOLECULAR ADSORBTION STUDY
YLABEL VOLUME
XLABEL PRESSURE
X2LABEL MODEL--V = VM * (1 - EXP(-C*P))
X3LABEL JJF6*CS9.NONLINEAR29    JJF6*DATA.MISRA1    4/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT V PRED VS P
LET S1=RESSD
.
FIT V = VM * (1-(1+C*P/2)**(-2))
LET S2=RESSD
.
FIT V = VM * (1-(1+2*C*P)**(-.5))
LET S3=RESSD
.
FIT V = VM*C*P*((1+C*P)**(-1))
LET S4=RESSD
.
PRINT S1 S2 S3 S4
```

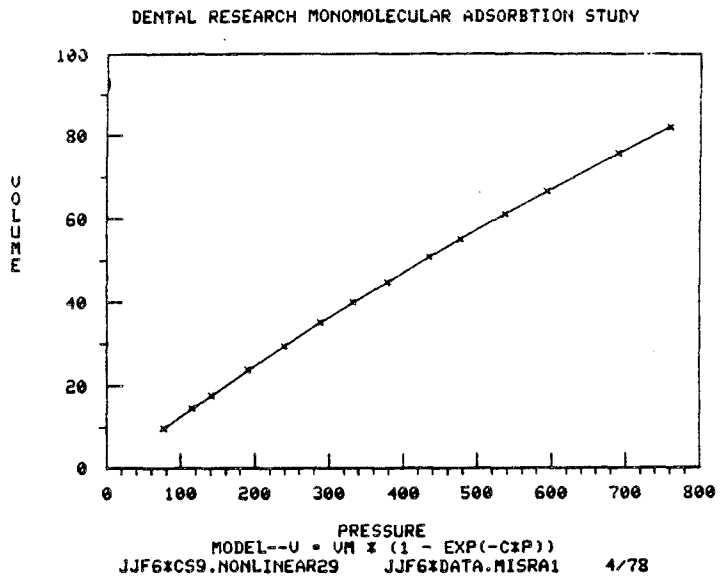


LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 14
MODEL-- V = VM * (1 - EXP(-C*P))
NO REPLICATION CASE

ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	*	PARAMETER ESTIMATES	
1--	.10000-01	.16396+01	*	.40000+03	.30000-03
2--	.50625-01	.99416+00	*	.36934+03	.33323-03
3--	.25312-01	.85428+00	*	.34023+03	.36523-03
4--	.56953-01	.68782+00	*	.32051+03	.39177-03
5--	.28477-01	.57803+00	*	.30338+03	.41701-03
6--	.42715-01	.52336+00	*	.28080+03	.45354-03
7--	.21357-01	.33959+00	*	.26578+03	.48445-03
8--	.32036-01	.24685+00	*	.25156+03	.51618-03
9--	.16018-01	.12916+00	*	.24493+03	.53399-03
10--	.80090-02	.10551+00	*	.23995+03	.54716-03
11--	.40045-02	.10188+00	*	.23900+03	.54999-03
12--	.20023-02	.10188+00	*	.23894+03	.55016-03

FINAL PARAMETER ESTIMATES		(APPROX. ST. DEV.)
1 VM	238.940	(2.681)
2 C	.550162-03	(.7208-05)

RESIDUAL STANDARD DEVIATION = .1018779697
RESIDUAL DEGREES OF FREEDOM = 12



```
LEAST SQUARES NON-LINEAR FIT
      SAMPLE SIZE N =       14
      MODEL-- V = VM * (1-(1+C*P/2)**(-2))
      NO REPLICATION CASE

ITERATION  CONVERGENCE   RESIDUAL  *  PARAMETER
 NUMBER      MEASURE     STANDARD  *  ESTIMATES
                         DEVIATION *
-----------------------------------*-----------
  1--       .10000-01    .31704+01 *   .23894+03   .55016-03
  2--       .33750-01    .20920+01 *   .29097+03   .44409-03
  3--       .16875-01    .31091+00 *   .31733+03   .41787-03
  4--       .84375-02    .16555+00 *   .33322+03   .39566-03
  5--       .42187-02    .79861-01 *   .33765+03   .39078-03
  6--       .21094-02    .79303-01 *   .33798+03   .39041-03
  7--       .10547-02    .79303-01 *   .33799+03   .39040-03
  8--       .40045-02    .79302-01 *   .33799+03   .39040-03

      FINAL PARAMETER ESTIMATES           (APPROX. ST. DEV.)
      1  VM                 337.993      (  3.136    )
      2  C                  .390396-03   (  .4222-05)

      RESIDUAL    STANDARD DEVIATION =          .0793014774
      RESIDUAL    DEGREES OF FREEDOM =          12
```

```
LEAST SQUARES NON-LINEAR FIT
      SAMPLE SIZE N =       14
      MODEL-- V = VM * (1-(1+2*C*P)**(-.5))
      NO REPLICATION CASE

ITERATION  CONVERGENCE   RESIDUAL  *  PARAMETER
 NUMBER      MEASURE     STANDARD  *  ESTIMATES
                         DEVIATION *
-----------------------------------*-----------
  1--       .10000-01    .59889+01 *   .33799+03   .39040-03
  2--       .50625-01    .39276+01 *   .44754+03   .28899-03
  3--       .25312-01    .86068+00 *   .51803+03   .26122-03
  4--       .37969-01    .58037+00 *   .56860+03   .23478-03
  5--       .18984-01    .20499+00 *   .59791+03   .22321-03
  6--       .94922-02    .14231+00 *   .62522+03   .21190-03
  7--       .47461-02    .60271-01 *   .63533+03   .20849-03
  8--       .23730-02    .58430-01 *   .63639+03   .20815-03

      FINAL PARAMETER ESTIMATES           (APPROX. ST. DEV.)
      1  VM                 636.420      (  4.623    )
      2  C                  .208139-03   (  .1760-05)

      RESIDUAL    STANDARD DEVIATION =          .0584296780
      RESIDUAL    DEGREES OF FREEDOM =          12
```

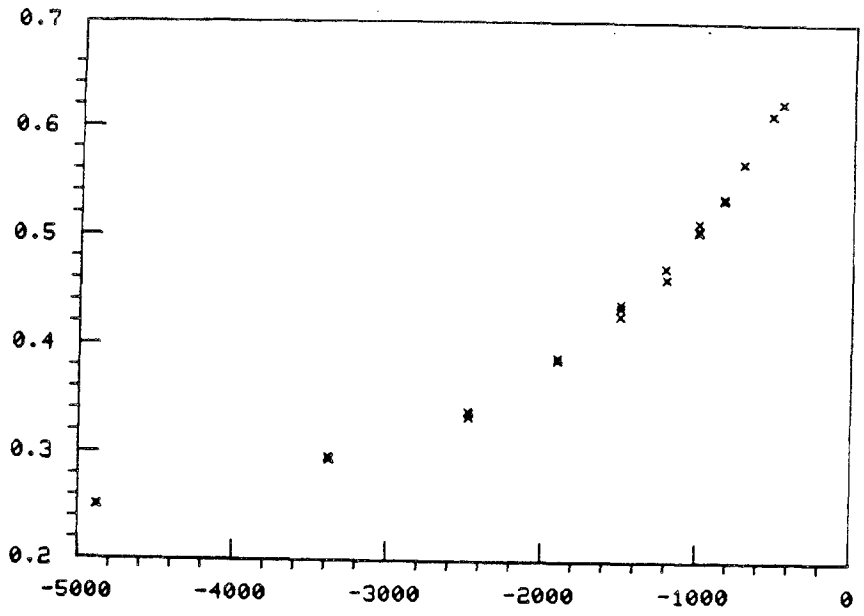
```
LEAST SQUARES NON-LINEAR FIT
      SAMPLE SIZE N =       14
      MODEL-- V = VM*C*P*((1+C*P)**(-1))
      NO REPLICATION CASE

ITERATION  CONVERGENCE   RESIDUAL  *  PARAMETER
 NUMBER      MEASURE     STANDARD  *  ESTIMATES
                         DEVIATION *
-----------------------------------*-----------
  1--       .10000-01    .26235+01 *   .63642+03   .20814-03
  2--       .22500-01    .23233+01 *   .49492+03   .24908-03
  3--       .11250-01    .56705+00 *   .45727+03   .28360-03
  4--       .56250-02    .13903+00 *   .43904+03   .30013-03
  5--       .28125-02    .68577-01 *   .43744+03   .30221-03
  6--       .14062-02    .68568-01 *   .43737+03   .30228-03

      FINAL PARAMETER ESTIMATES           (APPROX. ST. DEV.)
      1  VM                 437.365      (  3.617    )
      2  C                  .302277-03   (  .2912-05)

      RESIDUAL    STANDARD DEVIATION =          .0685681803
      RESIDUAL    DEGREES OF FREEDOM =          12
```

PARAMETERS AND CONSTANTS--

S1	--	.1018780+00
S2	--	.7930148-01
S3	--	.5842968-01
S4	--	.6856818-01

EXAMPLE 14

```
COMMENT EXAMPLE--LARRY ROSZMAN QUANTUM DEFECTS FOR SULFUR I ATOM
COMMENT MODEL   --ARCTANGENT
COMMENT NOTE    --FITTING A THEORETICAL MODEL

ECHO ON
HARDCOPY ON
BELL ON

READ JJF6*DATA.ROSZMAN1 X T
LET Q = X-SQRT(-109737.3/T)

CHARACTERS X
LINES BLANK
PLOT Q T

LET A = .2
LET B = -.00005
LET C = 200
LET D = -123
FIT Q = A-B*T-ATAN(C/(T-D))/3.14159

TITLE QUANTUM DEFECTS FOR SULFUR I ATOM
YLABEL QUANTUM DEFECT
XLABEL EXCITED STATE ENERGY
X2LABEL MODEL--Q = A - B*T - ATAN(C/(T-D))/3.14159
X3LABEL JJF6*CS9.NONLINEAR45    DATA.ROSZMAN1    11/17/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT Q PRED VS T

PLOT Q T AND
PLOT Q = A-B*T-ATAN(C/(T-D))/3.14159 FOR T = -5000 50 -400

YLABEL RESIDUALS
PLOT RES T
```



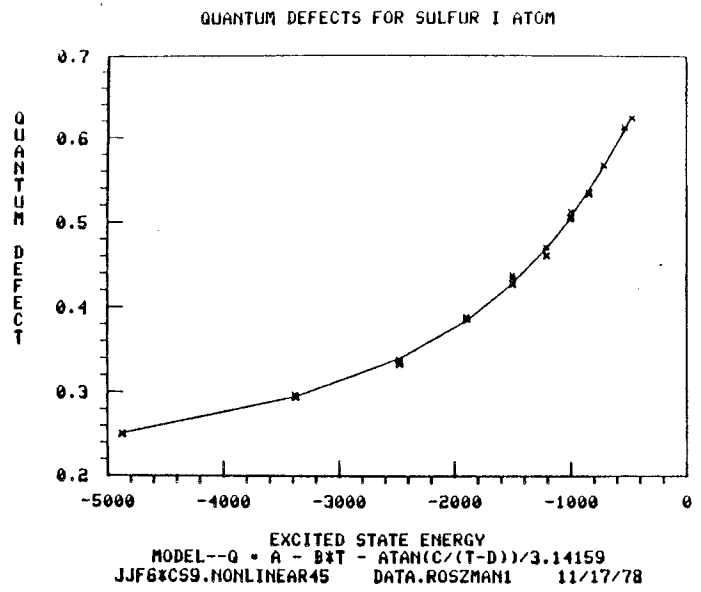
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 25
MODEL-- Q = A-B*T-ATAN(C/(T-D))/3.14159
NO REPLICATION CASE

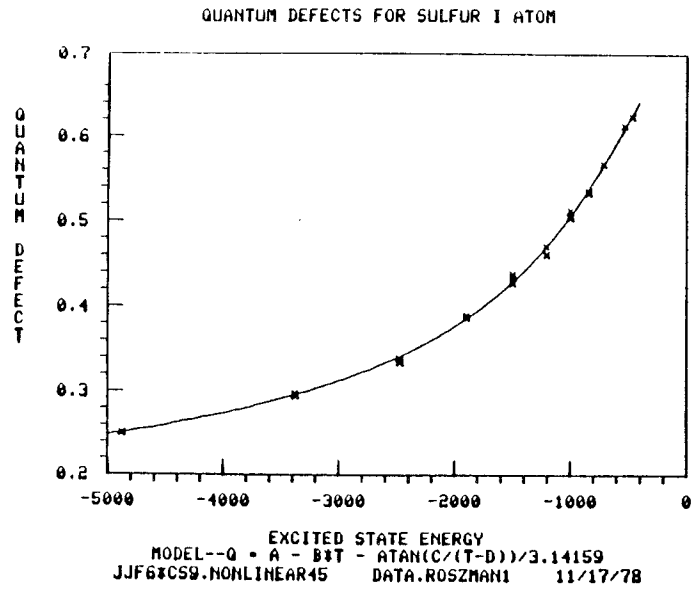
ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	*	PARAMETER ESTIMATES			
1--	.10000-01	.29229+00	*	.20000+00	-.50000-04	.20000+03	-.12300+03
2--	.50000-02	.64731-01	*	.29385+00	-.19971-04	.79684+03	.48209+03
3--	.25000-02	.40325-01	*	.14199+00	-.48900-06	.18723+04	.17000+03
4--	.12656-01	.35339-01	*	.22996+00	-.73756-05	.74204+03	-.21816+03
5--	.63281-02	.74600-02	*	.23390+00	-.10476-04	.10264+04	-.12025+03
6--	.31641-02	.51891-02	*	.20577+00	-.67880-05	.11940+04	-.15801+03
7--	.15820-02	.48543-02	*	.20194+00	-.61932-05	.12049+04	-.18133+03
8--	.79102-03	.48542-02	*	.20195+00	-.61919-05	.12046+04	-.18139+03

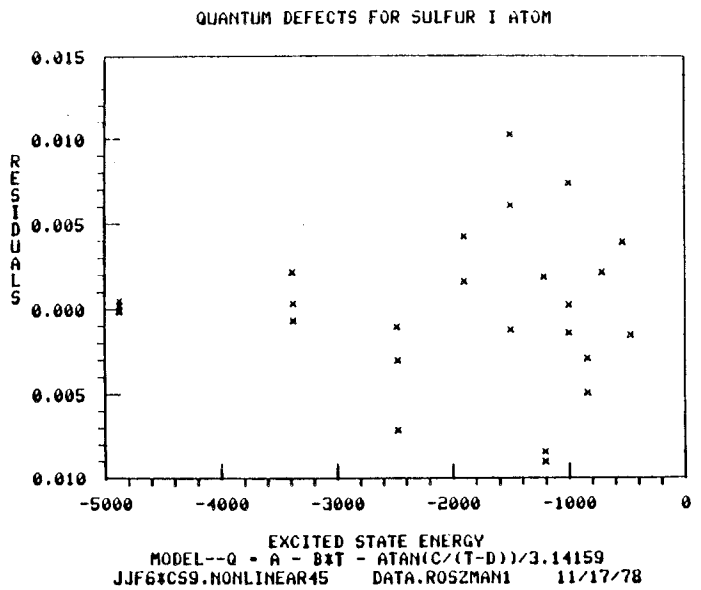
FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)

1	A	.201944	(.1927-01)
2	B	-.619161-05	(.3216-05)
3	C	1204.56	(74.64)
4	D	-181.394	(49.88)

RESIDUAL STANDARD DEVIATION = .0048542210
RESIDUAL DEGREES OF FREEDOM = 21

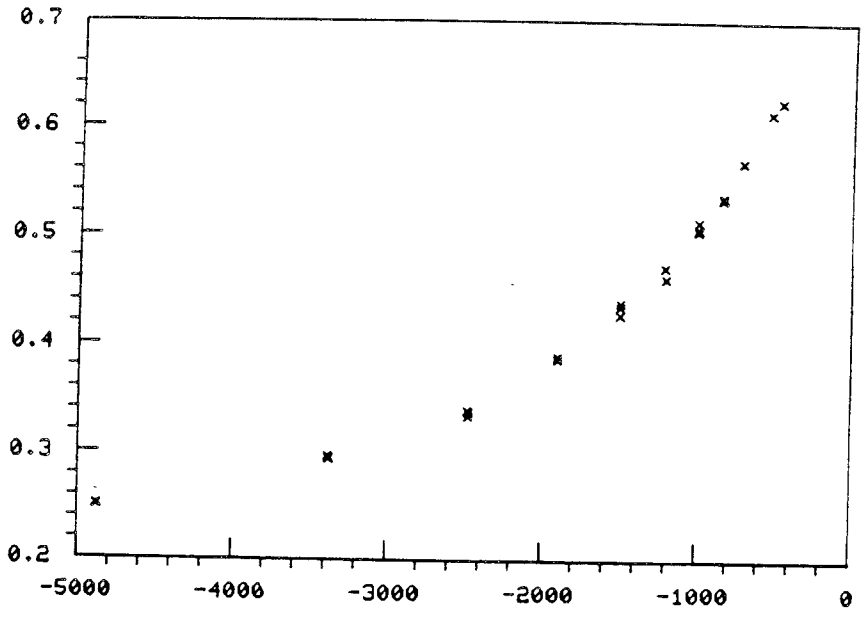




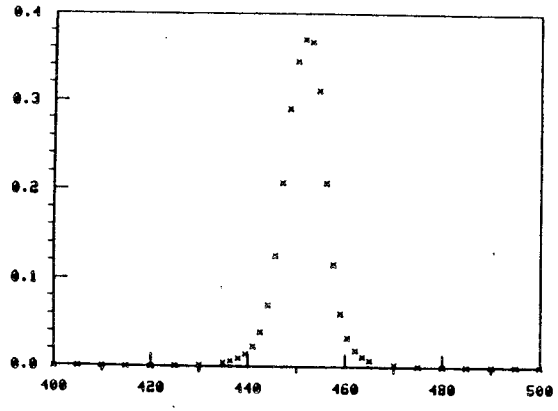


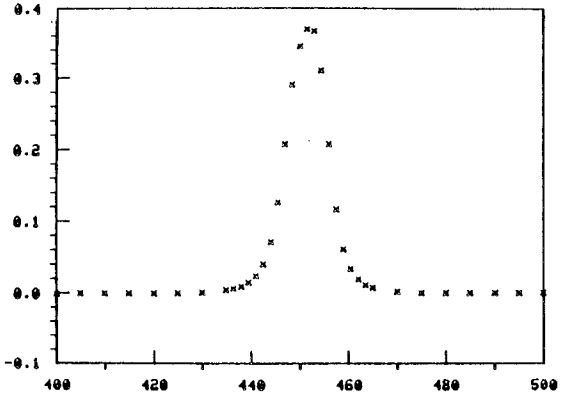
EXAMPLE 14

```
COMMENT EXAMPLE--LARRY ROSZMAN QUANTUM DEFECTS FOR SULFUR I ATOM
COMMENT MODEL   --ARCTANGENT
COMMENT NOTE    --FITTING A THEORETICAL MODEL

.
ECHO ON
HARDCOPY ON
BELL ON

.
READ JJF6*DATA.ROSZMAN1 X T
LET Q = X-SQRT(-109737.3/T)

.
CHARACTERS X
LINES BLANK
PLOT Q T

.
LET A = .2
LET B = -.00005
LET C = 200
LET D = -123
FIT Q = A-B*T-ATAN(C/(T-D))/3.14159

.
TITLE QUANTUM DEFECTS FOR SULFUR I ATOM
YLABEL QUANTUM DEFECT
XLABEL EXCITED STATE ENERGY
X2LABEL MODEL--Q = A - B*T - ATAN(C/(T-D))/3.14159
X3LABEL JJF6*CS9.NONLINEAR45    DATA.ROSZMAN1    11/17/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT Q PRED VS T

.
PLOT Q T AND
PLOT Q = A-B*T-ATAN(C/(T-D))/3.14159 FOR T = -5000 50 -400

.
YLABEL RESIDUALS
PLOT RES T
```



```
COMMENT EXAMPLE--KEN ECKERLE TRANSMITTANCE STUDY
COMMENT MODEL  --LORENTZIAN AND GAUSSIAN
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6%DATA.ECKERLE4 X Y
.
CHARACTERS X
LINES
PLOT Y X
.
YLIM -.1 .4
PLOT Y X
.
LET A = .3
LET MU1 = 450
LET B=4
FIT Y = A/(1+((X-MU1)/B)**2)
.
TITLE CIRCULAR INTERFERENCE FILTER STUDY
YLABEL TRANSMITTANCE
XLABEL WAVELENGTH
X2LABEL MODEL--Y = A / (1 + ((X-MU1)/B)**2)
X3LABEL JJF6%CS9.NONLINEAR26    JJF6%DATA.ECKERLE4    4/20/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT Y PRED VS X
LET PRED1 = PRED
LET S1=RESSD
.
LET D=1
LET E=10
LET F=.4
LET MU2=450
FIT Y = (D/E) * F * EXP(-0.5 * ((X-MU2)/E)**2)
.
X2LABEL MODEL--Y = (D/E) * F * EXP(-0.5 * ((X-MU2)/E)**2)
LINES BLANK DOTTED
PLOT Y PRED VS X
LET PRED2 = PRED
LET S2=RESSD
.
X2LABEL SOLID = LORENTZIAN; DOTTED = GAUSSIAN
CHARACTERS X BLANK BLANK
LINES BLANK SOLID DOT
PLOT Y PRED1 PRED2 VS X
.
LET MN=MIN(X)
LET MX=MAX(X)
LET INC=(MX-MN)/100
.
PLOT Y X AND
PLOT Y = A/(1+((X-MU1)/B)**2) FOR X = MN INC MX AND
PLOT Y = (D/E) * F * EXP(-0.5 * ((X-MU2)/E)**2) FOR X = MN INC MX
.
PRINT A B MU1 D E F MU2 S1 S2
```





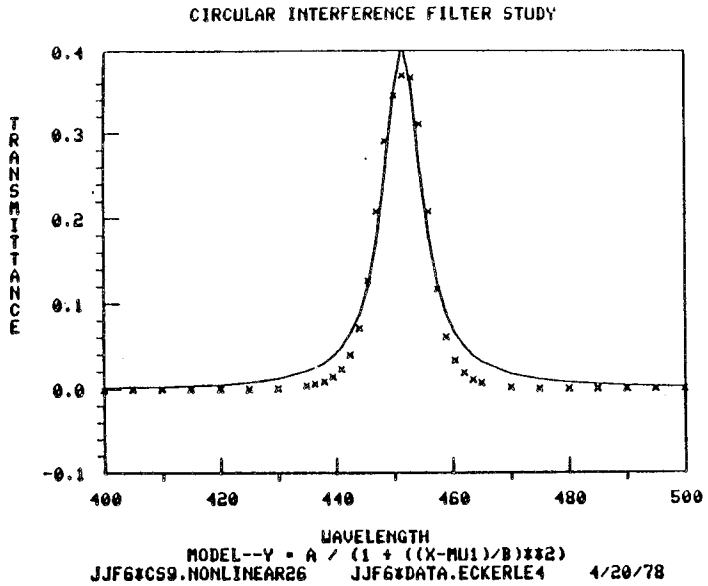
EXAMPLE 15

LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 35
MODEL-- Y = A/(1+((X-MU1)/B)**2)
NO REPLICATION CASE

ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	*	PARAMETER ESTIMATES		
1--	.10000-01	.55012-01	*	.30000+00	.45000+03	.40000+01
2--	.50000-02	.52846-01	*	.49220+00	.45292+03	.25342+01
3--	.19222+00	.48949-01	*	.27900+00	.45191+03	.42035+01
4--	.96108-01	.26227-01	*	.37642+00	.45112+03	.43148+01
5--	.48054-01	.24440-01	*	.43592+00	.45175+03	.34642+01
6--	.24027-01	.23333-01	*	.38152+00	.45147+03	.41820+01
7--	.12014-01	.22214-01	*	.41674+00	.45159+03	.37729+01
8--	.60068-02	.22115-01	*	.40203+00	.45152+03	.40177+01
9--	.30034-02	.22001-01	*	.41142+00	.45156+03	.38891+01
10--	.17319+00	.21999-01	*	.40751+00	.45155+03	.39504+01
11--	.86595-01	.21998-01	*	.40859+00	.45155+03	.39332+01
12--	.43298-01	.21996-01	*	.40850+00	.45155+03	.39360+01

	FINAL PARAMETER ESTIMATES		(APPROX. ST. DEV.)
1	A	.408500	(.1979-01)
2	MU1	451.547	(.2215)
3	B	3.93598	(.2589)

RESIDUAL STANDARD DEVIATION = .0219961645
RESIDUAL DEGREES OF FREEDOM = 32

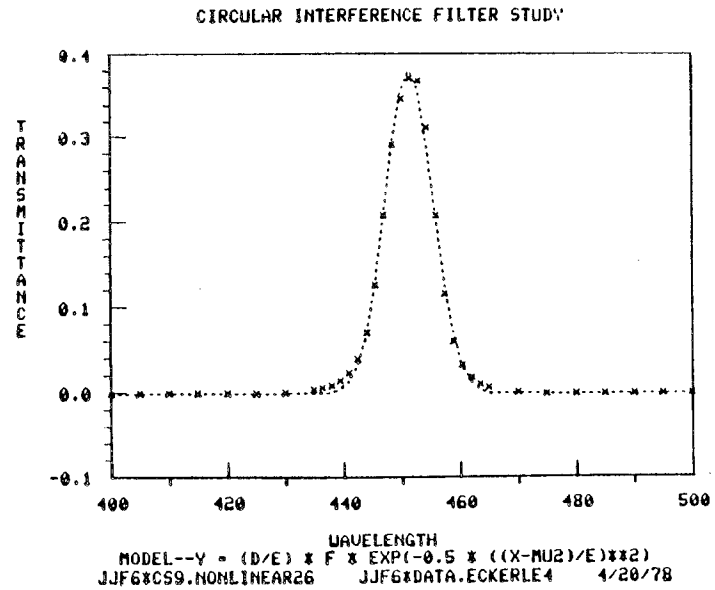


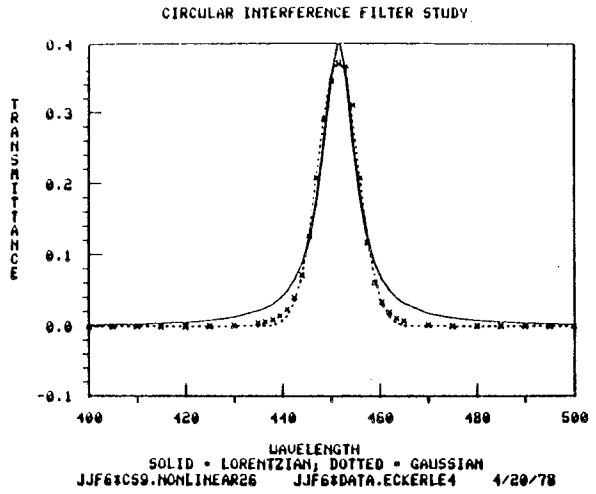
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 35
MODEL-- Y = (D/E) * F * EXP(-0.5 * ((X-MU2)/E)**2)
NO REPLICATION CASE

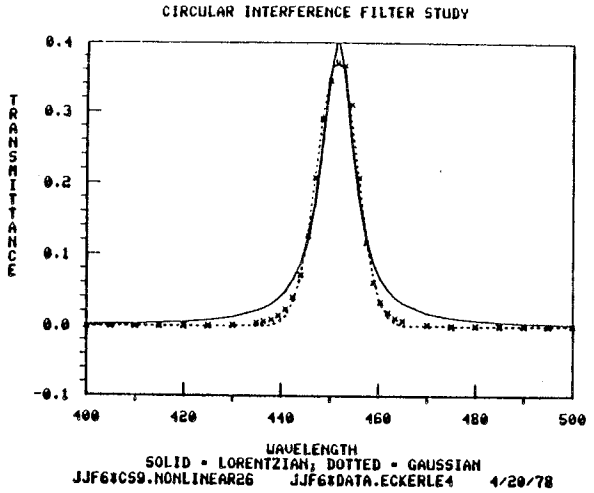
ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	*	PARAMETER ESTIMATES			
1--	.10000-01	.13025+00	*	.10000+01	.10000+02	.40000+00	.45000+03
2--	.29193+01	.86087-01	*	.13696+01	.44747+01	.54783+00	.44983+03
PARAMETER 3 IS LINEARLY DEPENDANT UPON PREVIOUS PARAMETERS							
PARAMETER WILL BE KEPT CONSTANT							
3--	.29193+01	.86087-01	*	.13696+01	.44747+01	.54783+00	.44983+03
4--	.14596+01	.74460-01	*	.15230+01	.41582+01	.54783+00	.45026+03
5--	.72982+00	.46686-01	*	.19160+01	.36249+01	.54783+00	.45093+03
6--	.36491+00	.19652-01	*	.24331+01	.35894+01	.54783+00	.45147+03
7--	.18246+00	.83489-02	*	.27378+01	.39311+01	.54783+00	.45156+03
8--	.91228-01	.69109-02	*	.28267+01	.40886+01	.54783+00	.45152+03
9--	.45614-01	.69051-02	*	.28375+01	.40931+01	.54783+00	.45152+03
10--	.22807-01	.69041-02	*	.28380+01	.40906+01	.54783+00	.45152+03

	FINAL PARAMETER ESTIMATES		(APPROX. ST. DEV.)
1	D	2.83794	(.2837-01)
2	E	4.09056	(.5387-01)
3	F	.547828	
4	MU2	451.515	(.6164-01)

RESIDUAL STANDARD DEVIATION = .0069040230
RESIDUAL DEGREES OF FREEDOM = 31



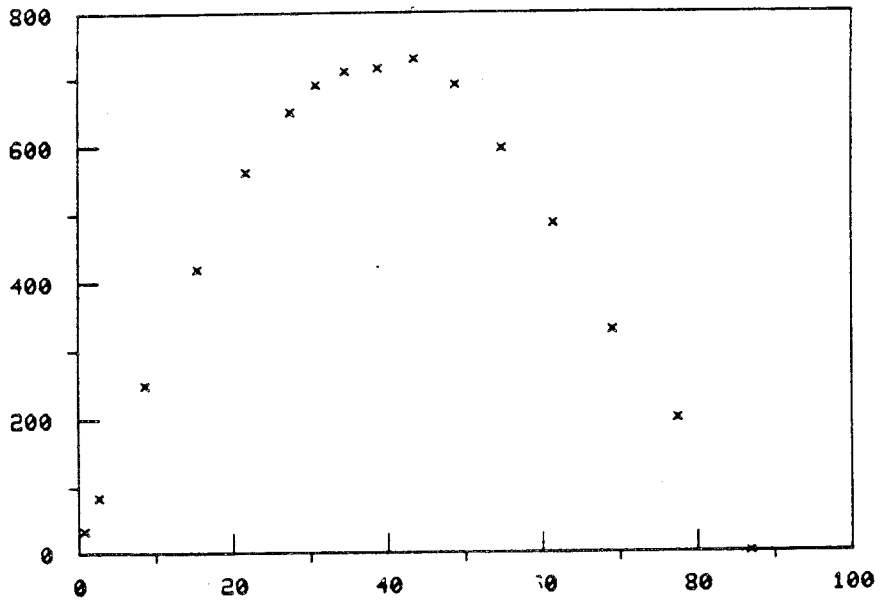


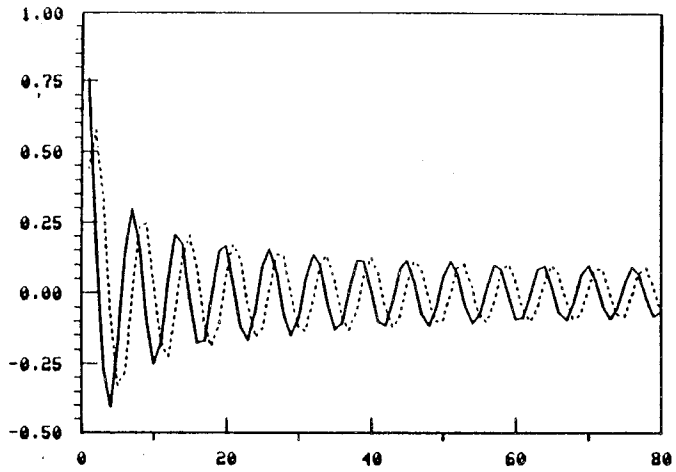


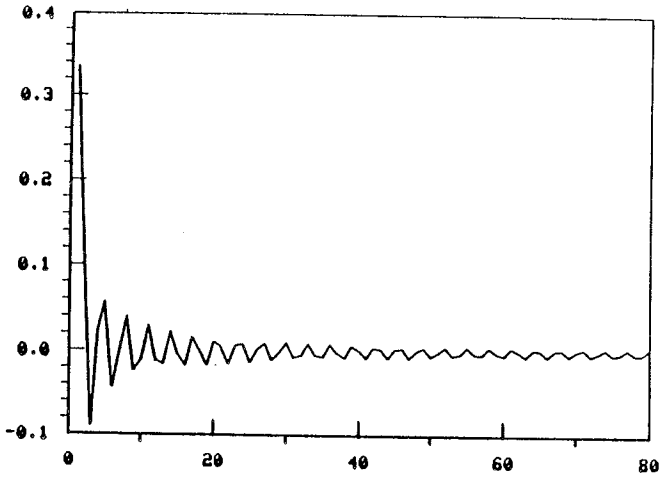
PARAMETERS AND CONSTANTS--

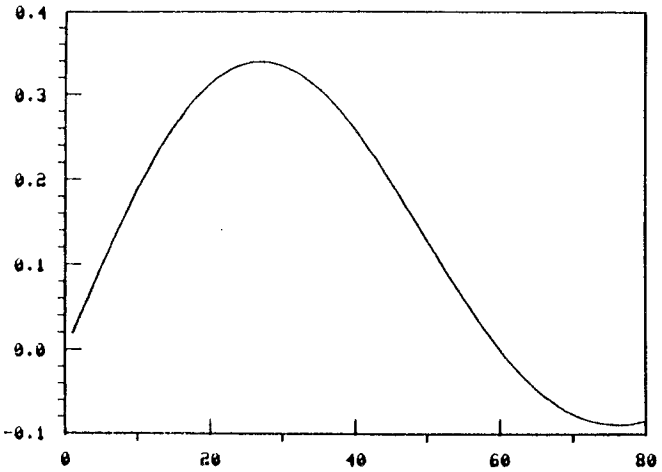
A	--	.4084996+00
B	--	.3935978+01
MU1	--	.4515468+03
D	--	.2837935+01
E	--	.4090563+01
F	--	.5478279+00
MU2	--	.4515153+03
S1	--	.2199616-01
S2	--	.6904023-02

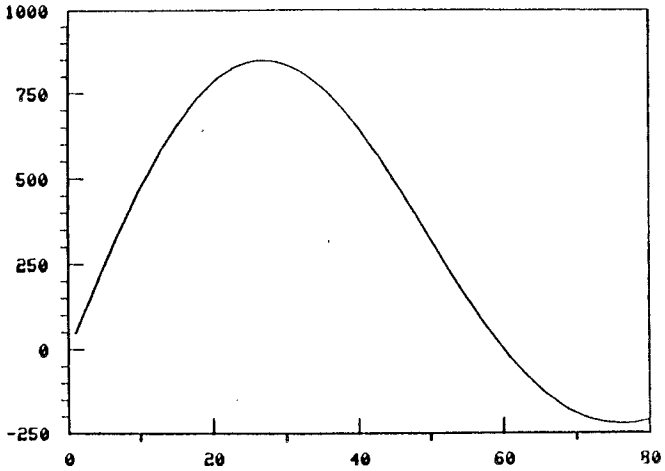
```
COMMENT EXAMPLE--FI                                    SION CALIBRATION CURVE
COMMENT MODEL   --PR                                   IONS
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6*DATA.HIGGINS1
NAME VOUT 1
NAME VAPP 3
.
CHARACTERS X
LINES
PLOT VOUT VAPP
.
CHARACTERS
LINES SOLID DOTTED
PLOT Y = BESS0(X) FOR X = 1 1 80 AND
PLOT Y = BESS1(X) FOR X = 1 1 80
PLOT Y = BESS0(X)*BESS1(X) FOR X = 1 1 80
PLOT Y = BESS0(.04*X)*BESS1(.04*X) FOR X = 1 1 80
PLOT Y = 2500*BESS0(.04*X)*BESS1(.04*X) FOR X = 1 1 80
.
LET A0 = 2500
LET A1 = .04
FIT VOUT = A0 * BESS0(A1*VAPP) * BESS1(A1*VAPP)
.
TITLE ULTRASONOVISION CALIBRATION
YLABEL VOLTAGE OUT
XLABEL VOLTAGE APPLIED
X2LABEL MODEL--Y = A0 * BESS0(A1*VAPP) * BESS1(A1*VAPP)
X3LABEL JJF6*CS9.NONLINEAR27    JJF6*DATA.HIGGINS1    2/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT VOUT PRED VS VAPP
.
YLABEL RESIDUALS
PLOT RES VAPP
```




1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
0
20
40
60
80


0.4
0.3
0.2
0.1
0.0
-0.1
0
20
40
60
80

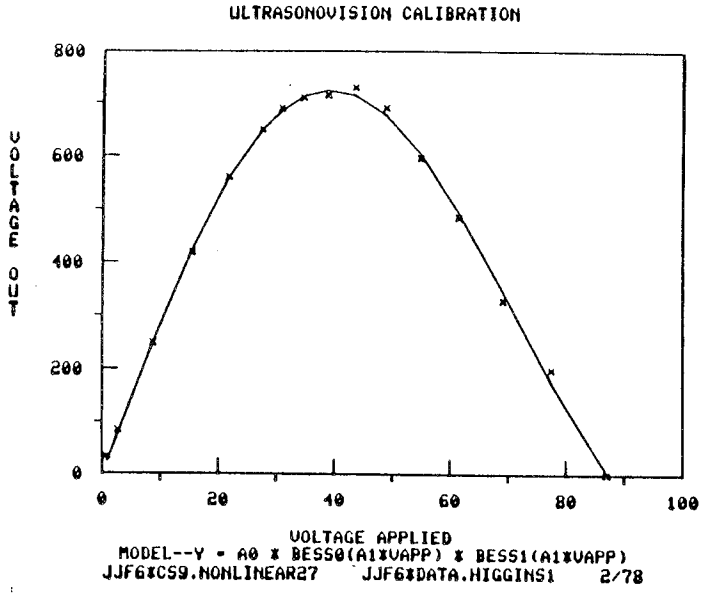

0.4
0.3
0.2
0.1
0.0
-0.1
0
20
40
60
80

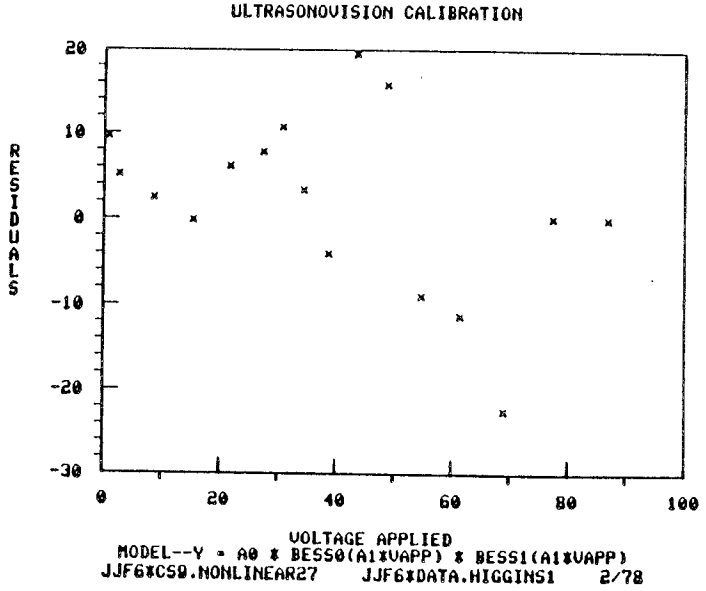

1000
750
500
250
0
-250
0
20
40
60
80

LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 16
MODEL-- VOUT = A0 * BESS0(A1*VAPP) * BESS1(A1*VAPP)
NO REPLICATION CASE

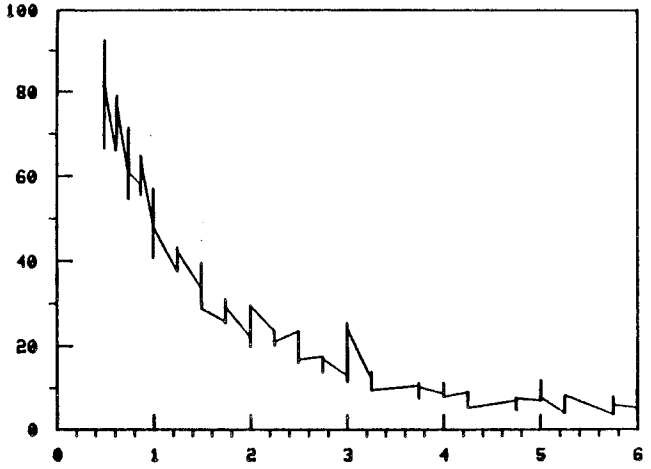
ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	*	PARAMETER ESTIMATES	
1--	.10000-01	.30272+03	*	.25000+04	.40000-01
2--	.50000-02	.13256+03	*	.17658+04	.30485-01
3--	.25000-02	.25082+02	*	.21089+04	.26768-01
4--	.12500-02	.11043+02	*	.21369+04	.27559-01
5--	.62500-03	.11036+02	*	.21382+04	.27554-01

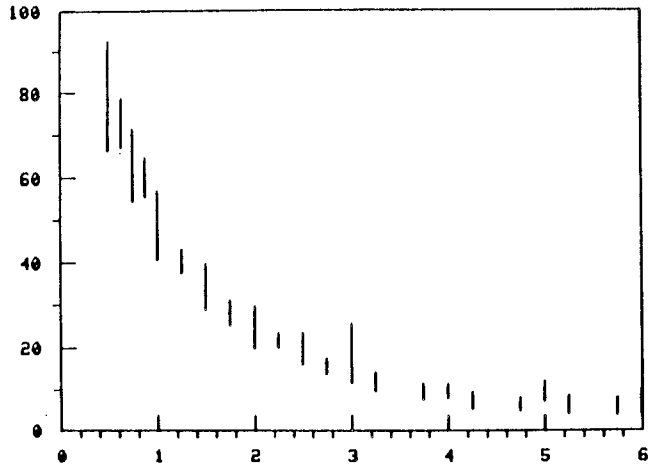
FINAL PARAMETER ESTIMATES			(APPROX. ST. DEV.)
1 A0		2138.24	(11.75)
2 A1		.275540-01	(.1038-03)
RESIDUAL	STANDARD DEVIATION =		11.0361322165
RESIDUAL	DEGREES OF FREEDOM =		14

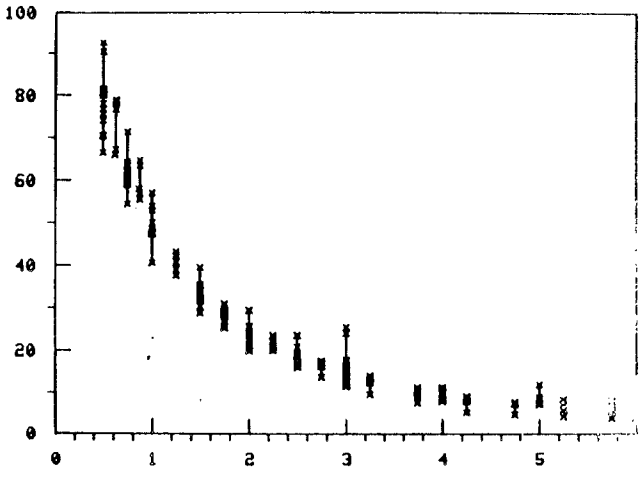




```
COMMENT EXAMPLE--DAN CHWIRUT ULTRASONIC REFERENCE BLOCK ANALYSIS
COMMENT MODEL   --EXPONENTIAL/LINEAR
COMMENT NOTE    --AUTOMATIC CHECK FOR REPLICATION

ECHO ON
HARDCOPY ON
BELL ON

READ JJF6%DATA.CHWIRUT1 Y X

PLOT Y X
PLOT Y X X
CHARACTERS X
PLOT Y X X

LET ALPHA=.15
LET A=.004
LET B = .01
FIT Y = EXP(-ALPHA%X)/(A+B%X)

TITLE ULTRASONIC REFERENCE BLOCK STUDY
YLABEL ULTRASONIC RESPONSE
XLABEL METAL DISTANCE
X2LABEL MODEL--Y = EXP(-ALPHA%X) / (A+B%X)
X3LABEL JJF6%CS9.NONLINEAR25     JJF6%DATA.CHWIRUT1     1/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT Y PRED VS X

YLABEL RESIDUALS
PLOT RES X
```

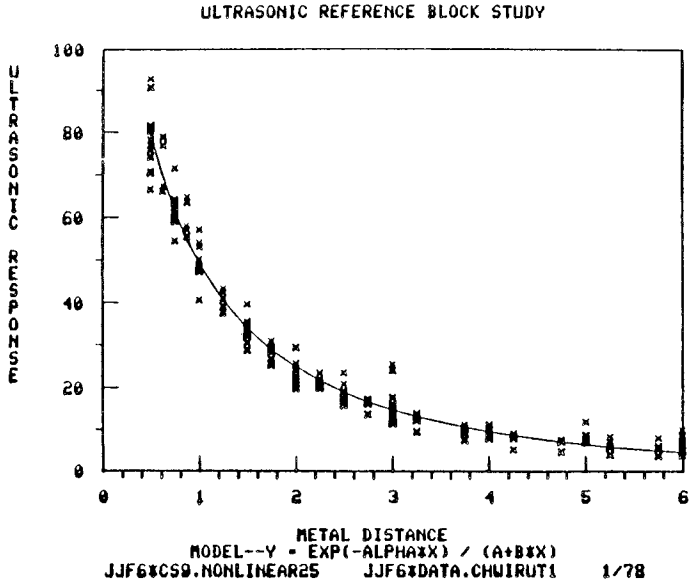


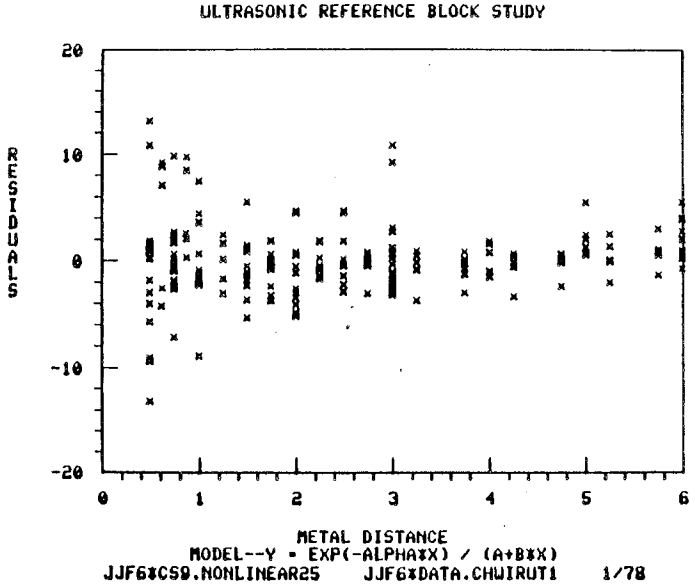




EXAMPLE 17

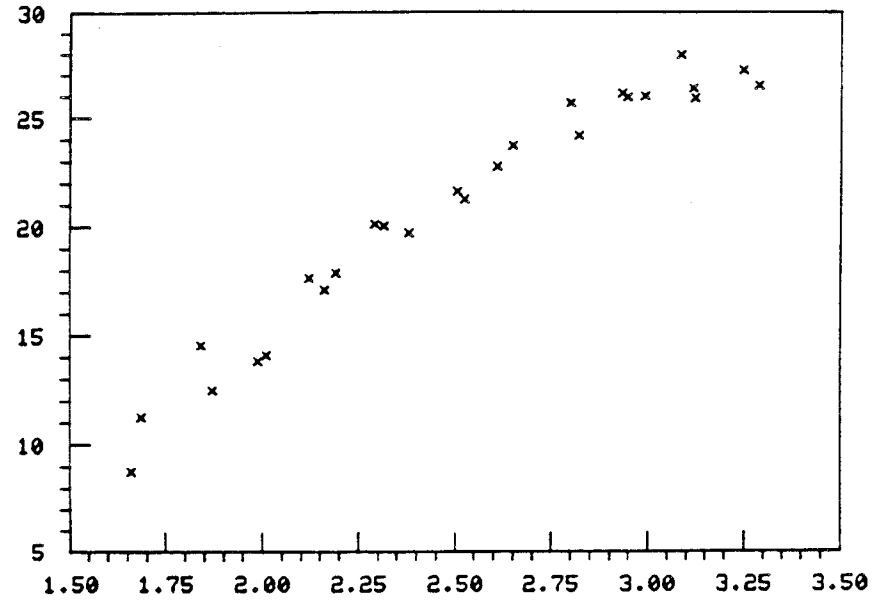
```
LEAST SQUARES NON-LINEAR FIT
      SAMPLE SIZE N =      214
      MODEL-- Y = EXP(-ALPHA*X)/(A+B*X)
      REPLICATION CASE
      REPLICATION STANDARD DEVIATION =        .3237755179+01
      REPLICATION DEGREES OF FREEDOM =          190
      NUMBER OF DISTINCT SUBSETS     =           24

ITERATION  CONVERGENCE  RESIDUAL  *  PARAMETER
 NUMBER      MEASURE    STANDARD  *  ESTIMATES
                        DEVIATION *
-----------------------------------*-----------
  1--       .10000-01    .10785+02 *   .15000+00   .40000-02   .10000-01
  2--       .50000-02    .37239+01 *   .18067+00   .55545-02   .10719-01
  3--       .25000-02    .33631+01 *   .19045+00   .61197-02   .10523-01
  4--       .12500-02    .33628+01 *   .19035+00   .61344-02   .10528-01

      FINAL PARAMETER ESTIMATES           (APPROX. ST. DEV.)
      1  ALPHA                .190307      (   .2208-01)
      2  A                    .613365-02   (   .3495-03)
      3  B                    .105298-01   (   .8033-03)

      RESIDUAL    STANDARD DEVIATION =          3.3627645671
      RESIDUAL    DEGREES OF FREEDOM =          211
      REPLICATION STANDARD DEVIATION =          3.2377551794
      REPLICATION DEGREES OF FREEDOM =          190
      LACK OF FIT F RATIO =     1.7909 = THE  97.8064% POINT OF THE
      F DISTRIBUTION WITH     21 AND    190 DEGREES OF FREEDOM
```





EXAMPLE 18

```
COMMENT EXAMPLE--H. S. LEW CONCRETE PULL-OUT BOND STRENGTH
COMMENT MODEL  --LINEAR/LINEAR
.
ECHO ON
HARDCOPY ON
BELL ON
.
SKIP 10
READ JJF6*DATA.LEW4 STREN MAT
.
CHARACTERS X
LINES
PLOT STREN MAT
.
LET STREN2(1)=11
LET STREN2(2)=22
LET STREN2(3)=26
LET MAT2(1)=1.7
LET MAT2(2)=2.6
LET MAT2(3)=3.2
EXACT 1/1 RATIONAL FIT STREN2 MAT2 STREN MAT
.
TITLE PULL-OUT BOND STRENGTH OF CONCRETE
YLABEL BOND STRENGTH
XLABEL MATURITY (= [TEMPERATURE(F) - 10] * ELAPSED DAYS)
X2LABEL EXACT LINEAR/LINEAR RATIONAL FIT
X3LABEL JJF6*CS9.NONLINEAR39    JJF6*DATA.LEW4    1/76
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT STREN PRED VS MAT
.
FIT STREN = (A0+A1*MAT)/(1+B1*MAT)
.
X2LABEL MODEL--STREN = (A0 + A1*MAT) / (1 + B1 * MAT)
PLOT STREN PRED VS MAT
.
YLABEL RESIDUALS
PLOT RES MAT
.
XLABEL
NORMAL PROBABILITY PLOT RES
```



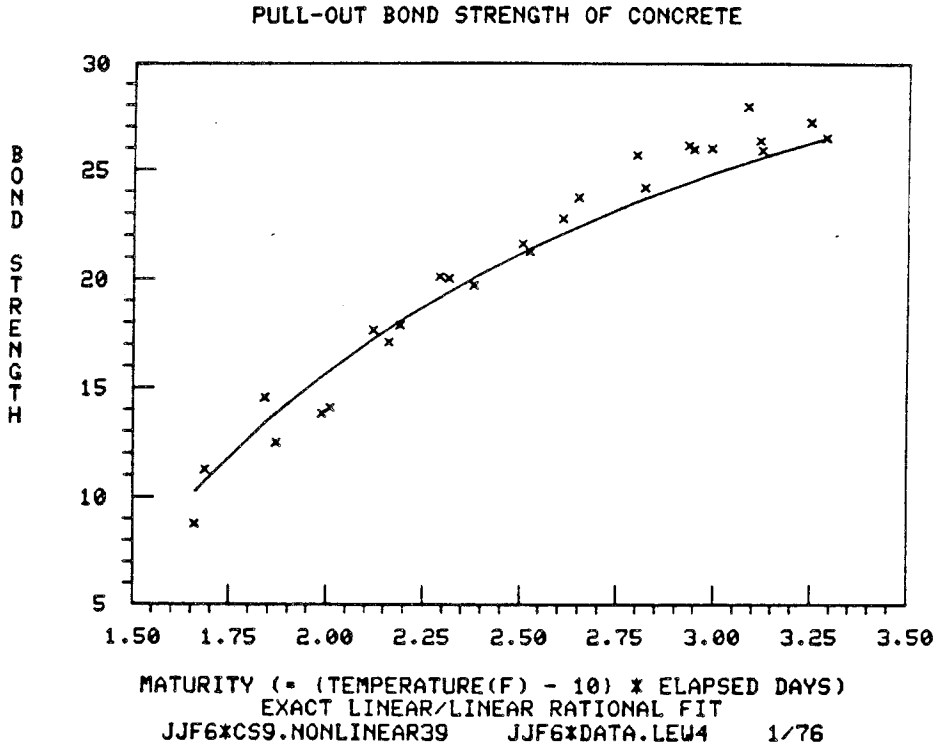
EXACT RATIONAL FUNCTION FIT

NUMBER OF POINTS IN FIRST SET	=	3
DEGREE OF NUMERATOR	=	1
DEGREE OF DENOMINATOR	=	1

NUMERATOR --A0 A1	=	-.55000179+03	.44000142+03
DENOMINATOR--B0 B1	=	.10000000+01	.10000033+02

APPLICATION OF EXACT-FIT COEFFICIENTS
TO SECOND PAIR OF VARIABLES--

NUMBER OF POINTS IN SECOND SET	=	26
NUMBER OF ESTIMATED COEFFICIENTS	=	3
RESIDUAL DEGREES OF FREEDOM	=	23
RESIDUAL SUM OF SQUARES	=	.35384189+02
RESIDUAL STANDARD DEVIATION (DENOM=N-P)	=	.12403399+01
AVERAGE ABSOLUTE RESIDUAL (DENOM=N)	=	.96846149+00
LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL	=	.25857794+01
LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL	=	-.17317243+01
LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL	=	.25857794+01

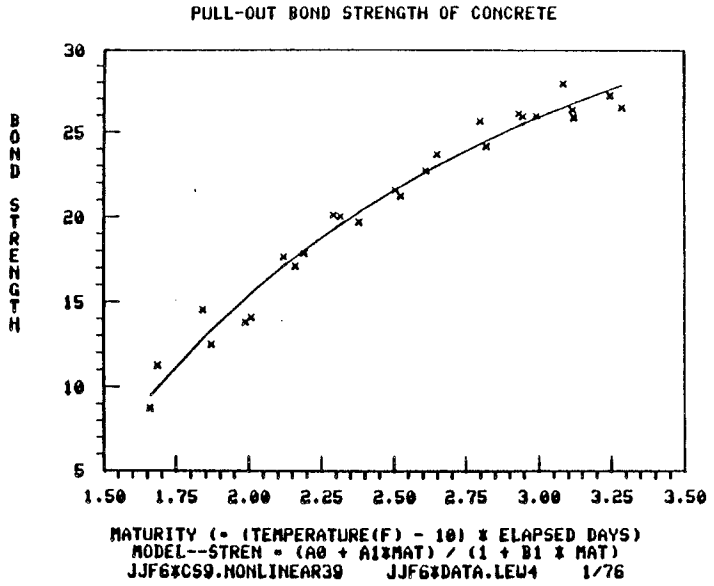


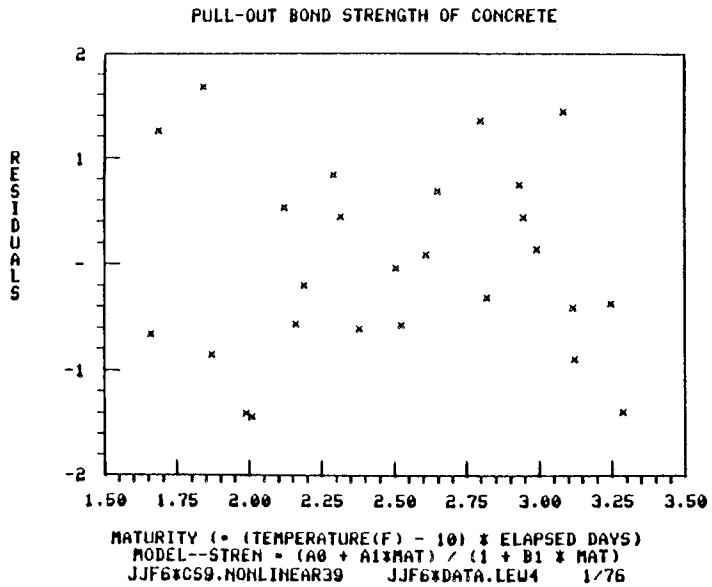
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 26
MODEL-- STREN = (A0+A1*MAT)/(1+B1*MAT)
NO REPLICATION CASE

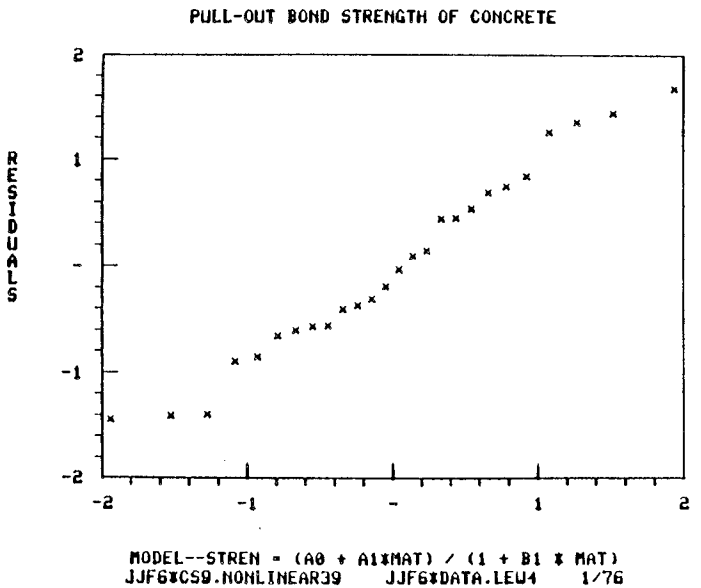
ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	PARAMETER ESTIMATES		
1--	.10000-01	.12403+01	-.55000+03	.44000+03	.10000+02
2--	.50000-02	.94361+00	-.57571+03	.43480+03	.90154+01
3--	.25000-02	.93787+00	-.53961+03	.40991+03	.85625+01
4--	.37500-02	.93750+00	-.48422+03	.36829+03	.76597+01
5--	.18750-02	.93678+00	-.43264+03	.32964+03	.68305+01
6--	.28125-02	.93619+00	-.35391+03	.27064+03	.55585+01
7--	.14062-02	.93478+00	-.29228+03	.22446+03	.45692+01
8--	.21094-02	.93419+00	-.22600+03	.17479+03	.35011+01
9--	.10547-02	.93282+00	-.19828+03	.15403+03	.30603+01
10--	.52734-03	.93265+00	-.18195+03	.14180+03	.27985+01
11--	.26367-03	.93253+00	-.18161+03	.14155+03	.27953+01

	FINAL PARAMETER ESTIMATES		(APPROX. ST. DEV.)
1	A0	-181.569	(267.7)
2	A1	141.515	(200.5)
3	B1	2.79461	(4.347)

RESIDUAL STANDARD DEVIATION = .9325327873
RESIDUAL DEGREES OF FREEDOM = 23

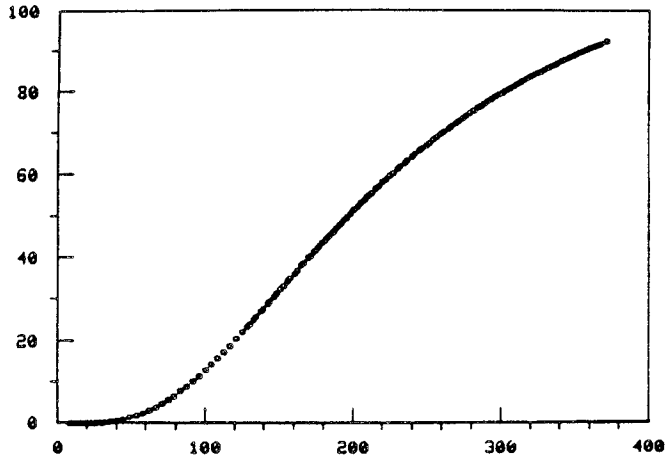






EXAMPLE 19

```
COMMENT EXAMPLE--DICK KIRBY SCANNING ELECTRON MICROSCOPE
COMMENT          LINE SPACING STANDARDS
COMMENT MODEL  --LINEAR/LINEAR AND QUADRATIC/QUADRATIC
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6*DATA.KIRBY2 X Y
.
CHARACTERS CIRCLE
LINES
PLOT Y X
.
LET ID = 0*Y
LET ID(1)=1
LET ID(75)=1
LET ID(152)=1
LET Y2=Y SUBSET ID 1
LET X2=X SUBSET ID 1
PACK Y2 X2 SUBSET ID 1
EXACT 1/1 RATIONAL FIT Y2 X2 Y X
.
TITLE SCANNING ELECTRON MICROSCOPE LINE SPACING STANDARDS
YLABEL Y
XLABEL X
X2LABEL EXACT FIT THROUGH 3 POINTS OF LINEAR/LINEAR MODEL
X3LABEL JJF6*CS9.NONLINEAR11    JJF6*DATA.KIRBY2    11/9/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT Y PRED VS X
.
FIT Y = (A0+A1*X)/(1+B1*X)
.
X2LABEL MODEL--Y = (A0 + A1*X) / (1 + B1*X)
PLOT Y PRED VS X
.
YLABEL RESIDUALS
PLOT RES X
.
LET ID(30)=1
LET ID(110)=1
LET Y2=Y SUBSET ID 1
LET X2=X SUBSET ID 1
PACK Y2 X2 SUBSET ID 1
EXACT 2/2 RATIONAL FIT Y2 X2 Y X
.
YLABEL Y
X2LABEL EXACT FIT THROUGH 5 POINTS OF QUADRATIC/QUADRATIC MODEL
PLOT Y PRED VS X
.
FIT Y = (A0+A1*X+A2*X*X)/(1+B1*X+B2*X*X)
.
X2LABEL MODEL--Y = (A0 + A1*X + A2*X*X) / (1 + B1*X + B2*X*X)
PLOT Y PRED VS X
.
YLABEL RESIDUALS
PLOT RES X
```



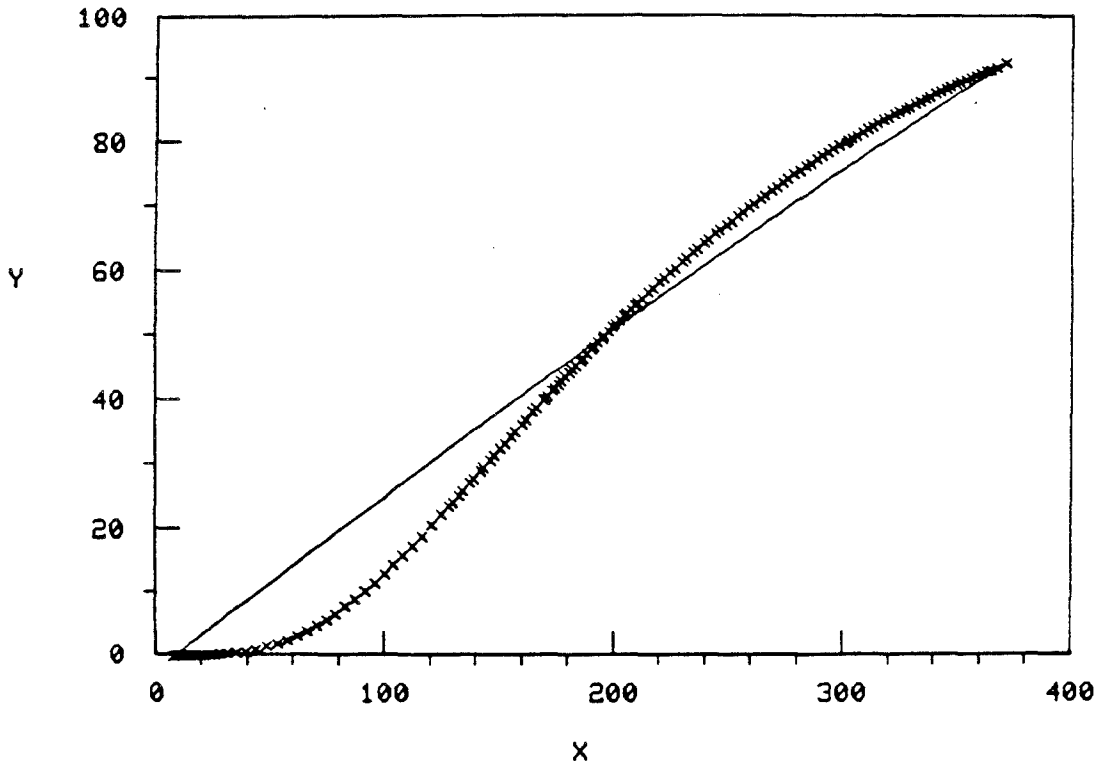
EXACT RATIONAL FUNCTION FIT

NUMBER OF POINTS IN FIRST SET	=	3	
DEGREE OF NUMERATOR	=	1	
DEGREE OF DENOMINATOR	=	1	
NUMERATOR --A0 A1	=	-.23937376+01	.27869345+00
DENOMINATOR--B0 B1	=	.10000000+01	.25954249-03

APPLICATION OF EXACT-FIT COEFFICIENTS
TO SECOND PAIR OF VARIABLES--

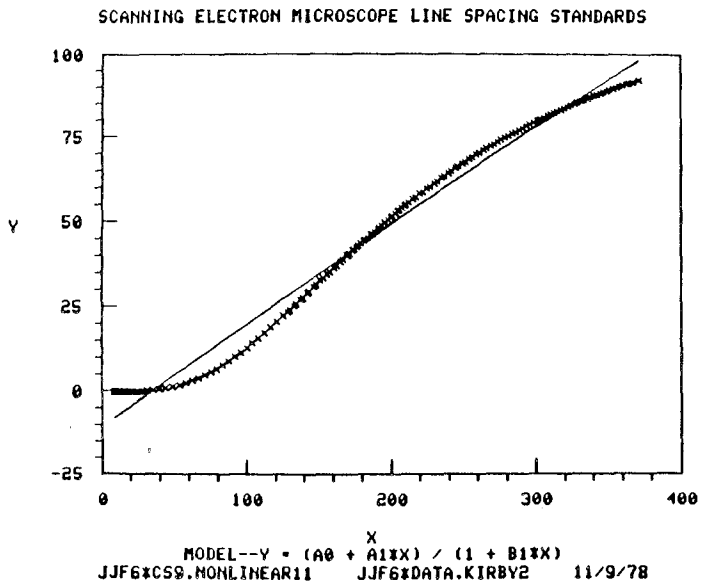
NUMBER OF POINTS IN SECOND SET	=	152
NUMBER OF ESTIMATED COEFFICIENTS	=	3
RESIDUAL DEGREES OF FREEDOM	=	149
RESIDUAL SUM OF SQUARES	=	.45225974+04
RESIDUAL STANDARD DEVIATION (DENOM=N-P)	=	.55093559+01
AVERAGE ABSOLUTE RESIDUAL (DENOM=N)	=	.42797687+01
LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL	=	.43819942+01
LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL	=	-.12615372+02
LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL	=	.12615372+02

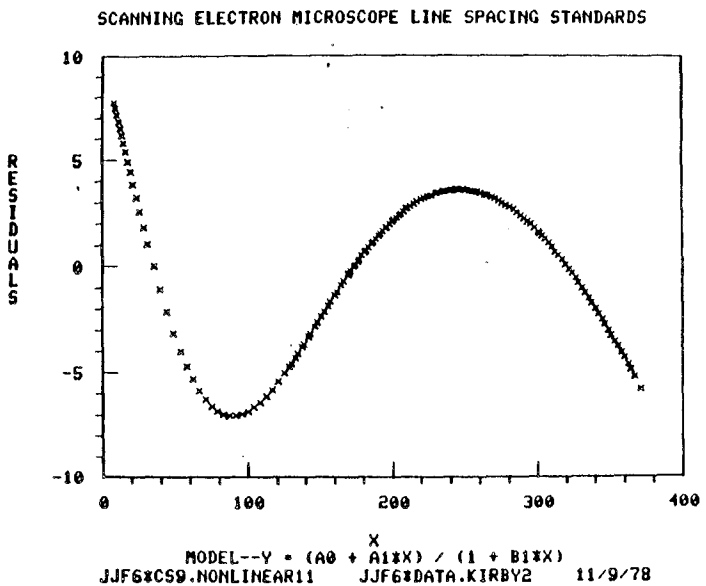
SCANNING ELECTRON MICROSCOPE LINE SPACING STANDARDS



EXACT FIT THROUGH 3 POINTS OF LINEAR/LINEAR MODEL
JJF6*CS9.NONLINEAR11 JJF6*DATA.KIRBY2 11/9/78

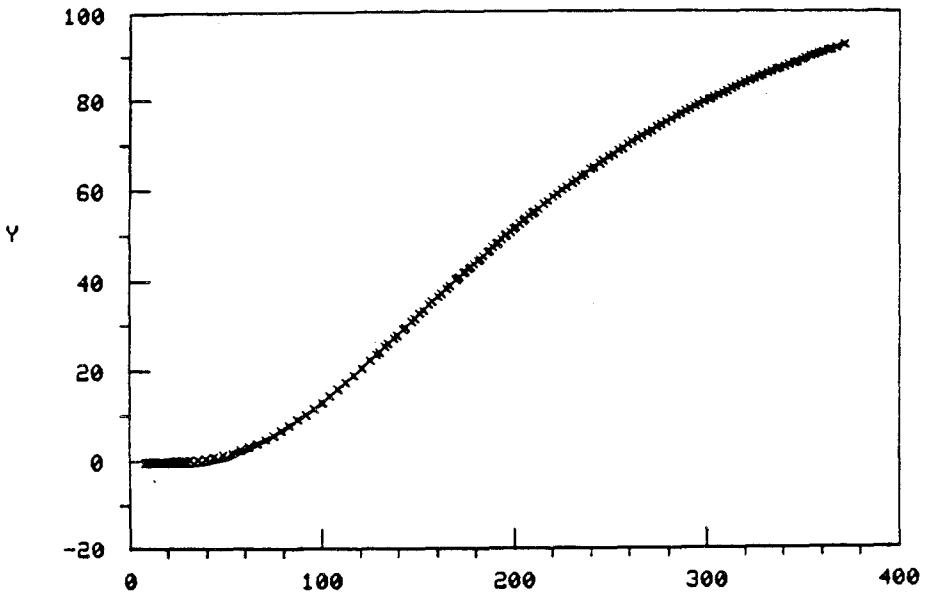
```
LEAST SQUARES NON-LINEAR FIT
      SAMPLE SIZE N =      152
      MODEL-- Y = (A0+A1*X)/(1+B1*X)
      REPLICATION CASE
      REPLICATION STANDARD DEVIATION =        .1202078687-01
      REPLICATION DEGREES OF FREEDOM =            1
      NUMBER OF DISTINCT SUBSETS     =          151

ITERATION  CONVERGENCE  RESIDUAL  *  PARAMETER
 NUMBER      MEASURE    STANDARD  *  ESTIMATES
                        DEVIATION *
-----------------------------------*-----------
  1--       .10000-01    .55094+01 *  -.23937+01    .27869+00    .25954-03
  2--       .50000-02    .36329+01 *  -.10120+02    .29728+00    .45735-04
  3--       .25000-02    .36250+01 *  -.10422+02    .30329+00    .11801-03
  4--       .12500-02    .36248+01 *  -.10331+02    .30203+00    .10565-03

      FINAL PARAMETER ESTIMATES           (APPROX. ST. DEV.)
      1  A0                  -10.3477      (  .9393    )
      2  A1                   .302264      (  .1016-01)
      3  B1                   .108003-03   (  .9839-04)

      RESIDUAL    STANDARD DEVIATION =          3.6248077750
      RESIDUAL    DEGREES OF FREEDOM =          149
      REPLICATION STANDARD DEVIATION =           .0120207869
      REPLICATION DEGREES OF FREEDOM =            1
      LACK OF FIT F RATIO = 91543.7461 = THE  99.7367% POINT OF THE
      F DISTRIBUTION WITH    148 AND      1 DEGREES OF FREEDOM
```





```
EXACT RATIONAL FUNCTION FIT
      NUMBER OF POINTS IN FIRST SET     =        5
      DEGREE OF NUMERATOR               =        2
      DEGREE OF DENOMINATOR             =        2

NUMERATOR  --A0  A1  A2          =        .125+01          -.170+00
 .304-02
DENOMINATOR--B0  B1  B2          =        .100+01          -.105-02
 .236-04

      APPLICATION OF EXACT-FIT COEFFICIENTS
      TO SECOND PAIR OF VARIABLES--

      NUMBER OF POINTS IN SECOND SET            =      152
      NUMBER OF ESTIMATED COEFFICIENTS          =        5
      RESIDUAL DEGREES OF FREEDOM               =      147

      RESIDUAL SUM OF SQUARES                   =    .23364094+02
      RESIDUAL STANDARD DEVIATION (DENOM=N-P)   =    .39867206+00
      AVERAGE ABSOLUTE RESIDUAL   (DENOM=N)     =    .17420260+00
      LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL  =    .14435167+01
      LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL  =   -.15705872+00
      LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL  =    .14435167+01
```

SCANNING ELECTRON MICROSCOPE LINE SPACING STANDARDS



EXACT FIT THROUGH 5 POINTS OF QUADRATIC/QUADRATIC MODEL
JJF6*CS9.NONLINEAR11 JJF6*DATA.KIRBY2 11/9/78

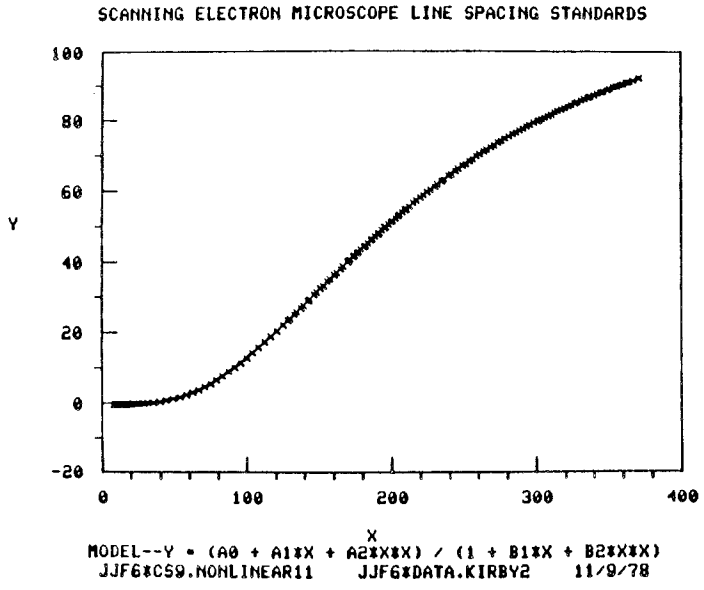
LEAST SQUARES NON-LINEAR FIT

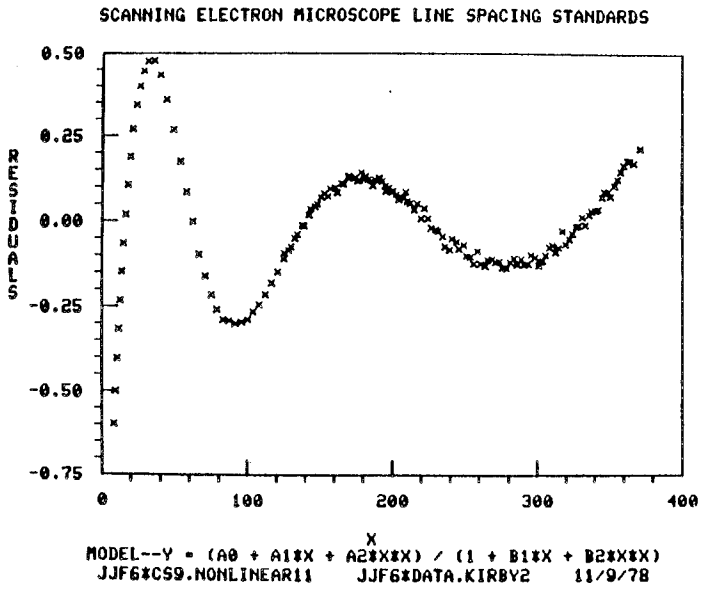
SAMPLE SIZE N = 152
MODEL-- Y = (A0+A1*X+A2*X*X)/(1+B1*X+B2*X*X)
REPLICATION CASE
REPLICATION STANDARD DEVIATION = .1202078687-01
REPLICATION DEGREES OF FREEDOM = 1
NUMBER OF DISTINCT SUBSETS = 151

ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	PARAMETER ESTIMATES				
1--	.10000-01	.39867+00	.12459+01	-.17018+00	.30363-02	-.10546-02	.23646-04
2--	.50000-02	.18319+00	.18670+01	-.15280+00	.27499-02	-.14969-02	.22306-04
3--	.25000-02	.17138+00	.15963+01	-.13709+00	.25808-02	-.17294-02	.21566-04
4--	.12500-02	.17107+00	.15629+01	-.13524+00	.25620-02	-.17530-02	.21483-04
5--	.62500-03	.17106+00	.15635+01	-.13528+00	.25625-02	-.17524-02	.21486-04

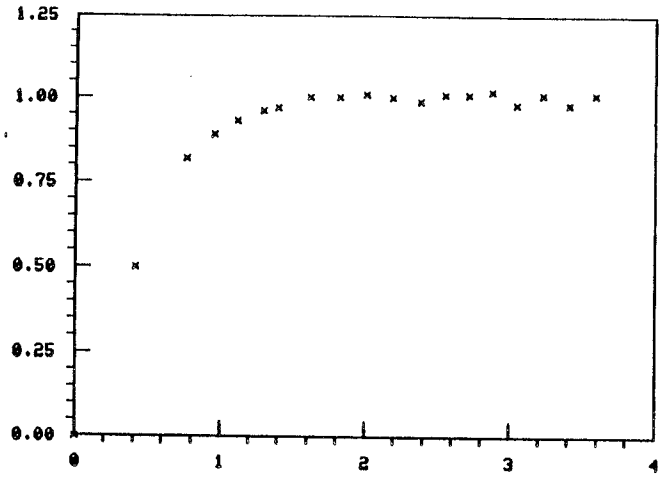
	FINAL PARAMETER ESTIMATES		(APPROX. ST. DEV.)
1	A0	1.56345	(.8621-01)
2	A1	-.135275	(.4119-02)
3	A2	.256242-02	(.4224-04)
4	B1	-.175250-02	(.5988-04)
5	B2	.214859-04	(.2052-06)

RESIDUAL STANDARD DEVIATION = .1710639019
RESIDUAL DEGREES OF FREEDOM = 147
REPLICATION STANDARD DEVIATION = .0120207869
REPLICATION DEGREES OF FREEDOM = 1
LACK OF FIT F RATIO = 203.8923 = THE 94.4264% POINT OF THE
F DISTRIBUTION WITH 146 AND 1 DEGREES OF FREEDOM



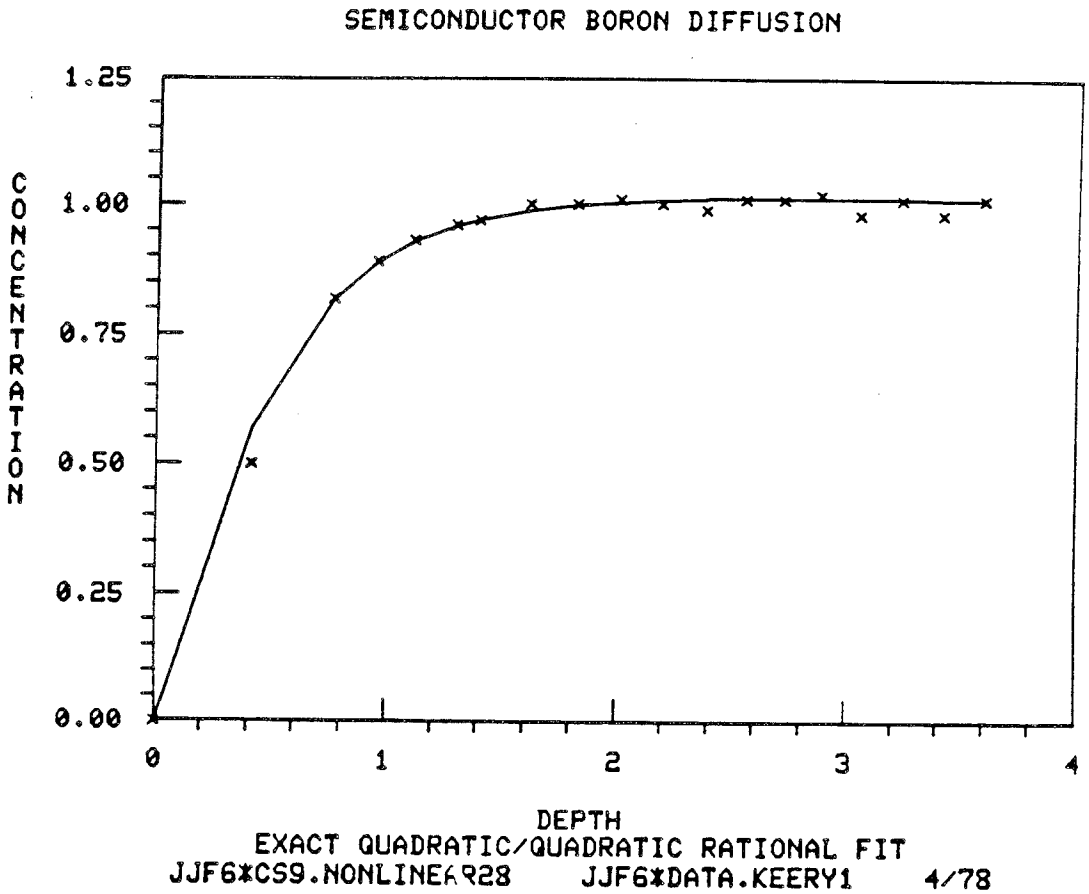


```
COMMENT EXAMPLE--BILL KEERY SEMICONDUCTOR DIFFUSION STUDY
COMMENT MODEL   --QUADRATIC/QUADRATIC
COMMENT NOTE    --EXACT RATIONAL FIT FOLLOWED BY LEAST SQUARES FIT
COMMENT NOTE    --A SECOND FIT RESTRICTED TO GO THROUGH ORIGIN
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6*DATA.KEERY1 Y X
.
CHARACTERS X
LINES
PLOT Y X
.
LET ID = 0*Y
LET ID(1)=1
LET ID(3)=1
LET ID(5)=1
LET ID(9)=1
LET ID(19)=1
LET Y2 = Y SUBSET ID 1
LET X2 = X SUBSET ID 1
PACK Y2 X2 SUBSET ID 1
EXACT 2/2 FIT Y2 X2 Y X
.
TITLE SEMICONDUCTOR BORON DIFFUSION
YLABEL CONCENTRATION
XLABEL DEPTH
X2LABEL EXACT QUADRATIC/QUADRATIC RATIONAL FIT
X3LABEL JJF6*CS9.NONLINEAR28    JJF6*DATA.KEERY1    4/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT Y PRED VS X
LET S1=RESSD
.
FIT Y = (A0+A1*X+A2*X**X)/(1+B1*X+B2*X**X)
.
X2LABEL MODEL--Y = (A0 + A1*X + A2*X**X) / (1 + B1*X + B2*X**X)
PLOT Y PRED VS X
LET S2=RESSD
.
LET MN=MIN(X)
LET MX=MAX(X)
LET DELTA=(MX-MN)/100
.
PLOT Y X AND
PLOT Y = (A0+A1*X+A2*X**X)/(1+B1*X+B2*X**X) FOR X = MN DELTA MX
.
FIT Y = (0+A1*X+A2*X**X)/(1+B1*X+B2*X**X)
.
X2LABEL MODEL--Y = (A1*X + A2*X**X) / (1 + B1*X + B2*X**X)
PLOT Y PRED VS X
LET S3=RESSD
.
PLOT Y X AND
PLOT Y = (A1*X+A2*X**X)/(1+B1*X+B2*X**X) FOR X = MN DELTA MX
.
PRINT S1 S2 S3
```



EXAMPLE 20

```
EXACT RATIONAL FUNCTION FIT
      NUMBER OF POINTS IN FIRST SET     =        5
      DEGREE OF NUMERATOR               =        2
      DEGREE OF DENOMINATOR             =        2

NUMERATOR  --A0  A1  A2          =        .000            .144+01
 .191+01
DENOMINATOR--B0  B1  B2          =        .100+01         .692+00
 .201+01

      APPLICATION OF EXACT-FIT COEFFICIENTS
      TO SECOND PAIR OF VARIABLES--

      NUMBER OF POINTS IN SECOND SET                =       19
      NUMBER OF ESTIMATED COEFFICIENTS              =        5
      RESIDUAL DEGREES OF FREEDOM                   =       14

      RESIDUAL SUM OF SQUARES                       =    .77758524-02
      RESIDUAL STANDARD DEVIATION (DENOM=N-P)       =    .23567308-01
      AVERAGE ABSOLUTE RESIDUAL   (DENOM=N)         =    .10378105-01
      LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL      =    .10173738-01
      LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL      =   -.70652820-01
      LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL      =    .70652820-01
```

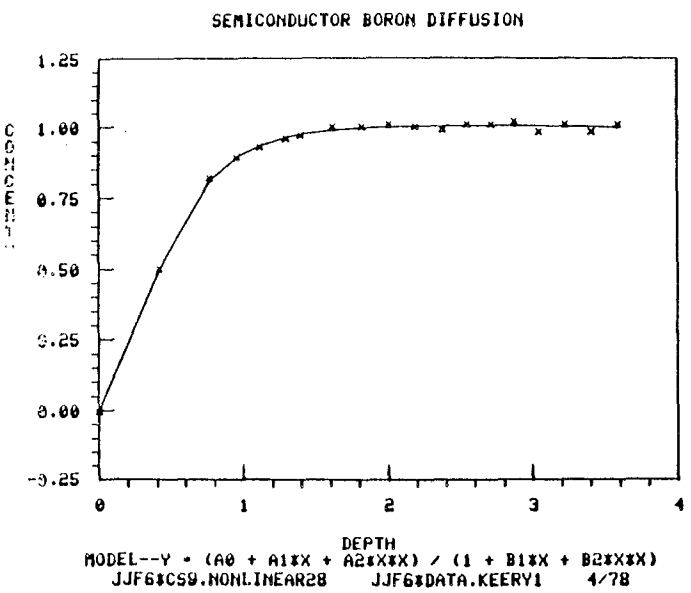


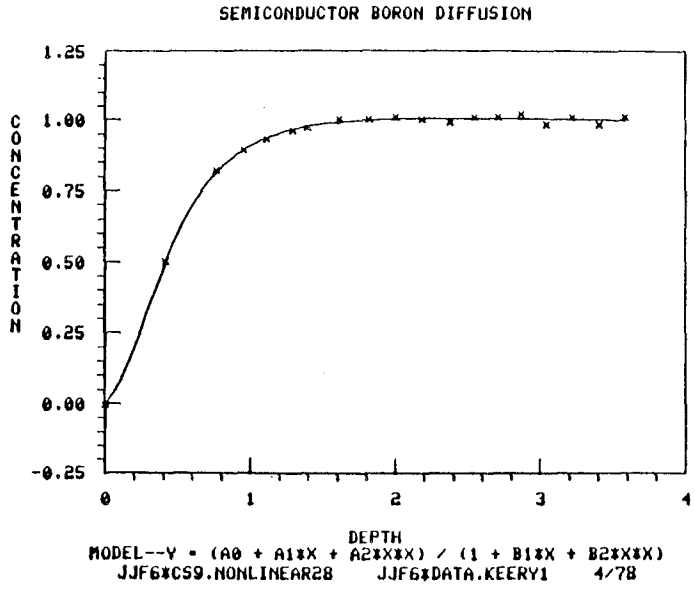
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 19
MODEL-- Y = (A0+A1*X+A2*X*X)/(1+B1*X+B2*X*X)
NO REPLICATION CASE

ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	*	PARAMETER ESTIMATES				
1--	.10000-01	.23567-01	*	.00000	.14353+01	.19057+01	.69152+00	.20124+01
2--	.50000-02	.12872-01	*	-.80934-03	.62511+00	.21116+01	-.26193+00	.22953+01
3--	.25000-02	.11996-01	*	-.23370-03	.57693+00	.24805+01	-.26810+00	.26497+01
4--	.12500-02	.11901-01	*	-.82112-04	.49801+00	.27022+01	-.32663+00	.28619+01
5--	.62500-03	.11875-01	*	.54263-03	.47626+00	.27544+01	-.34253+00	.29101+01
6--	.31250-03	.11872-01	*	-.91546-04	.47673+00	.27597+01	-.34203+00	.29148+01

	FINAL PARAMETER ESTIMATES		(APPROX. ST. DEV.)
1	A0	-.797385-04	(.1187-01)
2	A1	.476736	(.3359)
3	A2	2.75975	(.8703)
4	B1	-.342037	(.2764)
5	B2	2.91485	(.8478)

RESIDUAL STANDARD DEVIATION = .0118720974
RESIDUAL DEGREES OF FREEDOM = 14





LEAST SQUARES NON-LINEAR FIT

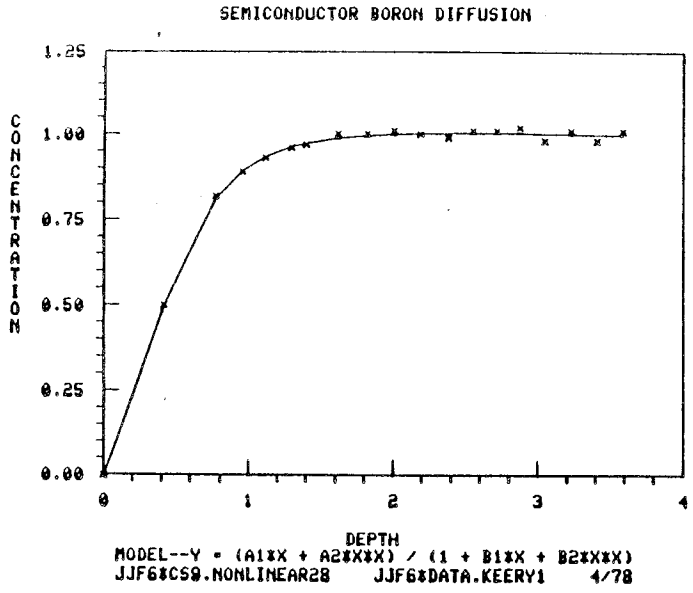
SAMPLE SIZE N = 19
MODEL-- Y = (0+A1*X+A2*X*X)/(1+B1*X+B2*X*X)
NO REPLICATION CASE

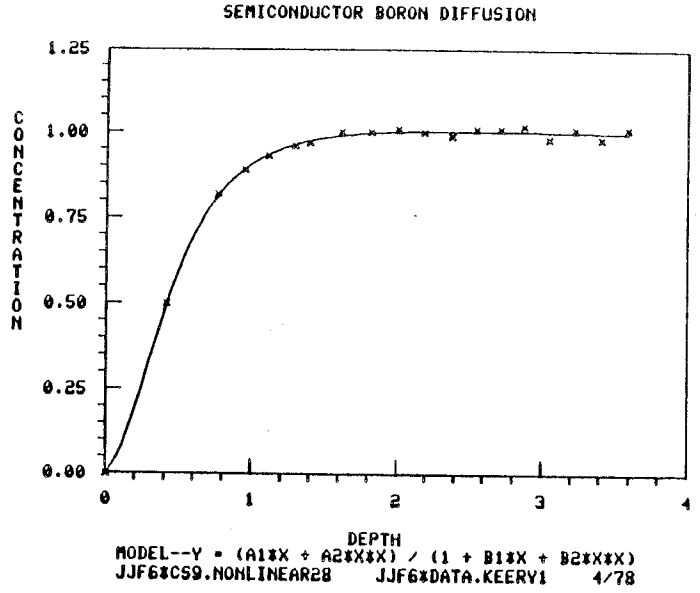
ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	PARAMETER ESTIMATES			
1--	.10000-01	.11470-01	.47674+00	.27598+01	-.34204+00	.29148+01

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)

1	A1	.476347	(.3207)
2	A2	2.75979	(.8407)
3	B1	-.342327	(.2642)
4	B2	2.91482	(.8190)

RESIDUAL STANDARD DEVIATION = .0114695737
RESIDUAL DEGREES OF FREEDOM = 15



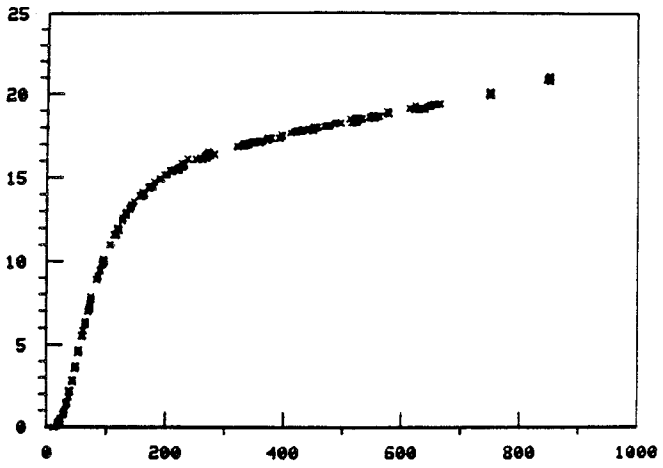


PARAMETERS AND CONSTANTS--

S1	--	.2356731-01
S2	--	.1187210-01
S3	--	.1146957-01

EXAMPLE 21

```
COMMENT EXAMPLE--TOM HAHN COPPER THERMAL EXPANSION STUDY
COMMENT MODEL  --QUADRATIC/QUADRATIC AND CUBIC/CUBIC
COMMENT NOTE   --TECHNIQUE FOR UPDATING AND IMPROVING
COMMENT          RATIONAL FIT MODEL
COMMENT NOTE   --EXACT RATIONAL FIT FOLLOWED BY LEAST SQUARES FIT
COMMENT NOTE   --A SECOND FIT RESTRICTED TO GO THROUGH ORIGIN
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6*DATA.HAHN1 X Y
.
CHARACTERS X
LINES
PLOT Y X
.
LET X2(1)=10
LET X2(2)=50
LET X2(3)=120
LET X2(4)=200
LET X2(5)=800
LET Y2(1)=0
LET Y2(2)=5
LET Y2(3)=12
LET Y2(4)=15
LET Y2(5)=20
EXACT 2/2 FIT Y2 X2 Y X
.
TITLE COEFFICIENT OF THERMAL EXPANSION (COPPER)
YLABEL COEF. OF THERMAL EXP.
XLABEL TEMPERATURE (DEGREES KELVIN)
X2LABEL EXACT QUADRATIC/QUADRATIC RATIONAL FIT
X3LABEL JJF6*CS9.NONLINEAR44    JJF6*DATA.HAHN1    11/17/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT Y PRED VS X
.
CHARACTERS BLANK ALL
LINES DOTTTED SOLID
PLOT Y PRED VS X
.
LET X2(6)=40
LET X2(7)=30
LET Y2(6)=3
LET Y2(7)=2
EXACT 3/3 RATIONAL FIT Y2 X2 Y X
.
X2LABEL EXACT CUBIC/CUBIC RATIONAL FIT
PLOT Y PRED VS X
.
FIT Y = (A0+A1*X+A2*X**X+A3*X**3)/(1+B1*X+B2*X**X+B3*X**3)
LET S1=RESSD
.
X2LABEL MODEL--Y = (A0+A1*X+A2*X**X+A3*X**3)/(1+B1*X+B2*X**X+B3*X**3)
PLOT Y PRED VS X
.
YLABEL RESIDUALS
PLOT RES X
.
XLABEL
NORMAL PROBABILITY PLOT RES
.
FIT Y = (A1*X+A2*X**X+A3*X**3)/(1+B1*X+B2*X**X+B3*X**3)
LET S2=RESSD
.
YLABEL COEF. OF THERMAL EXP.
XLABEL TEMPERATURE (DEGREES KELVIN)
X2LABEL MODEL--Y = (A1*X+A2*X**X+A3*X**3)/(1+B1*X+B2*X**X+B3*X**3)
PLOT Y PRED VS X
.
YLABEL RESIDUALS
PLOT RES X
.
XLABEL
NORMAL PROBABILITY PLOT RES
.
PRINT S1 S2
```



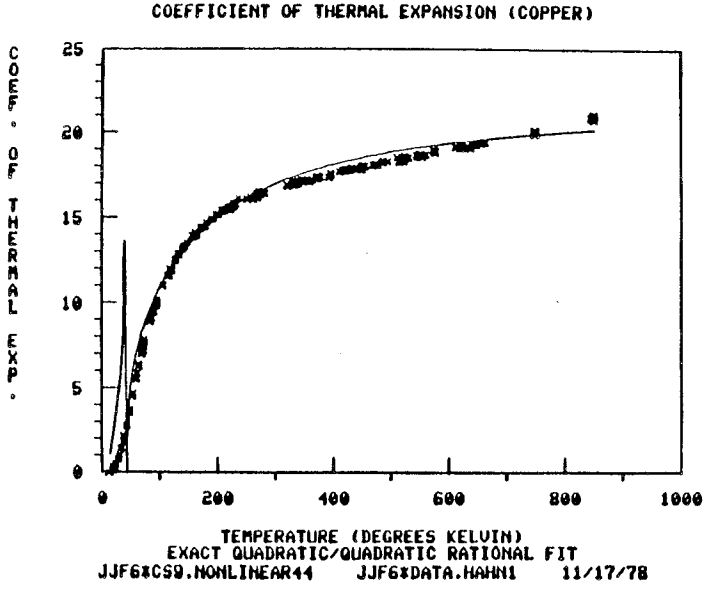
EXACT RATIONAL FUNCTION FIT

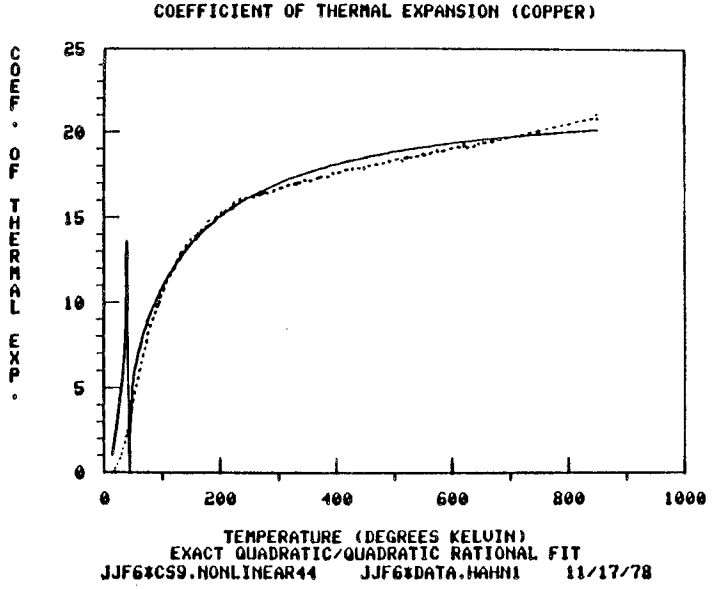
NUMBER OF POINTS IN FIRST SET	=	5
DEGREE OF NUMERATOR	=	2
DEGREE OF DENOMINATOR	=	2

NUMERATOR --A0 A1 A2	=	-.301+01	.369+00
-.683-02			
DENOMINATOR--B0 B1 B2	=	.100+01	-.112-01
-.306-03			

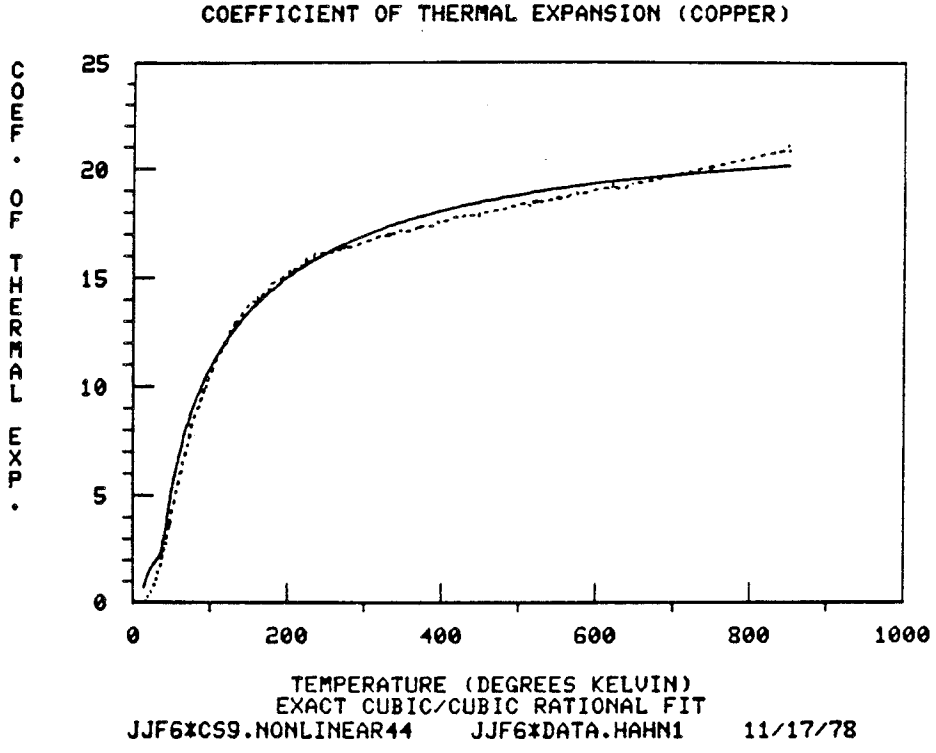
APPLICATION OF EXACT-FIT COEFFICIENTS
TO SECOND PAIR OF VARIABLES--

NUMBER OF POINTS IN SECOND SET	=	236
NUMBER OF ESTIMATED COEFFICIENTS	=	5
RESIDUAL DEGREES OF FREEDOM	=	231
RESIDUAL SUM OF SQUARES	=	.68716166+03
RESIDUAL STANDARD DEVIATION (DENOM=N-P)	=	.17247393+01
AVERAGE ABSOLUTE RESIDUAL (DENOM=N)	=	.82942525+00
LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL	=	.27054440+01
LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL	=	-.11427686+02
LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL	=	.11427686+02





```
 EXACT RATIONAL FUNCTION FIT
       NUMBER OF POINTS IN FIRST SET     =        7
       DEGREE OF NUMERATOR               =        3
       DEGREE OF DENOMINATOR             =        3

 NUMERATOR  --A0  A1  A2  A3       =   -.23235123+01     .35300880+00   -.
13832568-01    .17668116-03
 DENOMINATOR--B0  B1  B2  B3       =    .10000000+01    -.33957982-01    .
11009136-03    .79103667-05

       APPLICATION OF EXACT-FIT COEFFICIENTS
       TO SECOND PAIR OF VARIABLES--

       NUMBER OF POINTS IN SECOND SET            =      236
       NUMBER OF ESTIMATED COEFFICIENTS          =        7
       RESIDUAL DEGREES OF FREEDOM               =      229

       RESIDUAL SUM OF SQUARES                   =    .78287830+02
       RESIDUAL STANDARD DEVIATION (DENOM=N-P)   =    .58469500+00
       AVERAGE ABSOLUTE RESIDUAL   (DENOM=N)     =    .47006864+00
       LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL  =    .95732951+00
       LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL  =   -.13508249+01
       LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL  =    .13508249+01
```



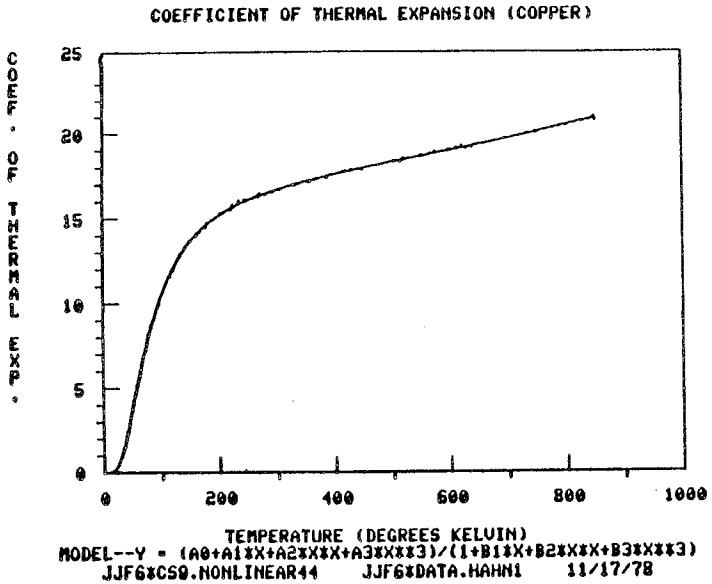
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 236
MODEL--Y = (A0+A1*X+A2*X**X+A3*X**3)/(1+B1*X+B2*X**X+B3*X**3)
REPLICATION CASE
REPLICATION STANDARD DEVIATION = .8131728601-01
REPLICATION DEGREES OF FREEDOM = 1
NUMBER OF DISTINCT SUBSETS = 236

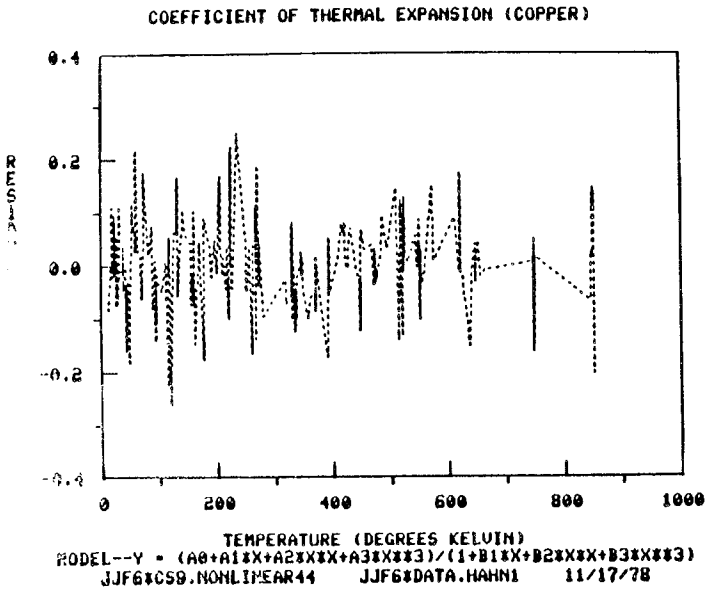
ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	PARAMETER ESTIMATES						
1--	.10000+01	.58470+00	-.23235+01	.35301+00	-.13833-01	.17668-03	-.33958-01	.81000-03	.78104-06
2--	.63300-02	.48298+00	-.25307+01	.32472+00	-.12198-01	.17714-03	-.22389-01	.82106-03	.81210-06
3--	.25400-02	.32987+00	-.17320+01	.88947+00	-.77378-02	.11616-03	-.17608-01	.10225-03	.52898-06
4--	.37500-02	.32066+00	-.67462+00	.56845-01	-.14382-02	.33765-04	-.16194-01	.13329-03	.14416-05
5--	.18750-02	.11796+00	.11942+01	-.12864+00	.36951-02	.13730-05	-.82924-02	.24279-03	-.71281-08
6--	.93760-03	.89555-01	.11688+01	-.13869+00	.42738-02	-.20518-05	-.66382-02	.24725-03	-.16223-06
7--	.46875-03	.83634-01	.11129+01	-.12558+00	.41481-02	-.15847-05	-.57260-02	.24300-03	-.13082-06
8--	.23438-03	.81958-01	.10822+01	-.12346+00	.40941-02	-.14472-05	-.57579-02	.24083-03	-.12421-06
9--	.11719-03	.81886-01	.10780+01	-.12279+00	.40883-02	-.14296-05	-.57605-02	.24062-03	-.12331-06
10--	.58594-04	.81884-01	.10785+01	-.12276+00	.40877-02	-.14270-05	-.57607-02	.24060-03	-.12322-06

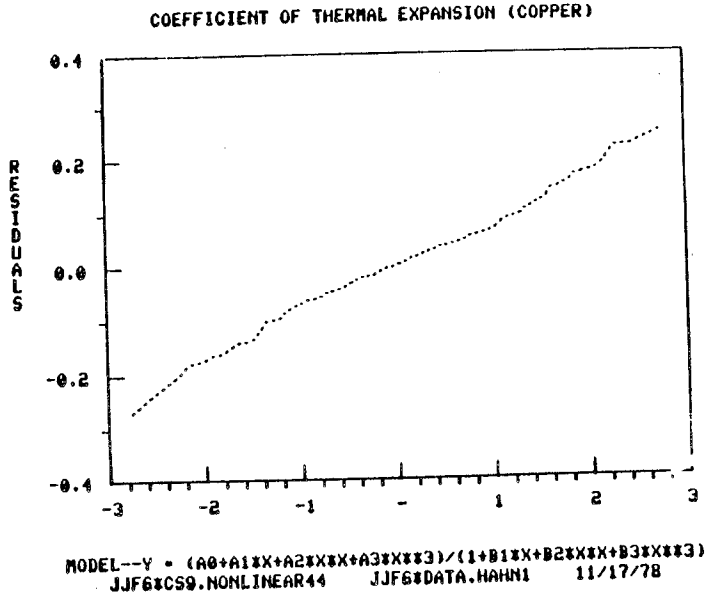
FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)

1	A0	1.07858	(.1795)
2	A1	-.122766	(.1196-01)
3	A2	.408778-02	(.2231-03)
4	A3	-.142786-05	(.2606-06)
5	B1	-.676885-02	(.2469-03)
6	B2	.240583-03	(.1052-04)
7	B3	-.123223-06	(.1810-07)

RESIDUAL STANDARD DEVIATION = .8118038480
RESIDUAL DEGREES OF FREEDOM = 229
REPLICATION STANDARD DEVIATION = .8131728860
REPLICATION DEGREES OF FREEDOM = 1
LACK OF FIT F RATIO = 1.0181 = THE 32.126% POINT OF THE F DISTRIBUTION WITH 229 AND 1 DEGREES OF FREEDOM







LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 236
MODEL--Y = (A1*X+A2*X*X+A3*X**3)/(1+B1*X+B2*X*X+B3*X**3)
REPLICATION CASE
REPLICATION STANDARD DEVIATION = .8131728601-01
REPLICATION DEGREES OF FREEDOM = 1
NUMBER OF DISTINCT SUBSETS = 236

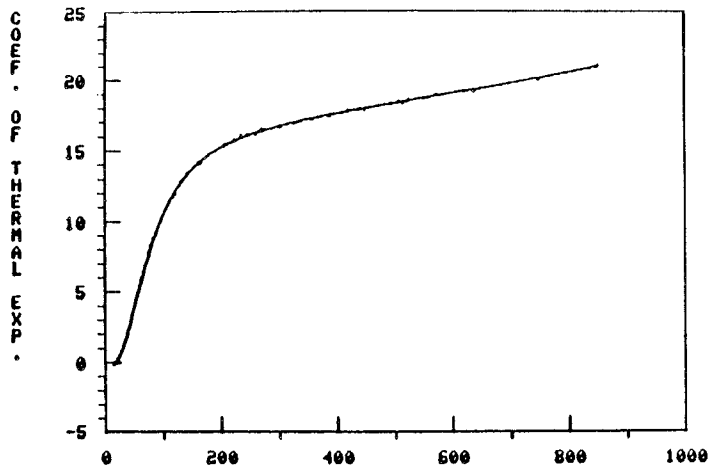
ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	A	PARAMETER ESTIMATES					
1--	.10000-01	.41429+00	A	-.12277+00	.40878-02	-.14279-05	-.57609-02	.84060-03	-.12322-06
2--	.60000-02	.11461+00	A	-.72578-01	.35767-02	-.13773-05	-.39626-02	.21113-03	-.11495-06
3--	.25800-02	.97798-01	A	-.68818-01	.31220-02	-.81652-06	-.46303-02	.19164-03	-.88638-07
4--	.12500-02	.96681-01	A	-.62423-01	.29211-02	-.41784-06	-.50839-02	.18413-03	-.78612-07
5--	.62500-03	.89248-01	A	-.61681-01	.28890-02	-.31131-06	-.51677-02	.18321-03	-.65703-07
6--	.31250-03	.89213-01	A	-.61682-01	.28848-02	-.29408-06	-.51806-02	.18311-03	-.65612-07

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)

1	A1	-.514920-01	(.2827-02)
2	A2	.288438-02	(.8400-04)
3	A3	-.292468-06	(.1916-06)
4	B1	-.618199-02	(.2263-03)
5	B2	.183496-03	(.3940-05)
6	B3	-.648213-07	(.8271-08)

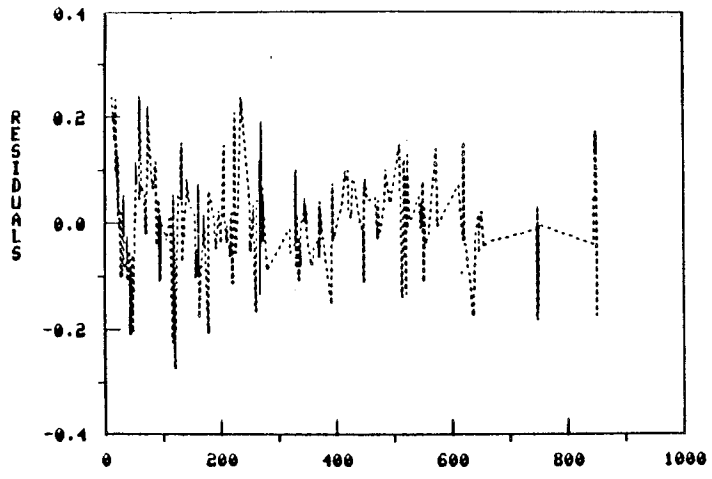
RESIDUAL STANDARD DEVIATION = .8892128153
RESIDUAL DEGREES OF FREEDOM = 230
REPLICATION STANDARD DEVIATION = .8813172868
REPLICATION DEGREES OF FREEDOM = 1
LACK OF FIT F RATIO = 1.2045 = THE 36.3168% POINT OF THE F DISTRIBUTION WITH 229 AND 1 DEGREES OF FREEDOM

COEFFICIENT OF THERMAL EXPANSION (COPPER)



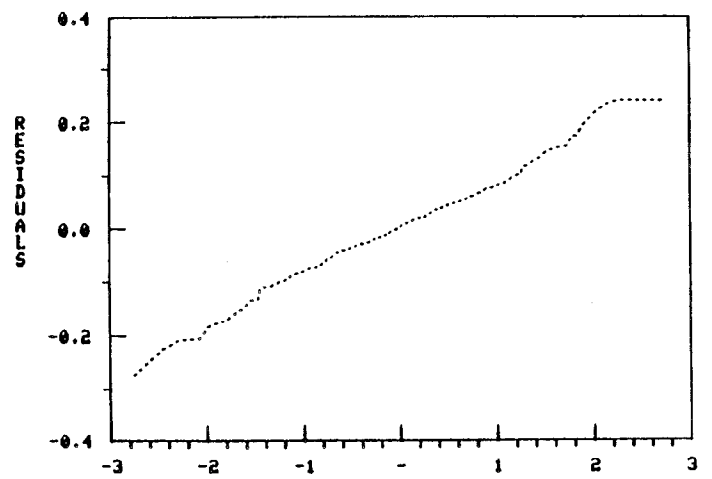
MODEL--Y = (A1*X+A2*X*X+A3*X**3)/(1+B1*X+B2*X*X+B3*X**3)
JJF6%CS9.NONLINEAR44 JJF6%DATA.HAHN1 11/17/78

COEFFICIENT OF THERMAL EXPANSION (COPPER)



MODEL--Y = (A1*X+A2*X*X+A3*X**3)/(1+B1*X+B2*X*X+B3*X**3)
JJF6%CS9.NONLINEAR44 JJF6%DATA.HAHN1 11/17/78

COEFFICIENT OF THERMAL EXPANSION (COPPER)

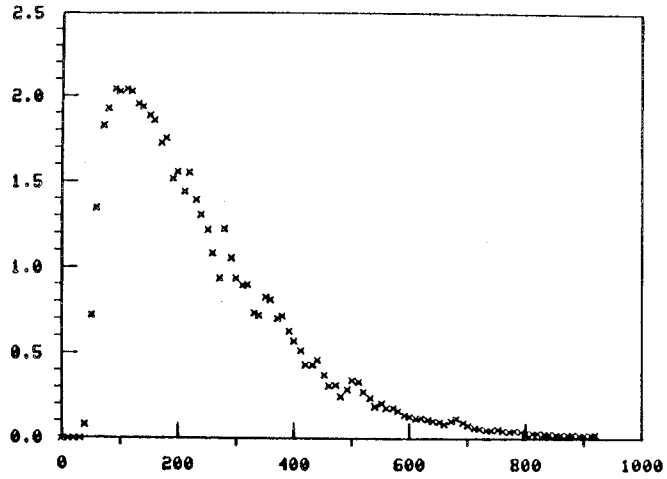


MODEL--Y = (A1*X+A2*X*X+A3*X**3)/(1+B1*X+B2*X*X+B3*X**3)
JJF6%CS9.NONLINEAR44 JJF6%DATA.HAHN1 11/17/78

PARAMETERS AND CONSTANTS--

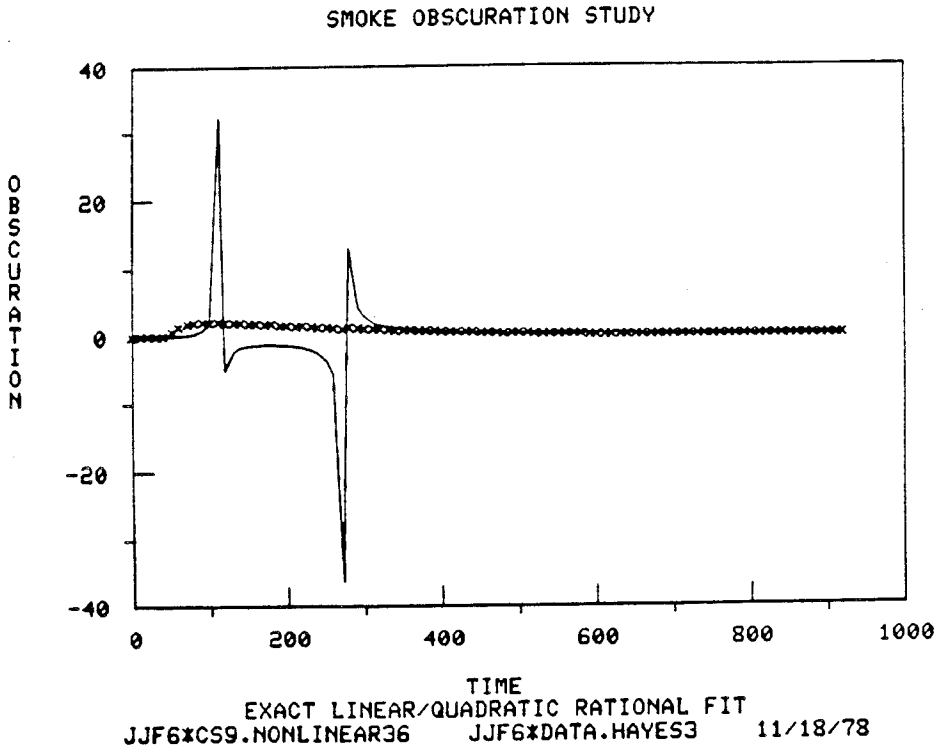
S1 -- .8180385-01
S2 -- .8921282-01

```
COMMENT EXAMPLE--WARREN HAYES SMOKE OBSCURATION STUDY
COMMENT MODEL   --LINEAR/QUADRATIC AND LINEAR/QUARTIC
COMMENT NOTE    --TECHNIQUE FOR UPDATING AND IMPROVING
COMMENT           RATIONAL FIT MODEL
COMMENT NOTE    --EXACT RATIONAL FIT FOLLOWED BY LEAST SQUARES FIT
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6*DATA.HAYES3 X ST Y
.
CHARACTERS X
LINES
PLOT Y X
.
LET X2(1)=0
LET X2(2)=100
LET X2(3)=400
LET X2(4)=800
LET Y2(1)=0
LET Y2(2)=2
LET Y2(3)=.5
LET Y2(4)=.1
EXACT 1/2 RATIONAL FIT Y2 X2 Y X
.
TITLE SMOKE OBSCURATION STUDY
YLABEL OBSCURATION
XLABEL TIME
X2LABEL EXACT LINEAR/QUADRATIC RATIONAL FIT
X3LABEL JJF6*CS9.NONLINEAR36    JJF6*DATA.HAYES3    11/18/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT Y PRED VS X
.
LET X2(5)=110
LET Y2(5)=2
LET X2(6)=270
LET Y2(6)=1
EXACT 1/4 RATIONAL FIT Y2 X2 Y X
.
X2LABEL EXACT LINEAR/QUARTIC RATIONAL FIT
PLOT Y PRED VS X
.
FIT Y = (A0+A1*X)/(1+B1*X+B2*X**2+B3*X**3+B4*X**4)
.
X2LABEL MODEL--Y = (A0+A1*X)/(1+B1*X+B2*X**2+B3*X**3+B4*X**4)
PLOT Y PRED VS X
.
YLABEL RESIDUALS
PLOT RES X
.
XLABEL
NORMAL PROBABILITY PLOT RES
```

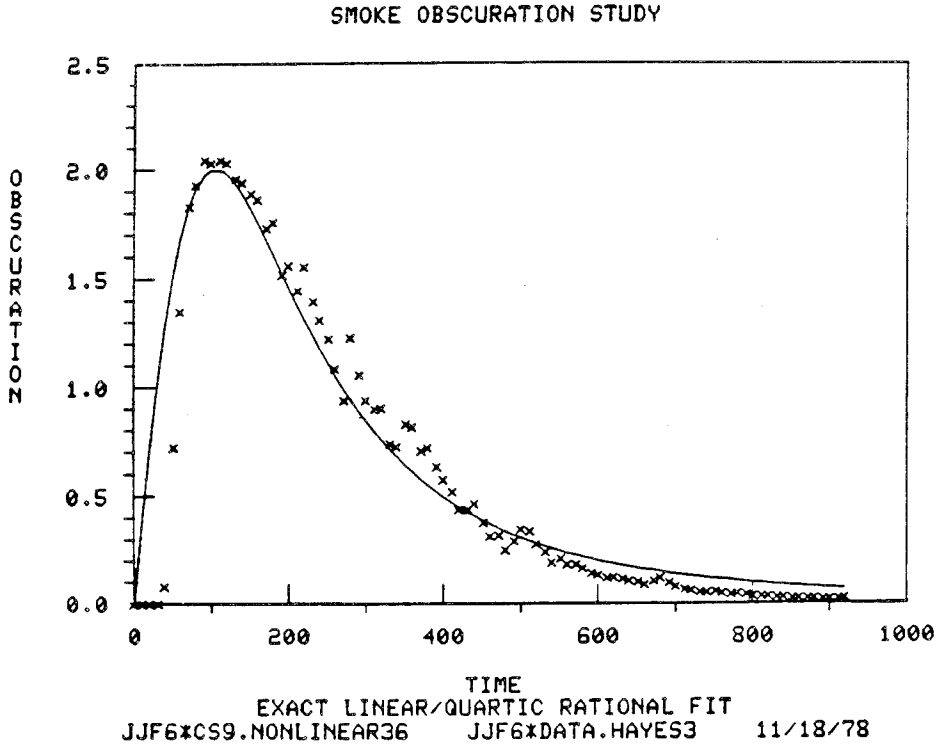


EXAMPLE 22

```
EXACT RATIONAL FUNCTION FIT
      NUMBER OF POINTS IN FIRST SET     =         4
      DEGREE OF NUMERATOR               =         1
      DEGREE OF DENOMINATOR             =         2

NUMERATOR  --A0  A1                =     .00000000        .14583332-02
DENOMINATOR--B0  B1  B2            =          .100+01         -.125-01
 .323-04

      APPLICATION OF EXACT-FIT COEFFICIENTS
      TO SECOND PAIR OF VARIABLES--

      NUMBER OF POINTS IN SECOND SET                =        93
      NUMBER OF ESTIMATED COEFFICIENTS              =         4
      RESIDUAL DEGREES OF FREEDOM                   =        89

      RESIDUAL SUM OF SQUARES                       =     .27069581+04
      RESIDUAL STANDARD DEVIATION (DENOM=N-P)       =     .55150031+01
      AVERAGE ABSOLUTE RESIDUAL   (DENOM=N)         =     .16479395+01
      LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL      =     .37217266+02
      LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL      =    -.30199815+02
      LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL      =     .37217266+02
```



```
 EXACT RATIONAL FUNCTION FIT
       NUMBER OF POINTS IN FIRST SET     =         6
       DEGREE OF NUMERATOR               =         1
       DEGREE OF DENOMINATOR             =         4

 NUMERATOR  --A0  A1                =     .00000000        .37565201-01
 DENOMINATOR--B0  B1  B2  B3  B4    =     .10000000+01     .20631021-02     .51
269280-04     .10805190-06     .51205132-09

       APPLICATION OF EXACT-FIT COEFFICIENTS
       TO SECOND PAIR OF VARIABLES--

       NUMBER OF POINTS IN SECOND SET                =        93
       NUMBER OF ESTIMATED COEFFICIENTS              =         6
       RESIDUAL DEGREES OF FREEDOM                   =        87

       RESIDUAL SUM OF SQUARES                       =    .46780170+01
       RESIDUAL STANDARD DEVIATION (DENOM=N-P)       =    .23188426+00
       AVERAGE ABSOLUTE RESIDUAL   (DENOM=N)         =    .11407338+00
       LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL      =    .27681495+00
       LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL      =   -.11952349+01
       LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL      =    .11952349+01
```

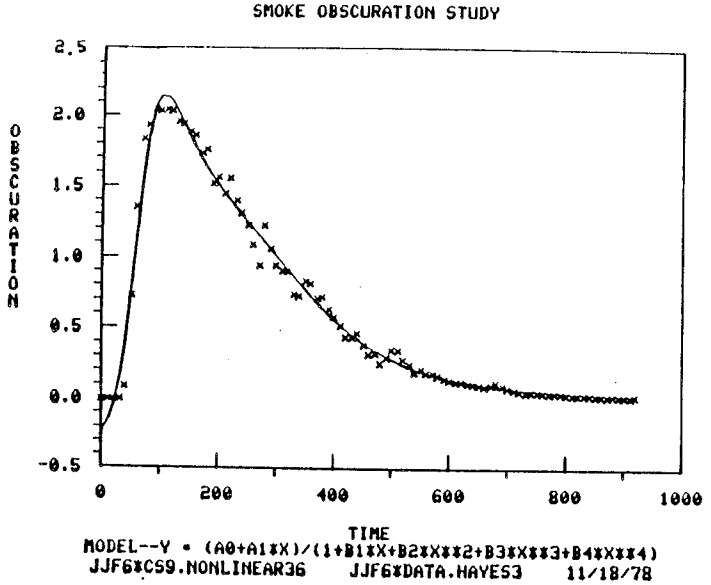


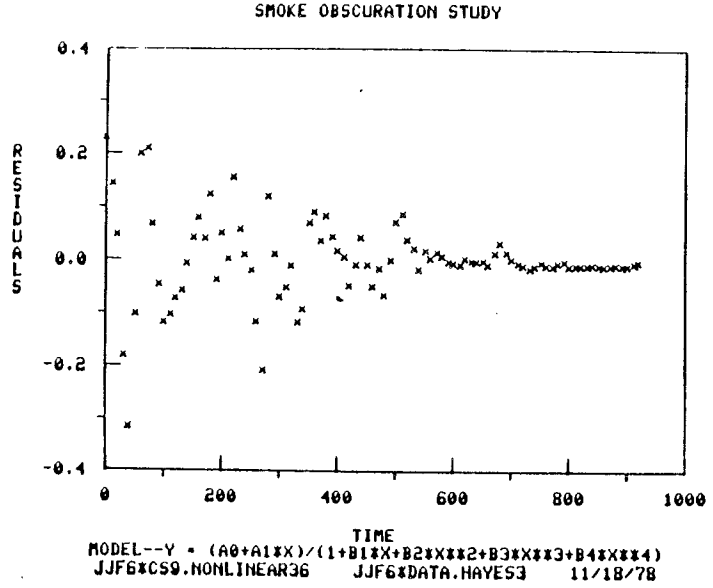
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 93
MODEL--Y = (A0+A1*X)/(1+B1*X+B2*X**2+B3*X**3+B4*X**4)
NO REPLICATION CASE

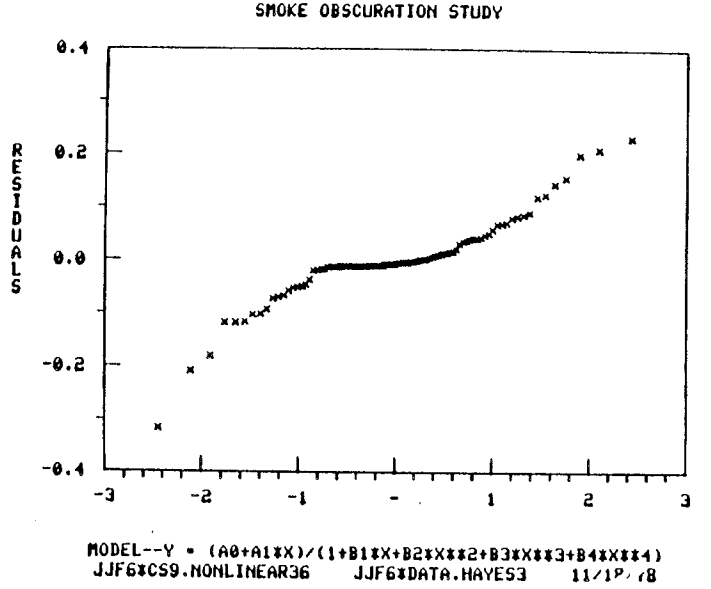
ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	#	PARAMETER ESTIMATES					
1--	.10000-01	.23188+00	#	.00000	.37565-01	.20631-02	.51269-04	.10805-06	.51205-09
2--	.75937-01	.23328+00	#	-.39031+00	.17405-01	-.52813-02	.24423-04	-.67498-08	.40611-09
3--	.37969-01	.13974+00	#	-.33554+00	.21664-01	-.49042-02	.39730-04	-.13344-07	.37219-09
4--	.18984-01	.11913+00	#	-.35523+00	.19998-01	-.97062-02	.89709-04	-.13140-06	.45032-09
5--	.94923-02	.10477+00	#	-.19193+00	.87006-02	-.21810-01	.20702-03	-.64294-06	.85069-09
6--	.47461-02	.96035-01	#	-.24154+00	.10466-01	-.21643-01	.21389-03	-.68935-06	.97690-09
7--	.23730-02	.80729-01	#	-.22237+00	.97144-02	-.22119-01	.21829-03	-.71728-06	.99606-09
8--	.11865-02	.79679-01	#	-.23198+00	.10049-01	-.21967-01	.21681-03	-.71189-06	.99236-09
9--	.59326-03	.79515-01	#	-.23069+00	.10023-01	-.21951-01	.21664-03	-.71062-06	.99064-09
10--	.29663-03	.79510-01	#	-.23043+00	.10014-01	-.21953-01	.21665-03	-.71053-06	.99052-09
11--	.14832-03	.79509-01	#	-.23046+00	.10015-01	-.21954-01	.21666-03	-.71058-06	.99061-09

	FINAL PARAMETER ESTIMATES		(APPROX. ST. DEV.)
1	A0	-.230448	(.4279-01)
2	A1	.100142-01	(.1132-02)
3	B1	-.219546-01	(.6089-03)
4	B2	.216863-03	(.9528-05)
5	B3	-.710602-06	(.4087-07)
6	B4	.990631-09	(.7576-10)

RESIDUAL STANDARD DEVIATION = .0795088410
RESIDUAL DEGREES OF FREEDOM = 87

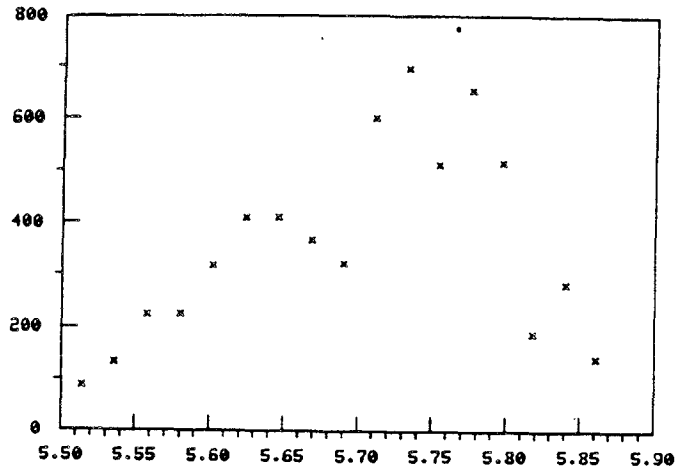






EXAMPLE 23

```
COMMENT EXAMPLE--GEORGE MULHOLLAND DOPPLER SPECTROMETER EXAMPLE
COMMENT MODEL   --LINEAR/QUADRATIC
COMMENT NOTE    --EXACT RATIONAL FIT FOLLOWED BY LEAST SQUARES FIT
.
ECHO ON
HARDCOPY ON
BELL ON
.
SKIP 1
READ JJF6*DATA.MULHOLLAND2 X Y
.
CHARACTERS X
LINES
PLOT Y X
.
LET X2(1)=5.55
LET X2(2)=5.65
LET X2(3)=5.75
LET X2(4)=5.85
LET Y2(1)=150
LET Y2(2)=400
LET Y2(3)=600
LET Y2(4)=200
EXACT 1/2 RATIONAL FIT Y2 X2 Y X
.
TITLE PARTICLE SIZE DISTRIBUTION (DOPPLER SPECTROMETER)
YLABEL DELTA N / DELTA D
XLABEL DIAMETER
X2LABEL EXACT LINEAR/QUADRATIC RATIONAL FIT
X3LABEL JJF6*CS9.NONLINEAR30    JJF6*DATA.MULHOLLAND2    7/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT Y PRED VS X
.
FIT Y = (A0+A1*X)/(1+B1*X+B2*X*X)

X2LABEL MODEL--Y = (A0 + A1*X) / (1 + B1*X + B2*X*X)
PLOT Y PRED VS X
.
LET MN=MIN(X)
LET MX=MAX(X)
LET DELTA=(MX-MN)/100
.
PLOT Y X AND
PLOT Y = (A0+A1*X)/(1+B1*X+B2*X*X) FOR X = MN DELTA MX
.
YLABEL RESIDUALS
PLOT RES X
.
XLABEL
NORMAL PROBABILITY PLOT RES
```



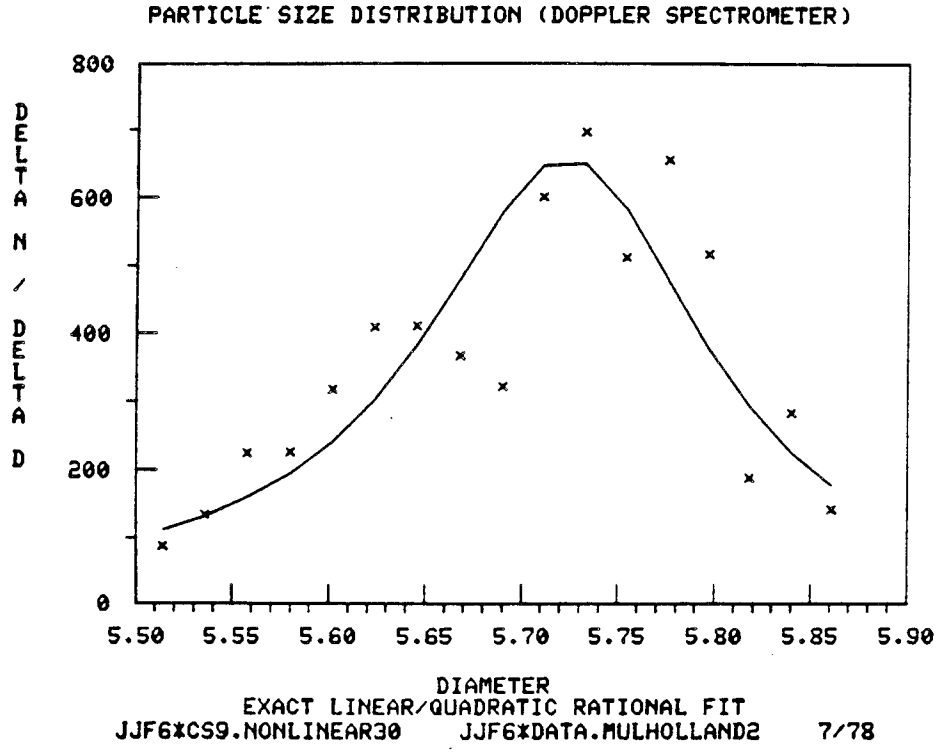
EXACT RATIONAL FUNCTION FIT

NUMBER OF POINTS IN FIRST SET	=	4
DEGREE OF NUMERATOR	=	1
DEGREE OF DENOMINATOR	=	2

NUMERATOR --A0 A1	=	.83234787+00	-.11806805+00
DENOMINATOR--B0 B1 B2	=	.100+01	-.349+00
.305-01			

APPLICATION OF EXACT-FIT COEFFICIENTS
TO SECOND PAIR OF VARIABLES--

NUMBER OF POINTS IN SECOND SET	=	17
NUMBER OF ESTIMATED COEFFICIENTS	=	4
RESIDUAL DEGREES OF FREEDOM	=	13
RESIDUAL SUM OF SQUARES	=	.17727561+06
RESIDUAL STANDARD DEVIATION (DENOM=N-P)	=	.11677579+03
AVERAGE ABSOLUTE RESIDUAL (DENOM=N)	=	.80742574+02
LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL	=	.17598337+03
LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL	=	-.25745164+03
LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL	=	.25745164+03

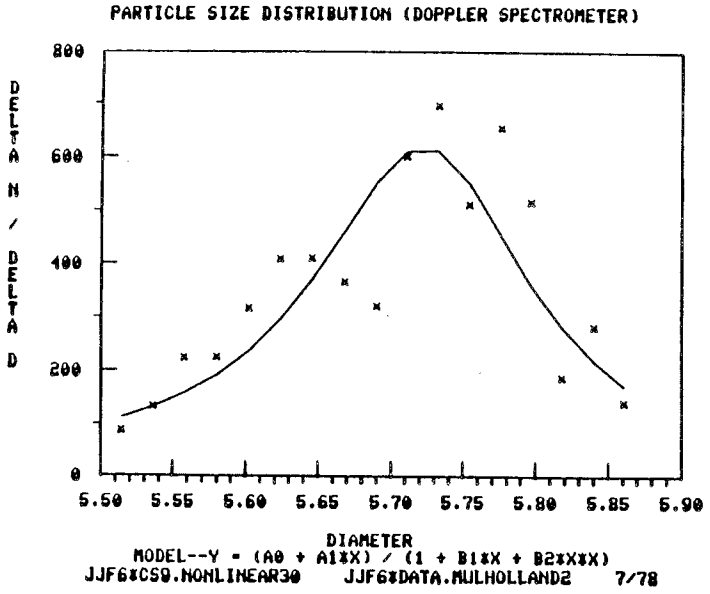


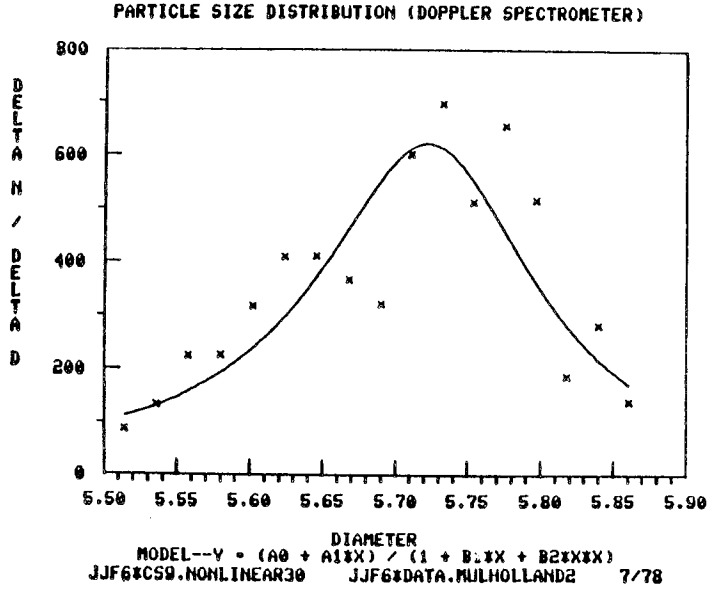
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 17
MODEL-- Y = (A0+A1*X)/(1+B1*X+B2*X*X)
NO REPLICATION CASE

ITERATION NUMBER	CONVERGENCE MEASURE	RESIDUAL STANDARD DEVIATION	*	PARAMETER ESTIMATES			
1--	.10303-01	.11677+03	*	.83235+00	-.11807+00	-.34921+00	.30495-01
2--	.57665+00	.11594+03	*	.83199+00	-.11810+00	-.34923+00	.30498-01
3--	.29193+01	.11576+03	*	.83209+00	-.11808+00	-.34923+00	.30498-01
4--	.24939+02	.11575+03	*	.83209+00	-.11808+00	-.34923+00	.30498-01

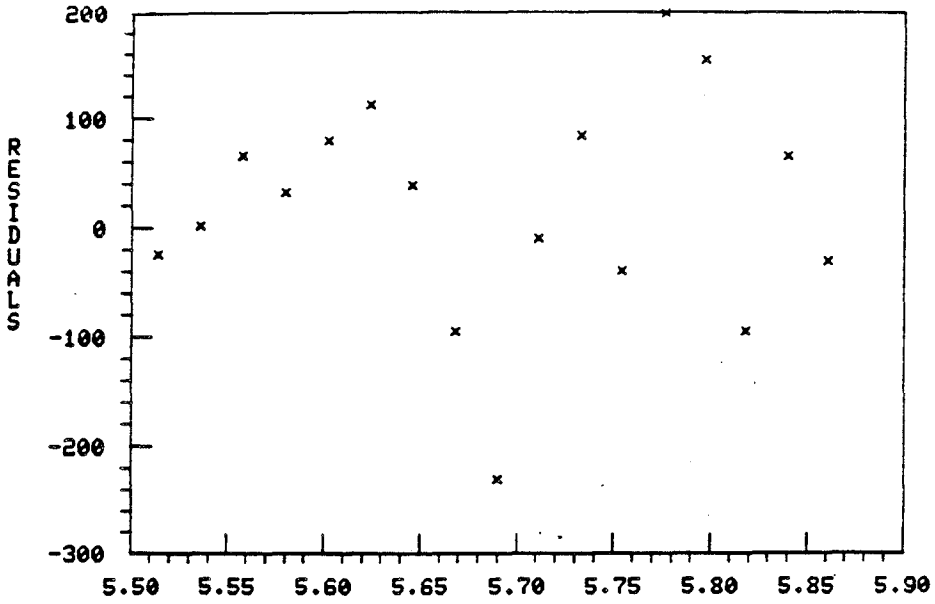
	FINAL PARAMETER ESTIMATES		(APPROX. ST. DEV.)
1	A0	.832090	(169.9)
2	A1	-.118081	(24.10)
3	B1	-.349229	(.4468)
4	B2	.304979-01	(.2350-01)

RESIDUAL STANDARD DEVIATION = 115.7529821396
RESIDUAL DEGREES OF FREEDOM = 13



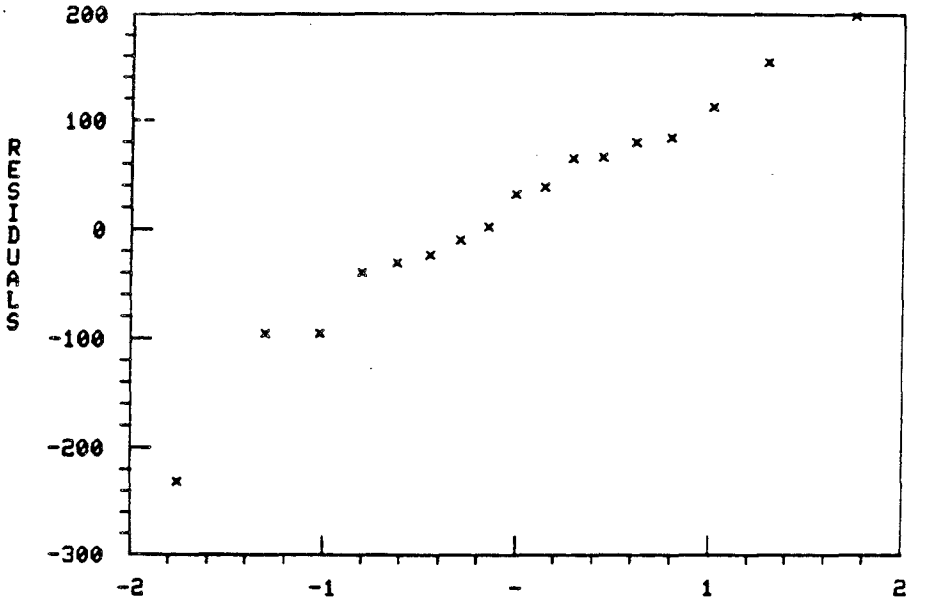


PARTICLE SIZE DISTRIBUTION (DOPPLER SPECTROMETER)



MODEL--Y = (A0 + A1*X) / (1 + B1*X + B2*X*X)
JJF6*CS9.NONLINEAR30 JJF6*DATA.MULHOLLAND2 7/78

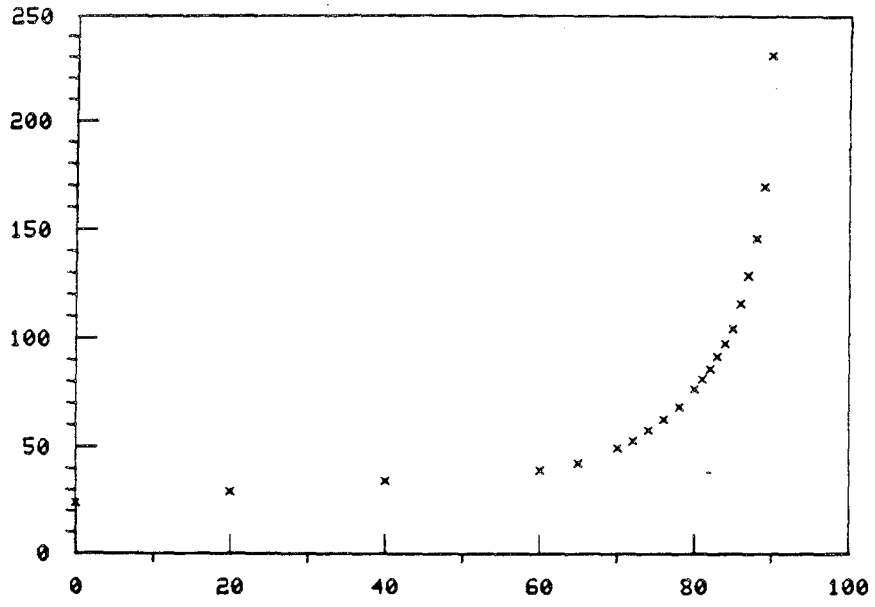
PARTICLE SIZE DISTRIBUTION (DOPPLER SPECTROMETER)



MODEL--Y = (A0 + A1*X) / (1 + B1*X + B2*X*X)
JJF6*CS9.NONLINEAR30 JJF6*DATA.MULHOLLAND2 7/78

EXAMPLE 24

```
COMMENT EXAMPLE--NEWT BREESE RESIDENTIAL TIME-TEMPERATURE CURVE
COMMENT MODEL   --CUBIC/CUBIC
COMMENT NOTE    --EXACT RATIONAL FIT FOLLOWED BY LEAST SQUARES FIT
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6*DATA.BREESE2
.
NAME TIME 1
NAME TEMP 2
.
CHARACTERS X
LINES
PLOT TEMP TIME
.
LET ID = 0*TEMP
LET ID(1)=1
LET ID(13)=1
LET ID(6)=1
LET ID(11)=1
LET ID(18)=1
LET ID(8)=1
LET ID(21)=1
LET TEMP2 = TEMP SUBSET ID 1
LET TIME2 = TIME SUBSET ID 1
PACK TEMP2 TIME2 SUBSET ID 1
EXACT 3/3 RATIONAL FIT TEMP2 TIME2 TEMP TIME
.
TITLE DERIVED FIRE EXPOSURE CURVE FIT (T=0,90)
YLABEL TEMPERATURE
X1LABEL TIME (S)
X2LABEL EXACT CUBIC/CUBIC RATIONAL FIT
X3LABEL JJF6*CS9.NONLINEAR21    JJF6*DATA.BREESE2    10/27/78
CHARACTER X BLANK
LINE BLANK SOLID
PLOT TEMP PRED VS TIME
.
LET A=MIN(TIME)
LET B=MAX(TIME)
LET DEL = (B-A)/100
PLOT TEMP TIME AND
PLOT Y = (A0+A1*X+A2*X**X+A3*X**3)/(1+B1*X+B2*X**X+B3*X**3) FOR X = A DEL B
```



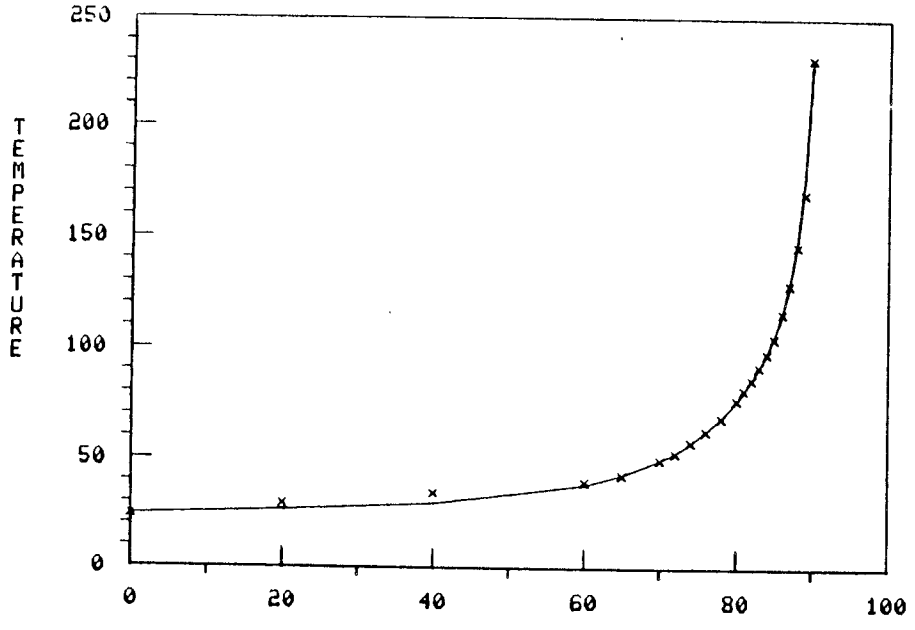
EXACT RATIONAL FUNCTION FIT

NUMBER OF POINTS IN FIRST SET	•	7			
DEGREE OF NUMERATOR	•	3			
DEGREE OF DENOMINATOR	•	3			
NUMERATOR --A0 A1 A2 A3	•	.25000000+02	-.78699218+00	.82258595-02	-.28315299-04
DENOMINATOR--B0 B1 B2 B3	•	.10000000+01	-.34418127-01	.39682391-03	-.15308051-05

APPLICATION OF EXACT-FIT COEFFICIENTS
TO SECOND PAIR OF VARIABLES--

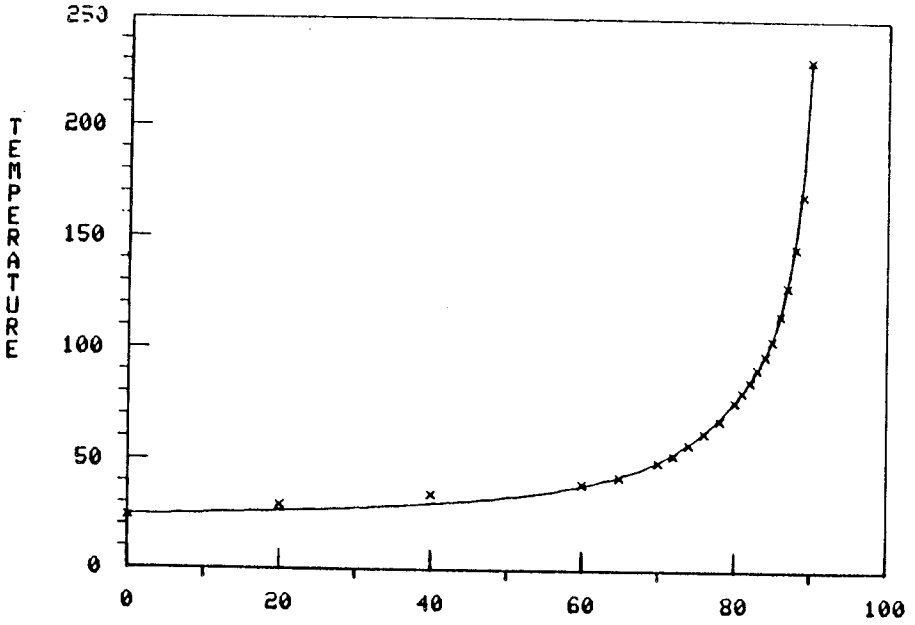
NUMBER OF POINTS IN SECOND SET	•	21
NUMBER OF ESTIMATED COEFFICIENTS	•	7
RESIDUAL DEGREES OF FREEDOM	•	14
RESIDUAL SUM OF SQUARES	•	.11272736+03
RESIDUAL STANDARD DEVIATION (DENOM=N-P)	•	.28375965+01
AVERAGE ABSOLUTE RESIDUAL (DENOM=N)	•	.11268669+01
LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL	•	.46076736+01
LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL	•	-.85434399+01
LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL	•	.85434399+01

DERIVED FIRE EXPOSURE CURVE FIT (T=0,90)



EXACT CUBIC/CUBIC RATIONAL FIT
JJF6*CS9.NONLINEAR21 JJF6*DATA.BREESE2 10/27/78

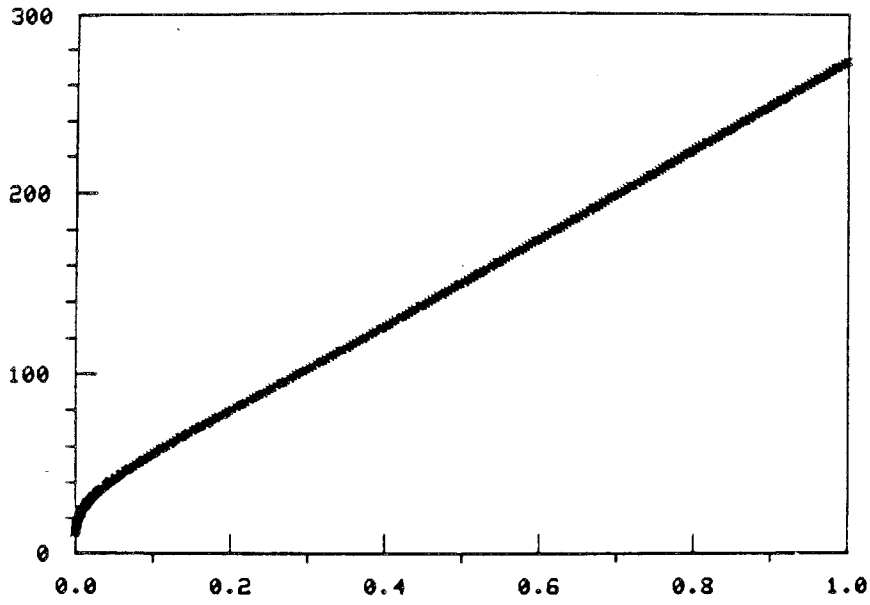
DERIVED FIRE EXPOSURE CURVE FIT (T=0,90)



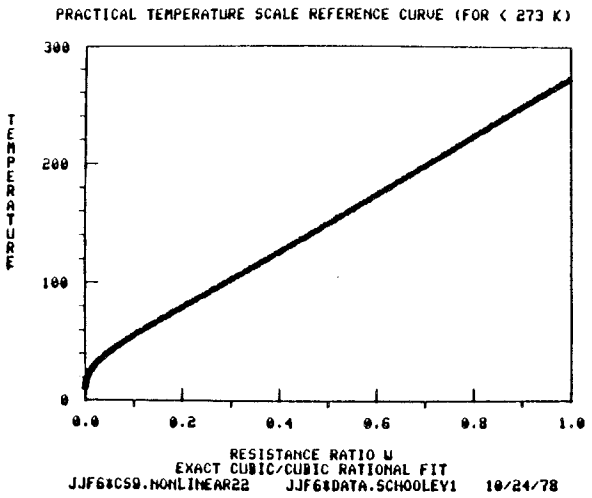
EXACT CUBIC/CUBIC RATIONAL FIT
JJF6*CS9.NONLINEAR21 JJF6*DATA.BREESE2 10/27/78

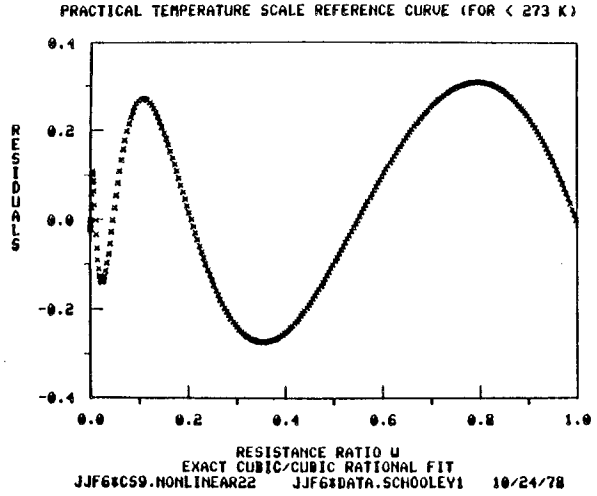
EXAMPLE 25

```
COMMENT EXAMPLE--PRACTICAL TEMPERATURE SCALE REFERENCE CURVE
COMMENT           FOR LESS THAN 273 DEGREES KELVIN
COMMENT MODEL   --CUBIC/CUBIC
COMMENT NOTE    --EXACT RATIONAL FIT FOLLOWED BY LEAST SQUARES FIT
COMMENT NOTE    --VALUE OF RESIDUAL PLOT
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6XDATA.SCHOOLEY1 T W
.
CHARACTERS X
LINES
PLOT T W
.
LET ID = 0*T
LET ID(1)=1
LET ID(5)=1
LET ID(15)=1
LET ID(30)=1
LET ID(70)=1
LET ID(150)=1
LET ID(261)=1
LET T2 = T SUBSET ID 1
LET W2 = W SUBSET ID 1
PACK T2 W2 SUBSET ID 1
.
EXACT 3/3 FIT T2 W2 T W
.
TITLE PRACTICAL TEMPERATURE SCALE REFERENCE CURVE (FOR < 273 K)
YLABEL TEMPERATURE
XLABEL RESISTANCE RATIO W
X2LABEL EXACT CUBIC/CUBIC RATIONAL FIT
X3LABEL JJF6XCS9.NONLINEAR22    JJF6XDATA.SCHOOLEY1    10/24/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT T PRED VS W
.
YLABEL RESIDUALS
PLOT RES W
```



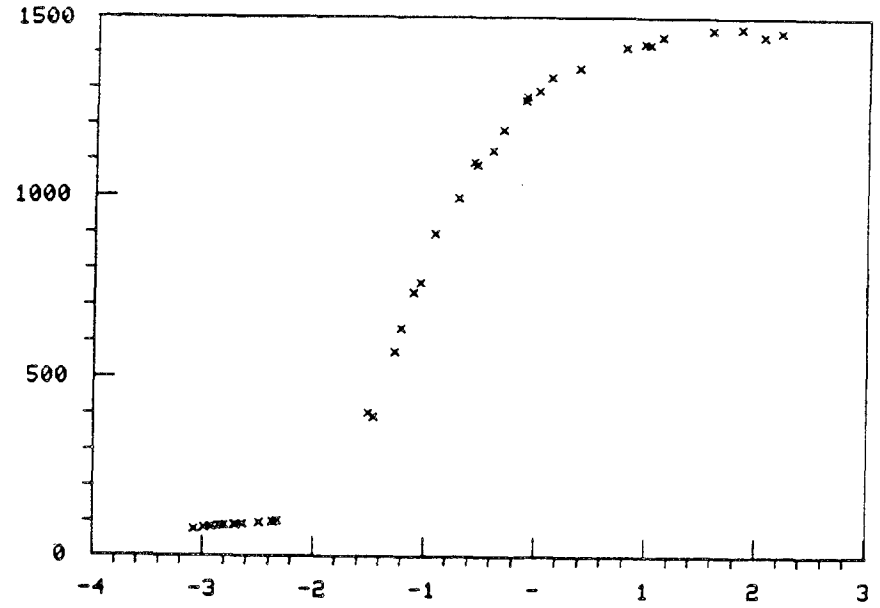
```
 EXACT RATIONAL FUNCTION FIT
       NUMBER OF POINTS IN FIRST SET    =        7
       DEGREE OF NUMERATOR              =        3
       DEGREE OF DENOMINATOR            =        3

 NUMERATOR  --A0  A1  A2  A3      =   -.22752360+02    .87906122+05    .
12248801+08    .65114699+08
 DENOMINATOR--B0  B1  B2  B3      =    .10000000+01    .53270117+04    .
29851157+06   -.20292635+05

       APPLICATION OF EXACT-FIT COEFFICIENTS
       TO SECOND PAIR OF VARIABLES--

       NUMBER OF POINTS IN SECOND SET          =      261
       NUMBER OF ESTIMATED COEFFICIENTS        =        7
       RESIDUAL DEGREES OF FREEDOM             =      254

       RESIDUAL SUM OF SQUARES                 =    .10105131+02
       RESIDUAL STANDARD DEVIATION (DENOM=N-P) =    .19945922+00
       AVERAGE ABSOLUTE RESIDUAL   (DENOM=N)   =    .17285277+00
       LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL =   .30939484+00
       LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL =  -.27073383+00
       LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL =   .30939484+00
```





EXAMPLE 26

```
COMMENT EXAMPLE--BOB THURBER SEMICONDUCTOR MOBILITY MODELLING
COMMENT MODEL   --CUBIC/CUBIC
COMMENT NOTE    --EXACT RATIONAL FIT FOLLOWED BY LEAST SQUARES FIT
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6*DATA.THURBER22 R N
.
LET LR=ALOG10(R)
LET Q  =  1.602*10**(-19)
LET Y = 1/(Q*R*N)
.
CHARACTERS X
LINES
PLOT Y LR
.
LET ID=0*N
LET ID(1) =1
LET ID(9) = 1
LET ID(12) = 1
LET ID(13) = 1
LET ID(21) = 1
LET ID(29) = 1
LET ID(37) = 1
.
LET Y2=Y SUBSET ID 1
LET LR2=LR SUBSET ID 1
PACK Y2 LR2 SUBSET ID 1
.
EXACT 3/3 RATIONAL FIT Y2 LR2 Y LR
.
TITLE PHOSPHORUS-DOPED SILICON   23 C
YLABEL ELECTRON MOBILITY
X1LABEL LOG(RHO)
X2LABEL EXACT CUBIC/CUBIC RATIONAL FIT
X3LABEL JJF6*CS9.NONLINEAR16    JJF6*DATA.THURBER22    11/7/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT Y PRED VS LR
.
FIT Y = (A0+A1*LR+A2*LR*LR+A3*LR**3)/(1+B1*LR+B2*LR*LR+B3*LR**3)
.
X2LABEL LEAST SQUARES CUBIC/CUBIC FIT
PLOT Y PRED VS LR
.
LET A = MIN(LR)
LET B = MAX(LR)
LET INC = (B-A)/100
PLOT Y LR AND
PLOT U = (A0+A1*U+A2*U*U+A3*U**3)/(1+B1*U+B2*U*U+B3*U**3) FOR U = A INC B
```



EXACT RATIONAL FUNCTION FIT

NUMBER OF POINTS IN FIRST SET	=	7
DEGREE OF NUMERATOR	=	3
DEGREE OF DENOMINATOR	=	3

NUMERATOR --A0 A1 A2 A3	=	.12877952+04	.14371486+04	.54575087+03 .68139794+02
DENOMINATOR--B0 B1 B2 B3	=	.10000000+01	.94009615+00	.38602169+00 .40002313-01

APPLICATION OF EXACT-FIT COEFFICIENTS
TO SECOND PAIR OF VARIABLES--

NUMBER OF POINTS IN SECOND SET	=	37
NUMBER OF ESTIMATED COEFFICIENTS	=	7
RESIDUAL DEGREES OF FREEDOM	=	30
RESIDUAL SUM OF SQUARES	=	.82915248+04
RESIDUAL STANDARD DEVIATION (DENOM=N-P)	=	.16624805+02
AVERAGE ABSOLUTE RESIDUAL (DENOM=N)	=	.96072464+01
LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL	=	.25774384+02
LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL	=	-.42453033+02
LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL	=	.42453033+02