

# **R Code for ‘A Practical Guide . . .’**

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2024-06-08

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# Preface

Code provided here is for the May 2024 text:

“A Practical Guide to Data Analysis Using R – An Example-Based Approach”, by  
John H Maindonald, W John Braun, and Jeffrey L Andrews.

Code that is shown in the text is filled out to include all code for graphs. In chapter 2 and later, the text includes code only for those graphs that are specifically targeted at the methodology under discussion.

The text has been available in digital format since early May 2024. It has been available in print in the UK from May 30, is due to be available in print in the USA in July, and in Australia and New Zealand in August 2024.

For details for the UK, see: [A Practical Guide to Data Analysis Using R – Cambridge University Press UK](#)

# 1 Chapter 1: Learning from data

```
library(knitr)
opts_chunk[['set']](fig.width=6, fig.height=6, comment=" ",
                    out.width="80%", fig.align="center", fig.show='hold',
                    size="small", ps=10, strip.white = TRUE,
                    tidy.opts = list(replace.assign=FALSE))

## xtras=TRUE    ## Set to TRUE to execute code 'extras'
xtras <- FALSE
library(knitr)
## opts_chunk[['set']](results="asis")
## opts_chunk[['set']](eval=FALSE)    ## Set to TRUE to execute main part of code
opts_chunk[['set']](eval=FALSE)
```

## Packages required (plus any dependencies)

latticeExtra (lattice is a dependency); DAAG; car; MASS; AICcmodavg; BayesFactor; boot; MPV; ggplot2; tidyR

Additionally, knitr and Hmisc are required in order to process the Rmd source file. The prettydoc package is by default used to format the html output.

## Chapter summary

### A note on terminology — variables, factors, and more!

### Section 1.1: Questions, and data that may point to answers

#### Subsection 1.1.1: A sample is a window into the wider population

```
## For the sequence below, precede with set.seed(3676)
set.seed(3696)
sample(1:9384, 12, replace=FALSE) # NB: `replace=FALSE` is the default
```

```
chosen1200 <- sample(1:19384, 1200, replace=FALSE)
```

```
## For the sequence below, precede with set.seed(366)
set.seed(366)
split(sample(seq(1:10)), rep(c("Control", "Treatment"), 5))
# sample(1:10) gives a random re-arrangement (permutation) of 1, 2, ..., 10
```

## Cluster sampling

### \*A note on with-replacement samples

```
sample(1:10, replace=TRUE)
## sample(1:10, replace=FALSE) returns a random permutation of 1,2,...10
```

## Subsection 1.1.2: Formulating the scientific question

### Example: a question about cuckoo eggs

```
library(latticeExtra) # Lattice package will be loaded and attached also
cuckoos <- DAAG::cuckoos
## Panel A: Dotplot without species means added
dotplot(species ~ length, data=cuckoos) ## `species ~ length` is a 'formula'
## Panel B: Box and whisker plot
bwplot(species ~ length, data=cuckoos)
## The following shows Panel A, including species means & other tweaks
av <- with(cuckoos, aggregate(length, list(species=species), FUN=mean))
dotplot(species ~ length, data=cuckoos, alpha=0.4, xlab="Length of egg (mm)") +
  as.layer(dotplot(species ~ x, pch=3, cex=1.4, col="black", data=av))
# Use `+` to indicate that more (another 'layer') is to be added.
# With `alpha=0.4`, 40% is the point color with 60% background color
# `pch=3`: Plot character 3 is '+'; `cex=1.4`: Default char size X 1.4
```

```
## Code
suppressPackageStartupMessages(library(latticeExtra, quietly=TRUE))
cuckoos <- DAAG::cuckoos
## For tidier labels replace ".", in several of the names, by a space
specnam <- with(cuckoos, sub(pattern=".", replacement=" ",
                             levels(species), fixed=TRUE))
# fixed=TRUE: "interpret "." as ".", not as a 'any single character'"
cuckoos <- within(cuckoos, levels(species) <- specnam)
## Panel A: Dotplot: data frame cuckoos (DAAG)
av <- with(cuckoos, aggregate(length, list(species=species), FUN=mean))
gphA <- dotplot(species ~ length, data=cuckoos, alpha=0.4) +
  as.layer(dotplot(species ~ x, pch=3, cex=1.4, col="black", data=av))
# alpha sets opacity. With alpha=0.4, 60% of the background shows through
# Enter `print(plt1)` or `plot(plt1)` or simply `plt1` to display the graph
## Panel B: Box plot
gphB <- bwplot(species ~ length, data=cuckoos)
update(c("A: Dotplot"=gphA, "B: Boxplot"=gphB), between=list(x=0.4),
       xlab="Length of egg (mm)")
## latticeExtra::c() joins compatible plots together.
## See `?latticeExtra::c`
```

### Subsection 1.1.3: Planning for a statistical analysis

Understand the data

Causal inference

What was measured? Is it the relevant measure?

Use relevant prior information in the planning stages

Subject area knowledge and judgments

The importance of clear communication

Data-based selection of comparisons



Models must be fit for their intended use

Subsection 1.1.4: Results that withstand thorough and informed challenge

Subsection 1.1.5: Using graphs to make sense of data

Graphical comparisons

Subsection 1.1.6: Formal model-based comparison

```
options(width=70)
cuckoos <- DAAG::cuckoos
av <- with(cuckoos, aggregate(length, list(species=species), FUN=mean))
setNames(round(av[["x"]],2), abbreviate(av[["species"]],10))

with(cuckoos, scale(length[species=="wren"], scale=FALSE))[,1]
```

## Section 1.2: Graphical tools for data exploration

Subsection 1.2.1: Displays of a single variable

```
library(latticeExtra, quietly=TRUE)
fossum <- subset(DAAG::possum, sex=="f")
femlen <- DAAG::bounce(fossum[["totlngth"]], d=0.1)
## Panel A
yaxpos <- c(0,5,10,15,20)/(5*nrow(fossum))
z <- boxplot(list(val = femlen), plot = FALSE)
gph1 <- bwplot(~femlen, ylim=c(0.55,2.75), xlim=c(70,100),
              scales=list(y=list(draw=FALSE)))+
  latticeExtra::layer(panel.rug(x,pch="|"))
legstat <- data.frame(x=c(z$out,z$stats), y=c(1.08, rep(1.3,5)),
  tx=c("Outlier?", "Smallest value", "lower quartile", "median",
    "upper quartile", "Largest value"),
  tx2= c("", "(outliers excepted)",rep("",3), "(no outliers)"))
gphA <- gph1+latticeExtra::layer(data=legstat,
  panel.text(x=x,y=y,labels=tx,adj=c(0,0.4),srt=90, cex=0.85),
  panel.text(x=x[c(2,6)]+0.75,y=c(1.125,1.38),labels=tx2[c(2,6)],
    adj=c(0,0.4),srt=90, cex=0.85))
```

```
## Panel B
gph2 <- densityplot(~femlen, ylim=c(0,0.108), xlim=c(70,100),
  plot.points=TRUE, pch="|", cex=1.75, ylab=c("", "Density"))
gph3 <- histogram(~femlen, ylim=c(0,0.108), type="density",
  scales=list(y=list(at=yaxpos, labels=c(0,5,10,15,20), col="gray40")),
  alpha=0.5, ylab="", breaks=c(75,80,85,90,95,100),
  col='transparent', border='gray40')
gph4 <- doubleYScale(gph2, gph3, use.style=FALSE, add.ylab2=FALSE)
gphB <- update(gph4, par.settings=list(fontsize = list(text=10, points=5)),
  scales=list(tck=c(0.5,0.5)))
update(c("B: Density curve, with histogram overlaid"=gphB,
  "A: Boxplot, with annotation added"=gphA, layout=c(1,2), y.same=F),
  as.table=TRUE, between=list(y=1.4),
  xlab="Total length of female possums (cm)")
```

```
fossum <- subset(DAAG::possum, sex=="f")
densityplot(~totlngth, plot.points=TRUE, pch="|", data=fossum) +
  layer_(panel.histogram(x, type="density", breaks=c(75,80,85,90,95,100)))
```

## Comparing univariate displays across factor levels

```
library(latticeExtra, quietly=TRUE)
fossum <- subset(DAAG::possum, sex=="f")
femlen <- DAAG::bounce(fossum[["totlngth"]], d=0.1)
## Panel A
yaxpos <- c(0,5,10,15,20)/(5*nrow(fossum))
z <- boxplot(list(val = femlen), plot = FALSE)
gph1 <- bwplot(~femlen, ylim=c(0.55,2.75), xlim=c(70,100),
  scales=list(y=list(draw=FALSE)))+
  latticeExtra::layer(panel.rug(x,pch="|"))
legstat <- data.frame(x=c(z$out,z$stats), y=c(1.08, rep(1.3,5)),
  tx=c("Outlier?", "Smallest value", "lower quartile", "median",
    "upper quartile", "Largest value"),
  tx2= c("", "(outliers excepted)", rep("",3), "(no outliers)"))
gphA <- gph1+latticeExtra::layer(data=legstat,
  panel.text(x=x,y=y,labels=tx,adj=c(0,0.4),srt=90, cex=0.85),
  panel.text(x=x[c(2,6)]+0.75,y=c(1.125,1.38),labels=tx2[c(2,6)],
    adj=c(0,0.4),srt=90, cex=0.85))
## Panel B
gph2 <- densityplot(~femlen, ylim=c(0,0.108), xlim=c(70,100),
  plot.points=TRUE, pch="|", cex=1.75, ylab=c("", "Density"))
```

```

gph3 <- histogram(~femlen, ylim=c(0,0.108), type="density",
  scales=list(y=list(at=yaxpos, labels=c(0,5,10,15,20), col="gray40")),
  alpha=0.5, ylab="", breaks=c(75,80,85,90,95,100),
  col='transparent',border='gray40')
gph4 <- doubleYScale(gph2, gph3, use.style=FALSE, add.ylab2=FALSE)
gphB <- update(gph4, par.settings=list(fontsize = list(text=10, points=5)),
  scales=list(tck=c(0.5,0.5)))
update(c("B: Density curve, with histogram overlaid"=gphB,
  "A: Boxplot, with annotation added"=gphA, layout=c(1,2), y.same=F),
  as.table=TRUE, between=list(y=1.4),
  xlab="Total length of female possums (cm)")

```

```

## Create boxplot graph object --- Simplified code
gph <- bwplot(Pop~totlngth | sex, data=possum)
## plot graph, with dotplot distribution of points below boxplots
gph + latticeExtra::layer(panel.dotplot(x, unclass(y)-0.4))

```

### Subsection 1.2.2: Patterns in univariate time series

```

layout(matrix(c(1,2)), heights=c(2.6,1.75))
measles <- DAAG::measles
## Panel A:
par(mgp=c(2.0,0.5,0))
plot(log10(measles), xlab="", ylim=log10 (c(1,5000*540)),
  ylab="Deaths", yaxt="n", fg="gray", adj=0.16)
londonpop <-ts(c(1088, 1258, 1504, 1778, 2073, 2491, 2921, 3336, 3881,
  4266, 4563, 4541, 4498, 4408), start=1801, end=1931, deltat=10)
points(log10(londonpop*500), pch=16, cex=.5)
ytiks1 <- c(1, 10, 100, 1000)
axis(2, at=log10(ytik1), labels=paste(ytik1), lwd=0, lwd.ticks=1)
abline(h=log10(ytik1), col = "lightgray", lwd=2)
par(mgp=c(-2,-0.5,0))
ytiks2 <- c(1000000, 5000000) ## London population in thousands
abline(h=log10(ytik2*0.5), col = "lightgray", lwd=1.5)
abline(v=seq(from=1650,to=1950,by=50), col = "lightgray", lwd = 1.5)
mtext(side=2, line=0.5, "Population", adj=1, cex=1.15, las=3)
axis(2, at=log10(ytik2*0.6), labels=paste(ytik2), tcl=0.3,
  hadj=0, lwd=0, lwd.ticks=1)
mtext(side=3, line=0.3, "A (1629-1939)", adj=0, cex=1.15)
##

```

```
## Panel B: window from 1840 to 1882
par(mgp=c(2.0,0.5,0))
plot(window(measles, start=1840, end=1882), xlab="",
ylab="Deaths    Pop (1000s)", ylim=c(0, 4200), fg="gray")
points(window(londonpop, start=1840, end=1882), pch=16, cex=0.5)
mtext(side=3, line=0.5, "B (1841-1881)", adj=0, cex=1.15)
```

### Subsection 1.2.3: Visualizing relationships between pairs of variables

### Subsection 1.2.4: Response lines (and/or curves)

```
par(pty="s")
plot(distance.traveled ~ starting.point, data=DAAG::modelcars, fg="gray",
xlim=c(0,12.5), xaxs="i", xlab = "Distance up ramp (cm)",
ylab="Distance traveled (cm)")
```

### Subsection 1.2.5: Multiple variables and times

```
## Apply function range to columns of data frame jobs (DAAG)
sapply(DAAG::jobs, range) ## NB: `BC` = British Columbia
```

```
## Panel A: Basic plot; all series in a single panel; use log y-scale
formRegions <- Ontario+Quebec+BC+Alberta+Prairies+Atlantic ~ Date
basicGphA <-
  xyplot(formRegions, outer=FALSE, data=DAAG::jobs, type="l", xlab="",
        ylab="Number of workers", scales=list(y=list(log="e")),
        auto.key=list(space="right", lines=TRUE, points=FALSE))
  ## `outer=FALSE`: plot all columns in one panel
## Panel B: Separate panels (`outer=TRUE`); sliced log scale
basicGphB <-
  xyplot(formRegions, data=DAAG::jobs, outer=TRUE, type="l", layout=c(3,2),
        xlab="", ylab="Number of workers",
        scales=list(y=list(relation="sliced", log=TRUE)))
# Provinces are in order of number of workers in Dec96
## Create improved x- and y-axis tick labels; will update to use
datelabpos <- seq(from=95, by=0.5, length=5)
datelabs <- format(seq(from=as.Date("1Jan1995", format="%d%b%Y"),
        by="6 month", length=5), "%b%y")
```

```

## Now create $y$-labels that have numbers, with log values underneath
ylabposA <- exp(pretty(log(unlist(DAAG::jobs[,-7])), 5))
ylabposA <- paste(round(ylabposA), "\n(", log(ylabposA), ")", sep="")
## Repeat, now with 100 ticks, to cover all 6 slices of the scale
ylabposB <- exp(pretty(log(unlist(DAAG::jobs[,-7])), 100))
ylabposB <- paste(round(ylabposB), "\n(", log(ylabposB), ")", sep="")
gphA <- update(basicGphA, scales=list(x=list(at=datelabpos, labels=datelabs),
                                     y=list(at=ylabposA, labels=ylabposA)))
gphB <- update(basicGphB, xlab="", between=list(x=0.25, y=0.25),
               scales=list(x=list(at=datelabpos, labels=datelabs),
                           y=list(at=ylabposB, labels=ylabposB)))
layout.list <- list(layout.heights=list(top.padding=0,
                                         bottom.padding=0, sub=0, xlab=0),
                    fontsize=list(text=8, points=5))
jobstheme <- modifyList(ggplot2like(pch=1, lty=c(4:6,1:3),
                                     col.line='black', cex=0.75), layout.list)
print(update(gphA, par.settings=jobstheme, axis=axis.grid,
             main=list("A: Same vertical log scale", y=0)),
       position=c(0.1, 0.615, 0.9, 1), newpage=TRUE)
print(update(gphB, par.settings=jobstheme, axis=axis.grid,
             main=list("B: Sliced vertical log scale", y=0)),
       position=c(0, 0, 1, 0.625), newpage=FALSE)

```

```

plot(c(1230, 1860), c(0, 10.5), axes=FALSE, bty="n",
     xlab="", ylab="", type="n", log="x")
xpoints <- c(1366, 1436, 1752, 1840)
axis(1, at=xpoints, labels=FALSE, tck=0.01, lty=1, lwd=0, lwd.ticks=1)
for(i in 1:4){
  axis(1, at=xpoints[i],
       labels=substitute(italic(a), list(a=paste(xpoints[i]))),
       line=-2.25, lty=0, cex=0.8, lwd=0, lwd.ticks=1)
  lines(rep(xpoints[i], 2), c(0, 0.15*par()[["cxy"]][2]), lty=1)
}
axpos <- 1250*cumprod(c(1, rep(1.2, 2)))
axis(1, at=c(axpos, 1840), labels=F, lwd.ticks=0)
lab <- round(axpos)
axis(1, at=axpos, labels=lab)
lab2 <- lapply(round(log2(xpoints), 3), function(x) substitute(2^a, list(a=x)))
axis(1, at=xpoints, labels=as.expression(lab2), line=-3.5, lwd=0)
labe <- lapply(format(round(log(xpoints), 3)), function(x) substitute(e^a, list(a=x)))
axis(1, at=xpoints, labels=as.expression(labe), line=-5, lwd=0)
lab10 <- lapply(round(log10(xpoints), 3), function(x) substitute(10^a, list(a=x)))

```

```

axis(1, at=xpoints, labels=as.expression(lab10), line=-6.5, lwd=0)
par(family="mono", xpd=TRUE)
axis(1, at=1220, labels="log=2", line=-3.5, hadj=0, lwd=0)
axis(1, at=1220, labels='log="e"', line=-5, hadj=0, lwd=0)
axis(1, at=1220, labels="log=10", line=-6.5, hadj=0, lwd=0)
wid2 <- strwidth("log=2")
par(family="sans")

```

### Subsection 1.2.6: \*Labeling technicalities

### Subsection 1.2.7: Graphical displays for categorical data

```

stones <- array(c(81,6,234,36,192,71,55,25), dim=c(2,2,2),
               dimnames=list(Success=c("yes","no"),
                             Method=c("open","ultrasound"), Size=c("<2cm", ">=2cm")))
margin12 <- margin.table(stones, margin=1:2)

```

```

byMethod <- 100*prop.table(margin12, margin=2)
pcGood <- 100*prop.table(stones, margin=2:3)["yes", , ]
dimnam <- dimnames(stones)
numOps <- margin.table(stones, margin=2:3)
opStats <- data.frame(Good=c(pcGood[1,],pcGood[2,]),
                     numOps=c(numOps[1,], numOps[2,]),
                     opType=factor(rep(dimnam[["Method"]],c(2,2))),
                     Size=rep(dimnam[["Size"]],2))
xlim <- range(opStats$Good)*c(0.65,1.015)
ylim <- c(0, max(opStats$numOps)*1.15)
plot(numOps~Good, data=opStats, type="h", lwd=4, xlim=xlim, ylim=ylim,
     fg="gray",col=rep(c("blue","red"),rep(2,2)),
     xlab="Success rate (%)", ylab="Number of operations")
# with(opStats, text(numOps~Good, labels=Size,
#                    col=rep(c('blue','red'),rep(2,2)),
#                    offset=0.25,pos=3, cex=0.75))
labpos <- lapply(split(opStats, opStats$Size),
                 function(x)apply(x[,1:2],2,function(z)c(z[1],mean(z),z[2])))
sizeNam <- names(labpos)
lapply(labpos, function(x)lines(x[, 'Good'],x[, 'numOps']+c(0,35,0),
                              type="c",col="gray"))
txtmid <- sapply(labpos, function(x)c(x[2, 'Good'],x[2, 'numOps']+35))
text(txtmid[1,]+c(-1.4,0.85),txtmid[2,],labels=sizeNam,col="gray40",

```

```

    pos=c(4,2), offset=0)
par(xpd=TRUE)
text(byMethod[1,1:2],rep(par()$usr[4],2)+0.5*strheight("^"), labels=c("^","^"),
     col=c("blue","red"),cex=1.2,srt=180)
text(byMethod[1,], par()$usr[4]+1.4*strheight("A"),
     labels=paste(round(byMethod[1,],1)),cex=0.85)
text(byMethod[1,1:2]+c(3.5,-3.5), rep(par()$usr[4],2)+2.65*strheight("A"),
     labels=c("All open","All ultrasound"), pos=c(2,4))
par(xpd=FALSE)
abline(h=100*(0:2),col="lightgray",lwd=0.5)
abline(v=10*(5:9),col="lightgray",lwd=0.5)
legend("topleft", col=c('blue','red'),lty=c(1,1), lwd=1, cex=0.9,
      y.intersp=0.75, legend=c("Open","Ultrasound"),bty="n",
      inset=c(-0.01,-0.01))

```

### Subsection 1.2.8: What to look for in plots

Outliers

Asymmetry of the distribution

Changes in variability

Clustering

Nonlinearity

Time trends in the data

## Section 1.3: Data Summary

### Subsection 1.3.1: Counts

```
## Table of counts example: data frame nswpsid1 (DAAG)
## Specify `useNA="ifany"` to ensure that any NAs are tabulated
tab <- with(DAAG::nswpsid1, table(trt, nodeg, useNA="ifany"))
dimnames(tab) <- list(trt=c("none", "training"), educ = c("completed", "dropout"))
tab
```

### Tabulation that accounts for frequencies or weights – the xtabs() function

```
gph <- lattice::bwplot(log(nassCDS$weight+1), xlab="Inverse sampling weights",
  scales=list(x=list(at=c(0,log(c(10^(0:5)+1))), labels=c(0,10^(0:5))))
update(gph, par.settings=DAAG::DAAGtheme(color=F, col.points='gray50'))
```

```
sampNum <- table(nassCDS$dead)
popNum <- as.vector(xtabs(weight ~ dead, data=nassCDS))
rbind(Sample=sampNum, "Total number"=round(popNum,1))
```

```
nassCDS <- DAAG::nassCDS
Atab <- xtabs(weight ~ airbag + dead, data=nassCDS)/1000
## Define a function that calculates Deaths per 1000
DeadPer1000 <- function(x)1000*x[2]/sum(x)
Atabm <- ftable(addmargins(Atab, margin=2, FUN=DeadPer1000))
print(Atabm, digits=2, method="compact", big.mark=",")
```

```
SAtab <- xtabs(weight ~ seatbelt + airbag + dead, data=nassCDS)
## SAtab <- addmargins(SAtab, margin=3, FUN=list(Total=sum)) ## Gdet Totals
SAtabf <- ftable(addmargins(SAtab, margin=3, FUN=DeadPer1000), col.vars=3)
print(SAtabf, digits=2, method="compact", big.mark=",")
```

```
FSAtab <- xtabs(weight ~ dvcat + seatbelt + airbag + dead, data=nassCDS)
FSAtabf <- ftable(addmargins(FSAtab, margin=4, FUN=DeadPer1000), col.vars=3:4)
print(FSAtabf, digits=1)
```

### Subsection 1.3.2: Summaries of information from data frames

```
## Individual vine yields, with means by block and treatment overlaid
kiwishade <- DAAG::kiwishade
kiwishade$block <- factor(kiwishade$block, levels=c("west","north","east"))
keyset <- list(space="top", columns=2,
```



```

text=list(c("Individual vine yields", "Plot means (4 vines)")),
points=list(pch=c(1,3), cex=c(1,1.35), col=c("gray40","black")))
panelfun <- function(x,y,...){panel.dotplot(x,y, pch=1, ...)
av <- sapply(split(x,y),mean); ypos <- unique(y)
lpoints(ypos~av, pch=3, col="black")}
dotplot(shade~yield | block, data=kiwishade, col="gray40", aspect=0.65,
        panel=panelfun, key=keyset, layout=c(3,1))
## Note that parameter settings were given both in the calls
## to the panel functions and in the list supplied to key.

```

```

## mean yield by block by shade: data frame kiwishade (DAAG)
kiwimeans <- with(DAAG::kiwishade,
                 aggregate(yield, by=list(block, shade), mean))
names(kiwimeans) <- c("block","shade","meanyield")
head(kiwimeans, 4) # First 4 rows

```

## The benefits of data summary – dengue status example

### Subsection 1.3.3: Measures of variation

#### Cuckoo eggs example

```

options(width=72)
## SD of length, by species: data frame cuckoos (DAAG)
z <- with(cuckoos, sapply(split(length,species), function(x)c(sd(x),length(x))))
print(setNames(paste0(round(z[1,],2)," (" ,z[2,]," )"),
              abbreviate(colnames(z),11)), quote=FALSE)

```

## Degrees of freedom

### Subsection 1.3.4: Inter-quartile range (IQR) and median absolute deviation (MAD)

### Subsection 1.3.5: A pooled standard deviation estimate

#### Elastic bands example

```

<>= sapply(DAAG::two65, function(x) c(Mean=mean(x), sd=sd(x))) |> round(2) @

```

### Subsection 1.3.6: Effect size

```
setNames(diff(c(ambient=244.1, heated=253.5))/c(sd=10.91), "Effect size")
```

Data are available in the data frame DAAG::two65.

```
vignette('effectsize', package='effectsize')
```

### Subsection 1.3.7: Correlation

```
set.seed(17)
x1 <- x2 <- x3 <- (11:30)/5
y1 <- x1 + rnorm(20, sd=0.5)
y2 <- 2 - 0.05 * x1 + 0.1 * ((x1 - 1.75))^4 + rnorm(20, sd=1.5)
y3 <- (x1 - 3.85)^2 + 0.015 + rnorm(20)
theta <- ((2 * pi) * (1:20))/20
x4 <- 10 + 4 * cos(theta)
y4 <- 10 + 4 * sin(theta) + rnorm(20, sd=0.6)
xy <- data.frame(x = c(rep(x1, 3), x4), y = c(y1, y2, y3, y4),
                 gp = factor(rep(1:4, rep(20, 4))))
xysplit <- split(xy, xy$gp)
rho <- sapply(xysplit, function(z)with(z,cor(x,y, method=c("pearson"))))
rhoS <- sapply(xysplit, function(z)with(z,cor(x,y, method=c("spearman"))))
rnam <- as.list(setNames(round(c(rho,rhoS),2), paste0("r",1:8)))
striplabs <- bquote(expression(paste(r==.(r1), "    ",r[s]==.(r5)),
                                paste(r==.(r2), "    ",r[s]==.(r6)),
                                paste(r==.(r3), "    ",r[s]==.(r7)),
                                paste(r==.(r4), "    ",r[s]==.(r8))), rnam)
xyplot(y ~ x | gp, data=xy, layout=c(4,1), xlab="", ylab="",
      strip=strip.custom(factor.levels=striplabs), aspect=1,
      scales=list(relation='free', draw=FALSE), between=list(x=0.5,y=0)
)
```

## Section 1.4: Distributions: quantifying uncertainty

### Subsection 1.4.1: Discrete distributions

```
## dbinom(0:10, size=10, prob=0.15)
setNames(round(dbinom(0:10, size=10, prob=0.15), 3), 0:10)
```

```
pbinom(q=4, size=10, prob=0.15)
```

```
qbinom(p = 0.70, size = 10, prob = 0.15)
## Check that this lies between the two cumulative probabilities:
## pbinom(q = 1:2, size=10, prob=0.15)
```

```
rbinom(15, size=4, p=0.5)
```

```
## dpois(x = 0:8, lambda = 3)
setNames(round(dpois(x = 0:8, lambda = 3),4), 0:8)
## Probability of > 8 raisins
## 1-ppois(q = 8, lambda = 3)      ## Or, ppois(q=8, lambda=3, lower.tail=FALSE)
```

```
1 - ppois(q = 8, lambda = 3)
ppois(q=8, lambda=3, lower.tail=FALSE) ## Alternative
1-sum(dpois(x = 0:8, lambda = 3))     ## Another alternative
```

```
raisins <- rpois(20, 3)
raisins
```

## Initializing the random number generator

```
set.seed(23286) # Use to reproduce the sample below
rbinom(15, size=1, p=0.5)
```

## Means and variances

### Subsection 1.4.2: Continuous distributions

```
z <- seq(-3,3,length=101)
plot(z, dnorm(z), type="l", ylab="Normal density",
     yaxs="i", bty="L", tcl=-0.3, fg="gray",
     xlab="Distance, in SDs, from mean", cex.lab=0.9)
```

```

polygon(c(z[z <= 1.0],1.0),c(dnorm(z[z <= 1.0]), dnorm(-3)), col="grey")
chh <- par()$cxy[2]
arrows(-1.8, 0.32, -0.25, 0.2, length=0.07, xpd=T)
cump <- round(pnorm(1), 3)
text(-1.8, 0.32+0.75*chh, paste("pnorm(1)\n", "=", cump), xpd=T, cex=0.8)

```

```

pnormExs <- c('pnorm(0)', 'pnorm(1)', 'pnorm(-1.96)', 'pnorm(1.96)',
'pnorm(1.96, mean=2)', 'pnorm(1.96, sd=2)')
Prob <- sapply(pnormExs, function(x)eval(parse(text=x)))
df <- as.data.frame(Prob)
df$Prob <- round(df$Prob,3)
print(df)

```

```

## Plot the normal density, in the range -3 to 3
z <- pretty(c(-3,3), 30) # Find ~30 equally spaced points
ht <- dnorm(z) # Equivalent to dnorm(z, mean=0, sd=1)
plot(z, ht, type="l", xlab="Normal variate", ylab="Density", yaxs="i")
# yaxs="i" locates the axes at the limits of the data

```

```

qnorm(.9) # 90th percentile; mean=0 and SD=1

```

```

## Additional examples:
setNames(qnorm(c(.5,.841,.975)), nm=c(.5,.841,.975))
qnorm(c(.1,.2,.3)) # -1.282 -0.842 -0.524 (10th, 20th and 30th percentiles)
qnorm(.1, mean=100, sd=10) # 87.2 (10th percentile, mean=100, SD=10)

```

## Different ways to represent distributions

### Generating simulated samples from the normal and other continuous distributions

```

options(digits=2) # Suggest number of digits to display
rnorm(10) # 10 random values from the normal distribution

```

```

mu <- 10
sigma <- 1
n <- 1
m <- 50
four <- 4
nrep <- 5

```

```

seed <- 21
totrows <- 1
if(is.null(totrows))
totrows <- floor(sqrt(nrep))
totcols <- ceiling(nrep/totrows)
z <- range(pretty(mu + (c(-3.4, 3.4) * sigma), 50))
xy <- data.frame(x=rep(0,nrep),y=rep(0,nrep),n=rep(n,nrep),
                 mm=rep(m,nrep),four=rep(four,nrep))
fac <- factor(paste("Simulation", 1:nrep),
              lev <- paste("Simulation", 1:nrep))
xlim<-z
## ylim<-c(0,dnorm(0)*sqrt(n))
ylim <- c(0,1)
xy <- split(xy,fac)
xy<-lapply(1:length(xy),function(i){c(as.list(xy[[i]]), list(xlim=xlim,
                    ylim=ylim))})
panel.mean <- function(data, mu = 10, sigma = 1, n2 = 1,
                        mm = 100, nrows, ncols, ...)
{
  vline <- function(x, y, lty = 1, col = 1)
  lines(c(x, x), c(0, y), lty = lty, col = col)
  n2<-data$n[1]
  mm<-data$mm[1]
  our<-data$four[1] ## Four characters in each unit interval of x
  nmid <- round(four * 4)
  nn <- array(0, 2 * nmid + 1)
  #####
  z <- mu+seq(from=-3.4*sigma, to=3.4*sigma, length=mm)
  atx<-pretty(z)
  qz <- pnorm((z - mu)/sigma)
  dz <- dnorm((z - mu)/sigma)
  chw <- sigma/four
  chh <- strheight("0")*0.75
  htfac <- (mm * chh)/four
  if(nrows==1&&ncols==1)
  lines(z, dz * htfac)
  if(nrows==1)axis(1,at=atx, lwd=0, lwd.ticks=1)
  y <- rnorm(mm, mu, sigma/sqrt(n2))
  pos <- round((y - mu)/sigma * four)
  for(i in 1:mm) {
    nn[nmid + pos[i]] <- nn[nmid + pos[i]] + 1
    xpos <- chw * pos[i]
  }
}

```

```

    text(mu + xpos, nn[nmid + pos[i]] * chh - chh/4, "x")
  }
}
DAAG::panelplot(xy, panel=panel.mean, totrows=totrows, totcols=totcols,
  oma=c(1.5, 0, rep(0.5,2)), fg='gray')

```

```

## The following gives a conventional histogram representations:
set.seed(21)          # Use to reproduce the data in the figure
df <- data.frame(x=rnorm(250), gp=rep(1:5, rep(50,5)))
lattice::histogram(~x|gp, data=df, layout=c(5,1))

```

```

runif(n = 20, min=0, max=1) # 20 numbers, uniform distn on (0, 1)
rexp(n=10, rate=3)          # 10 numbers, exponential, mean 1/3.

```

### Subsection 1.4.3: Graphical checks for normality

```

tab <- t(as.matrix(DAAG::pair65))
rbind(tab, "heated-ambient"=tab[1,]-tab[2,])

```

```

## Normal quantile-quantile plot for heated-ambient differences,
## compared with plots for random normal samples of the same size
plt <- with(DAAG::pair65, DAAG::qreference(heated-ambient, nrep=10, nrows=2))
update(plt, scales=list(tck=0.4), xlab="")

```

### Subsection 1.4.4: Population parameters and sample statistics

#### The sampling distribution of the mean

```

library(lattice)
## Generate n sample values; skew population
sampfun = function(n) exp(rnorm(n, mean = 0.5, sd = 0.3))
gph <- DAAG::sampdist(sampsize = c(3, 9, 30), seed = 23, nsamp = 1000,
  FUN = mean, sampvals=sampfun, plot.type = "density")
sampheme <- DAAG::DAAGtheme(color=FALSE)
print(update(gph, scales=list(tck=0.4), layout = c(3,1),
  par.settings=sampheme, main=list("A: Density curves", cex=1.25)),
  position=c(0,0.5,1,1), more=TRUE)
sampfun = function(n) exp(rnorm(n, mean = 0.5, sd = 0.3))

```

```

gph <- DAAG::sampdist(sampsize = c(3, 9, 30), seed = 23, nsamp = 1000,
                      FUN = mean, sampvals=sampfun, plot.type = "qq")
print(update(gph, scales=list(tck=0.4), layout = c(3,1),
             par.settings=samptHEME,
             main=list("B: Normal quantile-quantile plots", cex=1.25)),
       position=c(0,0,1,0.5))

```

**The standard error of the median**

**Simulation in learning and research**

**Subsection 1.4.5: The  $t$ -distribution**

```

x <- seq(from=-4.2, to = 4.2, length.out = 50)
ylim <- c(0, dnorm(0))
ylim[2] <- ylim[2]+0.1*diff(ylim)
h1 <- dnorm(x)
h3 <- dt(x, 3)
h8 <- dt(x,8)
plot(x, h1, type="l", xlab = "", xaxs="i", ylab = "", yaxs="i",
     bty="L", ylim=ylim, fg="gray")
mtext(side=3,line=0.5, "A: Normal (t8 overlaid)", adj=-0.2)
lines(x, h8, col="grey60")
mtext(side=1, line=1.75, "No. of SEMs from mean")
mtext(side=2, line=2.0, "Probability density")
chh <- par()$cxy[2]
topleft <- par()$usr[c(1,4)] + c(0, 0.6*chh)
legend(topleft[1], topleft[2], col=c("black","grey60"),
      lty=c(1,1), legend=c("Normal","t (8 d.f.)"), bty="n", cex=0.8)
plot(x, h1, type="l", xlab = "", xaxs="i",
     ylab = "", yaxs="i", bty="L", ylim=ylim, fg="gray")
mtext(side=3,line=0.5, "B: Normal (t3 overlaid)", adj=-0.2)
lines(x, h3, col="grey60")
mtext(side=1, line=1.75, "No. of SEMs from mean")
## mtext(side=2, line=2.0, "Probability density")
legend(topleft[1], topleft[2], col=c("black","grey60"),
      lty=c(1,1), legend=c("Normal","t (3 d.f.)"), bty="n", cex=0.8)
## Panels C and D
cump <- 0.975

```

```

x <- seq(from=-3.9, to = 3.9, length.out = 50)
ylim <- c(0, dnorm(0))
plotfun <- function(cump, dfun = dnorm, qfun=qnorm,
ytxt = "Probability density",
txt1="qnorm", txt2="", ...)
{
h <- dfun(x)
plot(x, h, type="l", xlab = "", xaxs="i", xaxt="n",
ylab = ytxt, yaxs="i", bty="L", ylim=ylim, fg="gray",
...)
axis(1, at=c(-2, 0), cex=0.8, lwd=0, lwd.ticks=1)
axis(1, at=c((-3):3), labels=F, lwd=0, lwd.ticks=1)
tailp <- 1-cump
z <- qfun(cump)
ztail <- pretty(c(z,4),20)
htail <- dfun(ztail)
polygon(c(z,z,ztail,max(ztail)), c(0,dfun(z),htail,0), col="gray")
text(0, 0.5*dfun(z)+0.08*dfun(0),
paste(round(tailp, 3), " + ", round(1-2*tailp,3),
"\n= ", round(cump, 3), sep=""), cex=0.8)
lines(rep(z, 2), c(0, dfun(z)))
lines(rep(-z, 2), c(0, dfun(z)), col="gray60")
chh <- par()$cxy[2]
arrows(z, -1.5*chh,z,-0.1*chh, length=0.1, xpd=T)
text(z, -2.5*chh, paste(txt1, "(", cump, txt2, ")"), "\n= ",
round(z,2), sep=""), xpd=T)
x1 <- z + .3
y1 <- dfun(x1)*0.35
y0 <- dfun(0)*0.2
arrows(-2.75, y0, -x1, y1, length=0.1, col="gray60")
arrows(2.75, y0, x1, y1, length=0.1)
text(-2.75, y0+0.5*chh, tailp, col="gray60")
text(2.75, y0+0.5*chh, tailp)
}
## ytxt <- "t probability density (8 d.f.)"
plotfun(cump=cump, cex.lab=1.05)
mtext(side=3, line=1.25, "C: Normal distribution", adj=-0.2)
ytxt <- "t probability density (8 d.f.)"
plotfun(cump=cump, dfun=function(x)dt(x, 8),
qfun=function(x)qt(x, 8),
ytxt="", txt1="qt", txt2="", 8", cex.lab=1.05)
mtext(side=3, line=1.25, "D: t distribution (8 df)", adj=-0.2)

```



```
qnorm(c(0.975,0.995), mean=0)    # normal distribution
qt(c(0.975, 0.995), df=8)        # t-distribution with 8 d.f.
```

#### Subsection 1.4.6: The likelihood, and maximum likelihood estimation

### Section 1.5: Simple forms of regression model

#### Subsection 1.5.1: Line or curve?

```
roller <- DAAG::roller
t(cbind(roller, "depression/weight ratio"=round(roller[,2]/roller[,1],2)))
```

#### Using models to predict

```
y <- DAAG::roller$depression
x <- DAAG::roller$weight
pretext <- c(reg = "A", lo = "B")
for(curve in c("reg", "lo")) {
  plot(x, y, xlab = "Roller weight (t)", xlim=c(0,12.75), fg="gray",
       ylab = "Depression in lawn (mm)", type="n")
  points(x, y, cex=0.8, pch = 4)
  mtext(side = 3, line = 0.25, pretext[curve], adj = 0)
  topleft <- par()$usr[c(1, 4)]
  chw <- strwidth("0"); chh <- strheight("0")
  points(topleft[1]+rep(0.75,2)*chw,topleft[2]-c(0.75,1.8)*chh,
        pch=c(4,1), col=c("black","gray40"), cex=0.8)
  text(topleft[1]+rep(1.2,2)*chw, topleft[2]-c(0.75,1.8)*chh,
       c("Data values", "Fitted values"),adj=0, cex=0.8)
  if(curve=="lo")
    text(topleft[1]+1.2*chw, topleft[2]-2.85*chh,"(smooth)", adj=0, cex=0.8)
  if(curve[1] == "reg") {
    u <- lm(y ~ -1 + x)
    abline(0, u$coef[1])
    yhat <- predict(u)
  }
  else {
    lawn.lm<-lm(y~x+I(x^2))
    yhat<-predict(lawn.lm)
    xnew<-pretty(x,20)
```

```

b<-lawn.lm$coef
ynew<-b[1]+b[2]*xnew+b[3]*xnew^2
lines(xnew,ynew)
}
here <- y < yhat
yyhat <- as.vector(rbind(y[here], yhat[here], rep(NA, sum(here))))
xx <- as.vector(rbind(x[here], x[here], rep(NA, sum(here))))
lines(xx, yyhat, lty = 2, col="gray")
here <- y > yhat
yyhat <- as.vector(rbind(y[here], yhat[here], rep(NA, sum(here))))
xx <- as.vector(rbind(x[here], x[here], rep(NA, sum(here))))
lines(xx, yyhat, lty = 1, col="gray")
n <- length(y)
ns <- min((1:n)[y - yhat >= 0.75*max(y - yhat)])
ypos <- 0.5 * (y[ns] + yhat[ns])
chw <- par()$cxy[1]
text(x[ns] - 0.25*chw, ypos, "+ve residual", adj = 1,cex=0.75, col="gray30")
points(x, yhat, pch = 1, col="gray40")
ns <- (1:n)[y - yhat == min(y - yhat)][1]
ypos <- 0.5 * (y[ns] + yhat[ns])
text(x[ns] + 0.4*chw, ypos, "-ve residual", adj = 0,cex=0.75,col="gray30")
}

```

**Which model is best — line or curve?**

### Subsection 1.5.2: Fitting models – the model formula

```

## Fit line - by default, this fits intercept & slope.
roller.lm <- lm(depression ~ weight, data=DAAG::roller)
## Compare with the code used to plot the data
plot(depression ~ weight, data=DAAG::roller)
## Add the fitted line to the plot
abline(roller.lm)

```

```

## For a model that omits the intercept term, specify
lm(depression ~ 0 + weight, data=roller) # Or, if preferred, replace `0` by `-1`

```

### Model objects

```
roller.lm <- lm(depression ~ weight, data=DAAG::roller)
names(roller.lm)      # Get names of list elements
```

```
coef(roller.lm)        # Extract coefficients
summary(roller.lm)     # Extract model summary information
coef(summary(roller.lm)) # Extract coefficients and SEs
fitted(roller.lm)      # Extract fitted values
predict(roller.lm)     # Predictions for existing or new data, with SE
                        # or confidence interval information if required.
resid(roller.lm)       # Extract residuals
```

```
roller.lm$coef          # An alternative is roller.lm[["coef"]]
```

```
print(summary(roller.lm), digits=3)
```

## Residual plots

```
## Normal quantile-quantile plot, plus 7 reference plots
DAAG::qqreference(residuals(roller.lm), nrep=8, nrows=2, xlab="")
```

## Simulation of regression data

```
roller.lm <- lm(depression ~ weight, data=DAAG::roller)
roller.sim <- simulate(roller.lm, nsim=20) # 20 simulations

with(DAAG::roller, matplot(weight, roller.sim, pch=1, ylim=range(depression)))
points(DAAG::roller, pch=16)
```

## Subsection 1.5.3: The model matrix in regression

```
model.matrix(roller.lm)
## Specify coef(roller.lm) to obtain the column multipliers.
```

## From straight line regression to multiple regression

```
mouse.lm <- lm(brainwt ~ lsize+bodywt, data=DAAG::litters)
coef(summary(mouse.lm))
```

## Section 1.6: Data-based judgments – frequentist, in a Bayesian world

### Subsection 1.6.1: Inference with known prior probabilities

```
## `before` is the `prevalence` or `prior`.
after <- function(prevalence, sens, spec){
  prPos <- sens*prevalence + (1-spec)*(1-prevalence)
  sens*prevalence/prPos}
## Compare posterior for a prior of 0.002 with those for 0.02 and 0.2
setNames(round(after(prevalence=c(0.002, 0.02, 0.2), sens=.8, spec=.95), 3),
  c("Prevalence=0.002", "Prevalence=0.02", "Prevalence=0.2"))
```

### Relating ‘incriminating’ evidence to the probability of guilt

### Subsection 1.6.2: Treatment differences that are on a continuous scale

```
## Use pipe syntax, introduced in R 4.1.0
sleep <- with(datasets::sleep, extra[group==2] - extra[group==1])
sleep |> (function(x)c(mean = mean(x), SD = sd(x), n=length(x)))( ) |>
  (function(x)c(x, SEM=x['SD']/sqrt(x['n'])))( ) |>
  setNames(c("mean","SD","n","SEM")) -> stats
print(stats, digits=3)
```

```
## Sum of tail probabilities
2*pt(1.580/0.389, 9, lower.tail=FALSE)
```

```
## 95% CI for mean of heated-ambient: data frame DAAG::pair65
t.test(sleep, conf.level=0.95)
```

### An hypothesis test

```
pt(4.06, 9, lower.tail=F)
```

The  $p$ -value probability relates to a decision process

### Subsection 1.6.3: Use of simulation with $p$ -values

```
eff2stat <- function(eff=c(.2,.4,.8,1.2), n=c(10,40), numreps=100,
                    FUN=function(x,N)pt(sqrt(N)*mean(x)/sd(x), df=N-1,
                                   lower.tail=FALSE)){
  simStat <- function(eff=c(.2,.4,.8,1.2), N=10, nrep=100, FUN){
    num <- N*nrep*length(eff)
    array(rnorm(num, mean=eff), dim=c(length(eff),nrep,N)) |>
      apply(2:1, FUN, N=N)
  }
  mat <- matrix(nrow=numreps*length(eff),ncol=length(n))
  for(j in 1:length(n)) mat[,j] <-
    as.vector(simStat(eff, N=n[j], numreps, FUN=FUN)) ## length(eff)*numrep
  data.frame(effsize=rep(rep(eff, each=numreps), length(n)),
             N=rep(n, each=numreps*length(eff)), stat=as.vector(mat))
}
```

```
set.seed(31)
df200 <- eff2stat(eff=c(.2,.4,.8,1.2), n=c(10, 40), numreps=200)
labx <- c(0.001,0.01,0.05,0.2,0.4,0.8)
gph <- bwplot(factor(effsize) ~ I(stat^0.25) | factor(N), data=df200,
              layout=c(2,1), xlab="P-value", ylab="Effect size",
              scales=list(x=list(at=labx^0.25, labels =labx)))
update(gph+latticeExtra::layer(panel.abline(v=labx[1:3]^0.25, col='lightgray')),
       strip=strip.custom(factor.levels=paste0("n=",c(10,40))),
       par.settings=DAAG::DAAGtheme(color=F, col.points="gray50"))
```

```
eff10 <- with(subset(df200, N==10&effsize==0.2), c(gt5pc=sum(stat>0.05), lohi=fivenum(stat)[
eff40 <- with(subset(df200, N==40&effsize==0.2), c(gt5pc=sum(stat>0.05), lohi=fivenum(stat)[
```

### Subsection 1.6.4: Power — minimizing the chance of false positives

```
tf1 <- rbind('R=0.2'=c(0.8*50, 0.05*250),
            'R=1'=c(0.8*150, 0.05*150),
            'R=5'=c(0.8*200, 0.05*50))
tf2 <- rbind(c('0.8 x50', '0.05x250'),
```

```

c('0.8x150', '0.05x150'),
c('0.8x250', '0.05 x50'))
tf <- cbind("True positives"=paste(tf2[,2],tf1[,2],sep="="),
"False positives"=paste(tf2[,1],tf1[,1],sep="="))
rownames(tf) <- rownames(tf1)
print(tf, quote=FALSE)

```

## Power calculations – examples

```

power.t.test(d=0.5, sig.level=0.05, type="one.sample", power=0.8)
pwr1 <- power.t.test(d=0.5, sig.level=0.005, type="one.sample", power=0.8)
pwr2 <- power.t.test(d=0.5, sig.level=0.005, type="two.sample", power=0.8)
## d=0.5, sig.level=0.005, One- and two-sample numbers
c("One sample"=pwr1$n, "Two sample"=pwr2$n)

```

```

effsize <- c(.05,.2,.4,.8,1.2); npairs <- c(10,20,40)
pwr0.05 <- matrix(nrow=length(effsize), ncol=length(npairs),
                  dimnames=list(paste0('ES=',effsize), paste0('n=',npairs)))
pwr0.005 <- matrix(nrow=length(effsize), ncol=length(npairs),
                  dimnames=list(paste0(effsize), paste0('n=',npairs)))
for(i in 1:length(effsize)) for(j in 1:length(npairs)){
  pwr0.05[i,j] <- power.t.test(n=npairs[j],d=effsize[i],sig.level=0.05,
                              type='one.sample')$power
  pwr0.005[i,j] <- power.t.test(n=npairs[j],d=effsize[i],sig.level=0.005,
                              type='one.sample')$power}
tab <- cbind(round(pwr0.05,4), round(pwr0.005,4))
tab[1:3,] <- round(tab[1:3,],3)
tab[5,3] <- '~1.0000'
tab[5,6] <- '~1.0000'

```

```
print(tab[,1:3], quote=F)
```

```
print(tab[,4:6], quote=F)
```

```

effsize <- c(.05,.2,.4,.8,1.2); npairs <- c(10,20,40)
pwr0.05 <- matrix(nrow=length(effsize), ncol=length(npairs),
                  dimnames=list(paste0('ES=',effsize), paste0('n=',npairs)))
pwr0.005 <- matrix(nrow=length(effsize), ncol=length(npairs),
                  dimnames=list(paste0(effsize), paste0('n=',npairs)))
for(i in 1:length(effsize)) for(j in 1:length(npairs)){

```

```

pwr0.05[i,j] <- power.t.test(n=npairs[j],d=effsize[i],sig.level=0.05,
                             type='one.sample')$power
pwr0.005[i,j] <- power.t.test(n=npairs[j],d=effsize[i],sig.level=0.005,
                              type='one.sample')$power}
tab <- cbind(round(pwr0.05,4), round(pwr0.005,4))
tab[1:3,] <- round(tab[1:3,],3)
tab[5,3] <- '~1.0000'
tab[5,6] <- '~1.0000'

```

## Positive Predictive Values

```

R <- pretty(0:3, 40)
postOdds <- outer(R/0.05,c(.8,.3,.08))
PPV <- as.data.frame(cbind(R,postOdds/(1+postOdds)))
names(PPV) <- c("R","p80","p30","p8")
key <- list(text = list(text=c("80% power","30% power", "8% power"), cex = 1.0),
            x = .6, y = .25, color=F)
gph <- lattice::xyplot(p80+p30+p8~R, data=PPV, lwd=2, type=c("l","g"),
                      xlab="Pre-study odds R", ylab="Post-study probability (PPV)")
update(gph, scales=list(tck=0.5), key=key)

```

### Subsection 1.6.5: The future for $p$ -values

How small a  $p$ -value is needed?

### Subsection 1.6.6: Reporting results

Is there an alternative that is more likely?

## Section 1.7: Information statistics and Bayesian methods with Bayes Factors

### Subsection 1.7.1: Information statistics – using likelihoods for model choice

```

## Calculations using mouse brain weight data
mouse.lm <- lm(brainwt ~ lsize+bodywt, data=DAAG::litters)
n <- nrow(DAAG::litters)
RSSlogLik <- with(mouse.lm, n*(log(sum(residuals^2)/n)+1+log(2*pi)))
p <- length(coef(mouse.lm))+1 # NB: p=4 (3 coefficients + 1 scale parameter)

```

```
k <- 2*n/(n-p-1)
c("AICc" = AICcmmodavg::AICc(mouse.lm), fromlogL=k*p-2*logLik(mouse.lm)[1],
  fromFit=k*p + RSSlogLik) |> print(digits=4)
```

## The sampling properties of the difference in AIC statistics

```
sim0vs1 <- function(mu=0, n=15, ntimes=200){
  a0 <- a1 <- numeric(ntimes)
  for(i in 1:ntimes){
    y <- rnorm(n, mean=mu, sd=1)
    m0 <- lm(y ~ 0); m1 <- lm(y ~ 1)
    a0[i] <- AIC(m0); a1[i] <- AIC(m1)
  }
  data.frame(a0=a0, a1=a1, diff01=a0-a1, mu=rep(paste0("mu=",mu)))
}
library(latticeExtra)
sim0 <- sim0vs1(mu=0)
sim0.5 <- sim0vs1(mu=0.5)
simboth <- rbind(sim0, sim0.5)
cdiff <- with(list(n=15, p=2), 2*(p+1)*p/(n-(p+1)-1))
xyplot(diff01 ~ a0 | mu, data=simboth, xlab="AIC(m0)", ylab="AIC(m0) - AIC(m1)") +
  latticeExtra::layer({panel.abline(h=0, col='red');
    panel.abline(h=cdiff, lwd=1.5, lty=3, col='red', alpha=0.5);
    panel.abline(h=-2, lty=2, col='red')})

tab <- rbind(c(with(sim0, sum(diff01>0))/200, with(sim0.5, sum(diff01>0))/200),
  c(with(sim0, sum(diff01>-cdiff))/200, with(sim0.5, sum(diff01>-cdiff))/200))
dimnames(tab) <- list(c("AIC: Proportion choosing m1",
  "AICc: Proportion choosing m1"),
  c("True model is m0", "True model is m1"))
tab
```

## Subsection 1.7.2: Bayesian methods with Bayes Factors

```
## Setting `scale=1/sqrt(2)` gives a mildly narrower distribution
print(c("pcauchy(1, scale=1)"=pcauchy(1, scale=1),
  "pcauchy(1, scale=1/sqrt(2))"=pcauchy(1, scale=1/sqrt(2))),
  quote=FALSE)
```



## The Cauchy prior with different choices of scale parameter

```
x <- seq(from=-4.5, to=4.5, by=0.1)
densMed <- dcauchy(x,scale=sqrt(2)/2)
densUltra <- dcauchy(x, scale=sqrt(2))
denn <- dnorm(x, sd=1)
plot(x,densMed, type='l', mgp=c(2,0.5,0), xlab="",
      ylab="Prior density", col="red", fg='gray')
mtext(side=1, line=2, expression("Effect size "=="phantom(0)*delta), cex=1.1)
lines(x, denn, col="blue", lty=2)
lines(x, densUltra,col=2, lty=2)
legend("topleft", title="Normal prior",
      y.intersp=0.8, lty=2, col="blue", bty='n', cex=0.8,
      legend=expression(bold('sd=1')))
legend("topright", title="Cauchy priors", y.intersp=0.8,
      col=c('red', 'red'),lty=c(1,2), cex=0.8,
      legend=c(expression(bold('medium')),
        expression(bold('ultrawide'))),bty="n")
mtext(side=3, line=0.25, adj=0, cex=1.15,
      expression("A: Alternative priors for "*delta=="frac(mu,sigma)))
## Panel B
pairedDiffs <- with(datasets::sleep, extra[group==2] - extra[group==1])
ttBF0 <- BayesFactor::ttestBF(pairedDiffs)
simpост <- BayesFactor::posterior(ttBF0, iterations=10000)
plot(density(simpост[, 'mu']), main="", xlab="", col="red",
      mgp=c(2,0.5,0), ylab="Posterior density", fg='gray')
mtext(side=1, line=2, expression(mu), cex=1.1)
abline(v=mean(pairedDiffs), col="gray")
mtext(side=3, line=0.5, expression("B: Posterior density for "*mu), adj=0, cex=1.15)

## Calculate and plot density for default prior - Selected lines of code
x <- seq(from=-4.5, to=4.5, by=0.1)
densMed <- dcauchy(x, scale=sqrt(2)/2)
plot(x, densMed, type='l')
## Panel B
pairedDiffs <- with(datasets::sleep, extra[group==2] - extra[group==1])
ttBF0 <- BayesFactor::ttestBF(pairedDiffs)
## Sample from posterior, and show density plot for mu
simpост <- BayesFactor::posterior(ttBF0, iterations=10000)
plot(density(simpост[, 'mu']))
```

## A thought experiment

```
tval <- setNames(qt(1-c(.05,.01,.005)/2, df=19), paste(c(.05,.01,.005)))
bf01 <- setNames(numeric(3), paste(c(.05,.01,.005)))
for(i in 1:3)bf01[i] <- BayesFactor::ttest.tstat(tval[i],n1=20, simple=T)
```

```
pairedDiffs <- with(datasets::sleep, extra[group==2] - extra[group==1])
ttBF0 <- BayesFactor::ttestBF(pairedDiffs)
ttBFwide <- BayesFactor::ttestBF(pairedDiffs, rscale=1)
ttBFultra <- BayesFactor::ttestBF(pairedDiffs, rscale=sqrt(2))
rscales <- c("medium"=sqrt(2)/2, "wide"=1, ultrawide=sqrt(2))
BF3 <- c(as.data.frame(ttBF0)[['bf']], as.data.frame(ttBFwide)[['bf']],
        as.data.frame(ttBFultra)[['bf']])
setNames(round(BF3,2), c("medium", "wide", "ultrawide"))
```

```
pval <- t.test(pairedDiffs)[['p.value']]
1/(-exp(1)*pval*log(pval))
```

### A null interval may make better sense

```
min45 <- round(0.75/sd(pairedDiffs),2) ## Use standardized units
ttBFint <- BayesFactor::ttestBF(pairedDiffs, nullInterval=c(-min45,min45))
round(as.data.frame(ttBFint)[['bf']],3)
```

```
bf01 <- as.data.frame(ttBFint)[['bf']]
```

### The effect of changing sample size

```
t2bfInterval <- function(t, n=10, rscale="medium", mu=c(-.1,.1)){
  null0 <- BayesFactor::ttest.tstat(t=t, n1=n, nullInterval=mu,
                                    rscale=rscale,simple=TRUE)
  alt0 <- BayesFactor::ttest.tstat(t=t, n1=n, nullInterval=mu, rscale=rscale,
                                   complement=TRUE, simple=TRUE)
  alt0/null0
}
##
## Calculate Bayes factors
pval <- c(0.05,0.01,0.001); nval <- c(4,6,10,20,40,80,160)
bfDF <- expand.grid(p=pval, n=nval)
pcol <- 1; ncol <- 2; tcol <- 3
bfDF[, 't'] <- apply(bfDF,1,function(x){qt(x[pcol]/2, df=x[ncol]-1,
```

```

other <- apply(bfDF,1,function(x)
  c(BayesFactor::ttest.tstat(t=x[tcoll], n1=x[ncol], rscale="medium",
    simple=TRUE),
  ## Now specify a null interval
  t2bfInterval(t=x[tcoll], n=x[ncol], mu=c(-0.1,0.1),rscale="medium")
))
bfDF <- setNames(cbind(bfDF, t(other)),
  c('p','n','t','bf','bfInterval'))

plabpos <- with(subset(bfDF, n==max(bfDF$n)), log((bf+bfInterval)/2))
gphA1 <- lattice::xyplot(log(bf)~log(n), groups=factor(p), data=bfDF,
  panel=function(x,y,...){
    lattice::panel.xyplot(x,y,type='b',...)}
ylabA <- 10^((-3):6/2)
scalesA <- list(x=list(at=log(nval), labels=nval),
  y=list(at=log(ylabA), labels=signif(ylabA,2)))
keyA <- list(corner=c(0.99,0.98), lines=list(col=c(1,1), lty=1:2),
  text=list(c('Point null at 0', "null=(-0.1,0.1)")))
ylim2 <- log(c(min(bfDF[['bfInterval']])-0.05,150))
gphA2 <- lattice::xyplot(log(bfInterval)~log(n), groups=factor(p), lty=2,
  xlim=c(log(3.5),log(max(nval)*3.25)), ylim=ylim2, data=bfDF,
  panel=function(x,y,...){
    panel.xyplot(x,y,type='b',...)
    panel.grid(h=-1,v=-1)
    panel.text(rep(log(max(nval*0.975)),3), plabpos,
      labels=c('p=0.05','0.01','0.001'), pos=4)
  },
  par.settings=DAAG::DAAGtheme(color=T),
  main="A: Bayes factor vs sample size",
  xlab="Sample size", ylab="Bayes factor", scales=scalesA, key=keyA)
## Panel B
bfDF[['eff']] = bfDF[['t']]/sqrt(bfDF[['n']])
ylabB <- 10^((-3):2/3)
scalesB= list(x=list(at=log(nval), labels=nval),
  y=list(at=log(ylabB), labels=signif(ylabB,2)))
keyB <- list(corner=c(0.98,0.975), lines=list(lty=1:3),
  points=list(pch=1:3), text=list(c('p=0.001','p=0.01','p=0.05'))))
gphB <- xyplot(log(eff)~log(n), groups=log(p), data=bfDF, pch=1:3, lty=1:3,
  type='b', xlab="Sample size", ylab="Effect size",
  par.settings=DAAG::DAAGtheme(color=T),
  main="B: Effect size vs sample size", key=keyB, scales=scalesB) +
  latticeExtra::layer(panel.grid(h=-1,v=-1))

```

```
plot(gphA2+latticeExtra::as.layer(gphA1), position=c(0, 0, 0.525, 1), more=T)
plot(gphB, position=c(0.52, 0, 1, 1), par.settings=DAAG::DAAGtheme(color=T))
```

## Different statistics give different perspectives

```
n1 <- BayesFactor::ttest.tstat(qt(0.00001, df=40), n1=40, simple=T)
n2 <- BayesFactor::ttest.tstat(qt(0.000001, df=40), n1=40, simple=T)
```

```
bf1 <- BayesFactor::ttest.tstat(qt(0.00001, df=40), n1=40, simple=T)
bf2 <- BayesFactor::ttest.tstat(qt(0.000001, df=40), n1=40, simple=T)
rbind("Bayes Factors"=setNames(c(bf1,bf2), c("p=0.00001","p=0.000001")),
      "t-statistics"=c(qt(0.00001, df=40), qt(0.000001, df=40)))
```

```
knitr::kable(matrix(c("A bare mention","Positive","Strong","Very strong"), nrow=1),
              col.names=c("1 -- 3", "3 -- 20", "20 -- 150", ">150"), align='c',
              midrule='', vline="")
```

## \*Technical details of the family of priors used in the BayesFactor package

## Section 1.8: Resampling methods for SEs, tests and confidence intervals

### Subsection 1.8.1: The one-sample permutation test

```
tab <- t(as.matrix(DAAG::pair65))
rbind(tab,"heated-ambient"=tab[1,]-tab[2,])
```

### Subsection 1.8.2: The two-sample permutation test

```
## First of 3 curves; permutation distribution of difference in means
two65 <- DAAG::two65
set.seed(47) # Repeat curves shown here
nsim <- 2000; dsims <- numeric(nsim)
x <- with(two65, c(ambient, heated))
n <- length(x); n2 <- length(two65$heated)
dbar <- with(two65, mean(heated)-mean(ambient))
for(i in 1:nsim){
```

```

mn <- sample(n,n2,replace=FALSE); dsims[i] <- mean(x[mn]) - mean(x[-mn]) }
plot(density(dsims), xlab="", main="", lwd=0.5, yaxs="i", ylim=c(0,0.08), bty="n")
abline(v=c(dbar, -dbar), lty=3)
pval1 <- (sum(dsims >= abs(dbar)) + sum (dsims <= -abs(dbar)))/nsim
mtext(side=3,line=0.25,
      text=expression(bar(italic(x))[2]-bar(italic(x))[1]), at=dbar)
mtext(side=3,line=0.25,
      text=expression(-(bar(italic(x))[2] - bar(italic(x))[1])), at=-dbar)
## Second permutation density
for(i in 1:nsim){
mn <- sample(n,n2,replace=FALSE)
dsims[i] <- mean(x[mn]) - mean(x[-mn])
}
pval2 <- (sum(dsims >= abs(dbar)) + sum (dsims <= -abs(dbar)))/nsim
lines(density(dsims),lty=2,lwd=1)
## Third permutation density
for(i in 1:nsim){
mn <- sample(n,n2,replace=FALSE)
dsims[i] <- mean(x[mn]) - mean(x[-mn])
}
pval3 <- (sum(dsims >= abs(dbar)) + sum (dsims <= -abs(dbar)))/nsim
lines(density(dsims),lty=3,lwd=1.25)
box(col="gray")
leg3 <- paste(c(pval1,pval2,pval3))
legend(x=20, y=0.078, title="P-values are", cex=1, xpd=TRUE,
      bty="n", lty=c(1,2,3), lwd=c(1,1,1,1.25), legend=leg3, y.intersp=0.8)

```

### Subsection 1.8.3: Estimating the standard error of the median: bootstrapping

```

## Bootstrap estimate of median of wren length: data frame cuckoos
wren <- subset(DAAG::cuckoos, species=="wren")[, "length"]
library(boot)
## First define median.fun(), with two required arguments:
##      data specifies the data vector,
##      indices selects vector elements for each resample
median.fun <- function(data, indices){median(data[indices])}
## Call boot(), with statistic=median.fun, R = # of resamples
set.seed(23)
(wren.boot <- boot(data = wren, statistic = median.fun, R = 4999))

```

## Subsection 1.8.4: Bootstrap estimates of confidence intervals

### Bootstrap 95% confidence intervals for the median

```
## Call the function boot.ci() , with boot.out=wren.boot  
boot.ci(boot.out=wren.boot, type=c("perc","bca"))
```

### The correlation coefficient

```
## Bootstrap estimate of 95% CI for `cor(chest, belly)`: `DAAG::possum`  
corr.fun <- function(data, indices)  
  with(data, cor(belly[indices], chest[indices]))  
set.seed(29)  
corr.boot <- boot(DAAG::possum, corr.fun, R=9999)
```

```
library(boot)  
boot.ci(boot.out = corr.boot, type = c("perc", "bca"))
```

### The bootstrap – parting comments

## Section 1.9: Organizing and managing work, and tools that can assist

### The RStudio Integrated Development Environment

#### Subsection 1.9.1: Reproducible reporting — the knitr package

## Section 1.10: The changing environment for data analysis

### Subsection 1.10.1: Models and machine learning

#### The limits of current machine learning systems

#### Traps in big data analysis

#### Of mice and machine learning — missing data values

#### Humans are not good intuitive statisticians

## Subsection 1.10.2: Replicability is the definitive check

To what extent is published work replicable? What is the evidence?

Some major replication studies

Replicability in pre-clinical cancer research

Studies where there may be strong social influences

The scientific study of scientific processes

Would lowering the  $p$ -value threshold help?

Peer review at the study planning stage

## Section 1.11: Further, or supplementary, reading

### Exercises (1\_12)

1.4

```
Animals <- MASS::Animals
manyMals <- rbind(Animals, sqrt(Animals), Animals^0.1, log(Animals))
manyMals$transgp <- rep(c("Untransformed", "Square root transform",
  "Power transform, lambda=0.1", "log transform"),
  rep(nrow(Animals),4))
manyMals$transgp <- with(manyMals, factor(transgp, levels=unique(transgp)))
lattice::xyplot(brain~body|transgp, data=manyMals,
  scales=list(relation='free'), layout=c(2,2))
```

1.5

```
with(Animals, c(cor(brain,body), cor(brain,body, method="spearman")))
with(Animals, c(cor(log(brain),log(body)),
  cor(log(brain),log(body), method="spearman")))
```

1.9

```
usableDF <- DAAG::cuckooohosts[c(1:6,8),]  
nr <- nrow(usableDF)  
with(usableDF, {  
  plot(c(clength, hlength), c(cbreadth, hbreadth), col=rep(1:2,c(nr,nr)))  
  for(i in 1:nr)lines(c(clength[i], hlength[i]), c(cbreadth[i], hbreadth[i]))  
  text(hlength, hbreadth, abbreviate(rownames(usableDF),8), pos=c(2,4,2,1,2,4,2))  
})
```

1.10

```
## Take a random sample of 100 values from the normal distribution  
x <- rnorm(100, mean=3, sd=5)  
(xbar <- mean(x))  
## Plot, against `xbar`, the sum of squared deviations from `xbar`  
lsfun <- function(xbar) apply(outer(x, xbar, "-")^2, 2, sum)  
curve(lsfun, from=xbar-0.01, to=xbar+0.01)
```

```
boxplot(avs, meds, horizontal=T)
```

1.15

```
x <- rpois(7, 78.3)  
mean(x); var(x)
```

1.16

```
nvals100 <- rnorm(100)  
heavytail <- rt(100, df = 4)  
veryheavytail <- rt(100, df = 2)  
boxplot(nvals100, heavytail, veryheavytail, horizontal=TRUE)
```

1.19

```
boxdists <- function(n=1000, times=10){  
  df <- data.frame(normal=rnorm(n*times), t=rt(n*times, 7),  
    sampnum <- rep(1:times, rep(n,times)))  
  lattice::bwplot(sampnum ~ normal+t, data=df, outer=TRUE, xlab="",  
    horizontal=T)  
}
```



1.20

```
a <- 1
form <- ~rchisq(1000,1)^a+rchisq(1000,25)^a+rchisq(1000,500)^a
lattice::qqmath(form, scales=list(relation="free"), outer=TRUE)
```

1.21

```
y <- rnorm(51)
ydep <- y1[-1] + y1[-51]
acf(y)      # acf plots `autocorrelation function'(see Chapter 6)
acf(ydep)
```

1.24

```
ptFun <- function(x,N)pt(sqrt(N)*mean(x)/sd(x), df=N-1, lower.tail=FALSE)
simStat <- function(eff=.4, N=10, nrep=200, FUN)
  array(rnorm(n=N*nrep*length(eff), mean=eff), dim=c(length(eff),nrep,N)) |>
  apply(2:1, FUN, N=N)
pval <- simStat(eff=.4, N=10, nrep=200, FUN=ptFun)
# Suggest a power transform that makes the distribution more symmetric
car::powerTransform(pval) # See Subsection 2.5.6
labx <- c(0.0001, 0.001, 0.005, 0.01, 0.05, 0.1, 0.25)
bwplot(~I(pval^0.2), scales=list(x=list(at=labx^0.2, labels=paste(labx))),
  xlab=expression("P-value (scale is " * p^{0.2} * ")") )
```

1.24a

```
pvalDF <- subset(df200, effsize==0.4 & N==10)$stat
plot(sort(pval^0.2), sort(pvalDF^0.2))
abline(0,1)
```

1.24c

```
## Estimated effect sizes: Set `FUN=effFun` in the call to `eff2stat()`
effFun <- function(x,N)mean(x)/sd(x)
# Try: `labx <- ((-1):6)/2`; `at = log(labx)`; `v = log(labx)`
## NB also, Bayes Factors: Set `FUN=BFfun` in the call to `eff2stat()`
BFfun <- function(x,N)BayesFactor::ttest.tstat(sqrt(N)*mean(x)/sd(x),
  n1=N, simple=T)
# A few very large Bayes Factors are likely to dominate the plots
```

1.27

```
(degC <- setNames(c(21,30,38,46),paste('rep',1:4)) )
```

1.27a

```
radonC <- tidyr::pivot_longer(MPV::radon, names_to='key',  
                             cols=names(degC), values_to='percent')  
radonC$temp <- degC[radonC$key]  
lattice::xyplot(percent ~ temp|factor(diameter), data = radonC)
```

```
matplot(scale(t(MPV::radon[,-1])), type="l", ylab="scaled residuals")
```

1.27d

```
radon.res <- aggregate(percent ~ diameter, data = radonC, FUN = scale,  
                        scale = FALSE)
```

1.30

```
diamonds <- ggplot2::diamonds  
with(diamonds, plot(carat, price, pch=16, cex=0.25))  
with(diamonds, smoothScatter(carat, price))
```

```
t2bfInterval <- function(t, n=10, rscale="medium", mu=c(-.1,.1)){  
  null0 <- BayesFactor::ttest.tstat(t=t, n1=n, nullInterval=mu,  
                                    rscale=rscale,simple=TRUE)  
  alt0 <- BayesFactor::ttest.tstat(t=t, n1=n, nullInterval=mu, rscale=rscale,  
                                   complement=TRUE, simple=TRUE)  
  alt0/null0  
}
```

```
pval <- c(0.05,0.01,0.001); nval <- c(10,40,160)  
bfDF <- expand.grid(p=pval, n=nval)  
pcol <- 1; ncol <- 2; tcol <- 3  
bfDF[, 't'] <- apply(bfDF,1,function(x){qt(x[pcol]/2, df=x[ncol]-1,  
other <- apply(bfDF,1,function(x)  
  c(BayesFactor::ttest.tstat(t=x[tcol], n1=x[ncol], rscale="medium",  
                             simple=TRUE),  
    BayesFactor::ttest.tstat(t=x[tcol], n1=x[ncol], rscale="wide",
```

```

                                simple=TRUE),
## Now specify a null interval
  t2bfInterval(t=x[tcol], n=x[ncol], mu=c(-0.1,0.1),rscale="medium"),
  t2bfInterval(t=x[tcol], n=x[ncol], mu=c(-0.1,0.1),rscale="wide")
))
bfDF <- setNames(cbind(bfDF, t(other)),
  c('p','n','t','bf','bfInterval'))

```

```

df <- data.frame(d = with(datasets::sleep, extra[group==2] - extra[group==1]))
library(statsr)
BayesFactor::ttestBF(df$d, rscale=1/sqrt(2)) # Or, `rscale="medium"`
# `rscale="medium"` is the default
bayes_inference(d, type='ht', data=df, statistic='mean', method='t', rscale=1/sqrt(2),
  alternative='twosided', null=0, prior_family = "JZS")
# Set `rscale=1/sqrt(2)` (default is 1.0)
# as for BayesFactor; gives same BF
# Compare with `prior_family = "JUI"` (`"JZS"` is the default),
# with (if not supplied) default settings
bayes_inference(d, type='ht', data=df, statistic='mean', method='t',
  alternative='twosided', null=0, prior_family = "JUI")

```

```

if(file.exists("/Users/johnm1/pkgs/PGRcode/inst/doc/")){
code <- knitr::knit_code$get()
txt <- paste0("\n## ", names(code),"\n", sapply(code, paste, collapse='\n'))
writeLines(txt, con="/Users/johnm1/pkgs/PGRcode/inst/doc/ch1.R")
}

```

## 2 Chapter 2: Generalizing from models

### Packages required (plus any dependencies)

DAAG MASS qra investr HistData BHH2 xtable BayesFactor boot zoo boot MCMCpack,  
Additionally, knitr and Hmisc are required in order to process the Rmd source file.

### Section 2.1 Model assumptions

#### Subsection 2.1.1: Inferences are never assumption free

#### Subsection 2.1.2: Has account been taken of all relevant effects?

```
## Tabulate by Admit and Gender
byGender <- 100*prop.table(margin.table(UCBAdmissions, margin=1:2), margin=2)
round(byGender,1)
```

```
## Admission rates, by department
pcAdmit <- 100*prop.table(UCBAdmissions, margin=2:3)["Admitted", , ]
round(pcAdmit,1)
```

```
applied <- margin.table(UCBAdmissions, margin=2:3)
pcAdmit <- 100*prop.table(UCBAdmissions, margin=2:3)["Admitted", , ]
byGender <- 100*prop.table(margin.table(UCBAdmissions,
margin=1:2), margin=2)
dimnam <- dimnames(UCBAdmissions)
mfStats <- data.frame(Admit=c(pcAdmit[1,],pcAdmit[2,]),
  Applicants=c(applied[1,], applied[2,]),
  mf=factor(rep(dimnam[['Gender']],c(6,6)),
  levels=dimnam[['Gender']]), Department=rep(dimnam[["Dept"]],2))
xlim <- c(0, max(mfStats$Admit)*1.025)
ylim <- c(0, max(mfStats$Applicants)*1.075)
plot(Applicants~Admit, data=mfStats, type="h",lwd=2, xlim=xlim, ylim=ylim,
  fg="gray", cex.lab=1.2, col=rep(c("blue","red"),rep(6,2)),
```

```

      xlab="UCB Admission rates (%)", ylab="Number of applicants")
pcA <- rbind(pcAdmit[1,], apply(pcAdmit,2, mean)+2, pcAdmit[2,], rep(NA,6))
pcA[2,3] <- pcA[2,3]+1
appA <- rbind(applied[1,], apply(applied,2, mean)+80,
              applied[2,], rep(NA,6))
deptNam <- dimnam[[3]]
for(j in 1: ncol(appA)) lines(pcA[,j], appA[,j], col="gray", lwd=0.8)
points(pcA[2,],appA[2,], pch=16, cex=1.1, col="white")
text(pcA[2,],appA[2,],deptNam, cex=0.85)
##
par(xpd=TRUE)
text(byGender[1,1:2], rep(par()$usr[4],2)+0.5*strheight("^"),
     labels=c("^","^"), col=c("blue","red"),cex=1.2,srt=180)
text(byGender[1,], par()$usr[4]+1.4*strheight("A"),
     labels=paste(round(byGender[1,],1)),cex=0.85)
text(byGender[1,1:2]+c(-3.5,3.5), rep(par()$usr[4],2)+2.65*strheight("A"),
     labels=c("All males","All females"), pos=c(4,2), cex=1.2)
par(xpd=FALSE)
abline(h=200*(0:4),col="lightgray",lty="dotted")
abline(v=20*(0:4),col="lightgray",lty="dotted")
legend("topleft", col=c('blue','red'),lty=c(1,1), lwd=0.75, cex=0.9,
      y.intersp=0.65, legend=c("Males","Females"),bty="n")

```

```

## Calculate totals, by department, of males & females applying
margin.table(UCBAdmissions, margin=2:3)

```

### Subsection 2.1.3: The limitations of models

### Subsection 2.1.4: Use the methodology that best suits the task in hand?

## Section 2.2: t-statistics, binomial proportions, and correlations

### Subsection 2.2.1: One- and two-sample t-tests

### Subsection 2.2.2: A two-sample comparison

```

stats2 <- sapply(DAAG::two65,
                 function(x) c(av=mean(x), sd=sd(x), n=length(x)))
pooledsd <- sqrt( sum(stats2['n',]*stats2['sd',]^2)/sum(stats2['n',]-1) )
stats2 <- setNames(c(as.vector(stats2), pooledsd),

```

```

      c('av1','sd1','n1','av2','sd2','n2','pooledsd'))
print(stats2, digits=4)

```

```

with(DAAG::two65, t.test(heated, ambient, var.equal=TRUE))

```

### When is pairing helpful?

```

titl <- paste("Second versus first member, for each pair. The first",
"\npanel is for the elastic band data. The second (from",
"\nDarwin) is for plants of the species Reseda lutea")
oldpar <- par(pty="s")
on.exit(par(oldpar))
DAAG::onesamp(dset = DAAG::pair65, x = "ambient", y = "heated",
  xlab = "Amount of stretch (ambient)",
  ylab = "Amount of stretch (heated)", fg='gray')
## Data set mignonette holds the Darwin (1877) data on Reseda lutea.
## Data were in 5 pots, holding 5,5,5,5,4 pairs of plants respectively.
DAAG::onesamp(dset = DAAG::mignonette, x = "self", y = "cross",
  xlab = "Height of self-fertilised plant", ylab =
  "Height of cross-fertilised plant", dubious = 0, cex=0.7, fg='gray')

```

### What if the standard deviations are unequal?

#### Subsection 2.2.3: The normal approximation to the binomial

#### Subsection 2.2.4: The Pearson or product-moment correlation

```

## Pearson correlation between `body` and `brain`: Animals
Animals <- MASS::Animals
rho <- with(Animals, cor(body, brain))
## Pearson correlation, after log transformation
rhoLogged <- with(log(Animals), cor(body, brain))
## Spearman rank correlation
rhoSpearman <- with(Animals, cor(body, brain, method="spearman"))
c(Pearson=round(rho,2), " Pearson:log values "=round(rhoLogged,2),
  Spearman=round(rhoSpearman,2))

```

## Section 2.3 Extra-binomial and extra-Poisson variation

```
maleDF <- data.frame(number=0:12, freq=unname(qra::malesINfirst12[["freq"]]))
N <- sum(maleDF$freq)
pihat <- with(maleDF, weighted.mean(number, freq))/12
probBin <- dbinom(0:12, size=12, prob=pihat)
rbind(Frequency=setNames(maleDF$freq, nm=0:12),
      binomialFit=setNames(probBin*N, nm=0:12),
      rawResiduals = maleDF$freq-probBin*N,
      SDbinomial=sqrt(probBin*(1-probBin)*N)) |>
  formatC(digits=2, format="fg") |> print(digits=2, quote=F, right=T)
```

```
set.seed(29)
rqres.plot(doBI, plot.type='all', type="QQ", main=""); box(col='white')
mtext(side=3, line=0.5, "A: Binomial, Q-Q", adj=0, cex=1.25)
rqres.plot(doBI, plot.type='all', type="wp", main=""); box(col='white')
## Plots C, D, E, F: Set object name; set `type="wp"` (C, E, F), or `type="QQ"` (D)
mtext(side=3, line=0.5, "B: Binomial, worm plot 1", adj=-0.05, cex=1.25)
rqres.plot(doBI, plot.type='all', type="wp", main=""); box(col='white')
mtext(side=3, line=0.5, "C: Binomial, worm plot 2", adj=-0.05, cex=1.25)
rqres.plot(doBB, plot.type='all', type="QQ", main="", ylab=''); box(col='white')
mtext(side=3, line=0.5, "D: BB model, Q-Q", adj=0, cex=1.25)
rqres.plot(doBB, plot.type='all', type="wp", main="", ylab=''); box(col='white')
mtext(side=3, line=0.5, "E: BB, worm plot 1", adj=0, cex=1.25)
rqres.plot(doBB, plot.type='all', type="wp", main="", ylab=''); box(col='white')
mtext(side=3, line=0.5, "F: BB, worm plot 2", adj=0, cex=1.25)
```

```
aicStat <- AIC(doBI, doBB)
rownames(aicStat) <-
  c(doBI="Binomial", doBB="Betabinomial")[rownames(aicStat)]
aicStat$dAIC <- with(aicStat, round(AIC-AIC[1],1))
aicStat
```

```
## Numbers of accidents in three months, with Poisson fit
machinists <- data.frame(number=0:8, freq=c(296, 74, 26, 8, 4, 4, 1, 0, 1))
N <- sum(machinists[["freq"]])
lambda <- with(machinists, weighted.mean(number, freq))
fitPoisson <- dpois(0:8, lambda)*sum(machinists[["freq"]])
rbind(Frequency=with(machinists, setNames(freq, number)),
      poissonFit=fitPoisson) |>
  formatC(digits=2, format="fg") |> print(quote=F, digits=2, right=T)
```

```

set.seed(23)
rqres.plot(doP0, plot.type='all', type="QQ", main=""); box(col='white')
## Repeat, changing the argument, for remaining plots
mtext(side=3, line=0.5, "A: Poisson, Q-Q plot", adj=0, cex=1.25)
rqres.plot(doP0, plot.type='all', type="wp", main="", ylab=''); box(col='white')
mtext(side=3, line=0.5, "B: Poisson, worm plot", adj=0, cex=1.25)
rqres.plot(doNBI, plot.type='all', type="wp", main="", ylab='')
mtext(side=3, line=0.5, "C: NBI, worm plot", adj=0, cex=1.25); box(col='white')

```

### Subsection 2.3.2: \*Technical details – extra-binomial or extra-Poisson variation

```

sigma <- exp(coef(doBB, "sigma"))
cat("Phi =", (1+12*sigma)/(1+sigma))

```

```

mu <- exp(coef(doNBI, "mu"))
sigma <- exp(coef(doNBI, "sigma"))
cat("Phi =", (1+sigma*mu))

```

## Section 2.4 Contingency tables

```

## 'Untreated' rows (no training) from psid3, 'treated' rows from nswdemo
nswpsid3 <- rbind(DAAG::psid3, subset(DAAG::nswdemo, trt==1))
degTAB <- with(nswpsid3, table(trt,nodeg))
# Code 'Yes' if completed high school; 'No' if dropout
dimnames(degTAB) <- list(trt=c("PSID3_males","NSW_male_trainees"),
                        deg =c("Yes","No"))
degTAB

```

```

# To agree with hand calculation below, specify correct=FALSE
chisq.test(degTAB, correct=FALSE)

```

### The mechanics of the chi-squared test

#### An example where a chi-squared test may not be valid



```
## Engine man data
engineman <- matrix(c(5,3,17,85), 2,2)
chisq.test(engineman)
```

## Rare and endangered plant species

```
fisher.test(engineman)
```

```
## Enter the data thus:
rareplants <- matrix(c(37,190,94,23, 59,23,10,141, 28,15,58,16), ncol=3,
  byrow=TRUE, dimnames=list(c("CC","CR","RC","RR"), c("D","W","WD")))
```

```
(x2 <- chisq.test(rareplants))
```

## Examination of departures from a consistent overall row pattern

```
## Expected values
x2$expected
```

```
options(digits=2)
## Standardized residuals
residuals(x2)
```

## Interpretation issues

## Section 2.5 Issues for Regression with a single explanatory variable

### Subsection 2.5.1: Iron slag example — check residuals with care!

```
leg <- c("A: Fitted line", "B: Residuals from line", "C: Variance check")
ord <- order(DAAG::ironslag[["magnetic"]])
ironslag <- DAAG::ironslag[ord,]
slagAlpha.lm <- lm(chemical~magnetic, data=ironslag)
resval <- residuals(slagAlpha.lm)
fitchem <- fitted(slagAlpha.lm)
sqrtabs2 <- sqrt(abs(resval))
plot(chemical~magnetic, xlab = "Magnetic", ylab = "Chemical",
```

```

    pch = 1, data=ironslag, fg="gray")
lines(fitchem~ironslag[["magnetic"]])
mtext(side = 3, line = 0.25, leg[1], adj=-0.1, cex=0.925)
scatter.smooth(resval~ironslag[["magnetic"]], lpars=list(col="red"), span=0.8,
               xlab = "Magnetic", ylab = "Residual", fg="gray")
mtext(side = 3, line = 0.25, leg[2], adj = -0.1, cex=0.925)
scatter.smooth(sqrtabs2 ~ fitchem, lpars=list(col="red"), span=0.8,
               xlab = "Predicted chemical", fg="gray",
               ylab = expression(sqrt(abs(residual))))
mtext(side = 3, line = 0.25, leg[3], adj = -0.1, cex=0.8)
## Diagnostics from fit using loess()
leg2 <- c("D: Smooth, using loess()",
          "E: Residuals from smooth",
          "F: Variance check")
slag.loess <- loess(chemical~magnetic, data=ironslag, span=0.8)
resval2 <- slag.loess[["residuals"]]
fitchem2 <- slag.loess[["fitted"]]
sqrtabs2 <- sqrt(abs(resval2))
plot(chemical~magnetic, xlab = "Magnetic", ylab = "Chemical",
     pch = 1, data=ironslag, fg="gray")
lines(fitchem2 ~ ironslag[["magnetic"]], col="red")
mtext(side = 3, line = 0.25, leg2[1], adj=-0.1, cex=0.925)
scatter.smooth(resval2~ironslag[["magnetic"]], span=0.8,
               lpars=list(col="red"),
               xlab = "Magnetic", ylab = "Residual", fg="gray")
mtext(side = 3, line = 0.25, leg2[2], adj = -0.1, cex=0.925)
scatter.smooth(sqrtabs2 ~ fitchem2, lpars=list(col="red"),
               span=0.8, xlab = "Predicted chemical", fg="gray",
               ylab = expression(sqrt(abs(residual))))
mtext(side = 3, line = 0.25, leg2[3], adj = -0.1, cex=0.925)

```

### Subsection 2.5.2: The analysis of variance table

```

roller.lm <- lm(depression ~ weight, data=DAAG::roller)
anova(roller.lm)

```

### Subsection 2.5.3: Outliers, influence, and robust regression

```
softbacks <- DAAG::softbacks
x <- softbacks[, "volume"]
y <- softbacks[, "weight"]
u <- lm(y ~ x)
yhat <- predict(u)
res <- resid(u)
r <- with(softbacks, cor(x, y))
xlim <- with(softbacks, range(volume))
xlim[2] <- xlim[2] + diff(xlim) * 0.08
plot(y ~ x, xlab = "Volume (cc)", xlim = xlim,
     data = softbacks, ylab = "Weight (g)", pch = 4,
     ylim = range(c(y, yhat)), cex.lab = 0.9, fg = "gray")
abline(u$coef[1], u$coef[2], lty = 1)
bottomright <- par()$usr[c(2, 3)]
chw <- par()$cxy[1]
chh <- par()$cxy[2]
z <- summary(u)$coef
btxt <- c(paste("a =", format(round(z[1, 1], 1))),
         " SE =", format(round(z[1, 2], 1))),
paste("b =", format(round(z[2, 1], 2)),
      " SE =", format(round(z[2, 2], 2))))
legend(bottomright[1], bottomright[2],
       legend = btxt, xjust = 1, yjust = 0, cex = 0.8, bty = "n")
```

```
softbacks.lm <- lm(weight ~ volume, data = DAAG::softbacks)
print(coef(summary(softbacks.lm)), digits = 3)
```

```
plot(softbacks.lm, fg = "gray",
     caption = c("A: Residuals vs Fitted", "B: Normal Q-Q",
                 "C: Scale-Location", "", "D: Resids vs Leverage"))
```

### Robust regression

### Subsection 2.5.4: Standard errors and confidence intervals

### Confidence intervals and tests for the slope

```
SEb <- coef(summary(roller.lm))[2, 2]
coef(roller.lm)[2] + qt(c(0.025,.975), 8)*SEb
```

## SEs and confidence intervals for predicted values

```
## Code to obtain fitted values and standard errors (SE, then SE.OBS)
fit.with.se <- predict(roller.lm, se.fit=TRUE)
fit.with.se$se.fit # SE
sqrt(fit.with.se[["se.fit"]]^2+fit.with.se$residual.scale^2) # SE.OBS
```

```
predict(roller.lm, interval="confidence", level=0.95)
predict(roller.lm, interval="prediction", level=0.95) # CI for a new observation
```

```
## Depression vs weight, with 95% pointwise bounds for both
## the fitted line and predicted values
investr::plotFit(roller.lm, interval="both", col.conf="red", fg="gray")
mtext(side=3,line=0.75, "A: Lawn roller data", cex=1.2, adj=-0.25)
## Male child vs father height, Galton's data
galtonMales <- subset(HistData::GaltonFamilies, gender=="male")
galton.lm <- lm(childHeight~father, data=galtonMales)
investr::plotFit(galton.lm, interval="both", col.conf="red", hide=FALSE,
                 col=adjustcolor('black',alpha=0.5), fg="gray")
mtext(side=3,line=0.75, "B: Son vs father heights", cex=1.2, adj=-0.25)
```

## Implications for design

```
panelci<-function(data,...)
{
  nrows<-list(...)$nrows
  ncols<-list(...)$ncols
  if(ncols==1)axis(2, lwd=0, lwd.ticks=1)
  if(ncols==1)axis(1, lwd=0, lwd.ticks=1) else
  axis(3, lwd=0, lwd.ticks=1)
  x<-data$stretch; y<-data$distance
  u <- lm(y ~ x)
  upred <- predict(u, interval="confidence")
  ci <- data.frame(fit=upred[, "fit"], lower=upred[, "lwr"], upper=upred[, "upr"])
  ord<-order(x)
  lines(x[ord], ci[["fit"]][ord], lty=1, lwd=2)
```

```

lines(lowess(x[ord], ci[["upper"]][ord]), lty=2, lwd=2, col="grey")
lines(lowess(x[ord], ci[["lower"]][ord]), lty=2, lwd=2, col="grey")
}
elastic1 <- DAAG::elastic1
elastic2 <- DAAG::elastic2
xy<-rbind(elastic2,elastic1)
nam <- c("Range of stretch 30-65 mm","Range of stretch 42-54 mm")
trial<-rep(nam, c(dim(elastic2)[1],dim(elastic1)[1]))
xlim<-range(elastic2$stretch)
ylim<-range(elastic2$distance)
xy<-split(xy,trial)
xy<-lapply(1:length(xy),function(i){c(as.list(xy[[i]]), list(xlim=xlim,
ylim=ylim))})
names(xy) <- nam
DAAG::panelplot(xy,panel=panelci,totrows=1,totcols=2,
                 par.strip.text=list(cex=.9), oma=c(4,4,2.5,2), fg='gray')
mtext(side = 2, line = 3.35, "Distance moved (cm)", cex=1.1, las=0)
mtext(side=1,line=3,"Amount of stretch (mm)", cex=1.1)

```

### Subsection 2.5.5: There are two regression lines!

```

## There are two regression lines!
pair65 <- DAAG::pair65
bothregs <- function(x=pair65[, "ambient"], y=pair65[, "heated"],
  xlab="Stretch (band at ambient)", ylab = "Stretch (heated band)", pch=16){
  plot(y ~ x, xlab = xlab, ylab = ylab, pch = pch, fg="gray")
  topleft <- par()$usr[c(1, 4)] + c(0.5, -0.5) * par()$cxy
  text(topleft[1], topleft[2], paste("r =", round(cor(x, y), 2)), adj = 0)
  u1 <- lm(y ~ x)
  abline(u1$coef[1], u1$coef[2])
  u2 <- lm(x ~ y)
  abline(- coef(u2)[1]/coef(u2)[2], 1/coef(u2)[2], lty = 2)
}
bothregs()
mtext(side = 3, line = 0.5, "A", adj = 0)
bothregs(x=trees[, "Girth"], y=trees[, "Height"],
  xlab="Girth (in)", ylab <- "Height (ft)", pch=16)
mtext(side = 3, line = 0.5, "B", adj = 0)

```

### An alternative to a regression line

## Subsection 2.5.6: Logarithmic and Power Transformations

```
## Logarithmic and Power Transformations
DAAG::powerplot(expr="sqrt(x)", xlab="")
DAAG::powerplot(expr="x^0.25", xlab="", ylab="")
DAAG::powerplot(expr="log(x)", xlab="", ylab="")
DAAG::powerplot(expr="x^2")
DAAG::powerplot(expr="x^4", ylab="")
DAAG::powerplot(expr="exp(x)", ylab="")
```

## General power transformations — Box-Cox and Yeo-Johnson

## Subsection 2.5.7: General forms of nonlinear response

## Subsection 2.5.8: Size and shape data – allometric growth

```
## Heart weight versus body weight, for 30 Cape fur seals.
g2.12 <- function()
{
  cfseal <- DAAG::cfseal
  x <- log(cfseal[, "weight"])
  y <- log(cfseal[, "heart"])
  ylim <- log(c(82.5, 1100))
  xlim <- log(c(17, 180))
  ylab <- "Heart weight (g, log scale)"
  xlab <- "Body weight (kg, log scale)"
  xtik <- c(20, 40, 80, 160)
  ytik <- c(100, 200, 400, 800)
  plot(x, y, xlab = xlab, ylab = ylab, axes = F, xlim =
    xlim, ylim = ylim, pch = 16, cex = 0.85, fg = "gray", cex.lab = 1.1)
  axis(1, at = log(xtik), labels = paste(xtik), lwd = 0, lwd.ticks = 1)
  axis(2, at = log(ytik), labels = paste(ytik), lwd = 0, lwd.ticks = 1)
  box(col = "gray")
  form1 <- formula(y ~ x)
  u <- lm(form1, data = cfseal)
  abline(u$coef[1], u$coef[2])
  usum <- summary(u)$coef
  options(digits = 3)
  print(usum)
}
```

```

cwh <- par()$cxy
eqn <- paste("log y =", round(usum[1, 1], 2), " [",
round(usum[1, 2], 2), "]" +", round(usum[2, 1], 3),
" [", round(usum[2, 2], 3), "]" log x")
mtext(side=3, line=1.15, eqn, adj = 0.4, cex = 0.8)
mtext(side=3, line=0.25, "(Values in square brackets are SEs)", adj = 0.4, cex = 0.8)
}
g2.12()

```

## The allometric growth equation

```

options(scipen=4)
cfseal.lm <- lm(log(heart) ~ log(weight), data=DAAG::cfseal)
print(coef(summary(cfseal.lm)), digits=4)

```

## Section 2.6 Empirical assessment of predictive accuracy

### Subsection 2.6.1: The training/test approach, and cross-validation

#### Cross-validation – a tutorial example

```

houseprices <- DAAG::houseprices
df <- DAAG::CVlm(houseprices, form.lm = formula(sale.price ~ area),m=3,printit=F,plotit=FALSE)
panelfun <- function(x,y,subscripts,groups, ...){
  lattice::panel.superpose(x,y,subscripts,groups, ...)
  lattice::panel.superpose(x,df[["cvpred"]],subscripts,groups,type="b", cex=0.5, ...)
}
gph <- lattice::xyplot(sale.price ~ area, groups=fold, data=df, pch=1:3, panel=panelfun)
parset <- DAAG::DAAGtheme(color=T, lty=1:3, pch=1:3, lwd=2)
keylist <- list(lines=TRUE, columns=3, between.columns=1.5, between=1, cex=0.85)
update(gph, par.settings=parset, auto.key=keylist)

```

```

set.seed(29)          # Generate results shown
rand <- sample(rep(1:3, length=15))
## sample() randomly permutes the vector of values 1:3
for(i in 1:3) cat(paste0(i,":"), (1:15)[rand == i],"\n")

```

```

houseprices <- DAAG::houseprices
row.names(houseprices) <- (1:nrow(houseprices))
DAAG::CVlm(houseprices, form.lm = formula(sale.price ~ area), plotit=FALSE)

```

```
## Estimate of  $\sigma^2$  from regression output
houseprices <- DAAG::houseprices
houseprices.lm <- lm(sale.price ~ area, houseprices)
summary(houseprices.lm)[["sigma"]]^2
```

### Subsection 2.6.2: Bootstrapping in regression

```
houseprices <- DAAG::houseprices
houseprices.lm <- lm(sale.price ~ area, houseprices)
print(coef(summary(houseprices.lm)), digits=2)
```

```
houseprices.fn <-
  function (houseprices, index,
            statfun=function(obj)coef(obj)[2]){
    house.resample <- houseprices[index, ]
    house.lm <- lm(sale.price ~ area, data=house.resample)
    statfun(house.lm)    # slope estimate for resampled data
  }
```

```
set.seed(1028)      # use to replicate the exact results below
library(boot)       # ensure that the boot package is loaded
## requires the data frame houseprices (DAAG)
(houseprices.boot <- boot(houseprices, R=999, statistic=houseprices.fn))
```

```
statfun1200 <- function(obj)predict(obj, newdata=data.frame(area=1200))
price1200.boot <- boot(houseprices, R=999, statistic=houseprices.fn,
  statfun=statfun1200)
boot.ci(price1200.boot, type="perc") # "basic" is an alternative to "perc"
```

```
set.seed(1111)
library(boot)
par(las=0)
houseprices2.fn<-function (houseprices,index){
  house.resample<-houseprices[index,]
  house.lm<-lm(sale.price~area,data=house.resample)
  houseprices$sale.price-predict(house.lm,houseprices)
  # resampled prediction errors
}
houseprices <- DAAG::houseprices
```



```

n<-nrow(houseprices)
R <- 199      ## Will obtain 199 estimates of prediction error
houseprices.lm<-lm(sale.price~area,data=houseprices)
houseprices2.boot<-boot(houseprices, R=R, statistic=houseprices2.fn)
house.fac<-factor(rep(1:n,rep(R,n)))
plot(house.fac,as.vector(houseprices2.boot$t),
      ylab="", xlab="House", fg="gray")
mtext(side=2, line=2, "Prediction Errors")
mtext(side = 3, line = 0.5, "A", adj = 0)
boot.se <- apply(houseprices2.boot$t,2,sd)
model.se <- predict.lm(houseprices.lm,se.fit=T)$se.fit
plot(boot.se/model.se, ylab="", xlab="House",pch=16, fg="gray")
mtext(side=2, line=2.0, "Ratio of SEs\nBootstrap to Model-Based", cex=0.9)
mtext(side = 3, line = 0.5, "B", adj = 0)
abline(1,0)

```

## Commentary

## Section 2.7 One- and two-way comparisons

### Subsection 2.7.1: One-way comparisons

```

tomato <- data.frame(Weight = c(1.5, 1.9, 1.3, 1.5, 2.4, 1.5,   # Water
                               1.5, 1.2, 1.2, 2.1, 2.9, 1.6,   # Nutrient
                               1.9, 1.6, 0.8, 1.15, 0.9, 1.6), # Nutrient+24D
                    trt = factor(rep(c("Water", "Nutrient", "Nutrient+24D"), c(6, 6, 6))))
## Make `Water` the first level of trt. In aov or lm calculations, it is
## then taken as the baseline or reference level.
tomato$trt <- relevel(tomato$trt, ref="Water")

```

```

## A: Weights of tomato plants (g)
library(lattice, quietly=TRUE)
gph <- stripplot(trt~Weight, aspect=0.35, scale=list(tck=0.6), data=tomato)
update(gph, scales=list(tck=0.4), cex=0.9, col="black", xlab="",
       main=list('A: Weights of tomato plants (g)', y=0, cex=1.1))

```

```

## B: Summarize comparison between LSD and Tukey's HSD graphically
tomato.aov <- aov(Weight ~ trt, data=tomato)
DAAG::onewayPlot(obj=tomato.aov)

```

```
title(main="B: LSD, compared with Tukey HSD", adj=0.1, outer=T,  
      line=-1.0, font.main=1, cex.main=1.25)
```

```
BHH2::anovaPlot(tomato.aov)
```

## The analysis of variance table

```
## Do analysis of variance calculations  
anova(tomato.aov)
```

## Other multiple comparison tests

### Subsection 2.7.2: Regression versus qualitative comparisons – issues of power

```
gph <- DAAG::simulateLinear(alpha=0.6, seed=17, aspect='iso')  
update(gph, par.settings=DAAG::DAAGtheme(color=FALSE, alpha=0.4))
```

### Subsection 2.7.3: \*Severe multiplicity — the false discovery rate

#### \*Microarrays and alternatives — technical note

#### The false discovery rate (FDR)

```
coralPval <- DAAG::coralPval  
pcrit <- c(0.05, 0.02, 0.01, 0.001)  
under <- sapply(pcrit, function(x)sum(coralPval<=x))
```

```
expected <- pcrit*length(coralPval)
```

```
fdrtab <- data.frame(Threshold=pcrit, Expected=expected,  
  Discoveries=under, FDR=round(expected/under, 4))  
print(xtable::xtable(fdrtab), include.rownames=FALSE, hline.after=FALSE)
```

```
fdr <- p.adjust(coralPval, method="BH")
```

```

fdrcrit <- c(0.05, 0.04, 0.02, 0.01)
under <- sapply(fdrcrit, function(x)sum(coralPval<=x))
setNames(under, paste(fdrcrit))

```

#### Subsection 2.7.4: Data with a two-way structure, i.e., two factors

```

par(fig=c(0.525,1,0,1), mgp=c(1.5,0.4,0))
lev <- c("F10", "NH4Cl", "NH4NO3", "F10 +ANU843",
        "NH4Cl +ANU843", "NH4NO3 +ANU843")
rice <- within(DAAG::rice, trt <- factor(trt, levels=lev))
with(rice, interaction.plot(fert, variety, ShootDryMass, fg="gray",
    legend = FALSE, xlab="Fertiliser", cex.lab=0.95, mex=0.65))
xleg <- par()$usr[2]
yleg <- par()$usr[4] - 0.72 * diff(par()$usr[3:4])
leginfo <- legend(xleg, yleg, bty = "n", legend = levels(rice$variety),
    col = 1, lty = 2:1, lwd=1, xjust = 1, cex = 0.8,
    y.intersp=0.8)$rect
text(leginfo$left + 0.5 * leginfo$w, leginfo$top, " variety",
    adj = 0.5, cex = 0.8)
mtext(side=3, line=0.65, cex=0.9, adj=-0.15, "B")
gph <- dotplot(trt ~ ShootDryMass, pch=1, cex=0.9, las=2,
    xlab="Shoot dry mass (g)", data=rice,
    panel=function(x,y,...){panel.dotplot(x,y,...)
    av <- sapply(split(x,y),mean);
    ypos <- unique(y)
    lpoints(ypos~av, pch=3, col="gray40", cex=1.25)},
    main=list("A", cex=0.88, just="left", x=0.1, y=-0.7, font=1))
pars <- DAAG::DAAGtheme(fontsize=list(text=9, points=6), color=FALSE)
print(update(gph, scales=list(tck=0.5), par.settings=pars, aspect=0.9),
    position=c(-0.065,0.0,0.6,1), newpage=FALSE)

```

### Subsection 2.7.5: Presentation issues

## Section 2.8 Data with a nested variation structure

### Subsection 2.8.1: Degrees of freedom considerations

### Subsection 2.8.2: General multi-way analysis of variance designs

## Section 2.9 Bayesian estimation – further commentary and approaches

### Subsection 2.9.1: Bayesian estimation with normal priors and likelihood

### Subsection 2.9.2: Further comments on Bayes Factors

### The Sellke calibration upper limit

### A note on the Bayesian Information Criterion

```
pval <- c(.05,.01,.001); np <- length(pval)
Nval <- c(4,6,10,20,40,80,160); nlen <- length(Nval)
## Difference in BIC statistics, interpreted as Bayes factor
t2BFbic <- function(p,N){t <- qt(p/2, df=N-1, lower.tail=FALSE)
  exp((N*log(1+t^2/(N-1))-log(N))/2)}
bicVal <- outer(pval, Nval, t2BFbic)
## Bayes factor, calculated using BayesFactor::ttest.tstat()
t2BF <- function(p, N){t <- qt(p/2, df=N-1, lower.tail=FALSE)
  BayesFactor::ttest.tstat(t=t, n1=N, simple=TRUE, rscale = "medium")}
BFval <- matrix(nrow=np, ncol=nlen)
for(i in 1:np)for(j in 1:nlen) BFval[i,j] <- t2BF(pval[i], Nval[j])
cfVal <- rbind(BFval, bicVal)[c(1,4,2,5,3,6),]
dimnames(cfVal) <- list(
  paste(rep(pval,rep(2,np)), rep(c("- from ttest.tstat", "- from BIC"),np)),
  paste0(c("n=",rep("",nlen-1)),Nval))
round(cfVal,1)
```

### Subsection 2.9.3: Bayesian regression estimation using the MCMCpack package

```
suppressPackageStartupMessages(library(MCMCpack))
roller.mcmc <- MCMCregress(depression ~ weight, data=DAAG::roller)
summary(roller.mcmc)
```

```
mat <- matrix(c(1:6), byrow=TRUE, ncol=2)
layout(mat, widths=rep(c(2,1.1),3), heights=rep(0.9,8))
# NB: widths & heights are relative
plot(roller.mcmc, auto.layout=FALSE, ask=FALSE, col="gray", fg="gray")
```

## Section 2.10: Recap

Regress  $y$  on  $x$ , or  $x$  on  $\sim y$ ?

## Section 2.11: Further reading

### Exercises (2.12)

2.2

```
## UCBAdmissions is in the datasets package
## For each combination of margins 1 and 2, calculate the sum
UCBtotal <- apply(UCBAdmissions, c(1,2), sum)
```

2.2b

```
apply(UCBAdmissions, 3, function(x)(x[1,1]*x[2,2])/(x[1,2]*x[2,1]))
```

2.3

```
tabA <- array(c(30,30,10,10,15,5,30,10), dim=c(2,2,2))
tabB <- array(c(30,30,20,10,10,5,20,25), dim=c(2,2,2))
```

2.5

```
z.transform <- function(r) .5*log((1+r)/(1-r))
z.inverse <- function(z) (exp(2*z)-1)/(exp(2*z)+1)
possum.fun <- function(data, indices) {
  chest <- data$chest[indices]
  belly <- data$belly[indices]
  z.transform(cor(belly, chest))}
possum.boot <- boot::boot(DAAG::possum, possum.fun, R=999)
z.inverse(boot.ci(possum.boot, type="perc")$percent[4:5])
# The 4th and 5th elements of the percent list element
# hold the interval endpoints. See ?boot.ci
```

2.11

```
with(pressure, MASS::boxcox(pressure ~ I(1/(temperature+273))))
```

2.14

```
"funRel" <-
function(x=leafshape$logpet, y=leafshape$loglen, scale=c(1,1)){
  ## Find principal components rotation; see Subsection 9.1.2
  ## Here (unlike 9.1.2) the interest is in the final component
  xy.prc <- prcomp(cbind(x,y), scale=scale)
  b <- xy.prc$rotation[,2]/scale
  c(bxy = -b[1]/b[2])      # slope of functional equation line
}
## Try the following:
leafshape <- DAAG::leafshape
funRel(scale=c(1,1))      # Take x and y errors as equally important
# Note that all lines pass through (mean(x), mean(y))
```

2.15

```
P <- rbind(
  c(1, 0, 0, 0, 0, 0),
  c(.5, 0, .5, 0, 0, 0),
  c(0, .5, 0, .5, 0, 0),
  c(0, 0, .5, 0, .5, 0),
  c(0, 0, 0, .5, 0, .5),
  c(0, 0, 0, 0, 0, 1))
dimnames(P) <- list(0:5,0:5)
P
```

```
Markov <- function(N=15, initial.value=1, transition=P, stopval=NULL)
{X <- numeric(N)
  X[1] <- initial.value + 1 # States 0:(n-1); subscripts 1:n
  n <- nrow(transition)
  for (i in 2:N){
    X[i] <- sample(1:n, size=1, prob=transition[X[i-1], ])
    if(length(stopval)>0)if(X[i] %in% (stopval+1)){X <- X[1:i]; break}}
  X - 1
}
# Set `stopval=c(0,5)` to stop when the player's fortune is $0 or $5
```

2.16

```
Pb <- rbind(  
  Sun = c(Sun=0.6, Cloud=0.2, Rain=0.2),  
  Cloud= c(0.2, 0.4, 0.4),  
  Rain= c(0.4, 0.3, 0.3))  
Pb
```

2.16b

```
plotmarkov <-  
  function(n=1000, width=101, start=0, transition=Pb, npanels=5){  
    xc2 <- Markov(n, initial.value=start, transition)  
    mav0 <- zoo::rollmean(as.integer(xc2==0), k=width)  
    mav1 <- zoo::rollmean(as.integer(xc2==1), k=width)  
    npanel <- cut(1:length(mav0), breaks=seq(from=1, to=length(mav0),  
      length=npanels+1), include.lowest=TRUE)  
    df <- data.frame(av0=mav0, av1=mav1, x=1:length(mav0), gp=npanel)  
    print(xyplot(av0+av1 ~ x | gp, data=df, layout=c(1,npanels), type="l",  
      par.strip.text=list(cex=0.65), auto.key=list(columns=2),  
      scales=list(x=list(relation="free"))))  
  }
```

```
if(file.exists("/Users/johnm1/pkgsrc/PGRcode/inst/doc/")){  
  code <- knitr::knit_code$get()  
  txt <- paste0("\n## ", names(code), "\n", sapply(code, paste, collapse='\n'))  
  writeLines(txt, con="/Users/johnm1/pkgsrc/PGRcode/inst/doc/ch2.R")  
}
```

## 3 Chapter 3: Multiple linear regression

### Packages required (plus any dependencies)

DAAG car MASS AICcmodavg leaps BayesFactor splines

Additionally, Hmisc and knitr are required in order to process the Rmd source file.

### Section 3.1 Basic ideas: the allbacks book weight data

```
allbacks <- DAAG::allbacks # Place the data in the workspace
allbacks.lm <- lm(weight ~ volume+area, data=allbacks)
print(coef(summary(allbacks.lm)), digits=2)
```

```
xlim <- range(allbacks$volume)
xlim <- xlim+c(-.075,.075)*diff(xlim)
## Plot of weight vs volume: data frame allbacks (DAAG)
plot(weight ~ volume, data=allbacks, pch=c(16,1)[unclass(cover)],
     lwd=1.25, xlim=xlim, fg="gray")
## unclass(cover) gives the integer codes that identify levels
## As text() does not accept the parameter data, use with()
## to specify the data frame.
with(allbacks, text(weight ~ volume, labels=paste(1:15), cex=0.75, offset=0.35,
     pos=c(2,4)[unclass(cover)]))
legend(x='topleft', pch=c(16,1), legend=c("hardback ", "softback"),
     horiz=T, bty="n", xjust=0.5, x.intersp=0.75, )
```

```
## Correlations between estimates -- model with intercept
round(summary(allbacks.lm, corr=TRUE)$correlation, 3)
```

```
out <- capture.output(summary(allbacks.lm,digits=2))
cat(out[15:17], sep='\n')
```



```
## 5% critical value; t-statistic with 12 d.f.  
qt(0.975, 12)
```

```
cat(out[5:7], sep='\n')
```

### Subsection 3.1.1: A sequential analysis of variance table

```
anova(allbacks.lm)
```

### Omission of the intercept term

```
## Show rows 1, 7, 8 and 15 only  
model.matrix(allbacks.lm)[c(1,7,8,15), ]  
## NB, also, code that returns the data frame used  
model.frame(allbacks.lm)
```

```
allbacks.lm0 <- lm(weight ~ -1+volume+area, data=allbacks)  
print(coef(summary(allbacks.lm0)), digits=2)
```

```
## Correlations between estimates -- no intercept  
print(round(summary(allbacks.lm0, corr=TRUE)$correlation, 3))
```

### Subsection 3.1.2: Diagnostic plots

```
allbacks.lm0 <- lm(weight ~ -1+volume+area, data=allbacks)  
plot(allbacks.lm0, caption=c('A: Resids vs Fitted', 'B: Normal Q-Q',  
  'C: Scale-Location', '', 'D: Resids vs Leverage'), cex.caption=0.85,  
  fg='gray')
```

```
## To show all plots in the one row, precede with  
par(mfrow=c(1,4))      # Follow with par(mfrow=c(1,1))
```

```
## The following has the default captions  
plot(allbacks.lm0)
```

```
allbacks.lm13 <- lm(weight ~ -1+volume+area, data=allbacks[-13, ])
print(coef(summary(allbacks.lm13)), digits=2)
```

## Section 3.2 The interpretation of model coefficients

### Subsection 3.2.1: Times for Northern Irish hill races

```
oldpar <- par(fg='gray20',col.axis='gray20',lwd=0.5,col.lab='gray20')
nihr <- within(DAAG:nihills, {mph <- dist/time; gradient <- climb/dist})
nihr <- nihr[, c("time", "dist", "climb", "gradient", "mph")]
varLabs <- c("\ntime\n(hours)", "\ndist\n(miles)", "\nclimb\n(feet)",
             "\ngradient\n(ft/mi)", "\nmph\n(mph)")
smoothPars <- list(col.smooth='red', lty.smooth=2, lwd.smooth=0.5, spread=0)
car::spm(nihr, cex.labels=1.2, regLine=FALSE, col='blue',
          oma=c(1.95,3,4, 3), gap=.25, var.labels=varLabs, smooth=smoothPars)
title(main="A: Untransformed scales:", outer=TRUE,
      adj=0, line=-1.0, cex.main=1, font.main=1)
## Panel B: Repeat with log(nihills) in place of nihills,
## and with variable labels suitably modified.
varLabs <- c("\ntime\n(log h)", "\ndist\n(log miles)", "\nclimb\n(log feet)",
             "\ngradient\n(log ft/mi)", "\nmph\n(log mph)")
car::spm(log(nihr), regLine=FALSE, col="blue", oma=c(1.95,2.5,4, 2.5),
          gap=.25, var.labels=varLabs, smooth=smoothPars)
title("B: Logarithmic scales", outer=TRUE,
      adj=0, line=-1.0, cex.main=1, font.main=1)
par(oldpar)
```

What is special about logarithmic transformations?

### Subsection 3.2.2: An equation that predicts dist/time

```
## Hold climb constant at mean on logarithmic scale
mphClimb.lm <- lm(mph ~ log(dist)+log(climb), data = nihr)
## Hold `gradient=climb/dist` constant at mean on logarithmic scale
mphGradient.lm <- lm(mph ~ log(dist)+log(gradient), data = nihr)
avRate <- mean(nihr$mph)
bClimb <- coef(mphClimb.lm)
```

```

constCl <- c(bClimb[1]+bClimb[3]*mean(log(nihr$climb)), bClimb[2])
bGradient <- coef(mphGradient.lm)
constSl <- c(bGradient[1]+bGradient[3]*mean((log(nihr$climb/nihr$dist))),
            bGradient[2])
# Use `dist` and `climb` as explanatory variables
coef(mphClimb.lm)
# Use `dist` and `gradient` as explanatory variables
coef(mphGradient.lm)

```

```

opar <- par(mfrow=c(1,2), mgp=c(2.25,0.5,0), mar=c(3.6,4.1,2.1,1.6))
lineCols <- c("red", adjustcolor("magenta",0.4))
yaxlab<-substitute(paste("Minutes per mile (Add ", ym, ")"), list(ym=round(avRate,2)))
car::crPlots(mphClimb.lm, terms = . ~ log(dist), xaxt='n',
            xlab="Distance", col.lines=lineCols, ylab=yaxlab)
axis(2, at=4:7, labels=paste(4:7))
labx <- c(4,8,16,32)
axis(1, at=log(2^(2:5)), labels=paste(2^(2:5)))
box(col="white")
mtext("A: Hold climb constant at mean value", adj=0,
      line=0.8, at=0.6, cex=1.15)
car::crPlots(mphGradient.lm, terms = . ~ log(dist), xaxt='n',
            xlab="Distance", col.lines=lineCols, ylab=yaxlab)
axis(1, at=log(2^(2:5)), labels=paste(2^(2:5)))
axis(2, at=4:7, labels=paste(4:7))
box(col="white")
mtext("B: Hold log(gradient) constant at mean", adj=0, line=0.8, at=0.6, cex=1.15)
par(opar)

```

```

summary(mphClimb.lm, corr=T)$correlation["log(dist)", "log(climb)"]
summary(mphGradient.lm, corr=T)$correlation["log(dist)", "log(gradient)"]

```

```

## Show the plots, with default captions
plot(mphClimb.lm, fg='gray')

```

```

plot(mphGradient.lm, caption=c('A: Resids vs Fitted', 'B: Normal Q-Q',
'C: Scale-Location', '', 'D: Resids vs Leverage'),
cex.caption=0.85, fg='gray')

```

### Subsection 3.2.3: Equations that predict log(time)

```
lognihr <- setNames(log(nihr), paste0("log", names(nihr)))
timeClimb.lm <- lm(logtime ~ logdist + logclimb, data = lognihr)
```

```
print(coef(summary(timeClimb.lm)), digits=2)
```

```
timeGradient.lm <- lm(logtime ~ logdist + loggradient, data=lognihr)
print(coef(summary(timeGradient.lm)), digits=3)
```

### Subsection 3.2.4: Book dimensions — the oddbooks dataset

```
oldpar <- par(fg='gray40',col.axis='gray20',lwd=0.5,col.lab='gray20')
## Code for Panel A
oddbooks <- DAAG::oddbooks
pairs(log(oddbooks), lower.panel=panel.smooth, upper.panel=panel.smooth,
      labels=c("log(thick)", "log(breadth)", "log(height)", "log(weight)"),
      gap=0.25, oma=c(1.95,1.95,4, 1.95), col='blue')
title(main="A: Columns from log(oddbooks)",
      outer=TRUE, adj=0, line=-1.0, cex.main=1.1, font.main=1)
## Panel B
oddothers <-
  with(oddbooks, data.frame(density = weight/(breadth*height*thick),
    area = breadth*height, thick=thick, weight=weight))
pairs(log(oddothers), lower.panel=panel.smooth, upper.panel=panel.smooth,
      labels=c("log(density)", "log(area)", "log(thick)", "log(weight)"),
      gap=0.5, oma=c(1.95,1.95,4, 1.95), col='blue')
title("B: Add density & area; omit breadth & height",
      outer=TRUE, adj=0, line=-1.0, cex.main=1.1, font.main=1)
par(oldpar)
```

```
lob3.lm <- lm(log(weight) ~ log(thick)+log(breadth)+log(height),
             data=oddbooks)
# coef(summary(lob3.lm))
```

```
lob2.lm <- lm(log(weight) ~ log(thick)+log(breadth), data=oddbooks)
coef(summary(lob2.lm))
```

```
lob0.lm <- lm(log(weight) ~ 1, data=oddbooks)
add1(lob0.lm, scope=~log(breadth) + log(thick) + log(height))
lob1.lm <- update(lob0.lm, formula=. ~ .+log(breadth))
```

```
round(rbind("lob1.lm"=predict(lob1.lm), "lob2.lm"=predict(lob2.lm),
           "lob3.lm"=predict(lob3.lm)),2)
```

```
oddbooks <- within(oddbooks, density <- weight/(thick*breadth*height))
lm(log(weight) ~ log(density), data=oddbooks) |> summary() |> coef() |>
  round(3)
```

```
## Code that the reader may care to try
lm(log(weight) ~ log(thick)+log(breadth)+log(height)+log(density),
   data=oddbooks) |> summary() |> coef() |> round(3)
```

### Subsection 3.2.5: Mouse brain weight example

```
oldpar <- par(fg='gray40',col.axis='gray20',lwd=0.5,col.lab='gray20')
litters <- DAAG::litters
pairs(litters, labels=c("lsize\n\n(litter size)", "bodywt\n\n(Body Weight)",
                      "brainwt\n\n(Brain Weight)"), gap=0.5, fg='gray',
      col="blue", oma=rep(1.95,4))
par(oldpar)
```

```
## Regression of brainwt on lsize
summary(lm(brainwt ~ lsize, data = litters), digits=3)$coef
## Regression of brainwt on lsize and bodywt
summary(lm(brainwt ~ lsize + bodywt, data = litters), digits=3)$coef
```

### Subsection 3.2.6: Issues for causal interpretation

#### Effects of lifestyle on health

The studies mostly agree. But what do they say?

#### Adjusting for confounders

## Section 3.3 Choosing the model, and checking it out

### Effects of lifestyle on health

The studies mostly agree. But what do they say?

#### Adjusting for confounders

```
oddbooks.lm <- lm((weight) ~ log(thick)+log(height)+log(breadth),
data=DAAG::oddbooks)
yterms <- predict(oddbooks.lm, type="terms")
```

#### Subsection 3.3.3: A more formal approach to the choice of transformation

```
## Use car::powerTransform
nihr <- within(DAAG::nihills, {mph <- dist/time; gradient <- climb/dist})
summary(car::powerTransform(nihr[, c("dist", "gradient")]), digits=3)
```

```
form <- mph ~ log(dist) + log(gradient)
summary(car::powerTransform(form, data=nihr))
```

#### The use of transformations — further comments

#### Subsection 3.3.4: Accuracy estimates, fitted values and new observations

```
lognihr <- log(DAAG::nihills)
names(lognihr) <- paste0("log", names(lognihr))
timeClimb.lm <- lm(logtime ~ logdist + logclimb, data = lognihr)
## Coverage intervals; use exp() to undo the log transformation
citimes <- exp(predict(timeClimb.lm, interval="confidence"))
## Prediction intervals, i.e., for new observations
pitimes <- exp(predict(timeClimb.lm, newdata=lognihr, interval="prediction"))
## fit ci:lwr ci:upr pi:lwr pi:upr
ci_then_pi <- cbind(citimes, pitimes[,2:3])
colnames(ci_then_pi) <- paste0(c("", rep(c("ci-", "pi-"), c(2,2))),
colnames(ci_then_pi))
```

```

## First 4 rows
print(ci_then_pi[1:4,], digits=2)

timeClimb2.lm <- update(timeClimb.lm, formula = . ~ . + I(logdist^2))
g3.10 <-
function(model1=timeClimb.lm, model2=timeClimb2.lm)
{
  ## Panel A
  citimes <- predict(model1, interval="confidence")
  ord <- order(citimes[, "fit"])
  citimes <- citimes[ord,]
  hat <- citimes[, "fit"]
  pitimes <- predict(model1, newdata=lognihr, interval="prediction")[ord,]
  logobs <- log(nihr[ord, "time"])
  xtiks <- pretty(exp(hat))
  ylim <- range(c(pitimes[, "lwr"], pitimes[, "upr"], logobs)-rep(hat, 3))
  logytiks <- pretty(ylim, 5)
  ytiks <- round(exp(logytiks), 2)
  xlim <- range(hat)
  plot(hat, citimes[, "lwr"]-hat, type="n", xlab = "Time (fitted)",
       ylab = "Difference from fit",
       xlim=xlim, ylim = ylim, xaxt="n", yaxt="n", fg="gray")
  mtext(side=3, line=0.75, adj=0, at=-2.0, "A: CIs and PIs: Mean, prediction")
  mtext(side=4, line=1.25, "exp(Difference from fit)", las=0)
  axis(1, at=log(xtiks), labels=paste(xtiks), lwd=0, lwd.ticks=1)
  axis(2, at=logytiks, las=1, lwd=0, lwd.ticks=1)
  axis(4, at=logytiks, labels=paste(ytiks), las=0, lwd=0, lwd.ticks=1)
  points(hat, logobs-hat, pch=16, cex=0.65)
  lines(hat, citimes[, "lwr"]-hat, col = "red")
  lines(hat, citimes[, "upr"]-hat, col = "red")
  lines(hat, pitimes[, "lwr"]-hat, col = "black")
  lines(hat, pitimes[, "upr"]-hat, col = "black")
  ## Panel B
  citimes2 <- predict(model2, interval="confidence")[ord,]
  plot(hat, citimes[, "lwr"]-hat, type="n", xlab = "Time (fitted)",
       ylab = "Difference from fit",
       xlim=xlim, ylim = ylim, xaxt="n", yaxt="n", fg="gray")
  mtext(side=3, line=0.75, adj=0, at=-2.0,
       "B: CIs for fit, compare two models")
  mtext(side=4, line=1.25, "exp(Difference from fit)", las=0)
  axis(1, at=log(xtiks), labels=paste(xtiks), lwd=0, lwd.ticks=1)
  axis(2, at=logytiks, las=1, lwd=0, lwd.ticks=1)

```

```
axis(4, at=logytiks, labels=paste(ytiks), las=0,, lwd=0, lwd.ticks=1)
points(hat, logobs-hat, pch=16, cex=0.65)
lines(hat, citimes[,"lwr"]-hat, col = "red")
lines(hat, citimes[,"upr"]-hat, col = "red")
hat2 <- citimes2[,"fit"]
lines(hat, citimes2[,"lwr"]-hat2, col = "blue", lty=2, lwd=1.5)
lines(hat, citimes2[,"upr"]-hat2, col = "blue", lty=2, lwd=1.5)
}
```

```
timeClimb2.lm <- update(timeClimb.lm, formula = . ~ . + I(logdist^2))
```

### Subsection 3.3.5: Choosing the model — deaths from Atlantic hurricanes

```
oldpar <- par(fg='gray20',col.axis='gray20',lwd=0.5,col.lab='gray20')
hurric <- DAAG::hurricNamed[,c("LF.PressureMB", "BaseDam2014", "deaths")]
thurric <- car::powerTransform(hurric, family="yjPower")
transY <- car::yjPower(hurric, coef(thurric, round=TRUE))
smoothPars <- list(col.smooth='red', lty.smooth=2, lwd.smooth=1, spread=0)
car::spm(transY, lwd=0.5, regLine=FALSE, oma=rep(2.5,4), gap=0.5,
          col="blue", smooth=smoothPars, cex.labels=1)
par(oldpar)
```

```
modelform <- deaths ~ log(BaseDam2014) + LF.PressureMB
powerT <- car::powerTransform(modelform, data=as.data.frame(hurric),
                              family="yjPower")
summary(powerT, digits=3)
```

```
deathP <- with(hurric, car::yjPower(deaths, lambda=-0.2))
power.lm <- MASS::rlm(deathP ~ log(BaseDam2014) + LF.PressureMB, data=hurric)
print(coef(summary(power.lm)),digits=2)
```

```
## Use (deaths+1)^(-0.2) as outcome variable
plot(power.lm, cex.caption=0.85, fg="gray",
      caption=list('A: Resids vs Fitted', 'B: Normal Q-Q', 'C: Scale-Location', '',
                   'D: Resids vs Leverage'))
```

### Subsection 3.3.6: Strategies for fitting models — suggested steps

#### Diagnostic checks



## Section 3.4 Robust regression, outliers, and influence

### Subsection 3.4.1: Making outliers obvious — robust regression

```
hills2000 <- DAAG::hills2000[,c("dist", "climb", "time")]
varLabels <- c("\ndist\n(log miles)", "\nclimb\n(log feet)", "\ntime\n(log hours)")
smoothPars <- list(col.smooth='red', lty.smooth=2, lwd.smooth=1, spread=0)
hills2000 <- DAAG::hills2000[,c("dist", "climb", "time")]
varLabels <- c("\ndist\n(log miles)", "\nclimb\n(log feet)", "\ntime\n(log hours)")
car::spm(log(hills2000), smooth=smoothPars, regLine=FALSE, cex.labels=1.5,
var.labels = varLabels, lwd=0.5, gap=0.5, oma=c(1.95,1.95,1.95,1.95))

## Panel A
lhills2k.lm <- lm(log(time) ~ log(climb) + log(dist), data = hills2000)
plot(lhills2k.lm, caption="", which=1, fg="gray", col=adjustcolor("black", alpha=0.8))
mtext(side=3, line=0.75, "A: Least squares (lm) fit", adj=0, cex=1.1)

## Panel B
lhills2k.lqs <- MASS::lqs(log(time) ~ log(climb) + log(dist), data = hills2000)
reres <- residuals(lhills2k.lqs)
refit <- fitted(lhills2k.lqs)
big3 <- which(abs(reres) >= sort(abs(reres), decreasing=TRUE)[3])
plot(reres ~ refit, xlab="Fitted values (resistant fit)",
ylab="Residuals (resistant fit)", col=adjustcolor("black", alpha=0.8), fg="gray")
lines(lowess(reres ~ refit), col=2)
text(reres[big3] ~ refit[big3], labels=rownames(hills2000)[big3],
pos=4-2*(refit[big3] > mean(refit)), cex=0.8)
mtext(side=3, line=0.75, "B: Resistant (lqs) fit", adj=0, cex=1.1)

## Show only the 2nd diagnostic plot, i.e., a normal Q-Q plot
## plot(lhills2k.lm, which=2)
```

Outliers, influential or not, should be taken seriously

### Subsection 3.4.2: Leverage, influence, and Cook's distance

\* Leverage and the hat matrix — technical details

```
round(unnamed(hatvalues(timeClimb.lm)),2)
```

## Influential points and Cook's distance

### Dynamic graphics

```
## Residuals versus leverages
nihills <- DAAG::nihills
timeClimb.lm <- lm(log(time) ~ log(dist) + log(climb), data = nihills)
plot(timeClimb.lm, which=5, add.smooth=FALSE, ps=9, sub.caption="",
      cex.caption=1.1, fg="gray")
## The points can alternatively be plotted using
## plot(hatvalues(model.matrix(timeClimb.lm)), residuals(timeClimb.lm))
```

```
## Residuals versus leverages
plot(timeClimb.lm, which=5, add.smooth=FALSE)
## The points can alternatively be plotted using
## plot(hatvalues(model.matrix(timeClimb.lm)), residuals(timeClimb.lm))
```

```
## This code is designed to be evaluated separately from other chunks
with(nihills, scatter3d(x=log(dist), y=log(climb), z=log(time), grid=FALSE,
                        point.col="black", surface.col="gray60",
                        surface.alpha=0.2, axis.scales=FALSE))
with(nihills, Identify3d(x=log(dist), y=log(climb), z=log(time),
                         labels=row.names(DAAG::nihills), minlength=8), offset=0.05)
## To rotate display, hold down the left mouse button and move the mouse.
## To put labels on points, right-click and drag a box around them, perhaps
## repeatedly. Create an empty box to exit from point identification mode.
```

## Influence on the regression coefficients

```
## Residuals versus leverages
nihills <- DAAG::nihills
timeClimb.lm <- lm(log(time) ~ log(dist) + log(climb), data = nihills)
plot(timeClimb.lm, which=5, add.smooth=FALSE, ps=9, sub.caption="",
      cex.caption=1.1, fg="gray")
## The points can alternatively be plotted using
## plot(hatvalues(model.matrix(timeClimb.lm)), residuals(timeClimb.lm))
```

### \*Additional diagnostic plots

```
## As an indication of what is available, try
car::influencePlot(allbacks.lm)
```

## Section 3.5 Assessment and comparison of regression models

### Subsection 3.5.1: \*AIC, AICc, BIC, and Bayes Factors for normal theory regression models

```
## Calculations using mouse brain weight data
mouse.lm <- lm(brainwt ~ lsize+bodywt, data=DAAG::litters)
mouse0.lm <- update(mouse.lm, formula = . ~ . - lsize)
```

```
aicc <- sapply(list(mouse0.lm, mouse.lm), AICcmodavg::AICc)
infstats <- cbind(AIC(mouse0.lm, mouse.lm), AICc=aicc,
                  BIC=BIC(mouse0.lm, mouse.lm)[,-1])
print(rbind(infstats, "Difference"=apply(infstats,2,diff)), digits=3)
```

```
library(lattice)
df <- data.frame(n=5:35, AIC=rep(2,31), BIC=log(5:35))
cfAICc <- function(n,p,d) 2*(p+d)*n/(n-(p+d)-1) - 2*p*n/(n-p-1)
df <- cbind(df, AICc12=cfAICc(5:35,1,1), AICc34=cfAICc(5:35,3,1))
labs <- sort(c(2^(0:6), 2^(0:6)*1.5))
xyplot(AICc12+AICc34+AIC+BIC ~ n, data=df, type='l', auto.key=list(columns=4),
       scales=list(y=list(log=T, at=labs, labels=paste(labs))),
       par.settings=simpleTheme(lty=c(1,1:3), lwd=2, col=rep(c('gray','black'), c(1,3)))))
```

### The functions drop1() and add1()

```
## Obtain AIC or BIC using `drop1()` or `add1()`
n <- nrow(DAAG::litters)
drop1(mouse.lm, scope=~lsize) # AIC, with/without `lsize`
drop1(mouse.lm, scope=~lsize, k=log(n)) # BIC, w/wo `lsize`
add1(mouse0.lm, scope=~bodywt+lsize) # AIC, w/wo `lsize`, alternative
```

### The use of Bayesfactor::lmBF to compare the two models

```
suppressPackageStartupMessages(library(BayesFactor))
bf1 <- lmBF(brainwt ~ bodywt, data=DAAG::litters)
bf2 <- lmBF(brainwt ~ bodywt+lsize, data=DAAG::litters)
bf2/bf1
```

```
## Relative support statistics
setNames(exp(-apply(infstats[, -1], 2, diff)/2), c("AIC", "AICc", "BIC"))
```

### Subsection 3.5.2: Using anova() to compare models — the ihills data

```
lognihr <- log(DAAG::nihills)
lognihr <- setNames(log(nihr), paste0("log", names(nihr)))
timeClimb.lm <- lm(logtime ~ logdist + logclimb, data = lognihr)
timeClimb2.lm <- update(timeClimb.lm, formula = . ~ . + I(logdist^2))
print(anova(timeClimb.lm, timeClimb2.lm, test="F"), digits=4)
```

```
print(anova(timeClimb.lm, timeClimb2.lm, test="Cp"), digits=3)
## Compare with the AICc difference
sapply(list(timeClimb.lm, timeClimb2.lm), AICcmodavg::AICc)
```

```
form1 <- update(formula(timeClimb.lm), ~ . + I(logdist^2) + logdist:logclimb)
addcheck <- add1(timeClimb.lm, scope=form1, test="F")
print(addcheck, digits=4)
```

### Subsection 3.5.3: Training/test approaches, and cross-validation

```
## Check how well timeClimb.lm model predicts for hills2000 data
timeClimb.lm <- lm(logtime ~ logdist + logclimb, data = lognihr)
logscot <- log(subset(DAAG::hills2000,
                     !row.names(DAAG::hills2000)=="Caerketton"))
names(logscot) <- paste0("log", names(hills2000))
scotpred <- predict(timeClimb.lm, newdata=logscot, se=TRUE)
trainVar <- summary(timeClimb.lm)[["sigma"]]^2
trainDF <- summary(timeClimb.lm)[["df"]][2]
mspe <- mean((logscot[, 'logtime']-scotpred[['fit']])^2)
mspeDF <- nrow(logscot)
```

```
pf(mspe/trainVar, mspeDF, trainDF, lower.tail=FALSE)
```

```
scot.lm <- lm(logtime ~ logdist+logclimb, data=logscot)
signif(summary(scot.lm)[['sigma']]^2, 4)
```

#### Subsection 3.5.4: Further points and issues

Patterns in the diagnostic plots – are they more than hints?

```
{r 3_18, eval=F}
```

What is the scatter about the fitted response

Model selection and tuning risks

Generalization to new contexts requires a random sample of contexts

What happens if we do not transform the hillrace data?

Are “errors in x” an issue?

### Section 3.6 Problems with many explanatory variables

#### Subsection 3.6.1: Variable selection issues

Variable selection – a simulation with random data

```
y <- rnorm(100)
## Generate a 100 by 40 matrix of random normal data
xx <- matrix(rnorm(4000), ncol = 40)
dimnames(xx) <- list(NULL, paste("X", 1:40, sep=""))
```

```
## ## Find the best fitting model. (The 'leaps' package must be installed.)
xx.subsets <- leaps::regsubsets(xx, y, method = "exhaustive", nvmax = 3, nbest = 1)
subvar <- summary(xx.subsets)$which[3,-1]
best3.lm <- lm(y ~ -1+xx[, subvar])
print(summary(best3.lm, corr = FALSE))
```

```
## DAAG::bestsetNoise(m=100, n=40)
best3 <- capture.output(DAAG::bestsetNoise(m=100, n=40))
cat(best3[9:14], sep='\n')
```

### The extent of selection effects – a detailed simulation:

```
oldpar <- par(fg='gray20',col.axis='gray20',lwd=0.5,col.lab='gray20')
set.seed(41)
library(splines)
DAAG::bsnVaryNvar(nvmax=3, nvar = 3:35, xlab="")
mtext(side=1, line=1.75, "Number selected from")
```

### Cross-validation that accounts for the variable selection process

#### \*Regularization approaches

### Subsection 3.6.2: Multicollinearity

#### An example – compositional data

```
data(Coxite, package="compositions") # Places Coxite in the workspace
# NB: Proceed thus because `Coxite` is not exported from `compositions`
coxite <- as.data.frame(Coxite)
```

```
oldpar <- par(fg='gray20',col.axis='gray20',lwd=0.5,col.lab='gray20', tcl=-0.25)
panel.cor <- function(x, y, digits = 3, prefix = "", cex.cor=0.8, ...)
{
  old.par <- par(usr = c(0, 1, 0, 1)); on.exit(par(old.par))
  r <- abs(cor(x, y))
  txt <- format(c(r, 0.123456789), digits = digits)[1]
  txt <- paste0(prefix, txt)
  if(missing(cex.cor)) cex.cor <- 0.8/strwidth(txt)
  text(0.5, 0.5, txt, cex = cex.cor * sqrt(r))
}
pairs(coxite, gap=0.4, col=adjustcolor("blue", alpha=0.9), upper.panel=panel.cor)
par(oldpar)
```

```
coxiteAll.lm <- lm(porosity ~ A+B+C+D+E+depth, data=coxite)
print(coef(summary(coxiteAll.lm)), digits=2)
```

```
coxiteAll.lm <- lm(porosity ~ A+B+C+D+E+depth, data=coxite)
coxite.hat <- predict(coxiteAll.lm, interval="confidence")
hat <- coxite.hat[, "fit"]
plot(porosity ~ hat, data=coxite, fg="gray", type="n", xlab="Fitted values",
     ylab="Fitted values, with 95% CIs\n(Points are observed porosities)",
     tcl=-0.35)
with(coxite, points(porosity ~ hat, cex=0.75, col="gray45"))
lines(hat, hat, lwd=0.75)
ord <- order(hat)
sebar <- function(x, y1, y2, eps=0.15, lwd=0.75){
  lines(rep(x,2), c(y1,y2), lwd=lwd)
  lines(c(x-eps,x+eps), rep(y1,2), lwd=lwd)
  lines(c(x-eps,x+eps), rep(y2,2), lwd=lwd)
}
q <- ord[round(quantile(1:length(hat), (1:9)/10))]
for(i in q)sebar(hat[i], coxite.hat[i, "lwr"], coxite.hat[i, "upr"])
coxiteAll.lm <- lm(porosity ~ A+B+C+D+E+depth, data=coxite)
coxite.hat <- predict(coxiteAll.lm, interval="confidence")
hat <- coxite.hat[, "fit"]
```

```
## Pointwise confidence bounds can be obtained thus:
hat <- predict(coxiteAll.lm, interval="confidence", level=0.95)
```

### Subsection 3.6.3: The variance inflation factor (VIF)

```
print(DAAG::vif(lm(porosity ~ A+B+C+D+depth, data=coxite)), digits=2)
```

```
b <- leaps::regsubsets(porosity ~ ., data=coxite, nvmax=4, method='exhaustive')
## The calculation fails for nvmax=5
inOut <- summary(b)[["which"]]
## Extract and print the coefficients for the four regressions
dimnam <- list(rep("",4),c("Intercept", colnames(coxite)[-7]))
cmat <- matrix(nrow=4, ncol=7, dimnames=dimnam)
for(i in 1:4)cmat[i, inOut[i,]] <- signif(coef(b, id=1:4)[[i]], 3)
outMat <- cbind(cmat, " " = rep(NA, 4),
as.matrix(as.data.frame(summary(b)[c("adjr2", "cp", "bic")]))))
print(signif(outMat, 3), na.print="")
```

```
BC.lm <- lm(porosity ~ B+C, data=coxite)
print(signif(coef(summary(BC.lm)), digits=3))
car::vif(BC.lm)
```

```
## Diagnostic plots can be checked thus:
plot(BC.lm, eval=xtras)
```

## Numbers that do not quite add up

```
coxiteR <- coxite
coxiteR[, 1:5] <- round(coxiteR[, 1:5])
coxiteR.lm <- lm(porosity ~ ., data=coxiteR)
print(coef(summary(coxiteR.lm)), digits=2)
print(DAAG::vif(lm(porosity ~ .-E, data=coxiteR)), digits=2)
```

## Remedies for multicollinearity

### Section 3.7 Errors in x

#### Measurement of dietary intake

#### Simulations of the effect of measurement error

```
gph <- DAAG::errorsINx(gpdiff=0, plotit=FALSE, timesSDx=(1:4)/2,
                      layout=c(5,1), print.summary=FALSE)[["gph"]]
parset <- DAAG::DAAGtheme(color=FALSE, alpha=0.6, lwd=2,
                          col.points=c("gray50","black"),
                          col.line=c("gray50","black"), lty=1:2)
update(gph, par.settings=parset)
```

## \*Two explanatory variables

### Two explanatory variables, one measured without error – a simulation



```

gph <- DAAG::errorsINx(gpdiff=1.5, timesSDx=(1:2)*0.8, layout=c(3,1),
print.summary=FALSE, plotit=FALSE)[["gph"]]
parset <- DAAG::DAAGtheme(color=FALSE, alpha=0.6, lwd=2,
                           col.points=c("gray50","black"),
                           col.line=c("gray50","black"), lty=1:2)
update(gph, par.settings=parset)

```

## An arbitrary number of variables

### \*The classical error model versus the Berkson error model

### Using missing value approaches to address measurement error

## Section 3.8 Multiple regression models – additional points

```

coef(lm(area ~ volume + weight, data=allbacks))
b <- as.vector(coef(lm(weight ~ volume + area, data=allbacks)))
c("_Intercept_"=-b[1]/b[3], volume=-b[2]/b[3], weight=1/b[3])

```

## Unintended correlations

### Subsection 3.8.2: Missing explanatory variables

```

gaba <- DAAG::gaba
gabalong <- stack(gaba["30", -match('min', colnames(gaba))])
gabalong$sex <- factor(rep(c("male", "female", "all"), rep(2,3)),
levels=c("female", "male", "all"))
gabalong$treatment <- factor(rep(c("Baclofen", "No baclofen"), 3),
levels=c("No baclofen", "Baclofen"))
gph <- lattice::stripplot(sex~values, groups=treatment, data=gabalong,
panel=function(x,y,...){
lattice::panel.stripplot(x,y,...)
lattice::ltext(x,y,paste(c(3,9,15,7,22,12)), pos=1, cex=0.8)
}, auto.key=list(space="right", points=TRUE, cex=0.8))
bw9 <- list(fontsize=list(text=9, points=5),
cex=c(1.5,1.5), pch=c(1,16))

```

```
update(gph, par.settings=parset,
xlab=list("Average reduction: 30 min vs 0 min", cex=1.0),
scales=list(cex=1.0, tck=0.35))
```

## Strategies

### Subsection 3.8.3: Added variable plots

```
yONx.lm <- lm(logtime ~ logclimb, data=lognihr)
e_yONx <- resid(yONx.lm)
print(coef(yONx.lm), digits=4)
```

```
zONx.lm <- lm(logdist ~ logclimb, data=lognihr)
e_zONx <- resid(zONx.lm)
print(coef(yONx.lm), digits=4)
```

```
ey_xONez_x.lm <- lm(e_yONx ~ 0+e_zONx)
e_yONxz <- resid(ey_xONez_x.lm)
print(coef(ey_xONez_x.lm), digits=4)
```

```
oldpar <- par(fg='gray')
## Code for added variable plots
logtime.lm <- lm(logtime ~ logclimb+logdist, data=lognihr)
car::avPlots(logtime.lm, lwd=1, terms="logdist", fg="gray")
mtext(side=3, line=0.5, "A: Added var: 'logdist'", col="black", adj=0, cex=1.15)
car::avPlots(logtime.lm, lwd=1, terms="logclimb", fg="gray")
mtext(side=3, line=0.5, "B: Added var: 'logclimb'", col="black", adj=0, cex=1.15)
par(oldpar)
```

```
## One call to show both plots
car::avPlots(timeClimb.lm, terms=~.)
```

```
## Alternative code for first plot
plot(e_yONx ~ e_zONx)
```

```
plot(yONx.lm, which=1, caption="", fg="gray")
mtext(side=3, line=0.5, "A: From 'logtime' on 'logclimb'", adj=0, cex=0.85)
plot(zONx.lm, which=1, caption="", fg="gray")
mtext(side=3, line=0.5, "B: From 'logdist' on 'logclimb'", adj=0, cex=0.85)
plot(ey_xONez_x.lm, which=1, caption="", fg="gray")
mtext(side=3, line=0.5, "C: From AVP", adj=-0, cex=0.85)
```

## Alternatives to Added Variable Plots

### \*Algebraic details

```
ab1 <- coef(yONx.lm)
ab2 <- coef(zONx.lm)
b2 <- coef(ey_xONez_x.lm)
b1 <- ab1[2] - b2*ab2[2]
a <- ab1[1] - b2*ab2[1]
```

```
coef(lm(logtime ~ logclimb + logdist, data=lognihr))
```

## Subsection 3.8.4: Nonlinear methods – an alternative to transformation?

```
nihr$climb.mi <- nihr$climb/5280
nihr.nls0 <- nls(time ~ (dist^alpha)*(climb.mi^beta), start =
                c(alpha = 0.68, beta = 0.465), data = nihr)
## plot(residuals(nihr.nls0) ~ log(predict(nihr.nls0)))
```

```
signif(coef(summary(nihr.nls0)),3)
```

```
nihr.nls <- nls(time ~ gamma + delta1*dist^alpha + delta2*climb.mi^beta,
start=c(gamma = .045, delta1 = .09, alpha = 1,
delta2=.9, beta = 1.65), data=nihr)
## plot(residuals(nihr.nls) ~ log(predict(nihr.nls)))
```

```
signif(coef(summary(nihr.nls)),3)
```

## Section 3.9: Recap

## Section 3.10: Further reading

### Exercises (3.11)

3.1

```
## ## Set up factor that identifies the `have' cities
cities <- DAAG::cities
cities$have <- with(cities, factor(REGION %in% c("ON","WEST"),
                                labels=c("Have-not","Have")))
```

```
gphA <- lattice::xyplot(POP1996~POP1992, groups=have, data=cities,
                        auto.key=list(columns=2))
gphB<-lattice::xyplot(log(POP1996)~log(POP1992), groups=have, data=cities,
                      auto.key=list(columns=2))
print(gphA, split=c(1,1,2,1), more=TRUE)
print(gphB, split=c(2,1,2,1))
```

```
cities.lm1 <- lm(POP1996 ~ have+POP1992, data=cities)
cities.lm2 <- lm(log(POP1996) ~ have+log(POP1992), data=cities)
```

3.8a

```
nihills.lm <- lm(time ~ dist+climb, data=DAAG::nihills)
nihillsX.lm <- lm(time ~ dist+climb+dist:climb, data=DAAG::nihills)
anova(nihills.lm, nihillsX.lm) # Use `anova()` to make the comparison
coef(summary(nihillsX.lm))     # Check coefficient for interaction term
drop1(nihillsX.lm)
```

3.11

```
log(time) ~ log(dist) + log(climb) ## lm model
time ~ alpha*dist + beta*I(climb^2) ## nls model
```

3.13

```

x1 <- runif(10)           # predictor which will be missing
x2 <- rbinom(10, 1, 1-x1)
  ## observed predictor, depends on missing predictor
y <- 5*x1 + x2 + rnorm(10,sd=.1) # simulated model; coef of x2 is positive
y.lm <- lm(y ~ factor(x2)) # model fitted to observed data
coef(y.lm)
y.lm2 <- lm(y ~ x1 + factor(x2)) # correct model
coef(y.lm2)

```

3.16

```

bomData <- DAAG::bomregions2021
nraw.lqs <- MASS::lqs(northRain ~ SOI + CO2, data=bomData)
north.lqs <- MASS::lqs(I(northRain^(1/3)) ~ SOI + CO2, data=bomData)
plot(residuals(nraw.lqs) ~ Year, data=bomData)
plot(residuals(north.lqs) ~ Year, data=bomData)

```

3.17f

```

socpsych <- subset(DAAG::repPsych, Discipline=='Social')
with(socpsych, scatter.smooth(T_r.R~T_r.0))
abline(v=.5)

```

```

soc.rlm <- MASS::rlm(T_r.R~T_r.0, data=subset(socpsych, T_r.0<=0.5))
## Look at summary statistics
termplot(soc.rlm, partial.resid=T, se=T)

```

```

plot(soc.rlm)

```

```

if(file.exists("/Users/johnm1/pkgsg/PGRcode/inst/doc/")){
code <- knitr::knit_code$get()
txt <- paste0("\n## ", names(code),"\n", sapply(code, paste, collapse='\n'))
writeLines(txt, con="/Users/johnm1/pkgsg/PGRcode/inst/doc/ch3.R")
}

```

## 4 Chapter 4: Exploiting the linear model framework

### Packages required (with dependencies)

DAAG effects mgcv splines scam MASS latticeExtra car WDI AICcmodavg ggplot2 kableExtra qgam patchwork

Additionally, Hmisc and knitr are required in order to process the Rmd source file.

Note the use of the ‘patchwork’ package to make it easy to place two ggplot2 plots side by side.

```
Hmisc::knitrSet(basename="exploit", lang='markdown', fig.path="figs/g", w=7, h=7)
oldopt <- options(digits=4, width=70, scipen=999)
library(knitr)
## knitr::render_listings()
opts_chunk[["set"]](cache.path='cache-', out.width="80%", fig.align="center",
                    fig.show='hold', formatR.arrow=FALSE, ps=10,
                    strip.white = TRUE, comment=NA, width=70,
                    tidy.opts = list(replace.assign=FALSE))
```

### Section 4.1 Levels of a factor – using indicator variables

#### Subsection 4.1.1: Example – sugar weight

```
sugar <- DAAG::sugar # Copy dataset 'sugar' into the workspace
## Ensure that "Control" is the first level
sugar[["trt"]] <- relevel(sugar[["trt"]], ref="Control")
options()[["contrasts"]] # Check the default factor contrasts
## If your output does not agree with the above, then enter
## options(contrasts=c("contr.treatment", "contr.poly"))
```

```
sugar.aov <- aov(weight ~ trt, data=sugar)
## To display the model matrix, enter: model.matrix(sugar.aov)
## Note the use of summary.lm(), not summary() or summary.aov()
round(signif(coef(summary.lm(sugar.aov)), 3), 4)
```

```
sem <- summary.lm(sugar.aov)$sigma/sqrt(3) # 3 results/trt
# Alternatively, sem <- 6.33/sqrt(2)
qtukey(p=.95, nmeans=4, df=8) * sem
```

### Subsection 4.1.2: Different choices for the model matrix when there are factors

```
contrasts(sugar$trt) <- 'contr.sum'
sugarSum.aov <- aov(weight ~ trt, data = sugar)
round(signif(coef(summary.lm(sugarSum.aov)), 3), 4)
```

```
dummy.coef(sugarSum.aov)
```

### Factor contrasts – further details

```
contrasts(sugar$trt) <- "contr.sum"
```

```
fish <- factor(1:3, labels=c("Trout","Cod","Perch"))
```

```
contr.treatment(fish)
# Base is "Trout"
```

```
contr.SAS(fish)
# Base is "Perch"
```

```
contr.sum(fish)
# Base is mean of levels
```

**\*Tests for main effects in the presence of interactions?**

## Section 4.2 Block designs and balanced incomplete block designs

### Subsection 4.2.1: Analysis of the rice data, allowing for block effects

```
rice <- DAAG::rice
ricebl.aov <- aov(ShootDryMass ~ Block + variety * fert, data=rice)
print(summary(ricebl.aov), digits=3)
```

```
round(signif(coef(summary.lm(ricebl.aov)), 3), 5)
with(summary.lm(ricebl.aov),
cat("Residual standard error: ", sigma, "on", df[2], "degrees of freedom"))
```

```
## AOV calculations, ignoring block effects
rice.aov <- aov(ShootDryMass ~ variety * fert, data=rice)
summary.lm(rice.aov)$sigma
```

```
ricebl.aov <- aov(ShootDryMass ~ factor(Block) + variety * fert, data=rice)
```

```
model.tables(ricebl.aov, type="means", se=TRUE, cterms="variety:fert")
```

### Subsection 4.2.2: A balanced incomplete block design

```
appletaste <- DAAG::appletaste
with(appletaste, table(product, panelist))
```

```
sapply(appletaste, is.factor) # panelist & product are factors
appletaste.aov <- aov(aftertaste ~ product + panelist, data=appletaste)
summary(appletaste.aov)
```

```
as.data.frame(effects::Effect("product", appletaste.aov, confidence.level=0.95))
```

```
## NB that 'product' was first term in the model formula
## Thus, the 1st 4 coefficients have the information required
coef(summary.lm(appletaste.aov))[1:4, ]
```

## Section 4.3 Fitting multiple lines



```
## Fit various models to columns of data frame leaftemp (DAAG)
leaftemp <- DAAG::leaftemp
leaf.lm1 <- lm(tempDiff ~ 1, data = leaftemp)
leaf.lm2 <- lm(tempDiff ~ vapPress, data = leaftemp)
leaf.lm3 <- lm(tempDiff ~ CO2level + vapPress, data = leaftemp)
leaf.lm4 <- lm(tempDiff ~ CO2level + vapPress +
               vapPress:CO2level, data = leaftemp)
```

```
anova(leaf.lm1, leaf.lm2, leaf.lm3, leaf.lm4)
```

```
print(coef(summary(leaf.lm3)), digits=2)
```

## Section 4.4 Methods for fitting smooth curves

### Subsection 4.4.1: Polynomial Regression

```
seedrates <- DAAG::seedrates
form2 <- grain ~ rate + I(rate^2)
# Without the wrapper function I(), rate^2 would be interpreted
# as the model formula term rate:rate, and hence as rate.
quad.lm2 <- lm(form2, data = seedrates)
## Alternative, using gam()
## quad.gam <- mgcv::gam(form2, data = seedrates)
```

```
suppressPackageStartupMessages(library(ggplot2))
```

```
## Use ggplot2 functions to plot points, line, curve, & 95% CIs
## library(ggplot2)
gph <- ggplot(DAAG::seedrates, aes(rate, grain)) +
  geom_point(aes(size=3), color='magenta') + xlim(c(25, 185))
colors <- c("Linear"="blue", "Quadratic"="red")
ggdat <- ggplot_build(gph + geom_smooth(aes(rate, grain, color="Linear"),
  method=lm, formula=y~poly(x, 2), fullrange=TRUE))$data[[2]]
gph1 <- gph + geom_smooth(aes(color="Linear"), method=lm, formula=y~x, fullrange=TRUE, fill='d
gph1 + geom_line(data = ggdat, aes(x = x, y = y, color="Quadratic"),
  linewidth=0.75) +
  geom_ribbon(data=ggdat, aes(x=x, y=y, ymin=ymin, ymax=ymax,
    color="Quadratic"), linewidth=0.75,
```

```

fill=NA, linetype=2, outline.type='both', show.legend=FALSE) +
scale_color_manual(values=colors, aesthetics = "color")+
theme(legend.position=c(.8,.78)) +
coord_cartesian(expand=FALSE) + xlab("Seeding rate (kg/ha)") +
  ylab("Grains per head") + labs(color="Model") +
guides(size='none',
        color = guide_legend(override.aes = list(fill="transparent") ) )
## detach("package:ggplot2")

```

```

quad.lm2 <- lm(grain ~ rate + I(rate^2), data = DAAG::seedrates)
print(coef(summary(quad.lm2)), digits=2)
cat("\nCorrelation matrix\n")
print(summary(quad.lm2, corr=TRUE)$correlation, digits=2)

```

### **\*An alternative formulation using orthogonal polynomials**

```

seedratesP.lm2 <- lm(grain ~ poly(rate,2), data = seedrates)
print(coef(summary(seedratesP.lm2)), digits=2)

```

```

## Alternative, using mgcv::gam()
seedratesP.gam <- mgcv::gam(grain ~ poly(rate,2), data = seedrates)

```

```

logseed.lm <- lm(log(grain) ~ log(rate), data=DAAG::seedrates)
coef(summary(logseed.lm))

```

```

## Use ggplot2 functions to plot points, line, curve, & 95% CIs
## library(ggplot2)
gph <- ggplot(DAAG::seedrates, aes(rate,grain)) +
  geom_point(size=3, color="magenta")+xlim(c(25,185))
colors <- c("Loglinear"="gray40", "Quadratic"="red")
ggdat <- ggplot_build(gph+geom_smooth(method=lm, formula=y~poly(x,2),
                                     fullrange=TRUE))$data[[2]]
ggln <- ggplot_build(gph+geom_smooth(method=lm,
                                     formula=log(y)~log(x),fullrange=TRUE))$data[[2]]
## Assign to gphA rather than (as in text) plotting at this point
gphA <- gph + geom_line(data = ggdat, aes(x = x, y = y, color="Quadratic"),
  linewidth=0.75) +
geom_ribbon(data=ggdat, aes(x=x,y=y, ymin=ymin, ymax=ymax, color="Quadratic"),
  linewidth=0.75, fill=NA, linetype=2, outline.type='both',
  show.legend=FALSE) +

```

```

geom_line(data = ggln, aes(x = x, y = exp(y), color="Loglinear"),
          linewidth = 0.75) +
geom_ribbon(data=ggln, aes(x=x,y=exp(y), ymin=exp(ymin), ymax=exp(ymax),
                          color="Loglinear"), fill=NA, linewidth=0.75, linetype=3,
          outline.type='both', show.legend=FALSE)+
scale_color_manual(values=colors, aesthetics = "color")+
coord_cartesian(expand=FALSE) +
xlab("Seeding rate (kg/ha)") + ylab("Grains per head") +
labs(title="A: Loglinear fit vs quadratic fit", color="Model") +
guides(size='none',
       color = guide_legend(override.aes = list(fill="transparent") ) ) +
theme(legend.position=c(.8,.78))
df <- data.frame(rate=rep(DAAG::seedrates$rate,2), res=c(resid(logseed.lm),
log(DAAG::seedrates$grain)-log(fitted(quad.lm2))),
Model=rep(c("Loglinear","Quadratic"),rep(nrow(DAAG::seedrates),2)))
## Assign to gphB rather than (as in text) plotting at this point
gphB <- ggplot(df, aes(x=rate, y=res, shape=Model,color=Model))+
geom_point(size=2.5) + scale_color_manual(values=colors) +
xlab("Seeding rate (kg/ha)") + ylab("Residuals on log scale") +
labs(title="B: Residuals") +
guides(size='none',
       color = guide_legend(override.aes = list(fill="transparent") ) ) +
theme(legend.position=c(.8,.78))
## Now take advantage of the magic of the 'patchwork' package
library(patchwork)
gphA+gphB
## detach("package:ggplot2")

```

```

aic <- AIC(quad.lm2, logseed.lm)
aic["logseed.lm",2] <- aic["logseed.lm",2] + sum(2*log(seedrates$grain))
round(aic,1)

```

```

seedrates<-DAAG::seedrates
quad.lm2 <- lm(grain ~ poly(rate,degree=2), data=seedrates)
ns.lm2 <- lm(grain ~ splines::ns(rate,df=2), data=seedrates)
tps.gam2 <- mgcv::gam(grain ~ s(rate, k=3, fx=T), data=seedrates)

```

```

mflist <- lapply(list(quad=quad.lm2, nsplines=ns.lm2, tps=tps.gam2), model.matrix)
mftab <- with(mflist, cbind(quad, nsplines, tps))
colnames(mftab) <- c("(Int)", "poly2.1", "poly2.2", "(Int)", "ns2.1", "ns2.2", "(Int)", "s3.1")
library(kableExtra)

```

```

linesep = c(' ', ' ', ' ', '\\\\addlinespace')
kbl(mftab, booktabs=TRUE, format='latex', toprule=FALSE,
format.args=list(justify="right", width=8)) |>
kable_styling(latex_options = c("scale_down", latex_options = "hold_position"), position='center')
add_header_above(c('poly(rate,2)' = 3, 'splines::ns(rate,df=2)' = 3, 's(rate, k=3, fx=T)' = 3),
align='c', monospace=rep(T,3))|>
add_header_above(c('lm: grain~' = 3, 'lm: grain~'=3, 'gam: grain~'=3),
align='c', monospace=rep(T,3), line=F)

```

## GAM models versus models fitted using lm()

### Alternative fits – what is the best choice?

```

## Load required packages
suppressPackageStartupMessages(library(splines))
suppressPackageStartupMessages(library(mgcv))

```

```

ohms.tp <- gam(kohms~s(juice, bs="tp"), data=fruitohms)
ohms.cs <- gam(kohms~s(juice, bs="cs"), data=fruitohms)
range(fitted(ohms.tp)-fitted(ohms.cs))

```

```
summary(ohms.tp)
```

```
summary(ohms.tpBIC)
```

### Subsection 4.4.3: The contributions of basis curves to the fit

### Subsection 4.4.4: Checks on the fitted model

```

## Printed output from `gam.check(ohms.tpBIC)`
cat(out, sep="\n")

```

### Subsection 4.3.5: Monotone curves

```

ohms.scam <- scam::scam(kohms ~ s(juice,bs="mpd"), data=fruitohms)
summary(ohms.scam)

```

```
AIC(ohms.scam, ohms.tp)
```

```
BIC(ohms.scam, ohms.tp)
```

#### Subsection 4.4.6: Different smooths for different levels of a factor

```
whiteside <- MASS::whiteside  
gas.gam <- gam(Gas ~ Insul+s(Temp, by=Insul), data=whiteside)
```

```
summary(gas.gam)
```

```
Box.test(resid(gas.gam)[whiteside$Insul=='Before'], lag=1)  
Box.test(resid(gas.gam)[whiteside$Insul=='After'], lag=1)
```

#### Subsection 4.4.7: The remarkable reach of mgcv and related packages

##### Departures from independence assumptions

#### Subsection 4.4.8: Multiple spline smoothing terms — dewpoint data

```
## GAM model -- `dewpoint` data  
dewpoint <- DAAG::dewpoint  
ds.gam <- gam(dewpt ~ s(mintemp) + s(maxtemp), data=dewpoint)  
plot(ds.gam, resid=TRUE, pch=".", se=2, cex=2, fg="gray")
```

##### Using residuals as a check for non-additive effects

```
library(lattice)  
## Residuals vs maxtemp, for different mintemp ranges  
mintempRange <- equal.count(dewpoint$mintemp, number=3)  
ds.xy <- xyplot(residuals(ds.gam) ~ maxtemp|mintempRange, data=dewpoint,  
               layout=c(3,1), scales=list(tck=0.5), aspect=1, cex=0.65,  
               par.strip.text=list(cex=0.75), type=c("p","smooth"),  
               xlab="Maximum temperature", ylab="Residual")  
ds.xy
```

### \*A smooth surface

```
## Fit surface
ds.tp <- gam(dewpt ~ s(mintemp, maxtemp), data=DAAG::dewpoint)
vis.gam(ds.tp, plot.type="contour") # gives a contour plot of the
# fitted regression surface
vis.gam(ds.gam, plot.type="contour") # cf, model with 2 smooth terms
```

### Subsection 4.4.9: Atlantic hurricanes that made landfall in the US

```
hurricNamed <- DAAG::hurricNamed
hurricS.gam <- gam(car::yjpPower(deaths, lambda=-0.2) ~
  s(log(BaseDam2014)) + s(LF.PressureMB),
  data=hurricNamed, method="ML")
anova(hurricS.gam)
```

```
plot(hurricS.gam, resid=TRUE, pch=16, cex=0.5, select=1, fg="gray")
mtext(side=3, line=1, "A: Term in log(BaseDam2014)", cex=1.0, adj=0, at=-3.75)
plot(hurricS.gam, resid=TRUE, pch=16, cex=0.5, select=2, fg="gray")
mtext(side=3, line=1, "B: Term in LF.PressureMB", cex=1.0, adj=0, at=878)
qqnorm(resid(hurricS.gam), main="", fg="gray")
mtext(side=3, line=1, "C: Q-Q plot of residuals", cex=1.0, adj=0, at=-4.25)
```

### An explanatory variable with an overly long-tailed distribution

```
hurricSlog1.gam <- gam(log(deaths+1) ~ s(log(BaseDam2014)), data=hurricNamed)
hurricSlog2.gam <- gam(log(deaths+1) ~ s(BaseDam2014), data=hurricNamed)
```

```
plot(hurricSlog1.gam, resid=TRUE, pch=16, cex=0.5, adj=0, fg="gray")
mtext(side=3, "A: Use log(BaseDam2014)", cex=1.4, adj=0, line=1, at=-3.15)
plot(hurricSlog2.gam, resid=TRUE, pch=16, cex=0.5, fg="gray")
mtext(side=3, "B: Use BaseDam2014", cex=1.4, adj=0, line=1, at=-28500)
```

### Subsection 4.4.10: Other smoothing methods

## Section 4.5 Quantile regression

```
## If necessary, install the 'WDI' package & download data
if(!file.exists("wdi.RData")){
  if(!is.element("WDI", installed.packages()[,1])) install.packages("WDI")
  inds <- c('SP.DYN.TFRT.IN', 'SP.DYN.LE00.IN', 'SP.POP.TOTL')
  indnams <- c("FertilityRate", "LifeExpectancy", "population")
  wdi2020 <- WDI::WDI(country="all", indicator=inds, start=2020, end=2020,
    extra=TRUE)
  wdi2020 <- na.omit(droplevels(subset(wdi2020, !region %in% "Aggregates"))))
  wdi <- setNames(wdi2020[order(wdi2020[, inds[1]]),inds], indnams)
  save(wdi, file="wdi.RData")
}
```

## 2020 World Bank data on fertility and life expectancy

```
load("wdi.RData") # Needs `wdi.RData` in working directory; see footnote
library(qgam)
wdi[, "ppop"] <- with(wdi, population/sum(population))
wdi[, "logFert"] <- log(wdi[, "FertilityRate"])
form <- LifeExpectancy ~ s(logFert)
## Panel A model
fit.qgam <- qgam(form, data=wdi, qu=.5)
## Panel B: Multiple (10%, 90% quantiles; unweighted, then weighted
fit19.mqgam <- mqgam(form, data=wdi, qu=c(.1,.9))
wtd19.mqgam <- mqgam(form, data=wdi, qu=c(.1,.9),
  argGam=list(weights=wdi[["ppop"]]))
```

```
hat50 <- cbind(LifeExpectancy=wdi[, "LifeExpectancy"], logFert=wdi[, "logFert"],
  as.data.frame(predict(fit.qgam, se=T)))
hat50 <- within(hat50, {lo <- fit-2*se.fit; hi <- fit+2*se.fit})
hat19 <- as.data.frame(matrix(nrow=nrow(wdi), ncol=4))
for(i in 1:2){hat19[[i]] <- qdo(fit19.mqgam, c(.1,.9)[i], predict)
  hat19[[i+2]] <- qdo(wtd19.mqgam, c(.1,.9)[i], predict) }
## NB, can replace `predict` by `plot`, or `summary`
colnames(hat19) <- c(paste0(rep(c('q','qwt'),c(2,2)), rep(c('10','90'),2)))
hat19 <- cbind(hat19, logFert=wdi[, "logFert"])
```

```
## Panel A: Fit with SE limits, 50% quantile
gphA <- xyplot(lo+fit+hi~logFert, data=hat50, lty=c(2,1,2), lwd=1.5, type='l') +
  latticeExtra::as.layer(xyplot(LifeExpectancy~logFert,
    data=hat50, pch='.', cex=2))
```

```
## Panel B: Multiple quantiles; unweighted and weighted fits
gph19 <- xyplot(q10+q90+qwt10+qwt90 ~ logFert, type="l",
               data=hat19, lty=rep(1:2,c(2,2)),lwd=1.5)
gphB <- xyplot(LifeExpectancy ~ logFert, data=wdi) + as.layer(gph19)
update(c("A: 50% curve, 2 SE limits"=gphA, "B: 0.1, 0.9 quantiles"=gphB,
        x.same=T, y.same=T), between=list(x=0.5),
       xlab="Fertility Rate", ylab="Life Expectancy",
       scales=list(x=list(at=log(2^((0:5)/2))), labels=round(2^((0:5)/2),1)),
       alternating=F),
       par.settings=DAAG::DAAGtheme(color=F, col='gray50', cex=2, pch='.'))
```

```
## Plots for the individual quantiles can be obtained thus:
## ## Panel A
plot(fit.qgam, shift=mean(predict(fit.qgam)))
## Panel B, 10% quantile
fitm10 <- qdo(fit19.mqgam, qu=0.1)
plot(fitm10, resid=T, shift=mean(predict(fitm10)),
     ylim=range(wdi$LifeExpectancy), cex=2)
wfitm10 <- qdo(wtd19.mqgam, qu=0.1)
plot(wfitm10, resid=T, shift=mean(predict(wfitm10)),
     ylim=range(wdi$LifeExpectancy), cex=2)
```

## Section 4.6: Further reading and remarks

### Exercises (4.7)

4.2

```
roller.lm <- lm(depression~weight, data=DAAG::roller)
roller.lm2 <- lm(depression~weight+I(weight^2), data=DAAG::roller)
```

4.4

```
toycars <- DAAG::toycars
lattice::xyplot(distance ~ angle, groups=factor(car), type=c('p','r'),
               data=toycars, auto.key=list(columns=3))
```

4.4a



```
parLines.lm <- lm(distance ~ 0+factor(car)+angle, data=toycars)
sepLines.lm <- lm(distance ~ factor(car)/angle, data=toycars)
```

4.4b

```
sepPol3.lm <- lm(distance ~ factor(car)/angle+poly(angle,3)[,2:3], data=toycars)
```

4.4c

```
sapply(list(parLines.lm, sepLines.lm, sepPol3.lm), AICcmodavg::AICc)
```

4.4e

```
setNames(sapply(list(parLines.lm, sepLines.lm, sepPol3.lm),
  function(x)summary(x)$adj.r.squared), c("parLines","sepLines","sepPol3"))
```

4,7

```
seedrates.lm <- lm(grain ~ rate + I(rate^2), data=seedrates)
seedrates.pol <- lm(grain ~ poly(rate,2), data=seedrates)
```

4.10a

```
geo.gam <- gam(thickness ~ s(distance), data=DAAG::geophones)
```

4.11

```
plot(DAAG::geophones$distance, acf(resid(geo.gam), lag.max=55)$acf)
Box.test(resid(geo.gam), lag=10)
Box.test(resid(geo.gam), lag=20)
Box.test(resid(geo.gam), lag=20, type="Ljung")
```

4.15

```
library(mgcv)
xy <- data.frame(x=1:200, y=arima.sim(list(ar=0.75), n=200))
df.gam <- gam(y ~ s(x), data=xy)
plot(df.gam, residuals=TRUE)
```

4.16

```

library(mgcViz)
ohms.tpBIC <- gam(kohms ~ s(juice, bs="tp"), data=fruitohms,
                  gamma=log(nrow(fruitohms))/2, method="REML")
ohms.gamViz <- mgcViz::getViz(ohms.tpBIC) # Convert to a `gamViz` object
g1 <- plot(sm(ohms.gamViz, 1)) # Graphics object for term 1 (of 1)
g1 + l_fitLine(colour = "red") + l_rug(mapping = aes(x=x, y=y), alpha = 0.4) +
    l_ciLine(mul = 2, colour = "blue", linetype = 2) + # Multiply SE by `mul`
    l_points(shape = 19, size = 1, alpha = 0.5)

```

4.16a

```

plot(sm(ohms.gamViz, 1), nsim = 20) + l_ciLine() + l_fitLine() + l_simLine()

```

4.16b

```

gam(Gas ~ Insul+s(Temp, by=Insul), data=whiteside) |>
  getViz() -> gas.gamViz
plot(sm(gas.gamViz,1), nsim = 20) + l_ciLine() + l_fitLine() + l_simLine()

```

```

if(file.exists("/Users/johnm1/pkgsg/PGRcode/inst/doc/")){
code <- knitr::knit_code$get()
txt <- paste0("\n## ", names(code),"\n", sapply(code, paste, collapse='\n'))
writeLines(txt, con="/Users/johnm1/pkgsg/PGRcode/inst/doc/ch4.R")
}

```

## 5 Chapter 5: Generalized linear models and survival analysis

### Packages required (with dependencies)

DAAG car mgcv colorspace HistData gamlss dplyr tidyr MASS ggplot2 latticeExtra qgam VGAM survival HistData

Additionally, knitr is required in order to process the Rmd source file.

### Section 5.1 Generalized linear models

#### Subsection 5.1.1: Linking the expected value to the covariate

```
## Simplified plot showing the logit link function
p <- (1:39)/40
logitp <- log(p/(1 - p))
plot(p, logitp, xlab = "Proportion", ylab = "logit(p)", type = "l", pch = 1)
```

```
par(las=0)
p <- seq(from=1, to=99, by=1)/100; n<- 150; eps=0.001
gitp <- log(p/(1 - p))
plot(p, gitp, xlab = "", ylab = "", type = "l", pch = 1,
     las=1, xlim=0:1, xaxs="i", fg="gray")
mtext(side = 1, line = 1.75, expression("Proportion "*pi))
mtext(side = 2, line = 1.75,
     expression("logit("pi*) = log(Odds)"))
mtext(side = 3, line = 0.5, "A: Logit link", adj=0, cex=1.0)
pval <- c(0.001, 0.01, 0.1, 0.5, 0.9, 0.99, 0.999)
par(mgp = c(2.5, 0.5, 0))
## axis(1, at=c(0,1), lwd=0, labels=c(0,1), xpd=TRUE)
axis(4, adj=0.075, at = log(pval/(1 - pval)), las=1,
     col="gray", labels = paste(pval), lwd=0, lwd.ticks=1)
seP <- sqrt(p*(1-p)/100)
plot(p, seP, xlab = "", ylab = "", type = "l", pch = 1,
```

```

las=1, xlim=0:1, xaxs="i", fg="gray")
## axis(1, at=c(0,1), lwd=0, labels=c(0,1), xpd=TRUE)
mtext(side = 1, line = 1.75, expression("Proportion "*pi))
mtext(side = 2, line = 2.25, expression("SD["*p*"], "*n*=100"))
mtext(side = 3, line = 0.5,
expression("B: SD["*p*"], "*n*=100"), adj=0, cex=1.0)
seLP <- (p*(1-p)/n)*((p+eps)*(1-p+eps))^-2
plot(p, seLP, xlab = "", ylab = "", type = "l", pch = 1,
las=1, xlim=0:1, xaxs="i", fg="gray")
## axis(1, at=c(0,1), lwd=0, labels=c(0,1), xpd=TRUE)
mtext(side = 1, line = 1.75, expression("Proportion "*pi))
mtext(side = 2, line = 1.75, expression("SD[logit("*p*")], "*n*=100"))
mtext(side = 3, line = 0.5, "C: SD[logit(p)]", adj=0, cex=1.0)

```

### Subsection 5.1.2: Noise terms need not be normal

### Subsection 5.1.3: Variation that is greater than binomial or Poisson

### Least squares versus logistic regression

### Subsection 5.1.4: Log odds in contingency tables

### Subsection 5.1.5: Logistic regression with a continuous explanatory variable

```

anestot <- aggregate(DAAG::anesthetic[, c("move","nomove")],
by=list(conc=DAAG::anesthetic$conc), FUN=sum)
## The column 'conc', because from the 'by' list, is then a factor.
## The next line recovers the numeric values
anestot$conc <- as.numeric(as.character(anestot[["conc"]]))
anestot$total <- apply(anestot[, c("move","nomove")], 1, sum)
anestot$prop <- anestot$nomove/anestot$total

```

```

par(mgp=c(2.5,.5,0))
anesthetic <- DAAG::anesthetic
z <- table(anesthetic$nomove, anesthetic$conc)
tot <- apply(z, 2, sum)
prop <- z[2, ]/(tot)
oprop <- sum(z[2, ])/sum(tot)
conc <- as.numeric(dimnames(z)[[2]])

```

```

par(las=0)
plot(conc, prop, xlab = "Concentration", ylab = "Proportion",
      xlim=c(0.5, 2.5), ylim = c(0, 1), pch = 16, axes=F)
axis(1, cex=0.9, lwd=0, lwd.ticks=1)
axis(2, at=c(0, 0.5, 1.0), cex=0.9, lwd=0, lwd.ticks=1)
axis(2, at=c(0.25, 0.75), cex=0.9, lwd=0, lwd.ticks=1)
box(col="gray")
chh <- par()$cxy[2]
chw <- par()$cxy[1]
text(conc - 0.3 * chw, prop-sign(prop-0.5)*chh/4, paste(tot),
      adj = 1, cex=0.65)
abline(h = oprop, lty = 2)

```

```

## Fit model directly to the 0/1 data in nomove
anes.glm <- glm(nomove ~ conc, family=binomial(link="logit"),
                data=DAAG::anesthetic)
## Fit model to the proportions; supply total numbers as weights
anes1.logit <- glm(prop ~ conc, family=binomial(link="logit"),
                  weights=total, data=anestot)

```

```

DAAG::sumry(anes.glm, digits=2)

```

### A note on model output

```

## To get coefficients, SEs, and associated statistics, specify:
print(coef(summary(anes.glm)), digits=2)
## Get full default output
summary(anes.glm, digits=2)

```

## Section 5.2 Logistic multiple regression

```

frogs <- DAAG::frogs

```

```

## Presence/absence information: data frame frogs (DAAGS)
suppressMessages(library(ggplot2))
p <- ggplot(frogs, aes(easting, northing)) +
  geom_point(size=3, alpha=0.25) + coord_fixed() +
  xlab("Meters east of reference point")+ylab("Meters north") +

```

```
theme(axis.title=element_text(size=11), axis.text=element_text(size=8))
p + geom_point(data=subset(frogs, pres.abs==1),
               aes(easting, northing), alpha=1, shape=3, col="white", size=1.5)
```

```
frogs <- within(frogs, {maxSubmin <- meanmax-meanmin
                       maxAddmin <- meanmax+meanmin})
```

## Implications for the Variance Inflation Factor

### Subsection 5.2.1: Choose explanatory terms, and fit model

```
## Find power transformations
useCols <- c('distance','NoOfPools','NoOfSites','avrain','maxAddmin','maxSubmin')
tfrogs <- car::powerTransform(frogs[,useCols], family="yjPower")
## Create, for later use, a matrix with variables transformed as suggested
transY <- car::yjPower(frogs[,useCols], coef(tfrogs, round=TRUE))
summary(tfrogs, digits=2)
```

```
frogs0.glm <- glm(formula = pres.abs ~ log(distance) + log(NoOfPools)+
                  sqrt(NoOfSites) + avrain + maxAddmin + maxSubmin,
                  family = binomial, data = frogs)
DAAG::sumry(frogs0.glm, digits=1)
```

```
## Check effect of omitting sqrt(NoOfSites) and avrain from the model
## ~ . takes the existing formula. Precede terms to be
## omitted by '-'. (For additions, precede with '+')
frogs.glm <- update(frogs0.glm, ~ . -sqrt(NoOfSites)-avrain)
frogsAlt.glm <- update(frogs.glm, ~ . -maxAddmin+altitude)
AIC(frogs0.glm, frogs.glm, frogsAlt.glm)
```

```
rbind(
'frogs0.glm'=coef(frogs0.glm)[c('log(distance)', 'log(NoOfPools)', 'maxAddmin', 'maxSubmin')],
'frogs.glm'=coef(frogs.glm)[c('log(distance)', 'log(NoOfPools)', 'maxAddmin', 'maxSubmin')]
)
coef(frogsAlt.glm)[c('log(distance)', 'log(NoOfPools)', 'altitude', 'maxSubmin')]
```

```
coef(summary(frogs.glm))
```

### Subsection 5.2.2: Fitted values

```
## Use of `predict()` and `fitted()` --- examples
fitted(frogs.glm)      # Fitted values' scale of response
predict(frogs.glm, type="response") # Same as fitted(frogs.glm)
predict(frogs.glm, type="link")      # Scale of linear predictor
## For approximate SEs, specify
predict(frogs.glm, type="link", se.fit=TRUE)
```

```
library(ggplot2)
frogs$Prob. <- fitted(frogs.glm)
frogs$presAbs <- factor(frogs$pres.abs)
p <- ggplot(frogs, aes(easting, northing, color=Prob.)) +
  geom_point(size=2, alpha=0.5) + coord_fixed() +
  xlab("Meters east of reference point")+ylab("Meters north") +
  theme(axis.title=element_text(size=9), axis.text=element_text(size=6))
p2 <- p+scale_color_gradientn(colours=colorspace::heat_hcl(10,h=c(0,-100),
  l=c(75,40), c=c(40,80), power=1)) +
  guides(fill=guide_legend(title=NULL))
p2 + geom_point(data=subset(frogs, presAbs==1),
  aes(easting, northing), alpha=1, shape=3, col="white", size=1)
```

### Subsection 5.2.3: Plots that show the contributions of explanatory variables

```
opar <- par(mgp=c(2.1,.4,0), mfrow=c(1,3))
Cholera <- HistData::Cholera
fitP2.glm <- glm(cholera_deaths ~ offset(log(popn)) + water +
  log(elevation+3) + poly(poor_rate,2) +I(elevation==350),
  data=Cholera, family=quasipoisson)
Cholera[["water"]] <- factor(Cholera[["water"]], labels=c("Battersea",
  "NewRiver","Kew"))
termplot(fitP2.glm, partial=T, se=TRUE, pch =1,
  ylabs=rep("Partial residual",3), terms='water', fg="gray")
axis(1, at=2, labels="NewRiver", lwd=0, line=0.75)
termplot(fitP2.glm, partial=T, se=TRUE, pch =1,
  ylabs=rep("Partial residual",3), terms='log(elevation + 3)', fg="gray")
```

```
termplot(fitP2.glm, partial=T, se=TRUE, pch =1,
         ylabs=rep("Partial residual",3), terms='poly(poor_rate, 2)', fg="gray")
par(opar)
```

#### Subsection 5.2.4: Cross-validation estimates of predictive accuracy

```
DAAG::CVbinary(frogs.glm)
```

```
set.seed(19)
frogs.acc <- frogs0.acc <- numeric(6)
for (j in 1:6){
  randsam <- sample(1:10, 212, replace=TRUE)
  ## Sample 212 values (one per pbervation) from 1:10
  frogs.acc[j] <- DAAG::CVbinary(frogs.glm, rand=randsam,
                                print.details=FALSE)$acc.cv
  frogs0.acc[j] <- DAAG::CVbinary(frogs0.glm, rand=randsam,
                                  print.details=FALSE)$acc.cv
}
print(rbind("frogs (all variables)" = frogs.acc,
            "frogs0 (selected variables)" = frogs0.acc), digits=3)
```

#### Subsection 5.2.5: Cholera deaths in London — 1849 to 1855

##### By air, or by water — the 1849 epidemic

```
opar <- par(mgp=c(2.1,.4,0), mfrow=c(1,3))
Cholera <- HistData::Cholera
fitP2.glm <- glm(cholera_deaths ~ offset(log(popn)) + water +
                 log(elevation+3) + poly(poor_rate,2) +I(elevation==350),
                 data=Cholera, family=quasipoisson)
Cholera[["water"]] <- factor(Cholera[["water"]], labels=c("Battersea",
                                                         "NewRiver", "Kew"))
termplot(fitP2.glm, partial=T, se=TRUE, pch =1,
         ylabs=rep("Partial residual",3), terms='water', fg="gray")
axis(1, at=2, labels="NewRiver", lwd=0, line=0.75)
termplot(fitP2.glm, partial=T, se=TRUE, pch =1,
         ylabs=rep("Partial residual",3), terms='log(elevation + 3)', fg="gray")
termplot(fitP2.glm, partial=T, se=TRUE, pch =1,
```



```
ylabs=rep("Partial residual",3), terms='poly(poor_rate, 2)', fg="gray")
par(opar)
```

## The 1854 epidemic — a natural experiment

### Section 5.3 Logistic models for categorical data – an example

```
## Create data frame from multi-way table UCBAmissions (datasets)
## dimnames(UCBAmissions) # Check levels of table margins
UCB <- as.data.frame.table(UCBAmissions["Admitted", , ], responseName="admit")
UCB$reject <- as.data.frame.table(UCBAmissions["Rejected", , ])$Freq
UCB$Gender <- relevel(UCB$Gender, ref="Male")
## Add further columns total and p (proportion admitted)
UCB$total <- UCB$admit + UCB$reject
UCB$pAdmit <- UCB$admit/UCB$total
```

```
UCB.glm <- glm(pAdmit ~ Dept*Gender, family=binomial, data=UCB, weights=total)
## Abbreviated `anova()` output:
anova(UCB.glm, test="Chisq") |>
  capture.output() |> tail(8) |> (\(x)x[-c(2,3)]()) |> cat(sep='\n')
```

```
round(signif(coef(summary(UCB.glm)),4), 3)
```

## Section 5.4 Models for counts — poisson, quasipoisson, and negative binomial

### Subsection 5.4.1: Data on aberrant crypt foci

```
par(pty="s")
plot(count ~ endtime, data=DAAG::ACF1, pch=16, fg="gray")
```

```
ACF.glm <- glm(formula = count ~ endtime + I(endtime^2),
               family = poisson(link="identity"), data = DAAG::ACF1)
DAAG::sumry(ACF.glm, digits=2)
```

```
unique(round(predict(ACF.glm),2))
```

```
sum(resid(ACF.glm, type="pearson")^2)/19
```

```
ACFq.glm <- glm(formula = count ~ endtime + I(endtime^2),  
family = quasipoisson, data = DAAG::ACF1)  
print(coef(summary(ACFq.glm)), digits=2)
```

```
sapply(split(residuals(ACFq.glm), DAAG::ACF1$endtime), var)
```

```
fligner.test(resid(ACFq.glm) ~ factor(DAAG::ACF1$endtime))
```

### Subsection 5.4.2: Moth habitat example

```
## Number of moths by habitat: data frame DAAG::moths  
moths <- DAAG::moths  
tab <- rbind(Number=table(moths[, 4]),  
             sapply(split(moths[, -4], moths$habitat), apply, 2, sum))
```

```
## Number of zero counts, by habitats  
with(droplevels(subset(moths, A==0)), table(habitat))
```

```
library(lattice)  
gph <- dotplot(habitat ~ A+P, data=DAAG::moths, xlab="Number of moths", outer=TRUE,  
              strip=strip.custom(factor.levels=paste("Number of species",c("A","B"))),  
              panel=function(x, y, ...){  
panel.dotplot(x,y, pch=1, ...)  
av <- sapply(split(x,y),mean)  
ypos <- factor(names(av), levels=names(av))  
lpoints(ypos~av, pch=3, col="gray45", cex=1.25)  
},  
key=list(text=list(c("Individual transects", "Mean")),  
points=list(pch=c(1,3), cex=c(1,1.25), col=c("black","gray45")),  
columns=2), scales=list(tck=0.5, alternating=1))  
bw9 <- list(fontsize=list(text=9, points=5))  
update(gph, par.settings=bw9)
```

```
Astats <- with(DAAG::moths, sapply(split(A, habitat),  
function(x)c(Amean=mean(x),Avar=var(x))))  
avlength <- with(DAAG::moths, sapply(split(meters, habitat), mean))  
round(rbind(Astats, avlen=avlength),1)
```

## A quasipoisson model

```
A.glm <- glm(A ~ habitat + log(meters), family=quasipoisson,  
data=DAAG::moths)  
DAAG::sumry(A.glm, digits=1)
```

```
subset(DAAG::moths, habitat=="Bank")
```

```
## Analysis with tighter convergence criterion  
A.glm <- update(A.glm, epsilon=1e-10)  
print(coef(summary(A.glm)), digits=2)
```

```
AfitSE <- predict(A.glm, se=TRUE)$se.fit  
cfSE <- with(DAAG::moths, c(AfitSE[habitat=="Bank"],  
range(AfitSE[habitat!="Bank"])))  
round(setNames(cfSE, c("SEbank", "SEotherMIN", "SEotherMAX")), digits=2)
```

## A more satisfactory choice of reference level

```
moths <- DAAG::moths  
moths$habitat <- relevel(moths$habitat, ref="Lowerside")  
Alower.glm <- glm(A ~ habitat + log(meters),  
family = quasipoisson, data = moths)  
print(coef(summary(Alower.glm)), digits=1)
```

## Subsection 5.4.3: Models with negative binomial errors

```
dframe <- data.frame(sigma1A =(Astats[2,]-Astats[1,])/Astats[1,]^2,  
sigma2A =(Astats[2,]-Astats[1,])/Astats[1,]^1,  
mu = Astats[1,], habitat=colnames(Astats))  
bw9 <- list(fontsize=list(text=9, points=5), pch=1:7)  
xyplot(sigma1A+sigma2A ~ mu, groups=habitat, outer=TRUE,  
data=subset(dframe,habitat!="Bank"),  
par.settings=bw9, auto.key=list(columns=4),  
strip=strip.custom(factor.levels=paste("Model",c("NBI","NBII"))),  
xlab="Mean number of species A moths",  
ylab=expression("Estimate of " * sigma))
```

```
library(gamlss, quietly=TRUE)
noBank <- subset(moths, habitat!='Bank')
mothsCon.lss <- gamlss(A ~ log(meters)+habitat, family=NBI(), data=noBank,
                      trace=F)
mothsVary.lss <- gamlss(A ~ log(meters)+habitat, family=NBI(),
                      sigma.formula=~habitat, trace=FALSE, data=noBank)
```

```
LR.test(mothsCon.lss, mothsVary.lss)
```

```
## mothsCon.lss <- gamlss(A ~ log(meters)+habitat,family=NBI(),data=noBank)
## summary(mothsCon.lss, type="qr") ## Main part of output
```

## Diagnostic plots

```
plot(mothsCon.lss, panel=panel.smooth)
```

## Use of the square root link function

```
Asqrt.lss <- gamlss(A ~ habitat + sqrt(meters), trace=FALSE,
                  family = NBI(mu.link='sqrt'), data = moths)
```

```
## Asqrt.lss <- gamlss(A ~ habitat + sqrt(meters),
##                    family = NBI(mu.link='sqrt'), data = moths)
## summary(Asqrt.lss, type="qr") ## Main part of output
out <- capture.output(summary(Asqrt.lss, digits=1))[-(3:10)]
cat(out, sep="\n")
```

## Subsection 5.4.4: Negative binomial versus alternatives — hurricane deaths

### Aside – a quasibinomial binomial fit

```
ordx <- with(DAAG::hurricNamed, order(BaseDam2014))
hurric <- DAAG::hurricNamed[ordx,]
# Ordering a/c values of BaseDam2014 simplifies later code
hurr.glm <- glm(deaths ~ log(BaseDam2014), family=quasipoisson, data=hurric)
plot(hurr.glm, col=adjustcolor('black', alpha=0.4),
     cex.caption=0.95, sub.caption=rep("",4), fg="gray")
```

## Negative binomial versus power transformed scale

### Fit a negative binomial (NBI) model

```
library(gamlss)
hurrib.gamlss <- gamlss::gamlss(deaths ~ log(BaseDam2014), family=NBI(),
                              data=hurric[-56,])
mures <- resid(hurrib.gamlss, what="mu")
zres <- resid(hurrib.gamlss, what="z-scores") ## equivalent normal quantiles

table(sign(mures))
```

### Fit linear model to power transformed response

```
hurrib.lm <- lm(car::yjPower(deaths,-0.2) ~ log(BaseDam2014), data=hurric[-56,])
## Use the following function to transform from power scale to log scale
powerTolog <- function(z, lambda)log(lambda*z+1)/lambda
## Calculate fitted values, and transform to log(deaths+1) scale
hatPower <- powerTolog(predict(hurrib.lm), lambda=-0.2)
resPower <- log(hurric[-56,"deaths"]+1) - hatPower

table(sign(resPower))
```

### Compare NBI and power transform fits with smoothed quantiles

```
library(qgam, quietly=TRUE)
hat68.8 <- predict(qgam(log(deaths+1) ~ s(log(BaseDam2014)), qu=.648,
                        data=hurric[-56,]))
hat40.9 <- predict(qgam(log(deaths+1) ~ s(log(BaseDam2014)), qu=.409,
                        data=hurric[-56,]))

xvar <- log(hurric$BaseDam2014)[-56]
plot(log(deaths+1) ~ log(BaseDam2014), data=hurric, xaxt="n", yaxt="n",
     cex=4, pch=".", fg="gray", col=adjustcolor("black",alpha.f=0.65),
     xlab="Damage, millions of US$ in 2014", ylab="Deaths")
axis(1, at=log(c(1,10,1000, 100000)),
     labels=paste(c(1,10,1000, 100000)), lwd=0, lwd.ticks=1)
axis(2, at=log(c(0,10,100,1000))+1),
     labels=paste(c(0,10,100,1000)), lwd=0, lwd.ticks=1)
```

```

## Negative binomial regression fitted values
hatNB <- fitted(hurrNB.gamlss)
lines(xvar, log(hatNB+1), col="blue", lty=2)
with(hurric, text(log(BaseDam2014)[56], log(deaths+1)[56], "Audrey", pos=3),
      cex=0.72)
## Show fit from power transform model
lines(xvar, hatPower, col="blue", lty=1)
## Show 68.8% and 40.1% fits from regression smooths
lines(hat68.8 ~ xvar, lty=2, col='red')
lines(hat40.9 ~ xvar, lty=1, col='red')
legend("topleft", col=rep(c('blue','red'),c(2,2)), lty=rep(2:1,2), cex=0.8,
      y.intersp=0.75, legend=c("Negative binomial fit","Power transform fit",
      "68.8% quantile", "40.9% quantile"), bty="n")
mtext(side=3, "A: Deaths vs damage", line=0.5, cex=1.15, adj=0)
## Quantile-quantile plot -- negative binomial model
qqnorm(zres, main="", fg="gray", cex=0.5,
      col=adjustcolor("black",alpha.f=0.65)); qqline(zres, col=2)
mtext(side=3, "B: Q-Q plot", line=0.5, cex=1.15, adj=0)

```

```

## a) Fitted and empirical centiles from hurrNB.gamlss
pc <- t(centiles.split(hurrNB.gamlss, xvar=log(hurric$BaseDam2014)[-56],
      cent=c(5,10,25,50,75,90,95), xcut.points=log(c(150, 1500)),
      plot=FALSE))
rownames(pc) <- c("up to 150M", "150M to 1500M", "above 1500M")
round(pc,2)

```

```

hurrP.gamlss <- gamlss(car::yjPower(deaths, -0.2) ~ log(BaseDam2014), data=hurric)

```

```

## Fitted and empirical centiles from hurrP.gamlss
pc <- t(centiles.split(hurrP.gamlss, xvar=log(hurric$BaseDam2014),
      cent=c(5,10,25,50,75,90,95),
      xcut.points=log(c(150, 1500)), plot=FALSE))
rownames(pc) <- c("up to 150M", "150M to 1500M", "above 1500M")
round(pc,2)

```

## Section 5.5 Fitting smooths

### Subsection 5.5.1: Handedness of first-class cricketers in the UK

```

tab <- with(DAAG::cricketer, table(left,dead))
colnames(tab) <- c('live','dead')
tab <- cbind(addmargins(tab, margin=2), prop.table(tab, margin=1))
tab

library(mgcv)
library(latticeExtra)
DAAG::cricketer |> dplyr::count(year, left, name="Freq") -> handYear
names(handYear)[2] <- "hand"
byYear <- tidyr::pivot_wider(handYear, names_from='hand', values_from="Freq")
hand.gam <- gam(cbind(left,right) ~ s(year), data=byYear, family=binomial)
const <- attr(predict(hand.gam, type='terms'), "constant")
  ## `const` is the mean on the scale of the linear predictor
plot(hand.gam, shift=const, trans=function(x)exp(x)/(1+exp(x)), ylim=c(.05,.4),
      xlab="", ylab="Proportion lefthanded", rug=FALSE, fg="gray",
      main=list("Proportion lefthanded, with 2SE limits",font=1,cex=1.2))
  ## Add `const`, then apply inverse link function.
  ## Plots estimated proportions (i.e., on the scale of the response)
with(byYear, points(year, I(left/(left+right)), cex=0.8, col="gray50"))
leftrt.gam <- gam(Freq ~ hand + s(year, by=factor(hand)), data=handYear,
                  family=poisson)
leftrt.pred <- predict(leftrt.gam, se=T, type='response')
handYear <- cbind(handYear, as.data.frame(leftrt.pred))
col2 <- DAAG::DAAGtheme(color=T)$superpose.symbol$col[c(2,2,1)]
gph.key <- list(space="top", columns=3, lines=list(lty=c(1,2,1), lwd=2, col=col2),
               text=list(c("left",expression(4.4*%"left"),"right")), cex=1.2)
gph <- xyplot(leftrt.pred$fit ~ year, groups=hand, ylab=list("Number born", cex=1.2),
              type="l", xlab="", data=handYear, key=gph.key, col=col2[c(3,1)], lwd=2)
gph1 <- xyplot(Freq~year, groups=hand, data=handYear, col=col2[c(3,1)])
gph2 <- xyplot(I(4.4*fit) ~ year, data=subset(handYear, hand=="left"),
               type="l", lty=2, lwd=2, col=col2[2])
update(gph+as.layer(gph1)+as.layer(gph2), par.settings=DAAG::DAAGtheme(color=TRUE),
       scales=list(cex=1.2))

```

## Section 5.6 Additional notes on generalized linear models

### Subsection 5.6.1: Residuals, and estimating the dispersion

#### Other choices of link function for binomial models

## Quasi models — estimating the dispersion

### Subsection 5.6.2: Standard errors and $z$ - or $t$ -statistics for binomial models

```
fac <- factor(LETTERS[1:4])
p <- c(73, 30, 11, 2)/500
n <- rep(500,4)
round(signif(coef(summary(glm(p ~ fac, family=binomial, weights=n))), 6), 6)
```

```
p <- c(0.001,0.002,(1:99)/100,0.998,0.999)
for(i in 1:3){
  link <- c("logit", "probit", "cloglog")[i]
  fun <- make.link(link)$linkfun
  x <- fun(p)
  u <- glm(p ~ x, family=binomial(link=link), weights=rep(1000,103))
  if (i==1)
    plot(x, hatvalues(u), type="l", ylab="Leverage", xaxt="n", fg='gray',
         yaxt="n",
         ylim=c(0, 0.0425), yaxs="i", xlab="Fitted proportion") else {
    phat <- predict(u, type="response")
    lines(log(phat/(1-phat)), hatvalues(u), type="l",
          col=c("black","black","gray")[i], lwd=0.75,
          lty=c(1,2,1)[i])
  }
}
pos=c(0.001,0.002, 0.005, 0.01,0.02,0.05,0.1,0.25,0.5,0.75,0.9,0.95,0.98,0.99, 0.995, 0.998,
sub1 <- seq(from=1,to=17, by=2)
sub3 <- seq(from=2,to=16, by=2)
axis(1, at=log(pos/(1-pos))[sub1], labels=paste(pos)[sub1],
     cex.axis=0.7, lwd=0, lwd.ticks=1)
axis(3, at=log(pos/(1-pos))[sub3], labels=paste(pos)[sub3],
     cex.axis=0.7, lwd=0, lwd.ticks=1)
axis(2, at=c(0,.01,.02,.03), cex.axis=.7, lwd=0, lwd.ticks=1)
legend("topleft", lty=c(1,2,1),
      legend=c("logit link", "probit link", "cloglog link"),
      col=c("black","black","gray"), bty="n", cex=0.8)
```



## Section 5.7 Models with an ordered categorical or categorical response

```
library(VGAM)
inhaler <- data.frame(freq=c(99,76,41,55,2,13),
  choice=rep(c("inh1","inh2"), 3),
  ease=ordered(rep(c("easy","re-read","unclear"), rep(2,3))))
inhaler1.vglm <- vglm(ease ~ 1, weights=freq, data=inhaler,
  cumulative(link="logitlink"), subset=inhaler$choice=="inh1")
inhaler2.vglm <- vglm(ease ~ 1, weights=freq, data=inhaler,
  cumulative(link="logitlink"), subset=inhaler$choice=="inh2")
```

```
## Inhaler 1
round(coef(summary(inhaler1.vglm)),3)
## Inhaler 2
round(coef(summary(inhaler2.vglm)),3)
```

```
inhaler.vglm <- vglm(ease ~ choice, weights=freq, data=inhaler,
  cumulative(link="logitlink", parallel=FALSE))
round(coef(summary(inhaler.vglm)),3)
```

```
inhalerP.vglm <- vglm(ease ~ choice, weights=freq, data=inhaler,
  cumulative(link="logitlink", parallel=TRUE))
round(coef(summary(inhalerP.vglm)),3)
```

```
pred <- predict(inhalerP.vglm, se.fit=TRUE, newdata=inhaler[1:2,])
colnames(pred$se.fit) <- paste("SE", colnames(pred$se.fit))
fitvals <- with(pred, cbind(fitted.values, se.fit))
colnames(fitvals) <- gsub('link', '', colnames(fitvals))
round(fitvals, 2)
```

```
d <- deviance(inhalerP.vglm) - deviance(inhaler.vglm)
## Refer to chi-squared distribution with 1 degree of freedom
c(Difference=d, "p-Value"=pchisq(3.416, df=1, lower.tail=FALSE))
```

### Subsection 5.7.2: Loglinear Models

## Section 5.8 Survival analysis

```
suppressMessages(library(survival))
```

```
df <- data.frame(x0 = c(1, 5, 1, 2, 14, 10, 12, 19)*30,
x1 = c(46, 58, 85, 67, 17, 85, 18, 42)*30,
fail = c(1, 0, 0, 1, 1, 0, 0, 1))
plot(c(0, 2610), c(0.65, 8.15), type = "n",
xlab = "Days from beginning of study",
ylab = "Subject number", axes = F)
## mtext(side = 1, line = 2.5, "Days from beginning of study", adj = 0.5)
m <- dim(df)[1]
par(las=2)
axis(2, at = (1:m), labels = paste((m:1)), lwd=0, lwd.ticks=1)
par(las=1)
abline(v = 600, lty = 4, col="gray40")
abline(v = 2550)
mtext(side = 3, line = 0.5, at = c(600, 2550),
text = c("\nEnd of recruitment",
"\nEnd of study"), cex = 0.9)
lines(rep((0:8) * 300, rep(3, 9)), rep(c(-0.4, -0.2, NA), 9),
xpd = T)
mtext(side = 1, line = 1.0, at = (0:8) * 300,
text = paste((0:8) * 300), adj = 0.5)
chw <- par()$cxy[1]
xx <- as.vector(t(cbind(df[, 1], df[, 2] - 0.25 * chw,
rep(NA, m))))
yy <- as.vector(t(cbind(matrix(rep(m:1, 2), ncol = 2),
rep(NA, m))))
lines(as.numeric(xx), as.numeric(yy))
points(df[, 1], m:1, pch = 16)
text(df[, 1]-0.25*chw, m:1, paste(df[,1]), pos=1, cex=0.75)
fail <- as.logical(df$fail)
points(df[fail, 2], (m:1)[fail], pch = 15)
points(df[!fail, 2], (m:1)[!fail], pch = 0)
text(df[, 2]+0.25*chw, m:1, paste(df[,2]), pos=1, cex=0.75)
par(xpd=TRUE)
legend(0, 11.5, pch = 16, legend = "Entry", y.intersp=0.15)
legend(1230, 11.5, pch = c(15, 0),
legend = c("Dead", "Censored"), ncol=2, y.intersp=0.15)
```

### Subsection 5.8.1: Analysis of the Aids2 data

```
str(MASS::Aids2, vec.len=2)
```

```
bloodAids <- subset(MASS::Aids2, T.categ=="blood")
bloodAids$days <- bloodAids$death-bloodAids$diag
bloodAids$dead <- as.integer(bloodAids$status=="D")
```

```
bloodAids <- subset(MASS::Aids2, T.categ=="blood")
bloodAids$days <- bloodAids$death-bloodAids$diag
bloodAids$dead <- as.integer(bloodAids$status=="D")
plot(survfit(Surv(days, dead) ~ sex, data=bloodAids),
     col=c(2,4), conf.int=TRUE, lty=1, fg="gray",
     xlab="Days from diagnosis", ylab="Survival probability")
legend("top", legend=levels(bloodAids$sex), lty=c(1,1),
     col=c(2,4), horiz=TRUE, bty="n")
```

```
## Pattern of censoring for male homosexuals
hsaids <- subset(MASS::Aids2, sex=="M" & T.categ=="hs")
hsaids$days <- hsaids$death-hsaids$diag
hsaids$dead <- as.integer(hsaids$status=="D")
table(hsaids$status, hsaids$death==11504)
```

```
hsaids <- subset(MASS::Aids2, sex=="M" & T.categ=="hs")
hsaids$days <- hsaids$death-hsaids$diag
hsaids$dead <- as.integer(hsaids$status=="D")
hsaids.surv <- survfit(Surv(days, dead) ~ 1, data=hsaids)
plot(hsaids.surv, col="gray", conf.int=F, tcl=-0.4, fg="gray")
par(new=TRUE)
plot(hsaids.surv, col=1, conf.int=F, mark.time=F, fg="gray",
     xlab="Days from diagnosis", ylab="Estimated survival probabality")
chw <- par()$cxy[1]
chh <- par()$cxy[2]
surv <- hsaids.surv$surv
xval <- c(200,700,1400,1900)
hat <- approx(hsaids.surv$time, surv, xout=xval)$y
for(i in 1:2) arrows(xval[i], hat[i], 0, hat[i],
                    length=0.05, col="gray")
lines(rep(xval[1],2), hat[1:2], col="gray")
##   lines(rep(xval[3],2), hat[3:4], col="gray")
```

```
## Offset triangle 1
chw <- par()$cxy[1]
lines(xval[c(1,2,1,1)]+650, hat[c(2,2,1,2)]+0.2,col="gray40")
xy1 <- c(mean(xval[c(1,1,2)]), mean(hat[c(1,2,2)]))
arrows(xy1[1], xy1[2], xy1[1]+650, xy1[2]+0.2, col="gray40", length=0.1)
text(xval[1]-0.1*chw+650, hat[1]+0.2,
paste(round(hat[1],2)), col="gray20",cex=0.75, adj=1)
text(xval[1]+650-0.1*chw, hat[2]+0.2,
paste(round(hat[2],2)), col="gray20",cex=0.75, adj=1)
text(mean(xval[1:2])+650, hat[2]+0.2-0.5*chh,
paste(round(diff(xval[1:2]))), col="gray20", cex=0.75)
text(xval[1]+650-0.5*chw, mean(hat[1:2]+0.2), paste(round(hat[1]-hat[2],3)),
srt=90, adj=0.5, col="gray20", cex=0.75)
```

#### Subsection 5.8.4: Hazard rates

#### Subsection 5.8.5: The Cox proportional hazards model

```
bloodAids.coxph <- coxph(Surv(days, dead) ~ sex, data=bloodAids)
print(summary(bloodAids.coxph), digits=6)
```

```
## Add `age` as explanatory variable
bloodAids.coxph1 <- coxph(Surv(days, dead) ~ sex+age, data=bloodAids)
```

```
bloodAids <- subset(MASS::Aids2,T.categ=="blood")
bloodAids <- within(bloodAids, {days <- death-diag
dead <- as.integer(status=="D")})
bloodAids.coxph <- coxph(Surv(days, dead) ~ sex, data = bloodAids)
plot(cox.zph(bloodAids.coxph), cex=0.75, bty="n")
box(col="gray")
```

```
cox.zph(bloodAids.coxph)
```

```
cricketer <- DAAG::cricketer
kia4.coxph <- coxph(Surv(life, kia) ~ left/poly(year,4),
data = cricketer, model=T)
kia6.coxph <- update(kia4.coxph, . ~ left/poly(year,6),
data = cricketer, model=T)
# Type `plot(cox.zph(kia6.coxph)` to plot the two graphs
```

```
# Perhaps check also `AIC(kia4.coxph, kia6.coxph)`
cox.zph(kia6.coxph)
```

```
plot(cox.zph(kia6.coxph), cex=0.75, bty="n")
box(col="gray")
```

## Section 5.9: Transformations for proportions and counts

## Section 5.10: Further reading

## Exercises (5.11)

5.1

```
inhibition <- rbind(
  conc =c(0.1,0.5, 1,10,20,30,50,70,80,100,150),
  no   = c(7,  1, 10, 9, 2, 9, 13, 1, 1,  4,  3),
  yes  = c(0,  0, 3, 4, 0, 6, 7, 0, 0,  1,  7)
)
colnames(inhibition) <- rep("", ncol(inhibition))
inhibition
```

```
if(file.exists("/Users/johnm1/pkgs/PGRcode/inst/doc/")){
  code <- knitr::knit_code$get()
  txt <- paste0("\n## ", names(code),"\n", sapply(code, paste, collapse='\n'))
  writeLines(txt, con="/Users/johnm1/pkgs/PGRcode/inst/doc/ch5.R")
}
```

## 6 Chapter 6: Time series models

### Packages required (plus any dependencies)

DAAG ggsi latticeExtra ggplot2 mice car forecast mgcv tseries

Additionally, knitr and Hmisc are required in order to process the Rmd source file.

### Section 6.1: Time series – some basic ideas

#### Subsection 6.1.1: Time series objects

```
class("lakeHuron")
```

```
[1] "character"
```

```
## Use `time()` to extract the `time` attribute  
range(time(LakeHuron))
```

```
[1] 1875 1972
```

```
## Use `window()` to subset a time series  
LHto1925 <- window(LakeHuron, from=1875, to=1925)
```

```
jobs <- DAAG::jobs  
names(jobs)
```

```
[1] "BC"          "Alberta"    "Prairies"   "Ontario"    "Quebec"     "Atlantic"   "Date"
```

```
allRegions <- ts(jobs[, -7]) # Create multivariate time series  
time(allRegions)           # Times run from 1 to 24
```

Time Series:

Start = 1

End = 24

Frequency = 1

```
[1] 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
```

```
allRegions <- ts(jobs[, -7], start=c(1995,1), frequency=12)
allRegions[, "BC"] # Extract jobs data for British Columbia
```

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1995	1752	1737	1765	1762	1754	1759	1766	1775	1777	1771	1757	1766
1996	1786	1784	1791	1800	1800	1798	1814	1803	1796	1818	1829	1840

```
jobsBC <- ts(jobs[, "BC"], start=c(1995,1), frequency=12)
# Obtain equivalent of `allRegions[, "BC"]` directly from `jobs` dataset
```

### Subsection 6.1.2: Preliminary graphical exploration

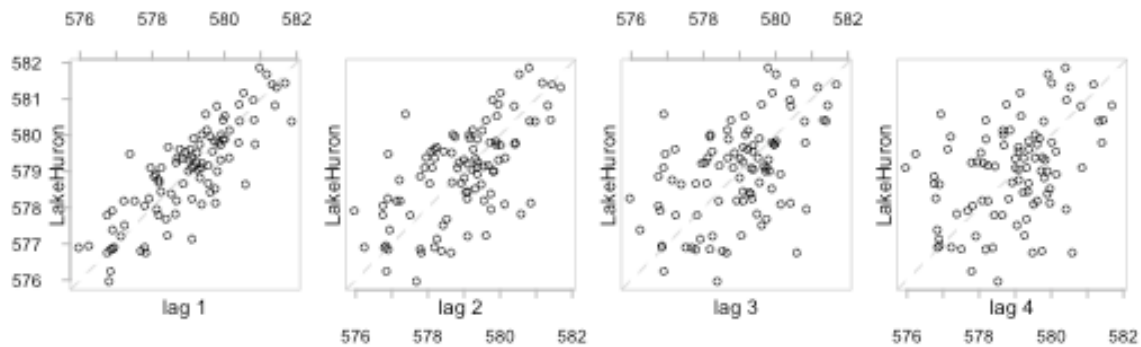


```
## Plot depth measurements: ts object LakeHuron (datasets)
plot(LakeHuron, ylab="depth (in feet)", xlab = "Time (in years)", fg="gray")
```

```
lag.plot(LakeHuron, lags=3, do.lines=FALSE)
```

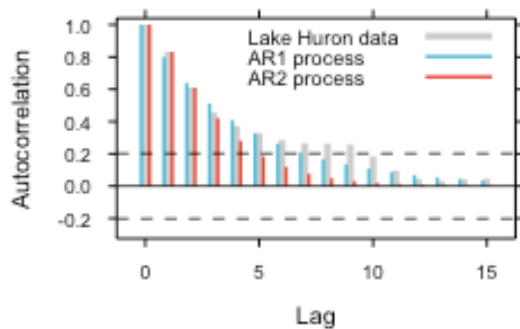
### Subsection 6.1.3: The autocorrelation and partial autocorrelation function

A: Lag plots

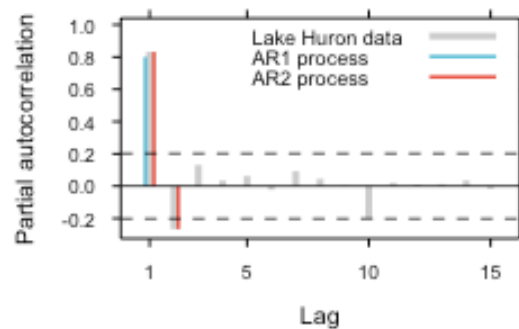


```
par(oma=c(0,0,1.5,0))
par(pty="s")
lag.plot(LakeHuron, set.lags=1:4, do.lines=F, oma=c(0,1.5,1.5,1.5),
fg="gray", layout=c(1,4), cex.lab=1.15, asp=1)
mtext(side=3, line=0.5, "A: Lag plots", adj=0, cex=0.85, outer=TRUE)
```

B: Autocorrelation -- Data vs AR processes



C: Partial autocorrelation -- Data vs AR processes



```
library(lattice)
col3 <- c("gray80", rev(ggsci::pal_npg()(2)))
lag.max <- 15
offset <- 0.18
```



```

ci95 <- 2/sqrt(length(LakeHuron))
ar2 <- ar(LakeHuron)
gph.key <- list(x=0.975, y=0.965, corner=c(1,1), columns=1, cex=0.85,
               text=list(c("Lake Huron data", "AR1 process", "AR2 process")),
               lines=list(lwd=c(3,1.5,1.5), col=col3, lend=2),
               padding.text=1)
parsetBC <- list(fontsize=list(text=8, points=5),
                 superpose.line=list(col=col3, lty=rep(1,3),
                                     lwd=c(3,1.5,1.5)))
parsetBC <- modifyList(parsetBC, list(grid.pars = list(lineend = "butt")))
lev3 <- factor(c("acfData", "acfAR1", "acfAR2"),
              levels=c("acfData", "acfAR1", "acfAR2"))
acfData <- acf(LakeHuron, main="", plot=FALSE, lag.max=lag.max)$acf
pacfData <- pacf(LakeHuron, main="", plot=FALSE, lag.max=lag.max)$acf
acfAR1 <- ARMAacf(ar=0.8, lag.max=lag.max)
acfAR2 <- ARMAacf(ar=ar2$ar, ma=0, lag.max=lag.max)
pacfAR1 <- ARMAacf(ar=0.8, lag.max=lag.max, pacf=TRUE)
pacfAR2 <- ARMAacf(ar=ar2$ar, ma=0, lag.max=lag.max, pacf=TRUE)
xy <- data.frame(acf=c(acfData, acfAR1, acfAR2),
                 Lag=c(rep(0:lag.max, 3))+rep(c(0, -offset, offset),
                                                rep(lag.max+1, 3)),
                 gp=rep(lev3, rep(lag.max+1, 3)))
gphB <- xyplot(acf ~ Lag, data = xy, groups=gp, type=c("h"),
              par.strip.text = list(cex = 0.85), lend=2, origin=0,
              ylim=c(-0.325, 1.04), key=gph.key, par.settings=parsetBC,
              panel=function(x,y,...){
                panel.xyplot(x,y,...)
                panel.abline(h=0, lwd=0.8)
                panel.abline(h=ci95, lwd=0.8, lty=2)
                panel.abline(h=-ci95, lwd=0.8, lty=2) } )
xyp <- data.frame(pacf=c(pacfData, pacfAR1, pacfAR2),
                 Lag=c(rep(1:lag.max, 3))+c(rep(c(0, -offset, offset),
                                                rep(lag.max, 3))), gp=rep(lev3, rep(lag.max, 3)))
gphC <- xyplot(pacf ~ Lag, data = xyp, groups=gp, type=c("h"),
              par.strip.text = list(cex = 0.85), lend=2,
              ylab = "Partial correlation", origin=0, ylim=c(-0.325, 1.04),
              key=gph.key, par.settings=parsetBC,
              panel=function(x,y,...){
                panel.xyplot(x,y,...)
                panel.abline(h=0, lwd=0.8)
                panel.abline(h=ci95, lwd=0.8, lty=2)
                panel.abline(h=-ci95, lwd=0.8, lty=2) } )

```

```

print(update(gphB, scales=list(alternating=FALSE, tck=0.5),
      ylab = "Autocorrelation",
      main=list("B: Autocorelation -- Data vs AR processes",
        font=1, x=0, y=0.25, just="left", cex=1)),
      pos=c(0,0,0.5,0.9))
print(update(gphC,
      scales=list(x=list(at=c(1,5,10,15)), alternating=FALSE, tck=0.5),
      ylab = "Partial autocorrelation",
      main=list("C: Partial autocorrelation -- Data vs AR processes",
        font=1, x=0, y=0.25, just="left", cex=1)),
      pos=c(0.5,0,1,0.9),newpage=FALSE)

```

```

acf(LakeHuron)
## pacf(LakeHuron) gives the plot of partial autocorrelations

```

### Subsection 6.1.4: Autoregressive (AR) models

#### The AR(1) model

```

## Yule-Walker autocorrelation estimate
LH.yw <- ar(LakeHuron, order.max = 1, method = "yw") # autocorrelation estimate
# order.max = 1 for AR(1) model
LH.yw$ar # autocorrelation estimate of alpha

```

```
[1] 0.8319112
```

```

## Maximum likelihood estimate
LH.mle <- ar(LakeHuron, order.max = 1, method = "mle")
LH.mle$ar # maximum likelihood estimate of alpha

```

```
[1] 0.837546
```

```
LH.mle$x.mean # estimated series mean
```

```
[1] 579.1141
```

```
LH.mle$var.pred # estimated innovation variance
```

```
[1] 0.5092867
```

## The general AR(p) model

```
ar(LakeHuron, method="mle")
```

Call:

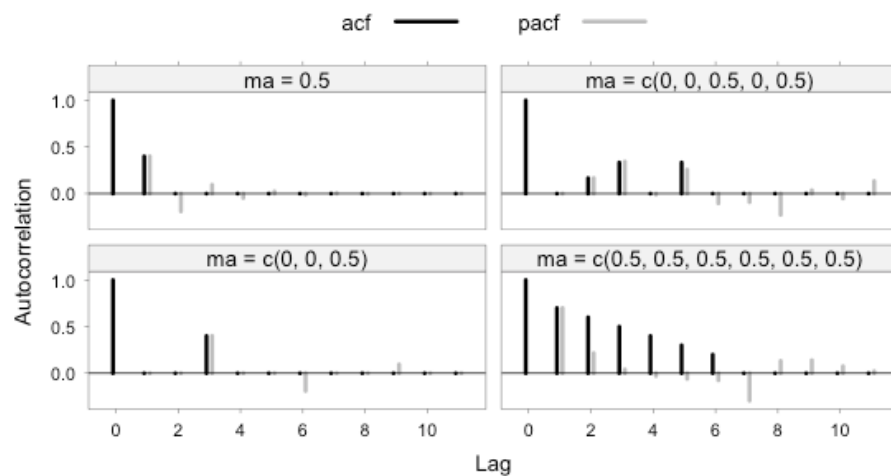
```
ar(x = LakeHuron, method = "mle")
```

Coefficients:

1	2
1.0437	-0.2496

Order selected 2     $\sigma^2$  estimated as 0.4788

## ~Moving average (MA) processes



## Subsection 6.1.5: ~Autoregressive moving average (ARMA) models – theory

## Subsection 6.1.6: Automatic model selection?

```
library(forecast, quietly=TRUE)
(aaLH <- auto.arima(LakeHuron, approximation=F, stepwise=F))
```

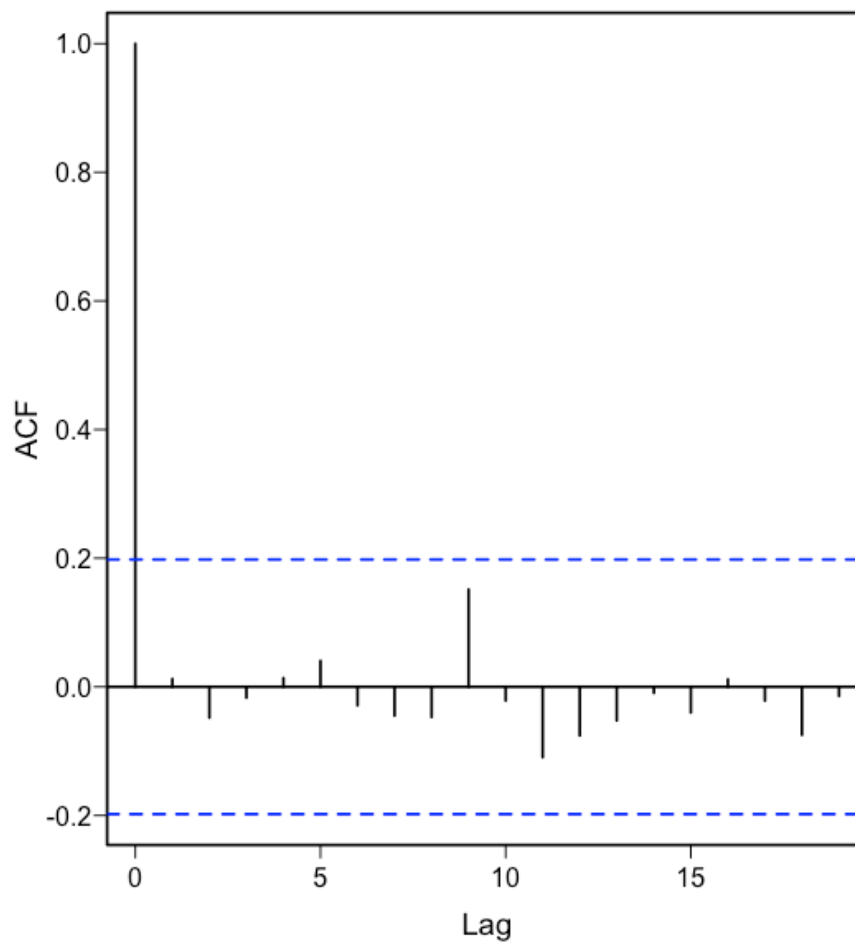
Series: LakeHuron  
ARIMA(2,1,1)

Coefficients:

	ar1	ar2	ma1
	0.9712	-0.2924	-0.9108
s.e.	0.1137	0.1030	0.0712

$\sigma^2 = 0.5003$ : log likelihood = -102.54  
AIC=213.07 AICc=213.51 BIC=223.37

```
## Check that model removes most of the correlation structure  
acf(resid(aaLH, type="innovation")) # `type="innovation"` is the default
```



```
auto.arima(LakeHuron)
```

Series: LakeHuron

ARIMA(0,1,0)

$\sigma^2 = 0.5588$ : log likelihood = -109.11

AIC=220.22 AICc=220.26 BIC=222.79

```
(aaLH0 <- auto.arima(LakeHuron, d=0, approximation=F, stepwise=F))
```

Series: LakeHuron

ARIMA(1,0,1) with non-zero mean

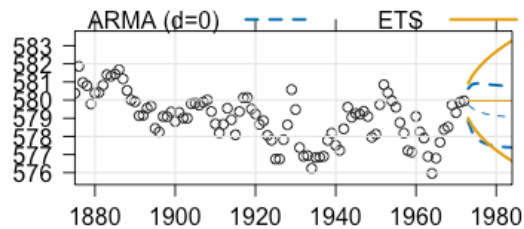
Coefficients:

	ar1	ma1	mean
	0.7449	0.3206	579.0555
s.e.	0.0777	0.1135	0.3501

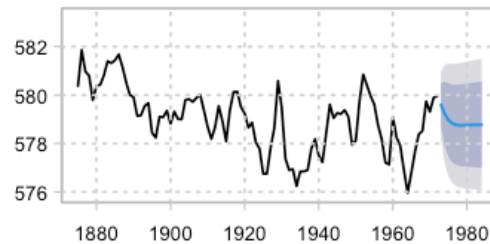
$\sigma^2 = 0.4899$ : log likelihood = -103.25

AIC=214.49 AICc=214.92 BIC=224.83

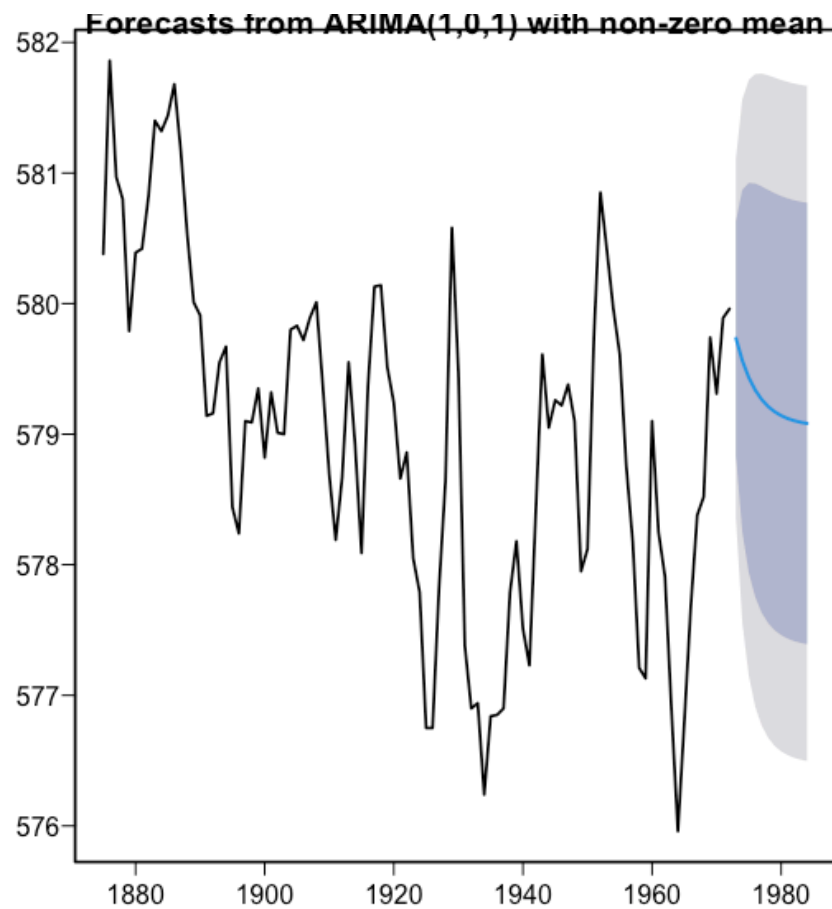
A: ARMA and ETS forecasts with 80% limits

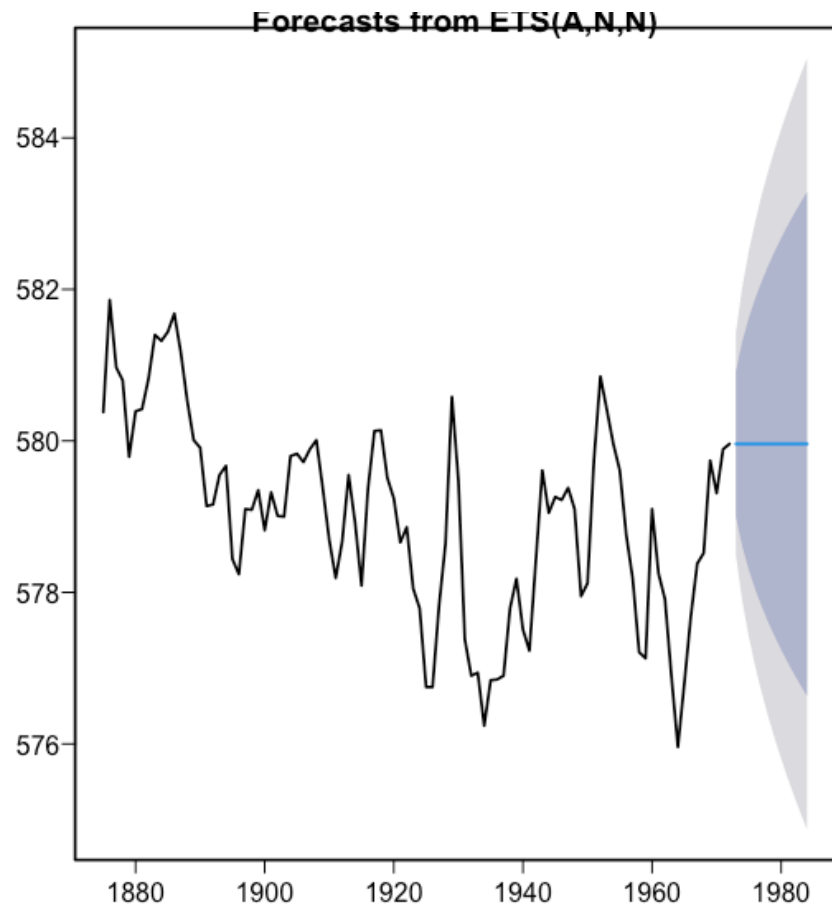


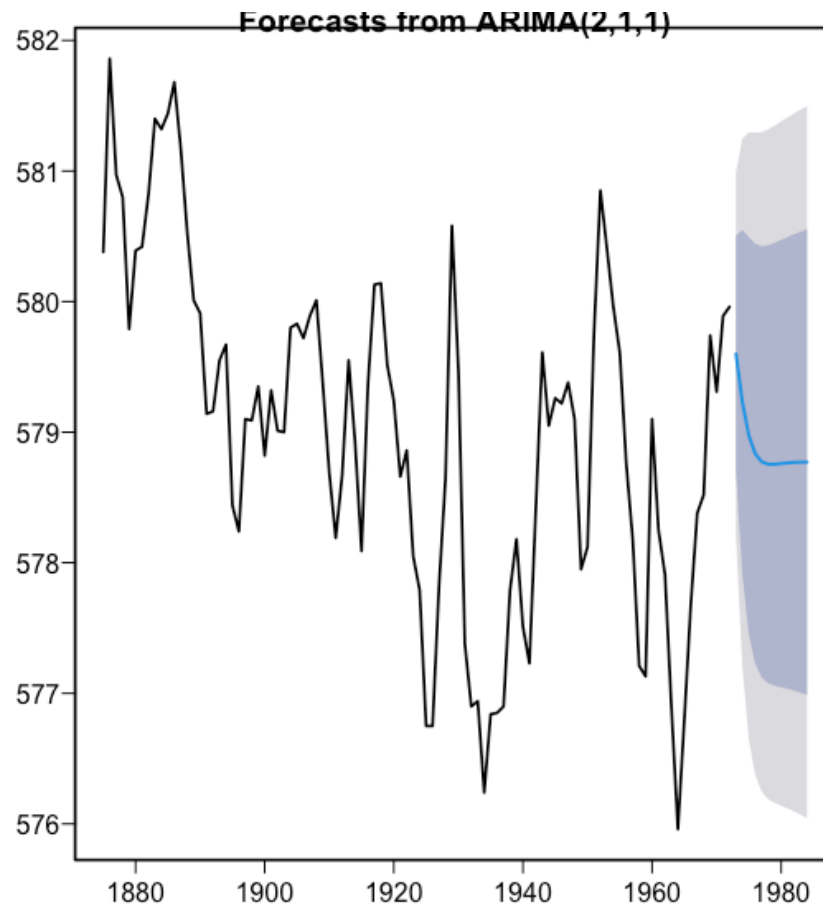
B: ARIMA forecast with 80% and 95% limits



```
plot(forecast(aaLH0, h=12)) ## `level=c(80,95)` is the default
fcETS <- forecast(LakeHuron, h=12)
plot(fcETS)
plot(forecast(aaLH, h=12, level=c(80,95))) # Panel B; ARIMA(2,1,1)
```







```
auto.arima(LakeHuron, d=0, max.Q=0, approximation=F, stepwise=F)
```

Series: LakeHuron  
ARIMA(1,0,1) with non-zero mean

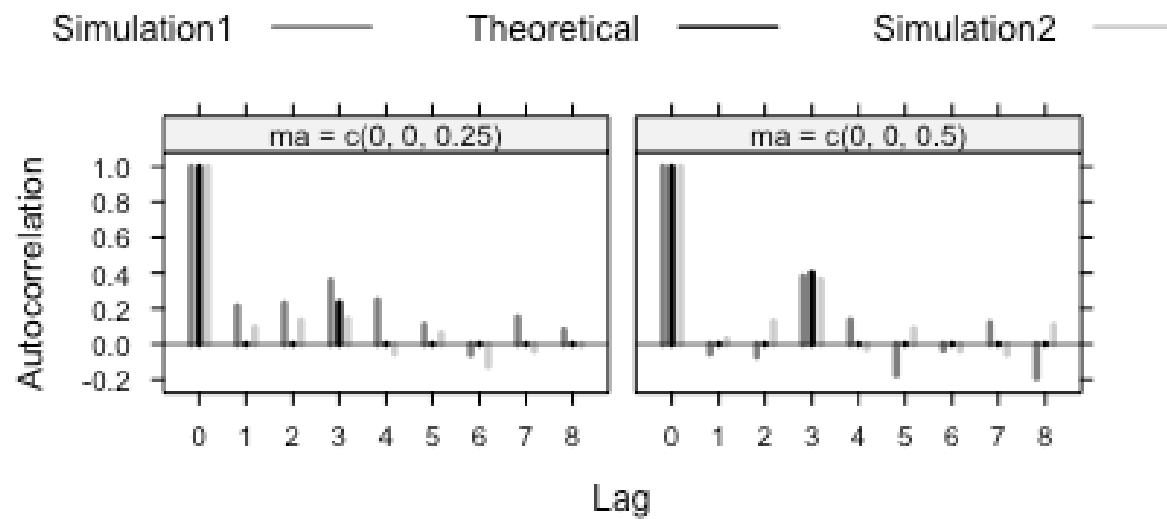
Coefficients:

	ar1	ma1	mean
	0.7449	0.3206	579.0555
s.e.	0.0777	0.1135	0.3501

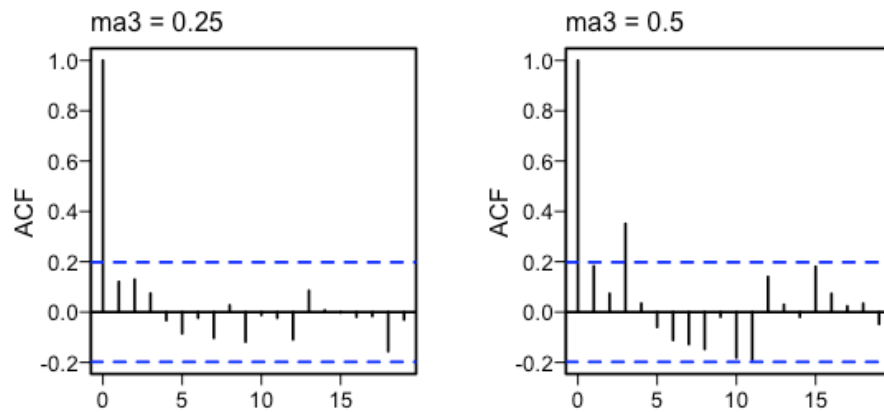
sigma<sup>2</sup> = 0.4899: log likelihood = -103.25  
AIC=214.49 AICc=214.92 BIC=224.83

**Use of simulation as a check**





```
oldpar <- par(mfrow=c(2,2), mar=c(3.1,4.6,2.6, 1.1))
for(i in 1:2){
  simts <- arima.sim(model=list(order=c(0,0,3), ma=c(0,0,0.25*i)), n=98)
  acf(simts, main="", xlab="")
  mtext(side=3, line=0.5, paste("ma3 =", 0.25*i), adj=0)
}
par(oldpar)
```



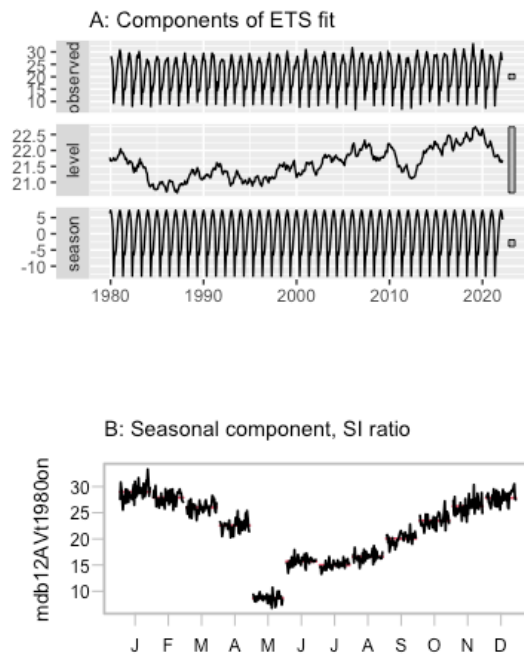
```
set.seed(29)          # Ensure that results are reproducible
estMAord <- matrix(0, nrow=4, ncol=20)
for(i in 1:4){
  for(j in 1:20){
    simts <- arima.sim(n=98, model=list(ma=c(0,0,0.125*i)))
    estMAord[i,j] <- auto.arima(simts, start.q=3)$arma[2] }
}
detectedAs <- table(row(estMAord), estMAord)
dimnames(detectedAs) <- list(ma3=paste(0.125*(1:4)),
  Order=paste(0:(dim(detectedAs)[2]-1)))
```

```
print(detectedAs)
```

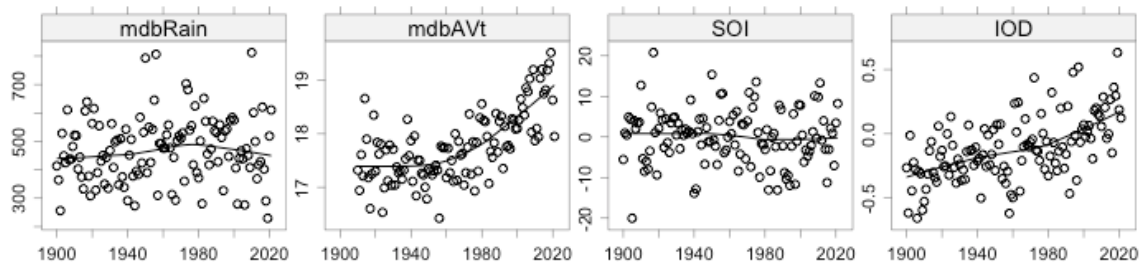
Order

ma3	0	1	2	3	4
0.125	18	2	0	0	0
0.25	11	3	0	6	0
0.375	7	2	2	7	2
0.5	1	1	0	13	5

### Subsection 6.1.7: Seasonal effects



```
suppressPackageStartupMessages(library(ggplot2))
mdb12AVt1980on <- window(DAAG::mdbAVtJtoD, c(1980,1))
AVt.ets <- ets(mdb12AVt1980on)
autoplot(AVt.ets, main="", fg="gray") +
  ggplot2::ggtitle("A: Components of ETS fit") +
  theme(plot.title = element_text(hjust=0, vjust=0.5, size=11))
monthplot(mdb12AVt1980on, col.base=2, fg="gray")
title("B: Seasonal component, SI ratio",
      font.main=1, line=1, adj=0, cex=1.25)
```

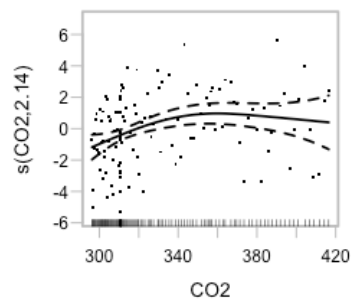


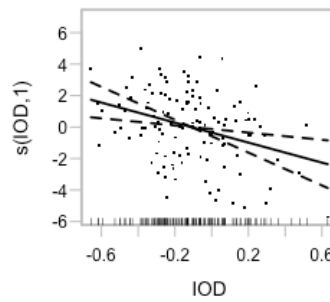
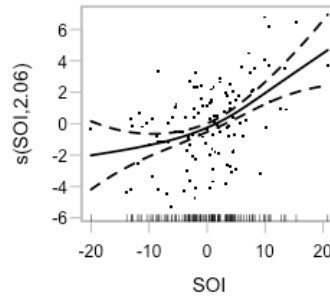
```
bomreg <- DAAG::bomregions2021
## Plot time series mdbRain, SOI, and IOD: ts object bomregions2021 (DAAG)
gph <- xyplot(ts(bomreg[, c("mdbRain", "mdbAVt", "SOI", "IOD")], start=1900),
              xlab="", type=c('p','smooth'), scales=list(alternating=rep(1,3)))
update(gph, layout=c(4,1), par.settings=DAAG::DAAGtheme(color=F))
```

```
suppressPackageStartupMessages(library(mgcv))
bomreg <- within(DAAG::bomregions2021, mdbrtRain <- mdbRain^0.5)
## Check first for a sequential correlation structure, after
## fitting smooth terms s(CO2), s(SOI), and s(IOD)
library(mgcv)
mdbrtRain.gam <- gam(mdbrtRain~s(CO2) + s(SOI) + s(IOD), data=bomreg)
auto.arima(resid(mdbrtRain.gam))
```

Series: resid(mdbrtRain.gam)  
ARIMA(0,0,0) with zero mean

sigma<sup>2</sup> = 4.424: log likelihood = -263.82  
AIC=529.64 AICc=529.67 BIC=532.44





```
plot(mdbrtRain.gam, residuals=T, cex=2, fg="gray")
## Do also `gam.check(mdbrtRain.gam)` (Output looks fine)
```

```
anova(mdbrtRain.gam)
```

Family: gaussian  
Link function: identity

Formula:  
mdbrtRain ~ s(CO2) + s(SOI) + s(IOD)

Approximate significance of smooth terms:

	edf	Ref.df	F	p-value
s(CO2)	2.144	2.673	4.345	0.01090
s(SOI)	2.058	2.637	12.581	2.72e-06
s(IOD)	1.000	1.000	9.696	0.00233

```
Box.test(resid(mdbrtRain.gam), lag=10, type="Ljung")
```

Box-Ljung test

```
data: resid(mdbrtRain.gam)
X-squared = 11.935, df = 10, p-value = 0.2894
```

```
## Examine normality of estimates of "residuals"
qqnorm(resid(mdbrtRain.gam))
```

### The mdbAVt series

```
mdbAVt.gam <- gam(mdbAVt ~ s(CO2)+s(SOI)+s(IOD), data=bomreg)
auto.arima(resid(mdbAVt.gam))
```

```
Series: resid(mdbAVt.gam)
ARIMA(0,0,0) with zero mean
```

```
sigma^2 = 0.1689: log likelihood = -59.33
AIC=120.67 AICc=120.7 BIC=123.39
```

```
anova(mdbAVt.gam)
```

```
Family: gaussian
Link function: identity
```

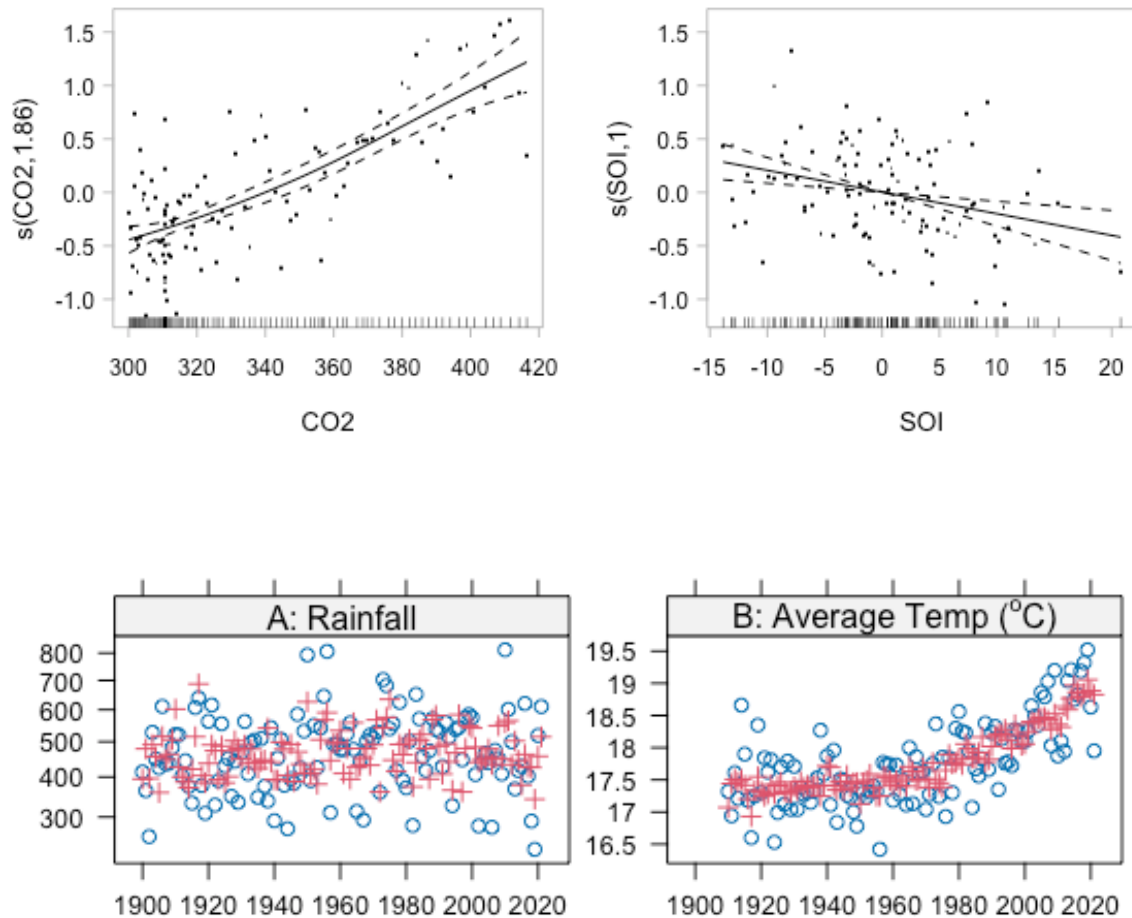
```
Formula:
mdbAVt ~ s(CO2) + s(SOI) + s(IOD)
```

Approximate significance of smooth terms:

	edf	Ref.df	F	p-value
s(CO2)	1.855	2.312	41.842	< 2e-16
s(SOI)	1.000	1.000	10.688	0.00145
s(IOD)	1.000	1.000	0.005	0.94403

```
mdbAVt1.gam <- gam(mdbAVt ~ s(CO2)+s(SOI), data=bomreg)
```

```
plot(mdbAVt1.gam, residuals=TRUE)
```



```
faclevs <- c("A: Rainfall", expression("B: Average Temp (\"^o*C)"))
fitrain <- fitted(mdbrtRain.gam)
fitAVt <- c(rep(NA,10), fitted(mdbAVt1.gam))
gph <- xyplot(mdbrtRain+mdbAVt~Year,data=bomreg, outer=T, xlab="", ylab="",
  scales=list(y=list(relation='free',
    at=list(sqrt((3:8)*100),(33:39)/2),
```

```

        labels=list((3:8)*100,(33:39)/2)), x=list(alternating=rep(1,2))),
        strip=strip.custom(factor.levels=faclevs))
gph + latticeExtra::as.layer(xyplot(fitrain+fitAVt~Year, outer=T,
                                   scales=list(y=list(relation='free')),
                                   data=bomreg, pch=3, col=2))

```

```

## Use `auto.arima()` to choose the ARIMA order:
aaFitC02 <- with(bomreg[-(1:10),], auto.arima(mdbAVt, xreg=cbind(CO2,SOI)))
## Try including a degree 2 polynomial term
aaFitpol2C02 <- with(bomreg[-(1:10),],
                    auto.arima(mdbAVt, xreg=cbind(poly(CO2,2),SOI)))
cbind(AIC(aaFitC02, aaFitpol2C02), BIC=BIC(aaFitC02, aaFitpol2C02))

```

	df	AIC	BIC	df	BIC
aaFitC02	7	125.3765		7	144.4060
aaFitpol2C02	5	129.0109		5	142.6033

### Subsection 6.1.8: The `gamm()` function, with a correlated errors model

```

SOI.gam <- gam(SOI~s(Year), data=bomreg)
auto.arima(resid(SOI.gam)) # sigma^2 = 43.4

```

Series: resid(SOI.gam)  
 ARIMA(0,0,2) with zero mean

Coefficients:

	ma1	ma2
	0.2070	-0.1557
s.e.	0.0943	0.1008

sigma^2 = 43.43: log likelihood = -402.2  
 AIC=810.41 AICc=810.61 BIC=818.82

```

## The following breaks the model into two parts -- gam and lme
SOI.gamm <- gamm(SOI~s(Year), data=bomreg)
res <- resid(SOI.gamm$lme, type="normalized")
auto.arima(res) # sigma^2 = 0.945

```



```
Series: res
ARIMA(0,0,2) with zero mean
```

```
Coefficients:
```

```
      ma1      ma2
      0.2070 -0.1557
s.e.  0.0943  0.1008
```

```
sigma^2 = 0.9446:  log likelihood = -168.68
AIC=343.37  AICc=343.57  BIC=351.78
```

```
## Extract scale estimate for `gam` component of SOI.gamm
summary(SOI.gamm$gam)[['scale']]      # 45.98
```

```
[1] 45.98091
```

```
# Note that 45.98 x .945 ~= 43.4
```

```
SOIma2.gamm <- gamm(SOI~s(Year), data=bomreg, correlation=corARMA(q=2))
coef(SOIma2.gamm$lme$modelStruct$corStruct, unconstrained = FALSE) # MA2 ests
```

```
      Theta1      Theta2
0.2070115 -0.1557351
```

```
SOIar2.gamm <- gamm(SOI~s(Year), data=bomreg, correlation=corARMA(p=2))
coef(SOIar2.gamm$lme$modelStruct$corStruct, unconstrained = FALSE) # AR2 ests
```

```
      Phi1      Phi2
0.2257966 -0.2097561
```

```
cbind(AIC(SOI.gam, SOIma2.gamm$lme, SOIar2.gamm$lme),
      BIC=BIC(SOI.gam, SOIma2.gamm$lme, SOIar2.gamm$lme)[,2])
```

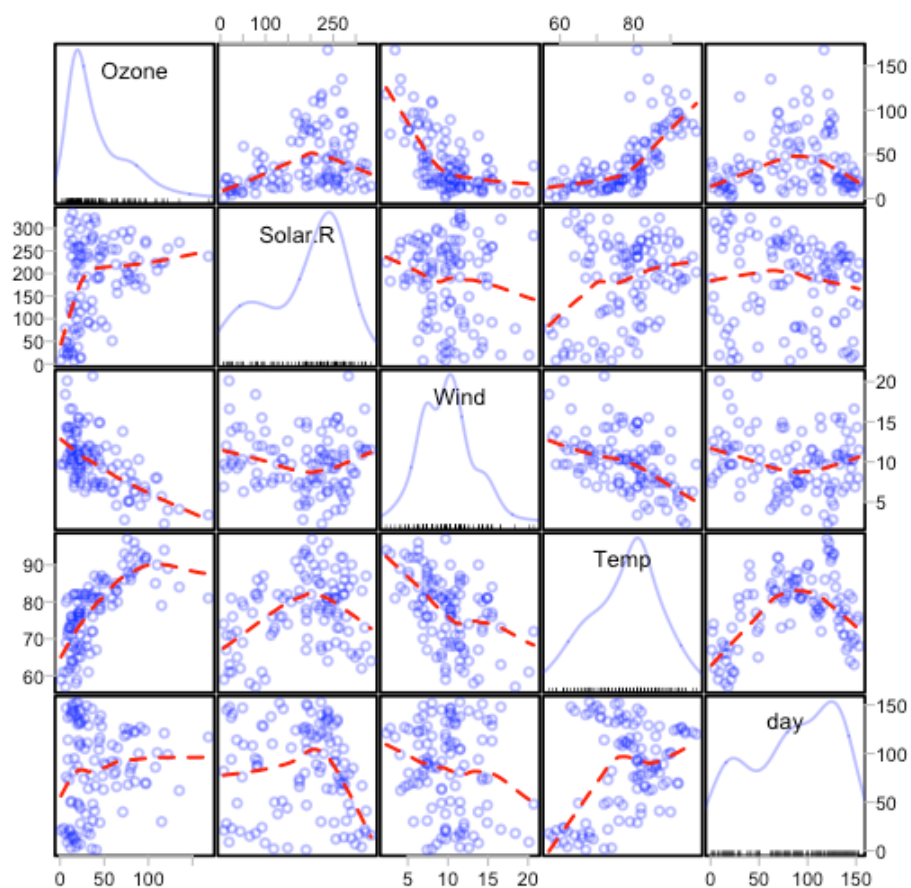
	df	AIC	BIC
SOI.gam	3	819.2646	827.6767
SOIma2.gamm\$lme	6	816.4090	833.2331
SOIar2.gamm\$lme	6	815.5117	832.3358

**The dataset airquality (153 days, New York, 1972)**

```
## Add time in days from May 1 to data.
airq <- cbind(airquality[, 1:4], day=1:nrow(airquality))
# Column 5 ('day' starting May 1) replaces columns 'Month' & 'Day')
## Check numbers of missing values # Solar.R:7; Ozone:37
mice::md.pattern(airq, plot=FALSE) # Final row has totals missing.
```

	Wind	Temp	day	Solar.R	Ozone	
111	1	1	1	1	1	0
35	1	1	1	1	0	1
5	1	1	1	0	1	1
2	1	1	1	0	0	2
	0	0	0	7	37	44

```
smoothPars <- list(col.smooth='red', lty.smooth=2, spread=0)
car::spm(airq, cex.labels=1.2, regLine=FALSE, oma=c(1.95,3,4,3), gap=.15,
         col=adjustcolor('blue', alpha.f=0.3), smooth=smoothPars, fg="gray")
```



```
car::powerTransform(gam(Ozone ~ s(Solar.R)+s(Wind)+s(Temp)+s(day), data=airq))
```

Estimated transformation parameter  
Y1  
0.2306506

```
airq$rt4Ozone <- airq$Ozone^0.25
```

```
Ozone.gam <- gam(rt4Ozone ~ s(Solar.R)+s(Wind)+s(Temp)+s(day), data=airq)
auto.arima(resid(Ozone.gam)) # Independent errors model appears OK
```

Series: resid(Ozone.gam)  
ARIMA(0,0,0) with zero mean

```
sigma^2 = 0.05897: log likelihood = -0.4  
AIC=2.79 AICc=2.83 BIC=5.5
```

```
## Check model terms  
anova(Ozone.gam) # For GAM models, this leaves out terms one at a time
```

```
Family: gaussian  
Link function: identity
```

```
Formula:  
rt4Ozone ~ s(Solar.R) + s(Wind) + s(Temp) + s(day)
```

Approximate significance of smooth terms:

	edf	Ref.df	F	p-value
s(Solar.R)	2.284	2.871	7.701	0.000159
s(Wind)	2.448	3.104	9.497	1.29e-05
s(Temp)	4.275	5.287	13.305	< 2e-16
s(day)	1.000	1.000	0.495	0.483507

```
## The term in `day` has no explanatory, and will be removed  
Ozone1.gam <- update(Ozone.gam, formula=rt4Ozone ~ s(Solar.R)+s(Wind)+s(Temp))
```

### Subsection 6.1.9: A calibration problem with time series errors

```
flakes <- DAAG::frostedflakes  
calib.arima <- with(flakes, auto.arima(IA400, xreg=Lab))  
calib.arima
```

```
Series: IA400  
Regression with ARIMA(0,0,1) errors
```

```
Coefficients:  
          ma1 intercept      xreg  
          0.3876      6.9235  0.8323  
s.e.    0.0859      2.5634  0.0679
```

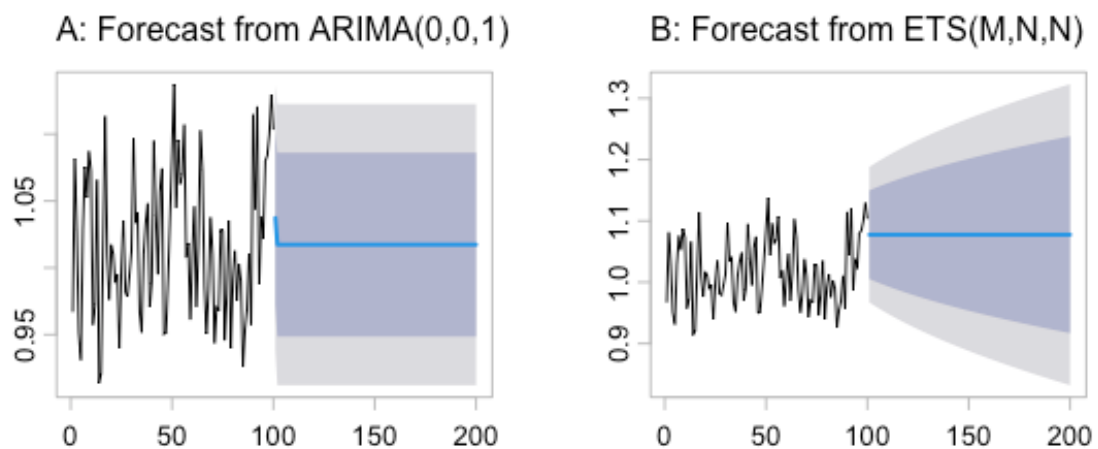
```
sigma^2 = 3.278: log likelihood = -199.81  
AIC=407.62 AICc=408.04 BIC=418.04
```

```
with(flakes, coef(auto.arima(IA400/Lab, approximation=F, stepwise=F)))
```

```
ma1 intercept
0.3718151 1.0173303
```

```
with(flakes, coef(auto.arima(IA400-Lab, approximation=F, stepwise=F)))
```

```
ma1 intercept
0.3838733 0.6201945
```



## Section 6.2: Nonlinear time series

```
x <- numeric(999) # x will contain the ARCH(1) errors
x0 <- rnorm(1)
for (i in 1:999){
  x0 <- rnorm(1, sd=sqrt(.25 + .95*x0^2))
  x[i] <- x0
}
```

```
suppressPackageStartupMessages(library(tseries))
garch(x, order = c(0, 1), trace=FALSE)
```

```
Call:
garch(x = x, order = c(0, 1), trace = FALSE)
```

```
Coefficient(s):
      a0      a1
0.2804  0.9633
```

## Section 6.3: Further reading

### Spatial statistics

### Other time series models and packages

## Section 6.4: Exercises

6.4

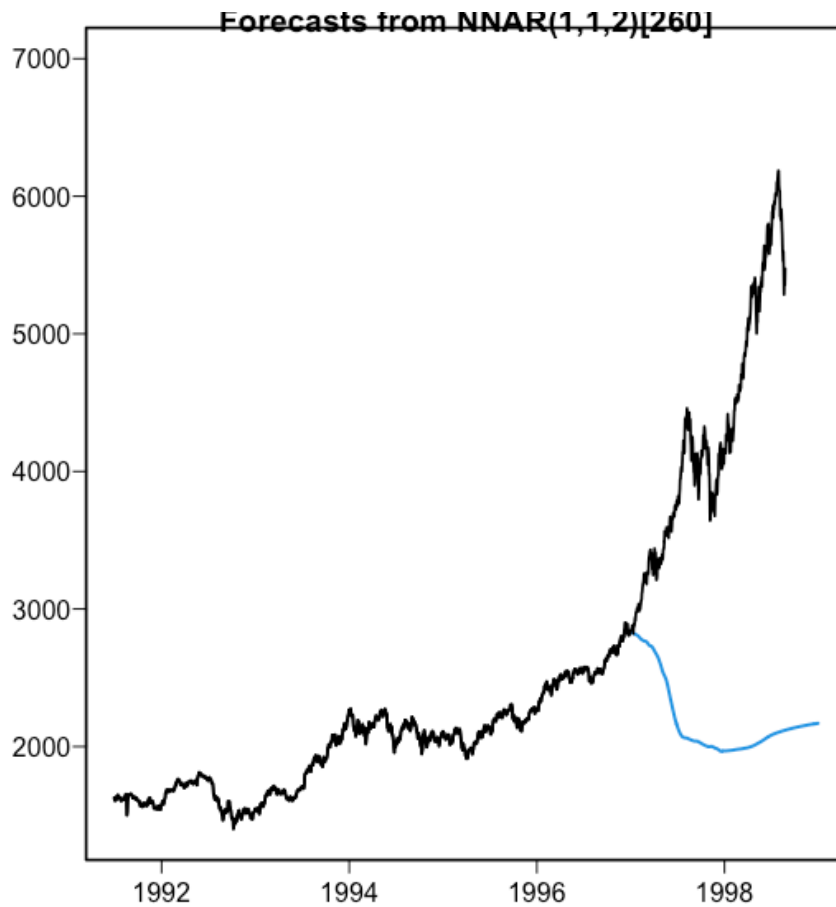
```
xx <- matrix(x, ncol=1000)
```

6.7

```
library(tseries)
data(ice.river)
river1 <- diff(log(ice.river[, 1]))
```

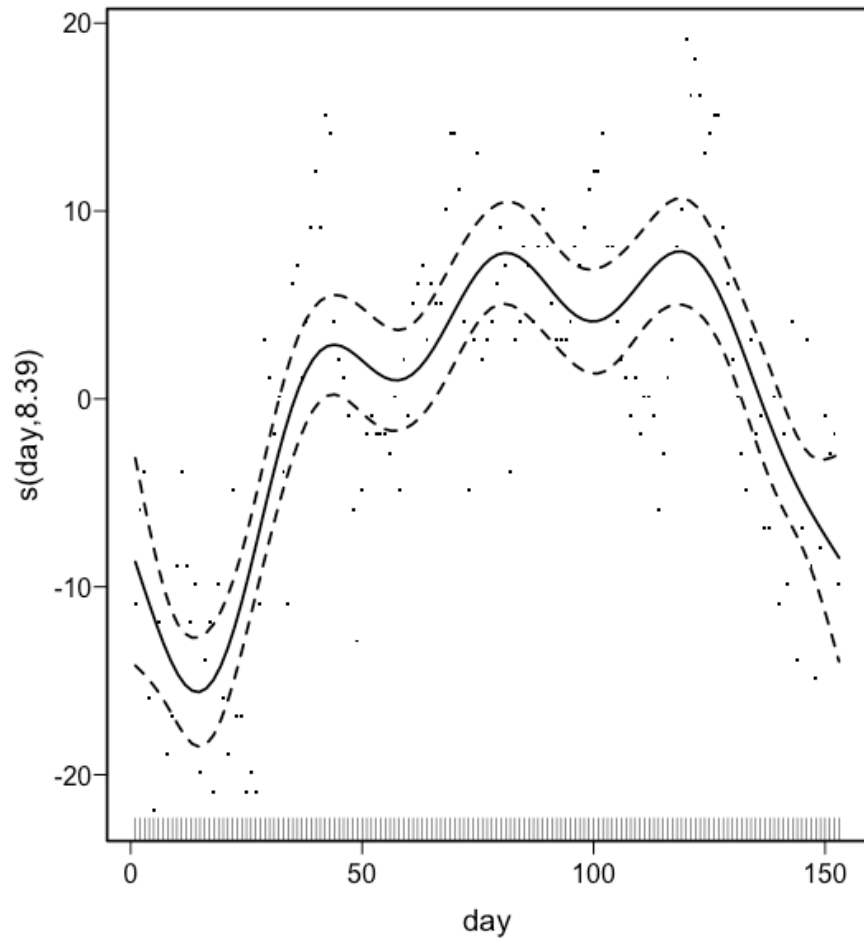
6.9

```
library(forecast)
Eu1 <- window(EuStockMarkets[,1], end = c(1996, 260))
Eu1nn <- nnetar(Eu1)
Eu1f <- forecast(Eu1nn, end=end(EuStockMarkets[,1]))
plot(Eu1f, ylim=c(1400, 7000))
lines(EuStockMarkets[,1])
```

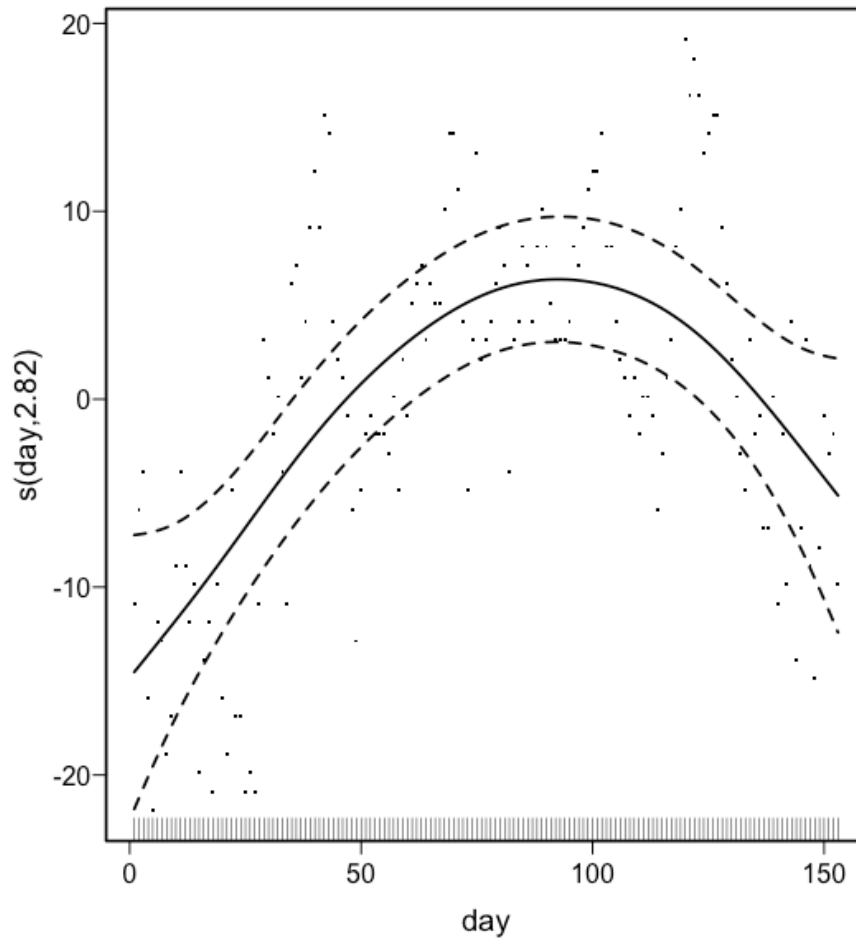


6.10a

```
airq <- cbind(airquality[, 1:4], day=1:nrow(airquality))
# Column 5 ('day' starting May 1) replaces columns 'Month' & 'Day'
library(mgcv)
temp.gam <- gam(Temp~s(day), data=airq)
tempAR1.gamm <- gamm(Temp~s(day), data=airq, correlation=corAR1())
plot(temp.gam, res=T, cex=2)
plot(tempAR1.gamm$gam, res=T, cex=2)
```







6.10b

```
(Phi <- coef(tempAR1.gamm$lme$modelStruct$corStruct, unconstrained = FALSE) )
Sigma <- sqrt(tempAR1.gamm$gam$sig2)
## Simulate an AR1 process with this parameter
AR1.sim <- arima.sim(model=list(ar=Phi), n=nrow(airq), sd=Sigma)
simSeries <- AR1.sim+fitted(tempAR1.gamm$gam)
plot(I(1:nrow(airq)), simSeries)
## Compare with initial series
plot(I(1:nrow(airq)), airq$Temp)
```

```
if(file.exists("/Users/johnm1/pkgs/PGRcode/inst/doc/")){
code <- knitr::knit_code$get()
txt <- paste0("\n## ", names(code), "\n", sapply(code, paste, collapse='\n'))
}
```

```
writeLines(txt, con="/Users/johnm1/pkg/PGRcode/inst/doc/ch6.R")  
}
```

## 7 Chapter 7: Multilevel models, and repeated measures

### Packages required (plus any dependencies)

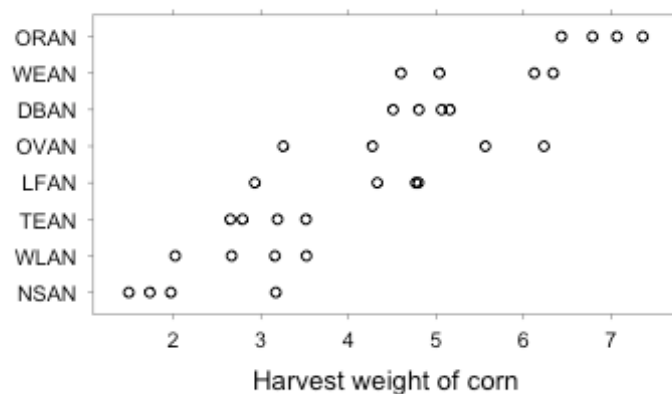
DAAG lme4 afex MASS utils devtools qra glmmTMB DHARMA MEMSS forecast splines  
gamlss plotrix nlme

Additionally, knitr and Hmisc are required in order to process the Rmd source file.

```
Hmisc::knitrSet(basename="mva", lang='markdown', fig.path="figs/g", w=7, h=7)
oldopt <- options(digits=4, formatR.arrow=FALSE, width=70, scipen=999)
library(knitr)
## knitr::render_listings()
opts_chunk[['set']](cache.path='cache-', out.width="80%", fig.align="center",
                    fig.show='hold', size="small", ps=10, strip.white = TRUE,
                    comment=NA, width=70, tidy.opts = list(replace.assign=FALSE))
```

### Section 7.1 Corn yield data — analysis using aov()

#### Corn yield measurements example



```
ant111b <- within(DAAG::ant111b, Site <- reorder(site, harvwt, FUN=mean))
gph <- lattice::stripplot(Site ~ harvwt, data=ant111b,
                          xlab="Harvest weight of corn")
update(gph, par.settings=DAAG::DAAGtheme(color=FALSE), scales=list(tck=0.5))
```

```
ant111b <- DAAG::ant111b
ant111b.aov <- aov(harvwt ~ 1 + Error(site), data=ant111b)
```

```
summary(ant111b.aov)
```

```
Error: site
      Df Sum Sq Mean Sq F value Pr(>F)
Residuals  7   70.3    10.1
```

```
Error: Within
      Df Sum Sq Mean Sq F value Pr(>F)
Residuals 24   13.9    0.578
```

## Interpreting the mean squares

### Details of the calculations

### Practical use of the analysis of variance results

### Random effects vs. fixed effects

### Nested factors – a variety of applications

### Subsection 7.1.1: A More Formal Approach

### Relations between variance components and mean squares

### Interpretation of variance components

### Intra-class correlation

## Section 7.2 Analysis using lme4::lmer()

```
library(lme4)
ant111b.lmer <- lmer(harvwt ~ 1 + (1 | site), data=ant111b)
```

```
## Note that there is no degrees of freedom information.
print(ant111b.lmer, ranef.comp="Variance")
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: harvwt ~ 1 + (1 | site)
Data: ant111b
REML criterion at convergence: 94.42
Random effects:
 Groups   Name      Variance
 site     (Intercept) 2.368
 Residual                0.578
Number of obs: 32, groups:  site, 8
Fixed Effects:
(Intercept)
      4.29
```

### The processing of output from lmer()

```
coef(summary(ant111b.lmer))
```

```
              Estimate Std. Error t value
(Intercept)    4.292      0.5604   7.659
```

### Fitted values and residuals in lmer()

```
s2W <- 0.578; s2L <- 2.37; n <- 4
sitemeans <- with(ant111b, sapply(split(harvwt, site), mean))
grandmean <- mean(sitemeans)
shrinkage <- (n*s2L)/(n*s2L+s2W)
## Check that fitted values equal BLUPs, and compare with site means
BLUP <- grandmean + shrinkage*(sitemeans - grandmean)
BLUP <- fitted(ant111b.lmer)[match(names(sitemeans), ant111b$site)]
BLUP <- grandmean + ranef(ant111b.lmer)$site[[1]]
```

```
rbind(BLUP=BLUP, sitemeans=sitemeans)
```

	DBAN	LFAN	NSAN	ORAN	OVAN	TEAN	WEAN	WLAN
BLUP	4.851	4.212	2.217	6.764	4.801	3.108	5.455	2.925
sitemeans	4.885	4.207	2.090	6.915	4.833	3.036	5.526	2.841

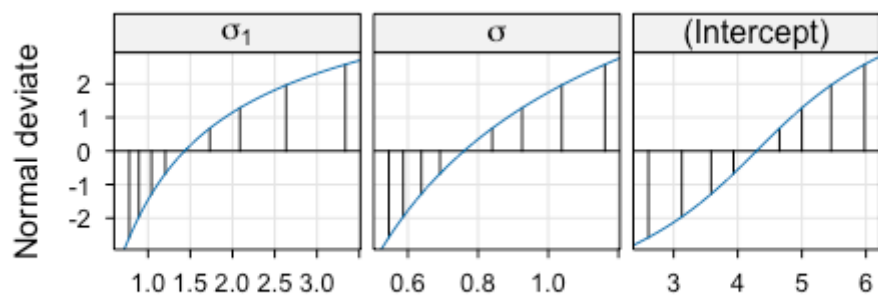
### \*Uncertainty in the parameter estimates — profile likelihood and alternatives

```
prof.lmer <- profile(ant111b.lmer)
CI95 <- confint(prof.lmer, level=0.95)
rbind("sigmaL^2"=CI95[1,]^2, "sigma^2"=CI95[2,]^2)
```

	2.5 %	97.5 %
sigmaL^2	0.7965	6.936
sigma^2	0.3444	1.079

```
CI95[3,]
```

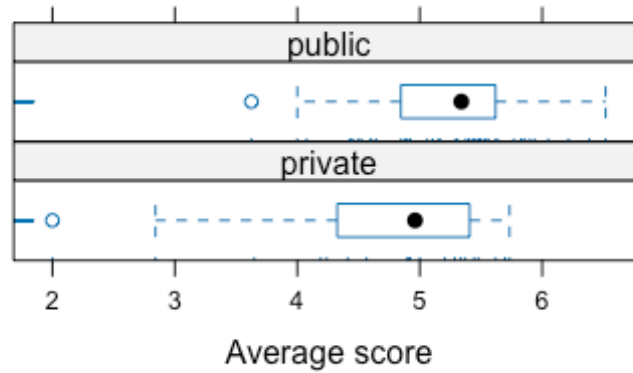
	2.5 %	97.5 %
	3.128	5.456



```
library(lattice)
gph <- xyplot(prof.lmer, conf=c(50, 80, 95, 99)/100,
              aspect=0.8, between=list(x=0.35))
update(gph, scales=list(tck=0.5), ylab="Normal deviate")
```

## Modeling more than two levels of random variation

### Section 7.3 Survey data, with clustering



```
## Means of like (data frame science: DAAG), by class
science <- DAAG::science
classmeans <- with(science, aggregate(like, by=list(PrivPub, Class), mean))
# NB: Class identifies classes independently of schools
#     class identifies classes within schools
names(classmeans) <- c("PrivPub", "Class", "avlike")
gph <- bwplot(~avlike|PrivPub, layout=c(1,2), xlab="Average score",
             panel=function(x,y,...){panel.bwplot(x,y,...)
             panel.rug(x,y,...)}, data=classmeans)
update(gph, scales=list(tcl=0.4))
```

#### Subsection 7.3.1: Alternative models

```
science <- DAAG::science
science.lmer <- lmer(like ~ sex + PrivPub + (1 | school) +
                    (1 | school:class), data = science,
                    na.action=na.exclude)
```

```
print(VarCorr(science.lmer), comp="Variance", digits=2)
```

Groups	Name	Variance
school:class	(Intercept)	0.32
school	(Intercept)	0.00
Residual		3.05

```
print(coef(summary(science.lmer)), digits=2)
```

	Estimate	Std. Error	t value
(Intercept)	4.72	0.162	29.1
sexm	0.18	0.098	1.9
PrivPubpublic	0.41	0.186	2.2

```
summary(science.lmer)$ngrps
```

school:class	school
66	41

```
science1.lmer <- lmer(like ~ sex + PrivPub + (1 | school:class),
  data = DAAG::science, na.action=na.exclude)
```

```
print(VarCorr(science1.lmer), comp="Variance", digits=3)
```

Groups	Name	Variance
school:class	(Intercept)	0.321
Residual		3.052

```
print(coef(summary(science1.lmer)), digits=2)
```

	Estimate	Std. Error	t value
(Intercept)	4.72	0.162	29.1
sexm	0.18	0.098	1.9
PrivPubpublic	0.41	0.186	2.2

```
opt <- options(contrasts=c("contr.sum","contr.poly"))
# Change is otherwise made as and if required for individual factors
# prior to fitting model, and a warning message is generated.
afex::mixed(like ~ sex + PrivPub + (1 | school:class), method="KR", type=2,
  data = na.omit(science), sig_symbols=rep("",4), progress=FALSE)
```



Mixed Model Anova Table (Type 2 tests, KR-method)

Model: like ~ sex + PrivPub + (1 | school:class)

Data: na.omit(science)

	Effect	df	F	p.value
1	sex	1, 1379.49	3.44	.064
2	PrivPub	1, 60.44	4.91	.030

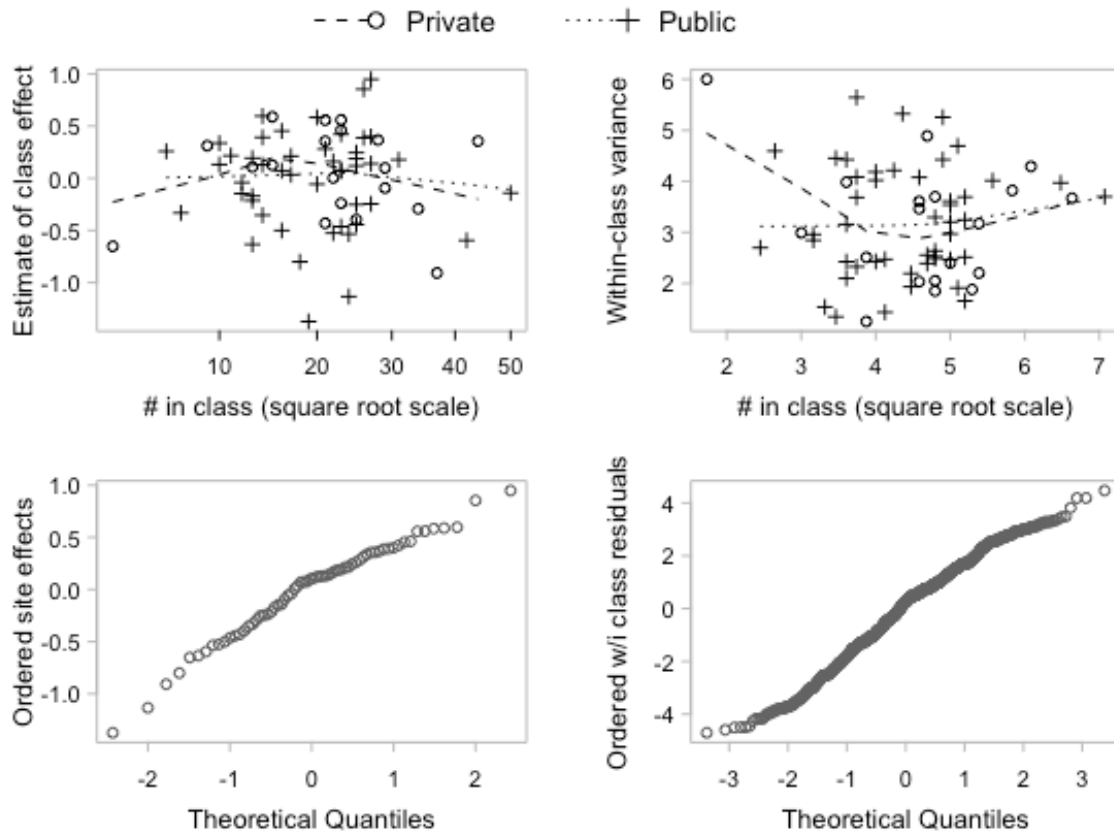
```
options(opt) # Reset to previous contrasts setting
```

### More detailed examination of the output

```
## Use profile likelihood
pp <- profile(science1.lmer, which="theta_")
# which="theta_": all random parameters
# which="beta_": fixed effect parameters
var95 <- confint(pp, level=0.95)^2
# Square to get variances in place of SDs
rownames(var95) <- c("sigma_Class^2", "sigma^2")
signif(var95, 3)
```

	2.5 %	97.5 %
sigma_Class^2	0.178	0.511
sigma^2	2.830	3.300

```
## Fit model and generate quantities that will be plotted
science1.lmer <- lmer(like ~ sex + PrivPub + (1 | school:class),
data = science, na.action=na.omit)
## Panel A: random site effects vs number in class
ranf <- ranef(obj = science1.lmer, drop=TRUE)[["school:class"]]
flist <- science1.lmer@flist[["school:class"]]
privpub <- science[match(names(ranf), flist), "PrivPub"]
num <- unclass(table(flist)); numlabs <- pretty(num)
## Panel B: Within class variance estimates vs numbers
res <- residuals(science1.lmer)
vars <- tapply(res, INDEX=list(flist), FUN=var)*(num-1)/(num-2)
## Panel C: Normal probability of random site effects (`ranf`)
## Panel D: Normal probability of residuals (`res`)
```



```
opar <- par(oma=c(0,0,1.5,0))
## Panel A: Plot effect estimates vs number
xlab12 <- "# in class (square root scale)"
plot(sqrt(num), ranf, xaxt="n", pch=c(1,3)[as.numeric(privpub)], cex=0.8,
      xlab=xlab12, ylab="Estimate of class effect", fg="gray")
lines(lowess(sqrt(num[privpub=="private"])),
      ranf[privpub=="private"], f=1.1, lty=2)
lines(lowess(sqrt(num[privpub=="public"])),
      ranf[privpub=="public"], f=1.1, lty=3)
axis(1, at=sqrt(numlabs), labels=paste(numlabs), lwd=0, lwd.ticks=1)
## Panel B: Within class variance estimates vs numbers
plot(sqrt(num), vars, pch=c(1,3)[unclass(privpub)], cex=0.8,
      xlab=xlab12, ylab="Within-class variance", fg="gray")
lines(lowess(sqrt(num[privpub=="private"])),
      as.vector(vars)[privpub=="private"], f=1.1, lty=2)
lines(lowess(sqrt(num[privpub=="public"])),
      as.vector(vars)[privpub=="public"], f=1.1, lty=3)
```

```
## Panel C: Normal quantile-quantile plot of site effects
qqnorm(ranf, ylab="Ordered site effects", cex=0.8, main="",
       col="gray40", fg="gray")
## Panel D: Normal quantile-quantile plot of residuals
qqnorm(res, ylab="Ordered w/i class residuals", cex=0.8, main="",
       col="gray40", fg="gray")
par(fig = c(0, 1, 0, 1), oma = c(0, 0, 0, 0), mar = c(0, 0, 0, 0),
    new = TRUE)
plot(0, 0, type = "n", bty = "n", xaxt = "n", yaxt = "n")
legend(x="top", legend=c("Private", "Public"), pch=c(1,3),
      lwd=c(1,1), lty=2:3, cex=1.25,
      xjust=0.5, yjust=0.8, horiz=TRUE, merge=FALSE, bty="n")
par(opar)
```

### Subsection 7.3.2: Instructive, though faulty, analyses

#### Ignoring class as the random effect

```
science2.lmer <- lmer(like ~ sex + PrivPub + (1 | school),
data = science, na.action=na.exclude)
print(coef(summary(science2.lmer)), digits=3)
```

	Estimate	Std. Error	t value
(Intercept)	4.738	0.163	29.00
sexm	0.197	0.101	1.96
PrivPubpublic	0.417	0.185	2.25

```
## NB: Output is misleading
print(VarCorr(science2.lmer), comp="Variance", digits=3)
```

Groups	Name	Variance
school	(Intercept)	0.166
Residual		3.219

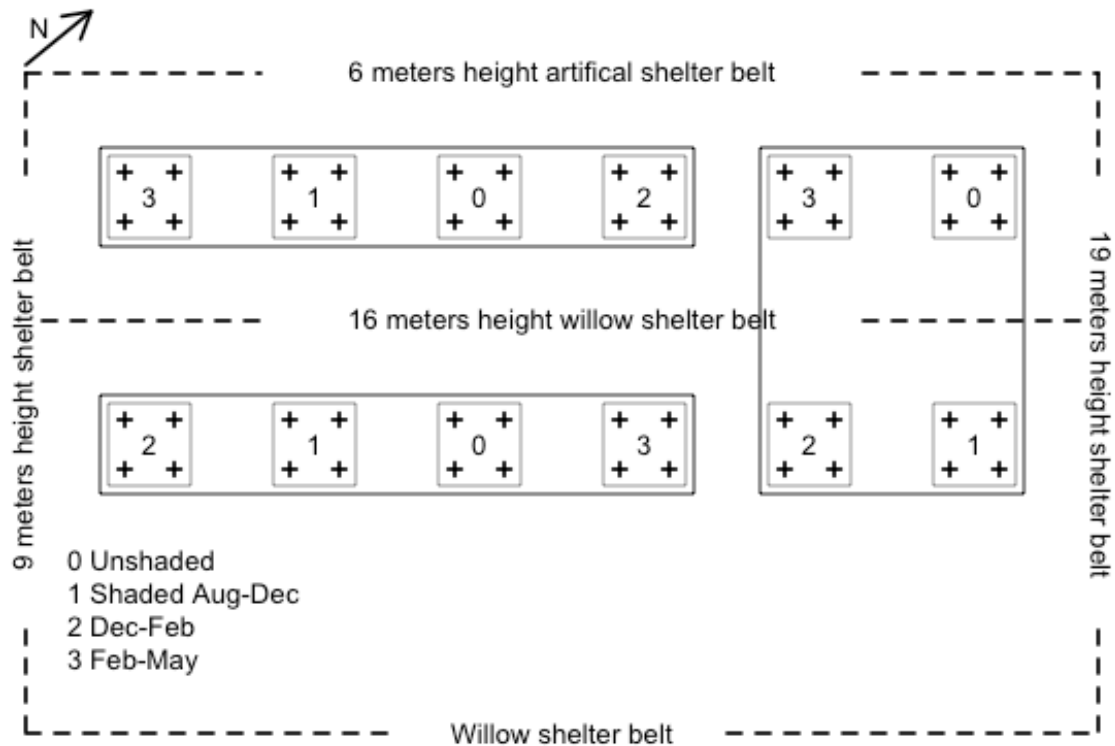
#### Ignoring the random structure in the data

```
## Faulty analysis, using lm
science.lm <- lm(like ~ sex + PrivPub, data=science)
round(coef(summary(science.lm)), digits=4)
```

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	4.7402	0.0996	47.616	0.0000
sexm	0.1509	0.0986	1.531	0.1261
PrivPubpublic	0.3951	0.1051	3.759	0.0002

### Subsection 7.3.3: Predictive accuracy

## Section 7.4 A multilevel experimental design



```
par(mar=rep(0.25,4))
MASS::eqscplot(c(0,13),c(4.0,13),type="n",xlab="",ylab="", asp=1, axes=F)
eps <- 0.1
suby <- 12
vines<-function(x=1,y=1,subp=0, suby=12){
  lines(c(y,y,y+1,y+1,y), suby-c(x,x+1,x+1,x,x),lwd=0.5)
  points(c(y+.2,y+.2,y+.8,y+.8),suby-c(x+.2,x+.8,x+.8,x+.2),pch=3,cex=0.65)
  text(y+.5,suby-(x+.5),paste(subp))
}
```

```

k<-0
for(i in c(1,3,5,7)){k<-k+1; vines(1,i,c(3,1,0,2)[k])}
k<-0
for(i in c(1,3,5,7)){k<-k+1; vines(4,i,c(2,1,0,3)[k])}
k <- 0
for(i in c(1,4,4,1)){k<-k+1
j<-c(9,9,11,11)[k]
vines(i,j,c(3,2,1,0)[k])
}
lines(c(2*eps,2.85,NA,10.15,13-2*eps), suby-c(3,3,NA,3,3),lty=2)
lines(c(0,2.85,NA,10.15,13),suby-c(0,0,NA,0,0),lty=2)
lines(c(0,4.5,NA,8.5,13),suby-c(8,8,NA,8,8),lty=2)
lines(rep(0,5),suby-c(0,1.25,NA,6.75,8),lty=2)
lines(rep(13,5),suby-c(0,1.25,NA,6.75,8),lty=2)
lines(c(9,9,12,12,9)+c(-eps,-eps,eps,eps,-eps),
      suby-(c(1,5,5,1,1)+c(-eps,eps,eps,-eps,-eps)), lwd=1)
lines(c(1,1,8,8,1)+c(-eps,-eps,eps,eps,-eps),
      suby-c(c(1,2,2,1,1)+c(-eps,eps,eps,-eps,-eps)), lwd=1)
lines(c(1,1,8,8,1)+c(-eps,-eps,eps,eps,-eps),
      suby-c(c(1,2,2,1,1)+3+c(-eps,eps,eps,-eps,-eps)), lwd=1)
text(6.5,suby,"6 meters height artifical shelter belt")
text(0,suby-4,"9 meters height shelter belt", srt=90)
text(13,suby-4,"19 meters height shelter belt", srt=-90)
text(6.5,suby-8,"Willow shelter belt")
text(0.5,suby-6.5,"0 Unshaded \n1 Shaded Aug-Dec \n2 Dec-Feb \n3 Feb-May", adj=0)
text(6.5,suby-3,"16 meters height willow shelter belt")
offset <- c(4.75, 4.75-sqrt(3)*0.5)/6
arrows(x0=0,y0=12.1, x1=0+offset[1], y1=12.1+offset[2], length=0.15)
text(0.17, 12.55,"N")

```

### Subsection 7.4.1: The analysis of variance (anova) table

```

## Analysis of variance: data frame kiwishade (DAAG)
kiwishade <- DAAG::kiwishade
kiwishade.aov <- aov(yield ~ shade + Error(block/shade),
data=kiwishade)
summary(kiwishade.aov)

```

Error: block

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Residuals	2	172	86.2		

Error: block:shade

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
shade	3	1395	465	22.2	0.0012
Residuals	6	126	21		

Error: Within

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Residuals	36	439	12.2		

### Subsection 7.4.2: Expected values of mean squares

```
model.tables(kiwishade.aov, type="means", cterms="shade")
```

Tables of means

Grand mean

96.53

shade

shade

	none	Aug2Dec	Dec2Feb	Feb2May
	100.20	103.23	89.92	92.77

```
## Calculate treatment means
with(kiwishade, sapply(split(yield, shade), mean))
```

	none	Aug2Dec	Dec2Feb	Feb2May
	100.20	103.23	89.92	92.77

### Subsection 7.4.3:\* The analysis of variance sums of squares breakdown

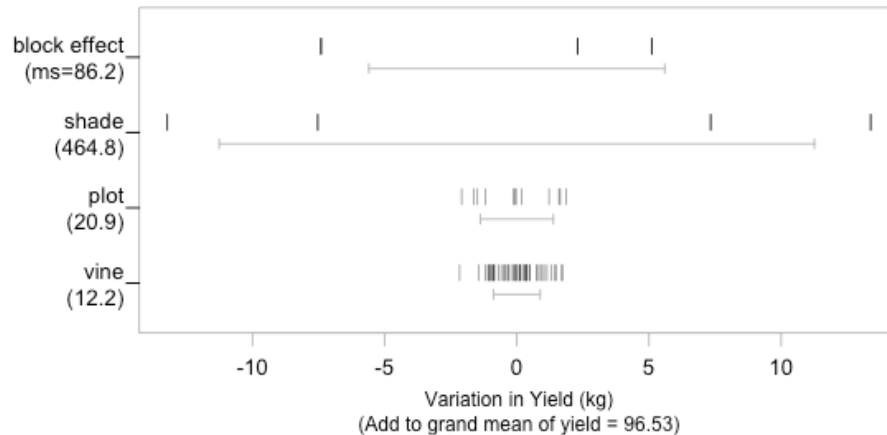
```
## For each plot, calculate mean, and differences from the mean
vine <- paste("vine", rep(1:4, 12), sep="")
vine1rows <- seq(from=1, to=45, by=4)
kiwivines <- unstack(kiwishade, yield ~ vine)
```

```

kiwimeans <- apply(kiwivines, 1, mean)
kiwivines <- cbind(kiwishade[vine1rows, c("block","shade")],
Mean=kiwimeans, kiwivines-kiwimeans)
kiwivines <- with(kiwivines, kiwivines[order(block, shade), ])
mean(kiwimeans)      # the grand mean

```

[1] 96.53



```

kiwishade <- DAAG::kiwishade
kiwimeans <- with(kiwishade, aggregate(yield,by=list(block,shade),mean))
names(kiwimeans) <- c("block","shade","meanyield")
plotmeans.lm <- lm(meanyield~block+shade, data=kiwimeans)
effects <- predict(plotmeans.lm, type="terms")
kiwishade.lm <- lm(yield ~ block*shade, data=kiwishade)
y <- c(effects[, "block"]/sqrt(2) * sqrt(16),
effects[, "shade"]/sqrt(3) * sqrt(12),
residuals(plotmeans.lm)/sqrt(6) * sqrt(4),
residuals(kiwishade.lm)/sqrt(12))
n <- rep(4:1, c(12, 12, 12, 48))
gps <- rep(c("block effect\n(ms=86.2)", "shade\n(464.8)",
"plot\n(20.9)", "vine\n(12.2)"), c(12, 12, 12, 48))
gps <- factor(gps, levels = rev(c("block effect\n(ms=86.2)",
"shade\n(464.8)", "plot\n(20.9)", "vine\n(12.2)")))
gps.sd <- sapply(split(y,gps), sd)
gps.av <- sapply(split(y,gps), mean)
plot(range(y), range(n)+c(-0.5, 0.5), xlab="", ylab="", yaxt="n", type="n", fg="gray")
text(y, n+0.15, "|", cex=0.8, col=adjustcolor("black", alpha.f=0.5))
un <- 1:4

```

```
sapply(un, function(j){lines(gps.av[j]+c(-gps.sd[j], gps.sd[j]),
rep(j-0.15,2), col="gray")
lines(rep(gps.av[j],2)-gps.sd[j],
j-0.15+c(-.06, .06), col="gray")
lines(rep(gps.av[j],2)+gps.sd[j],
j-0.15+c(-.06, .06), col="gray")
})
mtext(side=1,line=2.25, cex=0.9,
text="Variation in Yield (kg)\n(Add to grand mean of yield = 96.53)")
par(las=2)
axis(2, at=1:4, labels=levels(gps), lwd=0, lwd.ticks=1)
```

#### Subsection 7.4.4: The variance components

#### Subsection 7.4.5: The mixed model analysis

```
kiwishade.lmer <- lmer(yield ~ shade + (1|block) + (1|block:plot),
data=kiwishade)
# block:shade is an alternative to block:plot
```

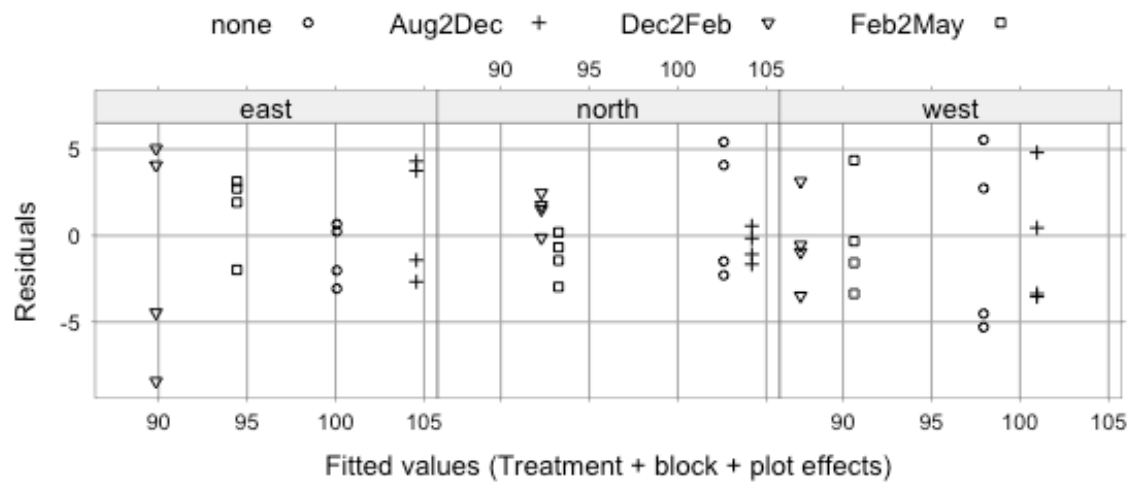
```
print(kiwishade.lmer, ranef.comp="Variance", digits=3)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: yield ~ shade + (1 | block) + (1 | block:plot)
Data: kiwishade
REML criterion at convergence: 252
Random effects:
Groups      Name      Variance
block:plot (Intercept)  2.19
block      (Intercept)  4.08
Residual                    12.18
Number of obs: 48, groups:  block:plot, 12; block, 3
Fixed Effects:
(Intercept)  shadeAug2Dec  shadeDec2Feb  shadeFeb2May
      100.20         3.03      -10.28        -7.43
```

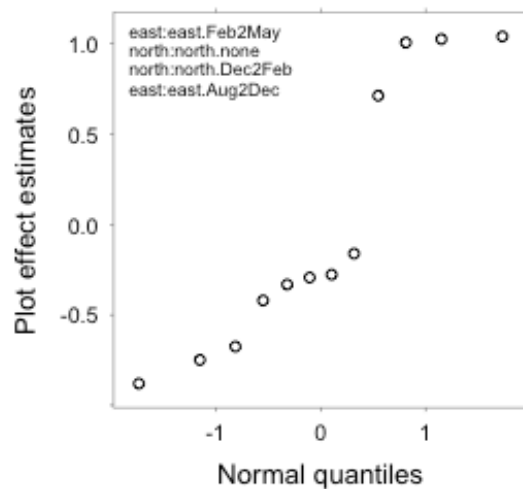
#### Residuals and estimated effects



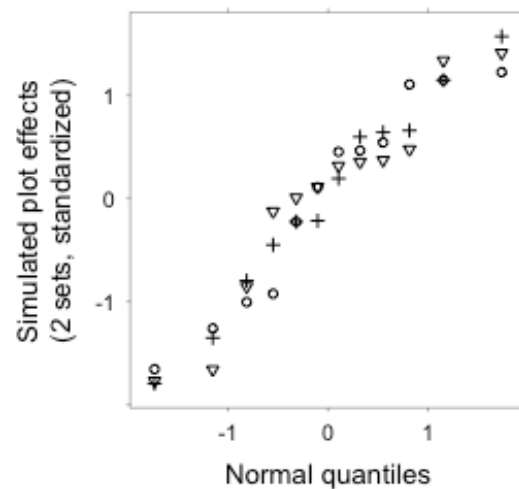
A: Standardized residuals after fitting block and plot effects



B: Normal Q-Q plot of plot effects



C: 3 simulations -- normal Q-Q plots



```
## Panel A
parsetA <- modifyList(DAAG::DAAGtheme(color=FALSE),
  list(layout.heights=list(key.top=2.25, axis.top=.75)))
if (!exists("kiwishade.lmer"))
kiwishade.lmer <- lme4::lmer(yield ~ shade + (1|block/shade), data=kiwishade)
pk1 <- xyplot(residuals(kiwishade.lmer) ~ fitted(kiwishade.lmer)|block,
  groups=shade, layout=c(3,1), par.strip.text=list(cex=1.0),
  data=kiwishade, grid=TRUE,
```

```

        xlab="Fitted values (Treatment + block + plot effects)",
        ylab="Residuals",
        main=list(label="A: Standardized residuals after fitting block and plot effects",
                  cex=1.05, x=0.01, y=0, font=1, just="left"),
        auto.key=list(space='top', points=TRUE, columns=4))
print(update(pk1, scales=list(x=list(alternating=TRUE), tck=0.35),
        par.settings=parsetA), position=c(0,.52,1,1))
## Panel B
ploteff <- ranef(kiwishade.lmer, drop=TRUE)[[1]]
nam <- names(sort(ploteff, decreasing=TRUE)[1:4])
parsetB <- modifyList(DAAG::DAAGtheme(color=FALSE),
        list(layout.heights=list(axis.top=0.85)))
pk2 <- qqmath(ploteff, ylab="Plot effect estimates",
        key=list(x=0, y=0.98, corner=c(0,1),
        text=list(labels=nam, cex=0.65)),
        main=list(label="B: Normal Q-Q plot of plot effects",
                  cex=1.05, x=0.01, y=0.25, font=1, just="left"),
        xlab="Normal quantiles")
print(update(pk2, scales=list(tck=0.35), par.settings=parsetB),
        newpage=FALSE, position=c(0,0,.5,.5))
## Panel C
simvals <- simulate(kiwishade.lmer, nsim=3, seed=23)
simranef <- function(y)ranef(lme4::refit(kiwishade.lmer, y))[[1]]
simeff <- apply(simvals, 2, function(y) scale(simranef(y)))
simeff <- as.data.frame(simeff)
pk3 <- qqmath(~ sim_1+sim_2+sim_3, data=simeff,
        ylab="Simulated plot effects\n(2 sets, standardized)",
        Qs=list(tck=0.35), aspect=1,
        main=list(label="C: 3 simulations -- normal Q-Q plots",
                  x=0.01, y=0.25, cex=1.05, font=1, just="left"),
        xlab="Normal quantiles")
print(update(pk3, scales=list(tck=0.35), par.settings=parsetB),
        newpage=FALSE, position=c(0.48,0,1,.5))

with(kiwishade, sapply(split(yield, shade), mean))

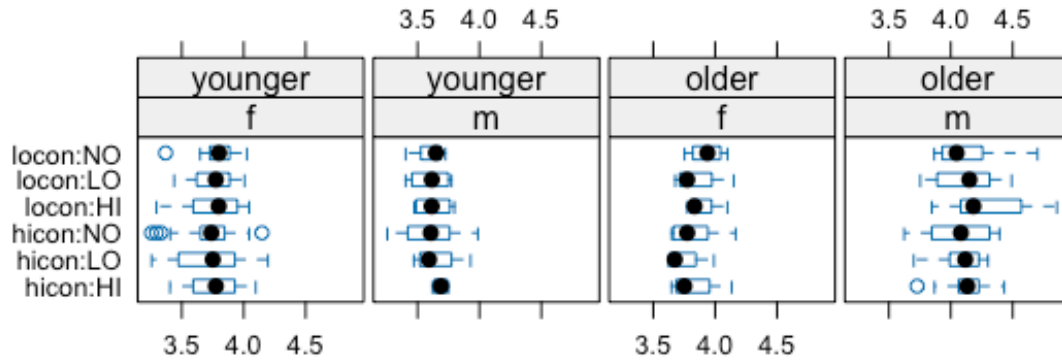
```

none	Aug2Dec	Dec2Feb	Feb2May
100.20	103.23	89.92	92.77

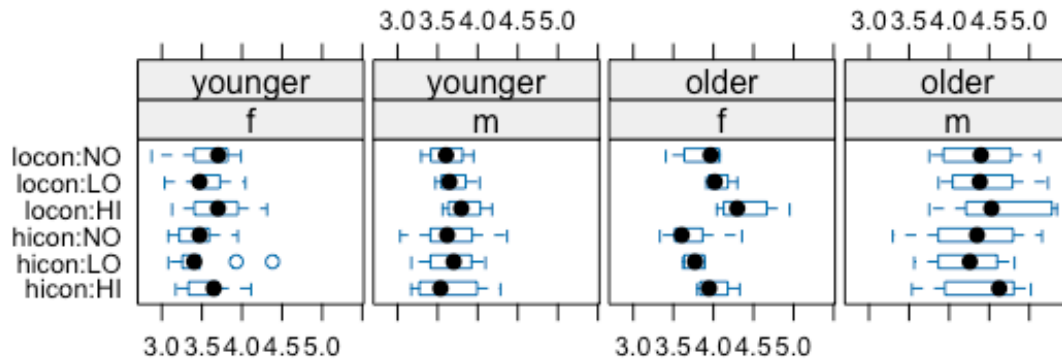
## Subsection 7.4.6: Predictive accuracy

## Section 7.5 Within and between subject effects

A: Boxplots for `log(csoa)`, by combinations of `target` and `tint`



B: Boxplots for `log(it)`, by combinations of `target` and `tint`



```
library(lattice)
tinting <- within(DAAG::tinting, targettint <- paste(target,toupper(tint),sep=':'))
gphA <- bwplot(targettint~log(csoa) | sex*agegp, data=tinting, layout=c(4,1), between=list(x=0.2))
mainA <- list("A: Boxplots for `log(csoa)`, by combinations of `target` and `tint`",
              font=1, x=0.125, y=0.125, cex=1.0, just='left')
gphB <- bwplot(targettint~log(it) | sex*agegp, data=tinting, layout=c(4,1), between=list(x=0.2))
mainB <- list("B: Boxplots for `log(it)`, by combinations of `target` and `tint`",
              font=1, x=0.125, y=0.125, cex=1.0, just='left')
print(update(gphA, main=mainA, xlab=""), position=c(0, 0.48, 1, 1.0), more=T)
print(update(gphB, main=mainB, xlab=""), position=c(0,0,1,0.52))
```

## Model fitting criteria

### Subsection 7.5.1: Model selection

```
## Capitalize tinting$agegp
levels(tinting$agegp) <- R.utils::capitalize(levels(tinting$agegp))
## Fit all interactions: data frame tinting (DAAG)
it3.lmer <- lmer(log(it) ~ tint*target*agegp*sex + (1 | id),
                 data=DAAG::tinting, REML=FALSE)
```

```
it2.lmer <- lmer(log(it) ~ (tint+target+agegp+sex)^2 + (1 | id),
                 data=DAAG::tinting, REML=FALSE)
```

```
it1.lmer <- lmer(log(it)~(tint+target+agegp+sex) + (1 | id),
                 data=DAAG::tinting, REML=FALSE)
```

```
anova(it1.lmer, it2.lmer, it3.lmer)
```

Data: DAAG::tinting

Models:

```
it1.lmer: log(it) ~ (tint + target + agegp + sex) + (1 | id)
it2.lmer: log(it) ~ (tint + target + agegp + sex)^2 + (1 | id)
it3.lmer: log(it) ~ tint * target * agegp * sex + (1 | id)
```

	npar	AIC	BIC	logLik	deviance	Chisq	Df	Pr(>Chisq)
it1.lmer	8	1.14	26.8	7.43	-14.9			
it2.lmer	17	-3.74	50.7	18.87	-37.7	22.88	9	0.0065
it3.lmer	26	8.15	91.5	21.93	-43.9	6.11	9	0.7288

```
## Code that gives the first four such plots, for the observed data
interaction.plot(tinting$tint, tinting$agegp, log(tinting$it))
interaction.plot(tinting$target, tinting$sex, log(tinting$it))
interaction.plot(tinting$tint, tinting$target, log(tinting$it))
interaction.plot(tinting$tint, tinting$sex, log(tinting$it))
```

### Subsection 7.5.2: Estimates of model parameters

```
it2.reml <- update(it2.lmer, REML=TRUE)
print(coef(summary(it2.reml)), digits=2)
```

	Estimate	Std. Error	t value
(Intercept)	3.6191	0.130	27.82
tint.L	0.1609	0.044	3.64
tint.Q	0.0210	0.045	0.46
targethicon	-0.1181	0.042	-2.79
agegpolder	0.4712	0.233	2.02
sexm	0.0821	0.233	0.35
tint.L:targethicon	-0.0919	0.046	-2.00
tint.Q:targethicon	-0.0072	0.048	-0.15
tint.L:agegpolder	0.1308	0.049	2.66
tint.Q:agegpolder	0.0697	0.052	1.34
tint.L:sexm	-0.0979	0.049	-1.99
tint.Q:sexm	0.0054	0.052	0.10
targethicon:agegpolder	-0.1389	0.058	-2.38
targethicon:sexm	0.0779	0.058	1.33
agegpolder:sexm	0.3316	0.326	1.02

```
# NB: The final column, giving degrees of freedom, is not in the
# summary output. It is our addition.
```

```
subs <- with(tinting, match(unique(id), id))
with(tinting, table(sex[subs], agegp[subs]))
```

	Younger	Older
f	9	4
m	4	9

## Section 7.6 A mixed model with a betabinomial error

### Subsection 7.6.1: The betabinomial distribution

```
## devtools::install_github("jhmaindonald/gra") # Use if not found on CRAN
```

### Notation

## Source of data

```
HawCon <- within(as.data.frame(qra::HawCon), {  
  trtGp <- gsub("Fly", "", paste0(CN,LifestageTrt))  
  trtGp <- factor(trtGp, levels=sort(unique(trtGp))[c(1,5,2,6,3,7,4,8)])  
  trtGpRep <- paste0(CN,LifestageTrt,":",RepNumber)  
  scTime <- scale(TrtTime) # Centering and scaling can help model fit  
})
```

```
## Load packages that will be used  
suppressMessages(library(lme4)); suppressMessages(library(glmmTMB))
```

```
library(splines)  
form <- cbind(Dead,Live)~0+trtGp/TrtTime+(1|trtGpRep)  
## Add the quadratic term from a degree 2 orthogonal polynomial  
form2s <- update(form, . ~ . + scale(scTime^2))  
## Scale "corrections" to reduce risk of potential numerical problems  
HCbb.cll <- glmmTMB(form, dispformula=~trtGp+ns(scTime,2),  
  family=glmmTMB::betabinomial(link="cloglog"), data=HawCon)  
HCbb2s.cll <- update(HCbb.cll, formula=form2s)  
HCbb.logit <- glmmTMB(form, dispformula=~trtGp+ns(scTime,2),  
  family=glmmTMB::betabinomial(link="logit"), data=HawCon)  
HCbb2s.logit <- update(HCbb.logit, formula=form2s)
```

```
summary(HCbb2s.cll)$coefficients$cond["scale(scTime^2)",] ## CLL
```

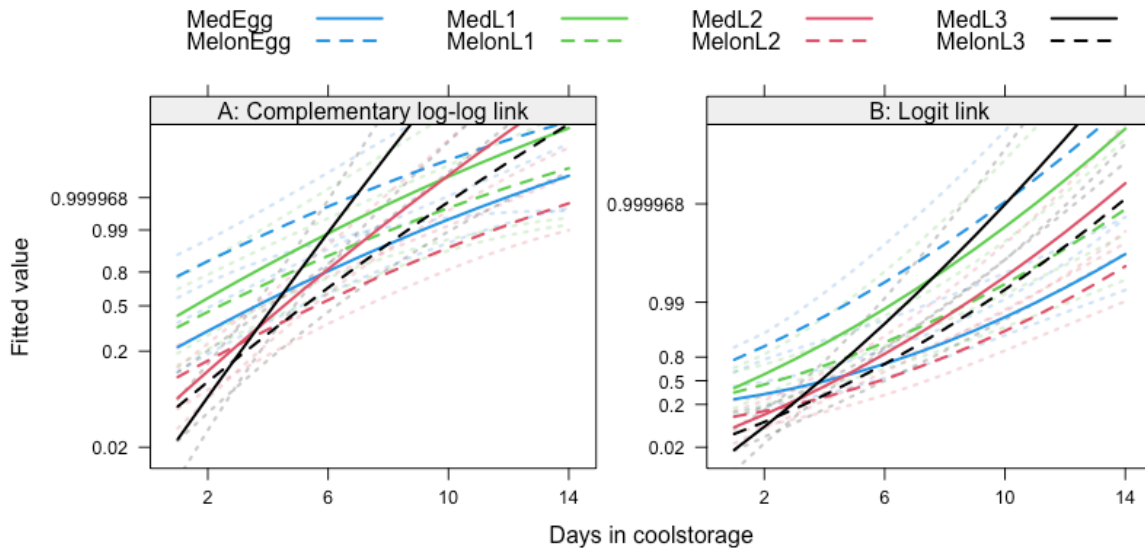
Estimate	Std. Error	z value	Pr(> z )
-0.09130	0.06143	-1.48627	0.13721

```
summary(HCbb2s.logit)$coefficients$cond["scale(scTime^2)",] ## Logit
```

Estimate	Std. Error	z value	Pr(> z )
0.43219	0.17766	2.43268	0.01499

```
AICtab <- AIC(BB=HCbb.cll,HCbb2s.cll,HCbb.logit,HCbb2s.logit)  
AICtab[['Details']] <- c(paste0('BB: Compl. log-log', c(' ', ' ', added curve')),  
  paste0('BB: Logit', c(' ', ' ', added curve')))  
AICtab[order(AICtab[["AIC"]]), ]
```

	df	AIC	Details
HCbb2s.c1l	28	709.9	BB: Compl. log-log
HCbb.c1l	27	710.0	BB: Compl. log-log, added curve
HCbb2s.logit	28	717.2	BB: Logit, added curve
HCbb.logit	27	721.7	BB: Logit



```

dat <- expand.grid(trtGp=factor(levels(HawCon$trtGp), levels=levels(HawCon$trtGp)),
  TrtTime=pretty(range(HawCon$TrtTime),15), trtGpRep=NA)
ab <- qra::getScaleCoef(HawCon$scTime)
dat$scTime <- with(dat,(TrtTime-ab[1])/ab[2])
hatClog <- predict(HCbb2s.c1l, newdata=dat)
hatClog <- predict(HCbb2s.c1l, se=T, newdata=dat)
hatLgt <- predict(HCbb2s.logit, newdata=dat)
hatLgt <- predict(HCbb2s.logit, se=T, newdata=dat)
dat2 <- cbind(rbind(dat,dat), link=rep(c('clog','logit'), rep(nrow(dat),2)))
dat2 <- within(dat2, {fit<-c(hatClog$fit,hatLgt$fit);
  se.fit<-c(hatClog$se.fit,hatLgt$se.fit);
  link=rep(c('clog','logit'), rep(nrow(dat),2))})
dat2 <- within(dat2, {lwr<-fit-2*se.fit; upr<-fit+2*se.fit})
library(lattice)
my.panel.bands <- function(x, y, upper, lower, fill, col,
  subscripts, ..., font, fontface)
{
  upper <- upper[subscripts]

```

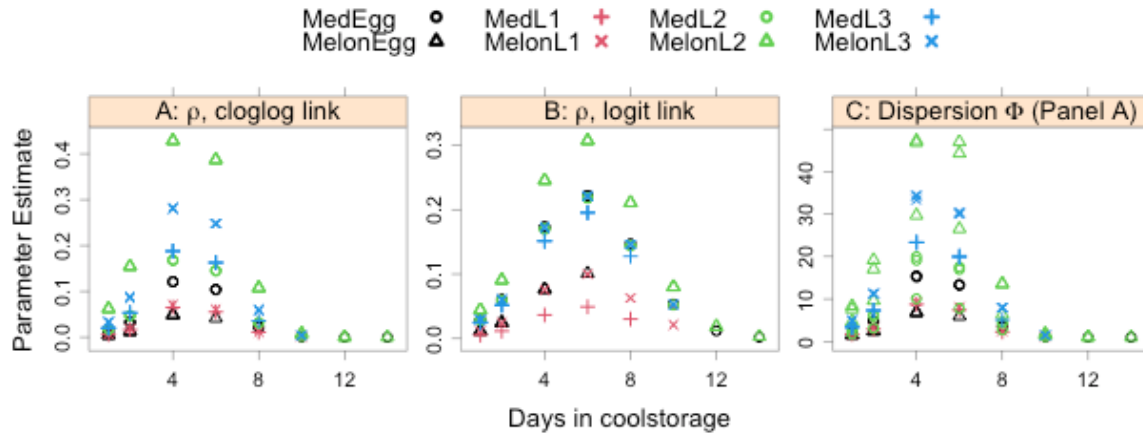
```

lower <- lower[subscripts]
panel.lines(x,lower, ...)
panel.lines(x,upper, ...)
}
panel2 <- function(x, y, ...){
panel.superpose(x, y, panel.groups = my.panel.bands, type='l', lty=3, alpha=0.25,...)
panel.xyplot(x, y, type='l', lwd=2, cex=0.6, ...)
}
parset <- simpleTheme(col=rep(4:1,rep(2,4)), lty=rep(1:2, 4), lwd=2)
## c('solid','1141')
p <- c(.02,.2,.5,.8, .99, .999968)
cloglog <- make.link('cloglog')$linkfun
logit <- make.link('logit')$linkfun
fitpos <- list(cloglog(p), logit(p))
lab <- paste(p)
lim <- list(cloglog(c(0.02, 0.99998)), logit(c(0.02, 0.99998)))
lim <- lapply(lim, function(x)c(x[1],x[1]+diff(x)*1.2))

gph <- xyplot(fit~TrtTime|link, outer=TRUE, data=dat2, groups=trtGp,
             upper = dat2$upr, lower = dat2$lwr, panel = panel2,
             xlab="Days in coolstorage", ylab="Fitted value",
             auto.key=list(text=levels(HawCon$trtGp), columns=4,
                           points=FALSE, lines=TRUE),
             par.settings=parset, layout=c(2,1),
             scales=list(x=list(at=c(2,6,10,14)),
                         y=list(relation='free',
                                at=fitpos, labels=lab, limits=lim), alternating=c(1,1)))
update(gph, strip=strip.custom(factor.levels=c("A: Complementary log-log link",
"B: Logit link")))

```



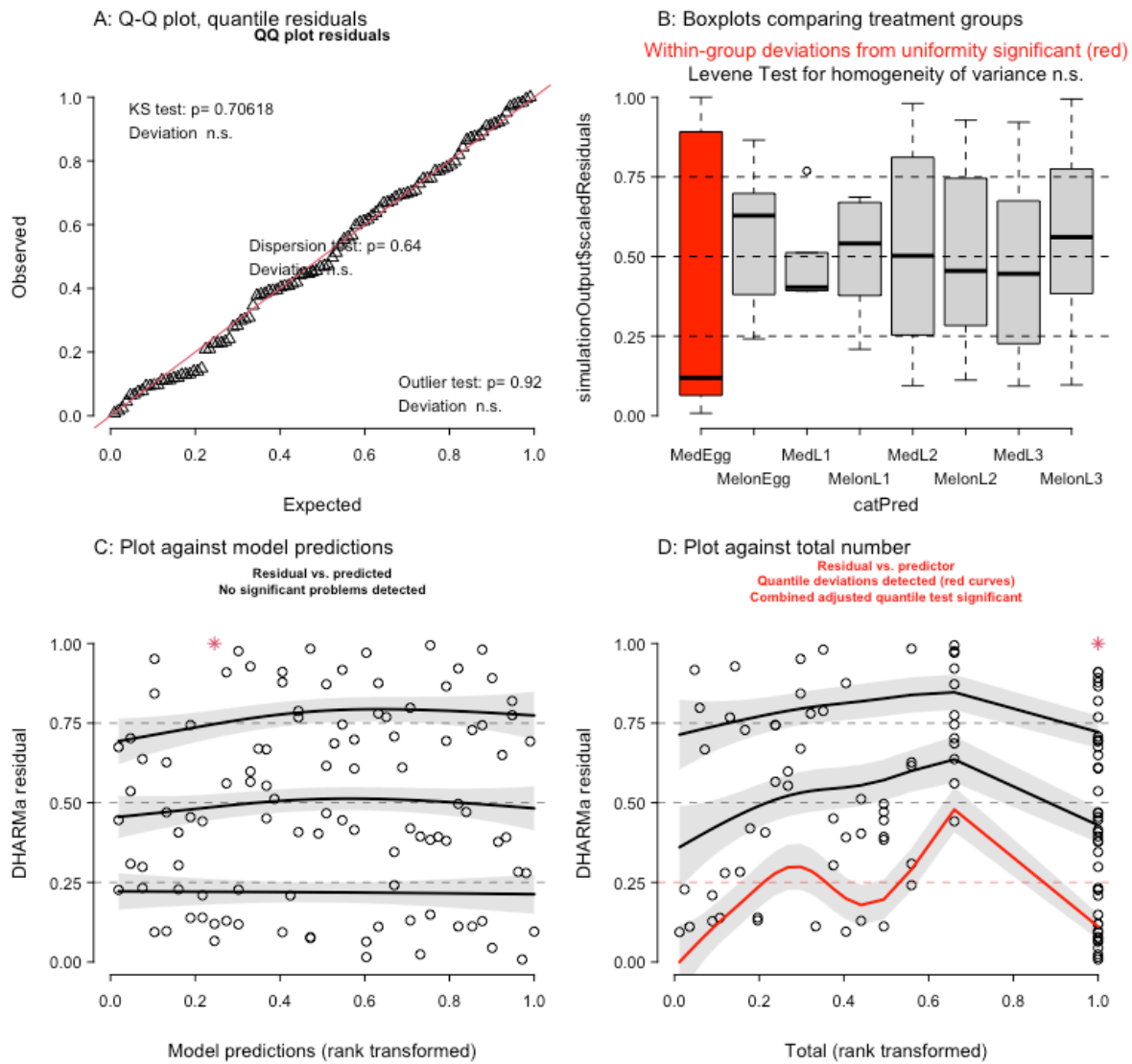


```

parset <- DAAG::DAAGtheme(color=TRUE, col.points=rep(1:4,rep(2,4)),
                           pch=rep(1:4,2), lwd=2)
HawCon$rho2clog <- qra::getRho(HCbb.cll)
HawCon$dispClog <- with(HawCon, 1+(Total-1)*rho2clog)
par(oma=c(0,0,2,0))
titles=c(expression("A: "*rho*", cloglog link"),expression("B: "*rho*", logit link"),
          expression("C: Dispersion "*Phi*" (Panel A)"))
library(lattice)
HawCon$rho2logit <- qra::getRho(HCbb.logit)
xyplot(rho2clog+rho2logit+dispClog ~ TrtTime, groups=trtGp, data=HawCon,
       outer=TRUE, between=list(x=0.25), par.settings=parset,
       scales=list(x=list(alternating=FALSE,at=c(4,8,12), labels=paste(c(4,8,12))),
                  y=list(relation='free',tick.number=4)),
       auto.key=list(columns=4, between.columns=2, between=1),
       xlab="Days in coolstorage", ylab="Parameter Estimate",
       strip=strip.custom(factor.levels=titles))

```

## Subsection 7.6.2: Diagnostic checks



```
par(oma=c(0,0,2,0.5))
## Code for plots, excluding main titles
set.seed(29)
simRef <- DHARMA::simulateResiduals(HCbb.c11, n=250, seed=FALSE)
DHARMA::plotQQunif(simRef)
title(main="A: Q-Q plot, quantile residuals", adj=0, line=2.5,
```

```

    font.main=1, cex.main=1.25)
DHARMA::plotResiduals(simRef, form=HawCon$trtGp)
axis(1, at=c(2,4,6,8), labels=levels(HawCon$trtGp)[c(2,4,6,8)],
     gap.axis=0, line=1, lwd=0)
title(main="B: Boxplots comparing treatment groups", adj=0, line=2.5,
      font.main=1, cex.main=1.25)
DHARMA::plotResiduals(simRef)
title(main="C: Plot against model predictions", adj=0, font.main=1, line=3.25,
      cex.main=1.25)
DHARMA::plotResiduals(simRef, form=HawCon$Total)
title(main="D: Plot against total number", adj=0, font.main=1, line=3.25,
      cex.main=1.25)

```

### Subsection 7.6.3: Lethal time estimates and confidence intervals

```

shorten <- function(nam, leaveout=c('trtGp','Fly',':')){
  for(txt in leaveout){
    nam <- gsub(txt,'', nam, fixed=TRUE)
  }
  nam
}

```

```

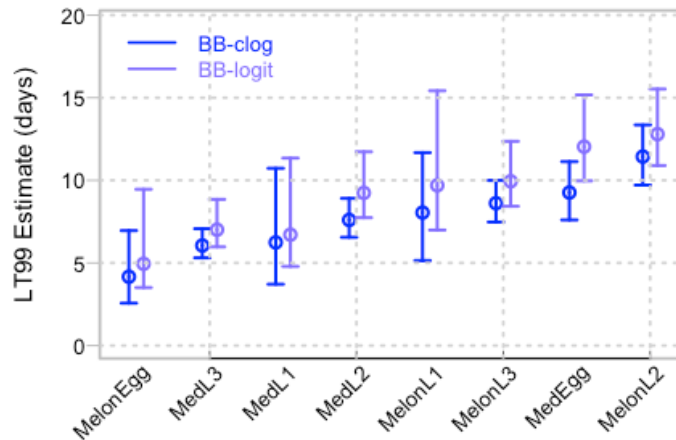
LTbb.cll <- qra::extractLT(p=0.99, obj=HCbb.cll, link="cloglog",
                        a=1:8, b=9:16, eps=0, df.t=NULL)[-2]
rownames(LTbb.cll) <- gsub("trtGp|Fly|:", '', rownames(LTbb.cll), perl=T)

```

```

LTbb.logit <- qra::extractLT(p=0.99, obj=HCbb.logit, link="logit",
                          a=1:8, b=9:16, eps=0, offset=0,
                          df.t=NULL)[-2]
rownames(LTbb.logit) <- shorten(rownames(LTbb.logit))

```



```
library(plotrix)
gpNam <- rownames(LTbb.cll)
ordEst <- order(LTbb.cll[,1])
col5 <- c("blue","lightslateblue","blueviolet",'gray50','gray80')
plotCI(1:8-0.1, y=LTbb.cll[ordEst,1], ui=LTbb.cll[ordEst,3],
       li=LTbb.cll[ordEst,2], lwd=2, col=col5[1], xaxt="n",
       fg="gray", xlab="", ylab="LT99 Estimate (days)",
       xlim=c(0.8,8.2), ylim=c(0,19.5))
plotCI(1:8+0.1, y=LTbb.logit[ordEst,1], ui=LTbb.logit[ordEst,3],
       li=LTbb.logit[ordEst,2], lwd=2, col=col5[2], xaxt="n", add=TRUE)
axis(1, labels=FALSE, tck=0.02, col.ticks="gray40")
text(x = 1:length(gpNam)+0.1,
     ## Move labels to just below bottom of chart.
     y = par("usr")[3] - 0.8,
     labels = gpNam[ordEst], ## Use names from the data list.
     xpd = NA, ## Change the clipping region
     srt = 45, ## Rotate the labels by 45 degrees.
     adj = 0.95) ## Adjust the labels to almost 100% right-justified
grid()
legend("topleft", legend=c("BB-clog", "BB-logit"),
      inset=c(0.01,0.01), lty=c(1,1), col=col5[1:2],
      text.col=col5[1:2], bty="n",y.intersp=0.85)
```

```
HCbin.cll <- glmmTMB(cbind(Dead,Live)~0+trtGp/TrtTime+(scTime|trtGp),
                    family=binomial(link="cloglog"), data=HawCon)
```

```
LTbin.cll <- gra::extractLT(p=0.99, obj=HCbin.cll,
                           a=1:8, b=9:16, eps=0, df.t=NULL)[,-2]
rownames(LTbin.cll) <- shorten(rownames(LTbin.cll))
```

## A warning against over-interpretation

## Section 7.7 Observation level random effects — the moths dataset

```
moths <- DAAG::moths
moths$transect <- 1:41 # Each row is from a different transect
moths$habitat <- relevel(moths$habitat, ref="Lowerside")
A.glmer <- glmer(A~habitat+sqrt(meters)+(1|transect),
family=poisson(link=sqrt), data=moths)
print(summary(A.glmer), show.resid=FALSE, correlation=FALSE, digits=3)
```

```
Generalized linear mixed model fit by maximum likelihood (Laplace
Approximation) [glmerMod]
Family: poisson ( sqrt )
Formula: A ~ habitat + sqrt(meters) + (1 | transect)
Data: moths
```

AIC	BIC	logLik	deviance	df.resid
212.6	229.7	-96.3	192.6	31

Random effects:

Groups	Name	Variance	Std.Dev.
transect	(Intercept)	0.319	0.564

Number of obs: 41, groups: transect, 41

Fixed effects:

	Estimate	Std. Error	z value	Pr(> z )
(Intercept)	1.7322	0.3513	4.93	0.00000082
habitatBank	-2.0415	0.9377	-2.18	0.029
habitatDisturbed	-1.0359	0.4071	-2.54	0.011
habitatNEsoak	-0.7319	0.4323	-1.69	0.090
habitatNWsoak	2.6787	0.5101	5.25	0.00000015
habitatSEsoak	0.1178	0.3923	0.30	0.764
habitatSWsoak	0.3900	0.5260	0.74	0.458
habitatUpperside	-0.3135	0.7549	-0.42	0.678
sqrt(meters)	0.0675	0.0631	1.07	0.285

```
suppressPackageStartupMessages(library(gamlss))
A1quasi.glm <- glm(A~habitat, data=moths, family=quasipoisson(link=sqrt))
Asqrt.lss <- gamlss(A ~ habitat + sqrt(meters), trace=FALSE,
                    family = NBI(mu.link='sqrt'), data = moths)
A1.glmer <- glmer(A~habitat+(1|transect), data=moths, family=poisson(link=sqrt))
```

```
Cglm <- coef(summary(A1quasi.glm))
Cglmer <- coef(summary(A1.glmer))
fitAll <- cbind("quasi-Coeff"=Cglm[-1,1], "quasi-SE"=Cglm[-1,2],
               "NBI-Coeff"=coef(Asqrt.lss)[2:8], "NBI-SE"=c(0.94,.41,.43,.51,.39,.53,.75),
               "glmer-Coeff"=Cglmer[-1,1], "glmer-SE"=Cglmer[-1,2])
rownames(fitAll) <- substring(rownames(fitAll),8)
round(fitAll, 2) # NB, all SEs are for the difference from 'Lowerside'
```

	quasi-Coeff	quasi-SE	NBI-Coeff	NBI-SE	glmer-Coeff	glmer-SE
Bank	-2.13	0.86	-2.19	0.94	-1.99	0.95
Disturbed	-1.07	0.41	-1.00	0.41	-1.13	0.40
NEsoak	-0.61	0.43	-0.80	0.43	-0.57	0.41
NWsoak	2.73	0.54	2.67	0.51	2.73	0.51
SEsoak	0.16	0.41	0.09	0.39	0.19	0.39
SWsoak	0.45	0.54	0.28	0.53	0.54	0.51
Upperside	0.23	0.45	-0.41	0.75	0.36	0.43

```
detach("package:glmmTMB", character.only=TRUE)
```

## Section 7.8 Repeated measures in time

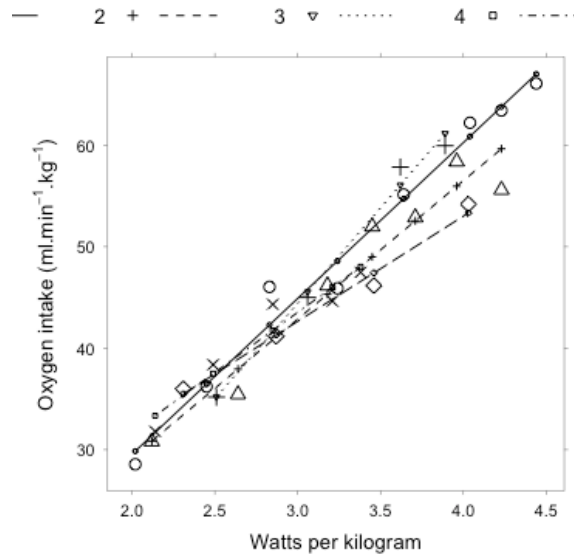
### The theory of repeated measures modeling

#### \*Correlation structure

### Different approaches to repeated measures analysis

#### Subsection 7.8.1: Example – random variation between profiles

```
humanpower1 <- DAAG::humanpower1
```



```
## Plot points and fitted lines
xyplot(o2 ~ wattsPerKg, groups=id, data=humanpower1,
       par.settings=DAAG::DAAGtheme(color=F), scales=list(tck=0.5),
       panel=function(x,y,subscripts,groups,...){
         yhat <- fitted(lm(y ~ groups*x))
         panel.superpose(x, y, subscripts, groups, pch=1:5, cex=1.2)
         panel.superpose(x, yhat, subscripts, groups, type="b", cex=0.5)
       },
       auto.key=list(columns=5, lines=T),
       xlab="Watts per kilogram",
       ylab=expression("Oxygen intake ("*ml.min^{-1}*kg^{-1}*")"))
```

### Separate lines for different athletes

```
## Calculate intercepts and slopes; plot Slopes vs Intercepts
## Uses the function lmList() from the lme4 package
humanpower1 <- DAAG::humanpower1
hp.lmList <- lmList(o2 ~ wattsPerKg | id, data=humanpower1)
coefs <- coef(hp.lmList)
round(coefs,3)
```

```
(Intercept) wattsPerKg
1          -1.155      15.35
```

2	1.916	13.65
3	-12.008	18.81
4	8.029	11.83
5	11.553	10.36

```
c("Correlation between intercept and slope"=cor(coefs[,1],coefs[,2]))
```

Correlation between intercept and slope  
-0.9975

### A random coefficients model

```
hp.lmer <- lmer(o2 ~ wattsPerKg + (wattsPerKg | id), data=humanpower1)
```

```
hp.lmer <- lmer(o2 ~ wattsPerKg + (0+wattsPerKg | id), data=humanpower1)
print(summary(hp.lmer), digits=3)
```

Linear mixed model fit by REML ['lmerMod']  
Formula: o2 ~ wattsPerKg + (0 + wattsPerKg | id)  
Data: humanpower1

REML criterion at convergence: 129.7

Scaled residuals:

Min	1Q	Median	3Q	Max
-1.9117	-0.8978	0.0598	0.7854	1.5382

Random effects:

Groups	Name	Variance	Std.Dev.
id	wattsPerKg	0.211	0.46
Residual		5.776	2.40

Number of obs: 28, groups: id, 5

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	1.299	2.220	0.59
wattsPerKg	14.204	0.715	19.88

Correlation of Fixed Effects:

	(Intr)
wattsPerKg	-0.936

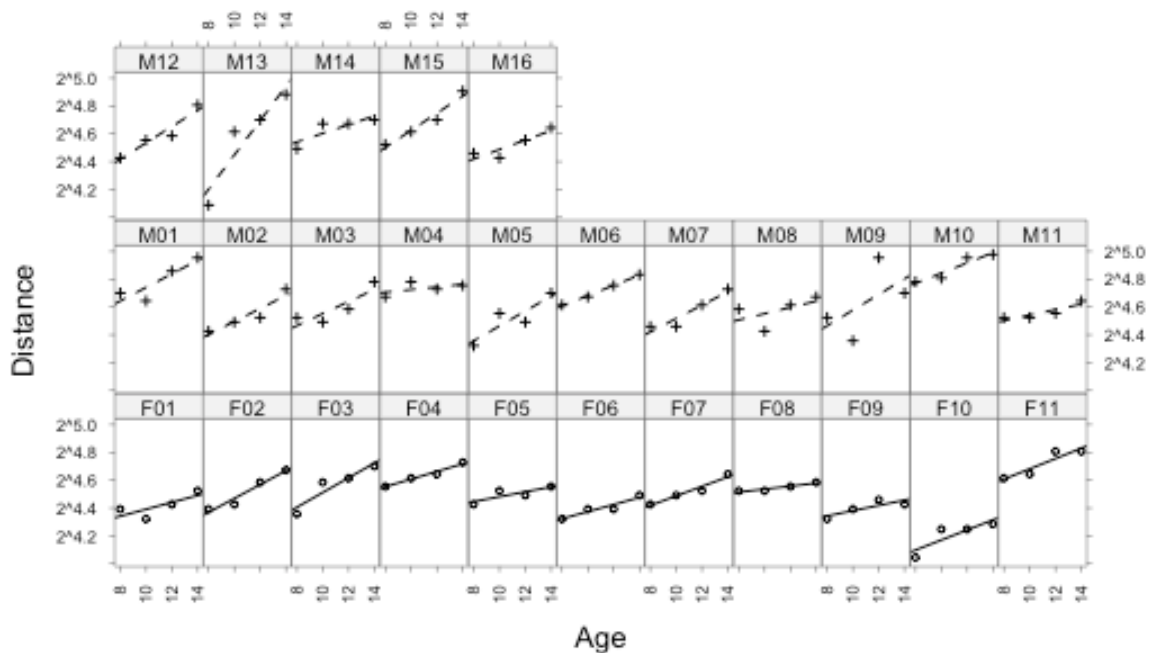


```
sort(coef(lmList(o2 ~ wattsPerKg | id, data=humanpower1))[,1])
```

```
[1] -12.008 -1.155 1.916 8.029 11.553
```

### Subsection 7.8.2: Orthodontic measurements on children

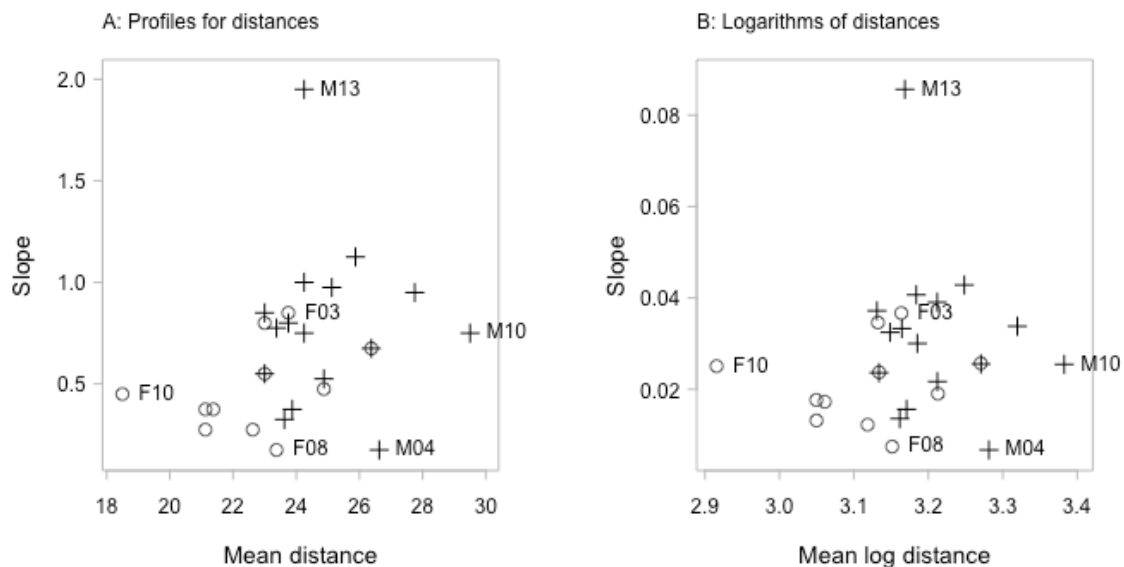
```
Orthodont <- MEMSS::Orthodont
Orthodont$logdist <- log(Orthodont$distance)
```



```
## Plot showing pattern of change for each of the 25 individuals
gph <- xyplot(distance ~ age | Subject, groups=Sex, data=Orthodont,
              scales=list(x=list(rot=90, tick.number=3), y=list(log=2, tck=0.5), type=c("p"
              layout=c(11,3))
update(gph, xlab=list("Age", cex=1.4), ylab=list("Distance", cex=1.4),
       par.settings=DAAG::DAAGtheme(color=FALSE, fontsize=list(text=7, points=4)))
```

## Preliminary data exploration

```
## Use lmList() to find the slopes
ab <- cbind(coef(lmList(distance ~ age | Subject, Orthodont)),
            coef(lmList(logdist ~ age|Subject, data=Orthodont)))
names(ab) <- c("a", "b","alog","blog")
## Obtain intercept at x=mean(x)=11, for each subject.
## (For each subject, this is independent of the slope)
ab <- within(ab, {ybar = a + b*11; b=b; ylogbar = alog + blog * 11;
                blog=blog; sex = substring(rownames(ab), 1,1)
            })
bySex <- sapply(split(ab, ab$sex), function(z)range(z$b))
extremes <- with(ab, ybar %in% range(ybar) | b %in% bySex)
```



```
fundiff <- function(x)range(x)+diff(range(x))*c(-0.015, 0.04)
lims <- sapply(subset(ab, select=c('ybar',"b","ylogbar","blog")), fundiff)
plot(b ~ ybar, col=c(F="gray40", M="black")[sex], data=ab,
     fg="gray", xlim=lims[, "ybar"], ylim=lims[, "b"],
     pch=c(F=1, M=3)[sex], xlab="Mean distance", ylab="Slope")
with(subset(ab, extremes), text(b ~ ybar,
                                labels=rownames(ab)[extremes], pos=4, xpd=TRUE))
mtext(side=3, line=0.75, "A: Profiles for distances", adj=0)
# Type 'qqnorm(ab$b)' to see the extent of M13's outlyingness
```

```
## For Panel B, repeat with logdist replacing distance
plot(blog ~ ylogbar, col=c(F="gray40", M="black")[sex], data=ab, fg="gray",
     pch=c(F=1, M=3)[sex], xlim=lims[, "ylogbar"], ylim=lims[, "blog"],
     xlab="Mean log distance", ylab="Slope")
with(subset(ab, extremes),
     text(blog ~ ylogbar, labels=rownames(ab)[extremes], pos=4, xpd=TRUE))
mtext(side=3, line=0.75, "B: Logarithms of distances", adj=0)
```

```
## Compare male slopes with female slopes
extreme.males <- match(c("M04", "M13"), rownames(ab))
with(ab[-extreme.males, ],
t.test(blog[sex=="F"], blog[sex=="M"], var.equal=TRUE))
```

### Two Sample t-test

```
data: blog[sex == "F"] and blog[sex == "M"]
t = -2.3, df = 23, p-value = 0.03
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.0160529 -0.0009191
sample estimates:
mean of x mean of y
 0.02115  0.02963
```

```
# Specify var.equal=TRUE, to allow comparison with anova output
```

### A random coefficients model

```
keep <- !(Orthodont$Subject%in%c("M04", "M13"))
Orthodont$scAge <- with(Orthodont, age-11) ## Center values of age
orthdiffx.lmer <- lmer(logdist ~ Sex * scAge + (scAge | Subject),
                      data=Orthodont, subset=keep)
```

```
rePCA(orthdiffx.lmer)
```

```
$Subject
Standard deviations (1, ..., p=2):
[1] 1.632 0.000
```

Rotation (n x k) = (2 x 2):

	[,1]	[,2]
[1,]	-0.99993	-0.01153
[2,]	-0.01153	0.99993

```
attr("class")  
[1] "prcomplst"
```

```
orthdiff.lmer <- lmer(logdist ~ Sex * scAge + (1 | Subject),  
                      data=Orthodont, subset=keep)  
summary(orthdiff.lmer)
```

Linear mixed model fit by REML ['lmerMod']  
Formula: logdist ~ Sex \* scAge + (1 | Subject)  
Data: Orthodont  
Subset: keep

REML criterion at convergence: -232

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.312	-0.510	0.014	0.543	3.945

Random effects:

Groups	Name	Variance	Std.Dev.
Subject	(Intercept)	0.00633	0.0796
Residual		0.00238	0.0488

Number of obs: 100, groups: Subject, 25

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	3.11451	0.02510	124.11
SexMale	0.09443	0.03354	2.82
scAge	0.02115	0.00329	6.42
SexMale:scAge	0.00849	0.00440	1.93

Correlation of Fixed Effects:

	(Intr)	SexMal	scAge
SexMale	-0.748		
scAge	0.000	0.000	
SexMal:scAg	0.000	0.000	-0.748

```

Orthodont2 <- droplevels(subset(Orthodont, keep))
opt <- options(contrasts=c("contr.sum","contr.poly"))
orthdiff.mixed <- afex::mixed(logdist ~ Sex * scAge + (1 | Subject), type=2,
                             method='S', data=Orthodont2)
  ## NB `type` refers to type of test, NOT `error` type.
options(opt)          # Reset to previous contrasts setting
orthdiff.mixed

```

Mixed Model Anova Table (Type 2 tests, S-method)

Model: logdist ~ Sex \* scAge + (1 | Subject)

Data: Orthodont2

	Effect	df	F	p.value
1	Sex	1, 23.00	7.93 **	.010
2	scAge	1, 73.00	140.72 ***	<.001
3	Sex:scAge	1, 73.00	3.72 +	.058

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '+' 0.1 ' ' 1

```

contrasts(Orthodont2[['Subject']]) <- 'contr.sum'
contrasts(Orthodont2[['Sex']]) <- 'contr.sum'

```

```

orthdiffa.lmer <- update(orthdiff.lmer, formula=. ~ . -Sex:scAge)
AIC(orthdiffa.lmer,orthdiff.lmer)

```

	df	AIC
orthdiffa.lmer	5	-227.4
orthdiff.lmer	6	-220.1

## Correlation between successive times

```

res <- resid(orthdiff.lmer)
Subject <- factor(Orthodont$Subject[keep])
orth.arma <- sapply(split(res, Subject),
                    function(x)forecast::auto.arima(x)$arma[c(1,6,2)])

```

```

orthsum <- apply(orth.arma,2,sum)
orth.arma[, orthsum>0]

```

	F08	M06
[1,]	1	1
[2,]	0	0
[3,]	1	1

### Fitting a sequential correlation structure

```
library(nlme)
keep <- !(Orthodont$Subject%in%c("M04","M13"))
Orthodont2 <- droplevels(subset(Orthodont,keep))
orthdiff.lme <- lme(logdist ~ Sex * scAge, random = ~1|Subject,
                    cor=corCAR1(0.1, form=~1|Subject), data=Orthodont2)
## For AR1 models `phi` is the sequential correlation estimate
orthdiff.lme$modelStruct$corStruct
```

Correlation structure of class corCAR1 representing  
 Phi  
 0.0000000003311

**\*The variance for the difference in slopes**

## Section 7.9: Further notes on multilevel models

**Subsection 7.9.1: Sources of variation – complication or focus of interest?**

**Subsection 7.9.2: Predictions from models with a complex error structure**

**Consequences from assuming an overly simplistic error structure**

**Subsection 7.9.3: An historical perspective on multilevel models**

**Subsection 7.9.4: Meta-analysis**

**Subsection 7.9.5: Functional data analysis**

**Subsection 7.9.6: Error structure in explanatory variables**

### Exercises (7.12)

7.1

```

n.omit <- 2
take <- rep(TRUE, 48)
take[sample(1:48,2)] <- FALSE
kiwishade.lmer <- lmer(yield ~ shade + (1|block) + (1|block:plot),
                      data = kiwishade, subset=take)
vcov <- VarCorr(kiwishade.lmer)
print(vcov, comp="Variance")

```

Groups	Name	Variance
block:plot	(Intercept)	2.35
block	(Intercept)	3.28
Residual		12.45

7.6

```

cult.lmer <- lmer(ct ~ Cultivar + Dose + factor(year) +
                 (-1 + Dose | gp), data = DAAG::sorption, REML=TRUE)
cultdose.lmer <- lmer(ct ~ Cultivar/Dose + factor(year) +
                    (-1 + Dose | gp), data = DAAG::sorption, REML=TRUE)

```

```

if(file.exists("/Users/johnm1/pkgsg/PGRcode/inst/doc/")){
code <- knitr::knit_code$get()
txt <- paste0("\n## ", names(code), "\n", sapply(code, paste, collapse='\n'))
writeLines(txt, con="/Users/johnm1/pkgsg/PGRcode/inst/doc/ch7.R")
}

```

## 8 Chapter 8: Tree-based Classification and Regression

### Packages required (plus any dependencies)

DAAG latticeExtra plot rpart rpart.plot MASS ggplot2 car randomForest

Additionally, knitr and Hmisc are required in order to process the Rmd source file.

```
Hmisc::knitrSet(basename="treebased", lang='markdown', fig.path="figs/g", w=7, h=7)
oldopt <- options(digits=4, formatR.arrow=FALSE, width=70, scipen=999)
library(knitr)
## knitr::render_listings()
opts_chunk[['set']](cache.path='cache-', out.width="80%", fig.align="center",
                    fig.show='hold', size="small", ps=10, strip.white = TRUE,
                    comment=NA, width=70, tidy.opts = list(replace.assign=FALSE))
```

```
suppressPackageStartupMessages(library(latticeExtra))
```

### When are tree-based methods appropriate?

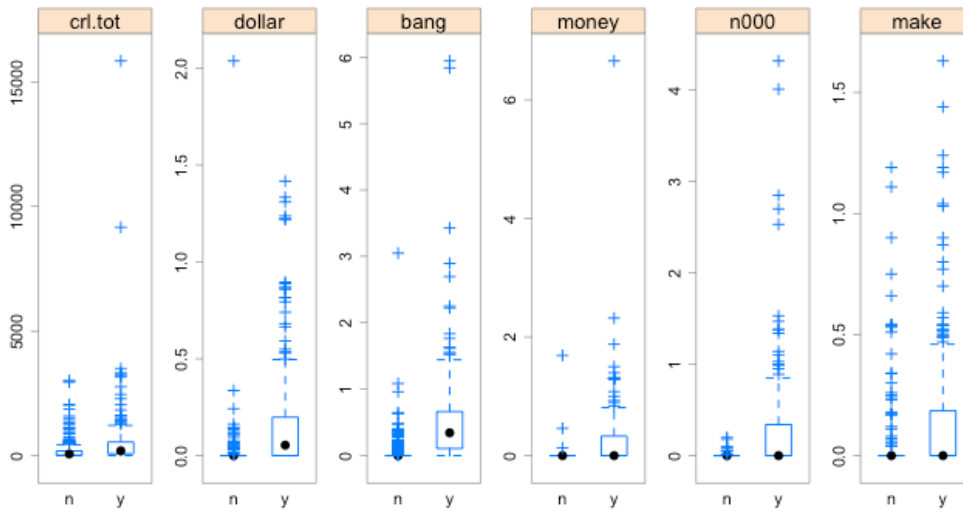
#### Section 8.1: Tree-based methods — uses and basic notions

Examples that will be used to demonstrate the methodology

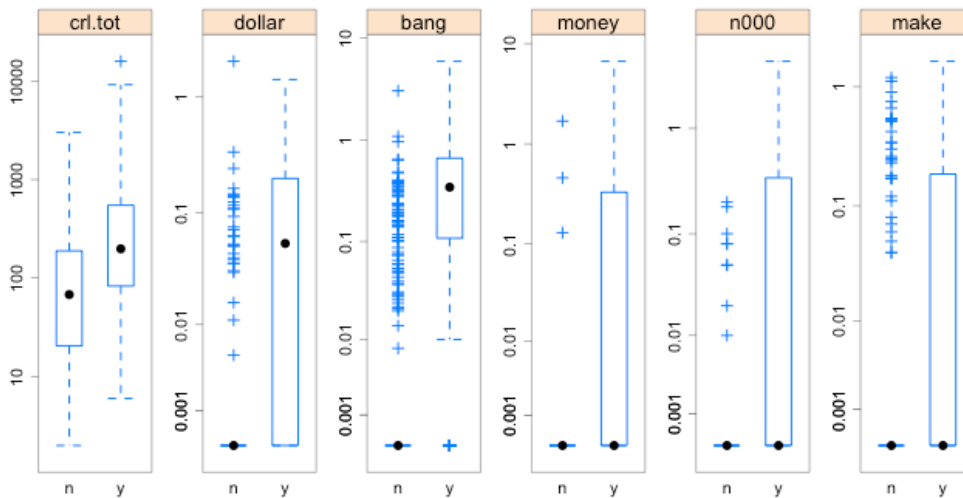


### Subsection 8.1.1: Detecting email spam— an initial look

A: Raw data values



B: Boxplots, using  $\log(x + 1)$  scale



```
nam <- c("crl.tot", "dollar", "bang", "money", "n000", "make")
nr <- sample(1:dim(DAAG::spam7)[1], 500)
yesno <- DAAG::spam7$yesno[nr]
spam7a <- DAAG::spam7[nr, c(nam, "yesno")]
formab <- as.formula(paste(paste(nam, collapse='+'), '~ yesno'))
```

```

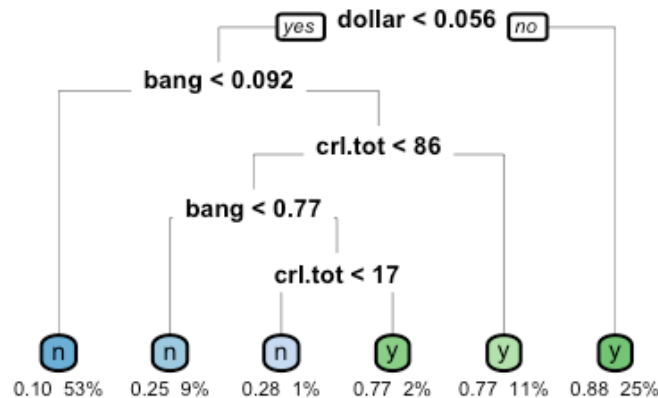
spamtheme <- DAAG::DAAGtheme(color = TRUE, pch=3)
lattice::bwplot(formab, data=spam7a, outer=T, horizontal=F, layout=c(7,1),
  scales=list(relation='free'), ylab="", par.settings=spamtheme,
  between=list(x=0.5),
  main=list("A: Raw data values", y=1.0, font=1, cex=1.25))
spam7b <- cbind(log(spam7a[,-7]+0.001), yesno=spam7a[,7])
yval <-c(0.001, 0.001,0.01,0.1,1,10,100,1000,10000)
lattice::bwplot(formab, data=spam7b, outer=T, horizontal=F, layout=c(7,1),
  scales=list(relation='free',
  y=list(at=log(yval+0.001), labels=yval, rot=90)),
  ylab="", par.settings=spamtheme, between=list(x=0.5),
main=list(expression("B: Boxplots, using \"*log(x+001)*\" scale"),
  y=1.0, font=1, cex=1.25))

```

```

## Obtain 500-row sample; repeat the first plot (of crl.tot)
spam.sample <- spam7[sample(seq(1,4601), 500, replace=FALSE), ]
boxplot(split(spam.sample$crl.tot, spam.sample$yesno))

```



```

suppressMessages(library(rpart))
set.seed(31) ## Reproduce tree shown in text
spam.rpart <- rpart(formula = yesno ~ crl.tot + dollar + bang + money + n000 +
  make, method="class", model=TRUE, data=DAAG::spam7)
rpart.plot::rpart.plot(spam.rpart, type=0, under=TRUE, branch.lwd=0.4,
  nn.lwd=0.4, box.palette="auto", tweak=1.25)

```

```
printcp(spam.rpart, digits=3)
```

Classification tree:

```
rpart(formula = yesno ~ crl.tot + dollar + bang + money + n000 +  
      make, data = DAAG::spam7, method = "class", model = TRUE)
```

Variables actually used in tree construction:

```
[1] bang    crl.tot dollar
```

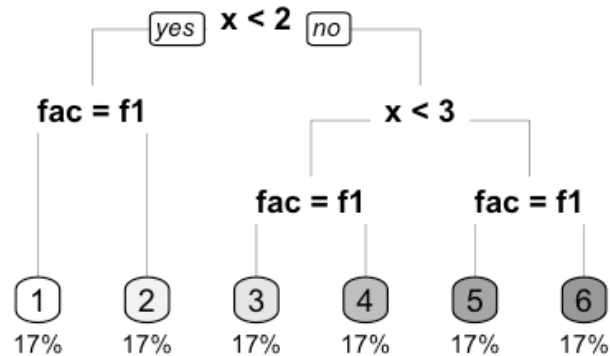
Root node error: 1813/4601 = 0.394

n= 4601

	CP	nsplit	rel	error	xerror	xstd
1	0.4766	0	1.000	1.000	0.0183	
2	0.0756	1	0.523	0.547	0.0154	
3	0.0116	3	0.372	0.388	0.0135	
4	0.0105	4	0.361	0.384	0.0134	
5	0.0100	5	0.350	0.382	0.0134	

### Subsection 8.1.2: Choosing the number of splits

## Section 8.2: Splitting criteria, with illustrative examples



```

tree.df <- data.frame(fac = factor(rep(c('f1','f2'), 3)),
x = rep(1:3, rep(2, 3)), Node = 1:6)
u.tree <- rpart(Node ~ fac + x, data = tree.df,
               control = list(minsplit = 2, minbucket = 1, cp = 1e-009))
rpart.plot::rpart.plot(u.tree, type=0, under=TRUE, branch.lwd=0.25,
                      nn.lwd=0.25, box.palette="Grays", tweak=1.6)

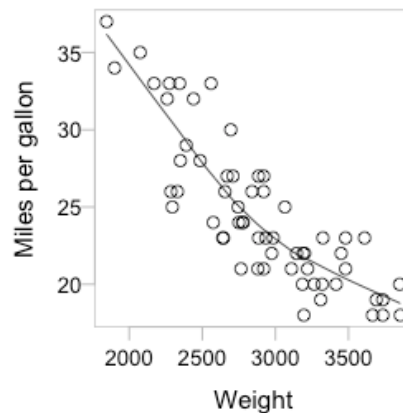
```

## Choosing the split— regression trees

### Subsection 8.2.1: Within and between sums of squares

### Subsection 8.2.2: Choosing the split— classification trees

### Subsection 8.2.3: Tree-based regression versus loess regression smoothing



```

u.lo <- loess(Mileage~Weight, data = car.test.frame, span = 2/3)
plot(Mileage~Weight, data=car.test.frame, xlab = "Weight",
     ylab = "Miles per gallon", sub = "", fg="gray")
xy <- with(car.test.frame, loess.smooth(Weight, Mileage))
ord<-order(xy$x)
lines(xy$x[ord],xy$y[ord])

```

```

## loess fit to Mileage vs Weight: data frame car.test.frame (rpart)
with(rpart::car.test.frame, scatter.smooth(Mileage ~ Weight))

```



```
par(fig=c(0, 0.32, 0,1))
set.seed(37)
car.tree <- rpart(Mileage ~ Weight, data = car.test.frame)
rpart.plot::rpart.plot(car.tree, type=0, under=TRUE,
                        box.palette="Grays", tweak=1.05)
par(fig=c(0.3,1, 0,1), new=TRUE)
set.seed(37)
car2.tree <- rpart(Mileage~Weight, data=car.test.frame, control =
                  list(minsplit = 10, minbucket = 5, cp = 0.0001))
rpart.plot::rpart.plot(car2.tree, type=0, under=TRUE,
                        box.palette="auto", tweak=1.05)
```

```
## Panel A: Split criteria were left a their defaults
car.tree <- rpart(Mileage ~ Weight, data = car.test.frame)
rpart.plot::rpart.plot(car.tree, type=0, under=TRUE)
## Panel B: Finer grained splits
car2.tree <- rpart(Mileage ~ Weight, data=car.test.frame, method="anova",
                  control = list(minsplit = 10, minbucket = 5, cp = 0.0001))
## See `?rpart::rpart.control` for details of control options.
```

```
dat <- data.frame(Weight=seq(from=min(car.test.frame$Weight),
to=max(car.test.frame$Weight)))
pred <- predict(car.tree, newdata=dat)
pred2 <- predict(car2.tree, newdata=dat)
lwr <- dat$Weight[c(1,diff(pred)) != 0]
upr <- dat$Weight[c(diff(pred),1) != 0]
xy2 <- with(car.test.frame, loess.smooth(Weight, Mileage, evaluation=2011))
lwrL0 <- xy2$y[c(1,diff(pred)) != 0]
```

```
uprL0 <- xy2$y[c(diff(pred),1) != 0]
round(rbind(lwr,upr,lwrL0,uprL0,
pred[c(diff(pred),1)!=0],pred2[c(diff(pred),1)!=0]),1)
```

	723	903	1243	2011
lwr	1845.0	2568.0	2748.0	3088.0
upr	2567.0	2747.0	3087.0	3855.0
lwrL0	36.2	27.1	25.1	22.3
uprL0	27.1	25.1	22.3	18.8
	30.9	25.6	23.8	20.4
	28.9	25.6	24.1	18.7

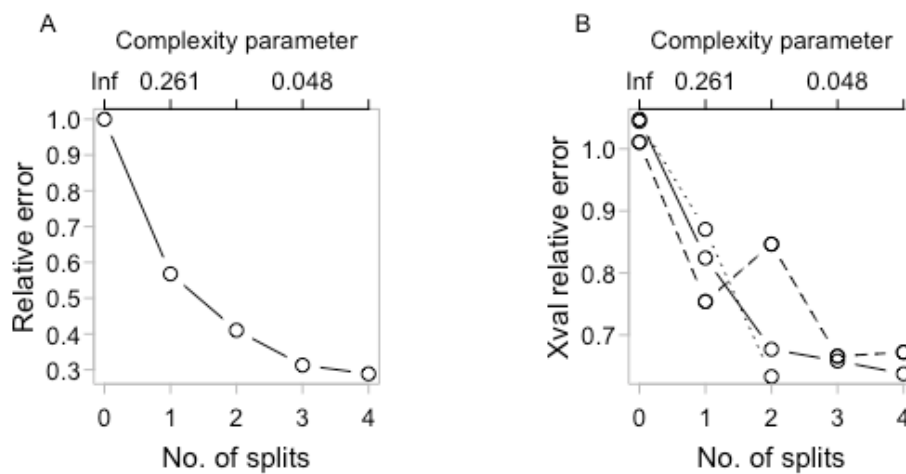
#### Subsection 8.2.4: Predictive accuracy, and the cost-complexity tradeoff

#### Subsection 8.2.5: Cross-validation

#### Subsection 8.2.6: The cost-complexity parameter

```
vignette("longintro", package="rpart")
```

#### Subsection 8.2.7: Prediction error versus tree size



## Section 8.3: The practicalities of tree construction – two examples

### Subsection 8.3.1: Data for female heart attack patients

```
mifem <- DAAG::mifem
summary(mifem)      # data frame mifem (DAAG)
```

outcome	age	yr onset	premi	smstat	diabetes
live:974	Min. :35.0	Min. :85.0	y :311	c :390	y :248
dead:321	1st Qu.:57.0	1st Qu.:87.0	n :928	x :280	n :978
	Median :63.0	Median :89.0	nk: 56	n :522	nk: 69
	Mean :60.9	Mean :88.8		nk:103	
	3rd Qu.:66.0	3rd Qu.:91.0			
	Max. :69.0	Max. :93.0			
highbp	hichol	angina	stroke		
y :813	y :452	y :472	y : 153		
n :406	n :655	n :724	n :1063		
nk: 76	nk:188	nk: 99	nk: 79		

```
set.seed(29)      # Make results reproducible
mifem.rpart <- rpart(outcome ~ ., method="class", data = mifem, cp = 0.0025)
```

```
## Tabular equivalent of Panel A from `plotcp(mifem.rpart)`
printcp(mifem.rpart, digits=3)
```

Classification tree:

```
rpart(formula = outcome ~ ., data = mifem, method = "class",
      cp = 0.0025)
```

Variables actually used in tree construction:

```
[1] age      angina    diabetes hichol    premi    smstat    stroke
[8] yr onset
```

Root node error: 321/1295 = 0.248

n= 1295

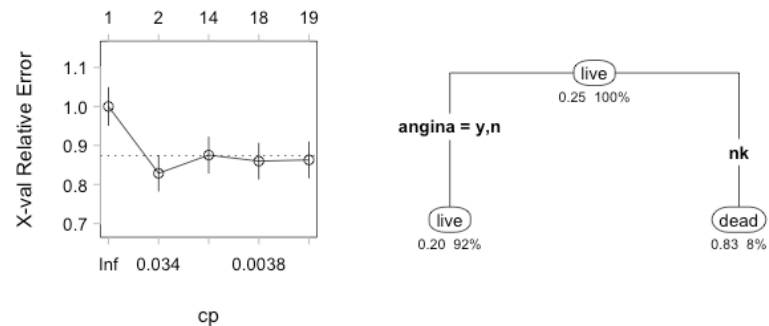
	CP	nsplit	rel error	xerror	xstd
1	0.20249	0	1.000	1.000	0.0484
2	0.00561	1	0.798	0.829	0.0453
3	0.00467	13	0.717	0.875	0.0462
4	0.00312	17	0.698	0.860	0.0459
5	0.00250	18	0.695	0.863	0.0460

```
cat(c(". . .", capture.output(printcp(mifem.rpart, digits=3))[-(1:9)]),
    sep="\n")
```

. . .  
Root node error: 321/1295 = 0.248

n= 1295

	CP	nsplit	rel error	xerror	xstd
1	0.20249	0	1.000	1.000	0.0484
2	0.00561	1	0.798	0.829	0.0453
3	0.00467	13	0.717	0.875	0.0462
4	0.00312	17	0.698	0.860	0.0459
5	0.00250	18	0.695	0.863	0.0460



```
plotcp(mifem.rpart, fg="gray", tcl=-0.25)
mifemb.rpart <- prune(mifem.rpart, cp=0.01) ## Prune tree back to 2 leaves
rpart.plot::rpart.plot(mifemb.rpart, under=TRUE, type=4,
    box.palette=0, tweak=1.0)
```



### Subsection 8.3.2: The one-standard-deviation rule

### Subsection 8.3.3: Printed Information on Each Split

```
print(mifemb.rpart)
```

n= 1295

```
node), split, n, loss, yval, (yprob)
      * denotes terminal node
```

```
1) root 1295 321 live (0.7521 0.2479)
  2) angina=y,n 1196 239 live (0.8002 0.1998) *
  3) angina=nk 99 17 dead (0.1717 0.8283) *
```

```
set.seed(59)
spam7a.rpart <- rpart(formula = yesno ~ crl.tot + dollar + bang +
                      money + n000 + make, method="class", cp = 0.002,
                      model=TRUE, data = DAAG::spam7)
```

```
printcp(spam7a.rpart, digits=3)
```

Classification tree:

```
rpart(formula = yesno ~ crl.tot + dollar + bang + money + n000 +
      make, data = DAAG::spam7, method = "class", model = TRUE,
      cp = 0.002)
```

Variables actually used in tree construction:

```
[1] bang    crl.tot  dollar  money   n000
```

Root node error: 1813/4601 = 0.394

n= 4601

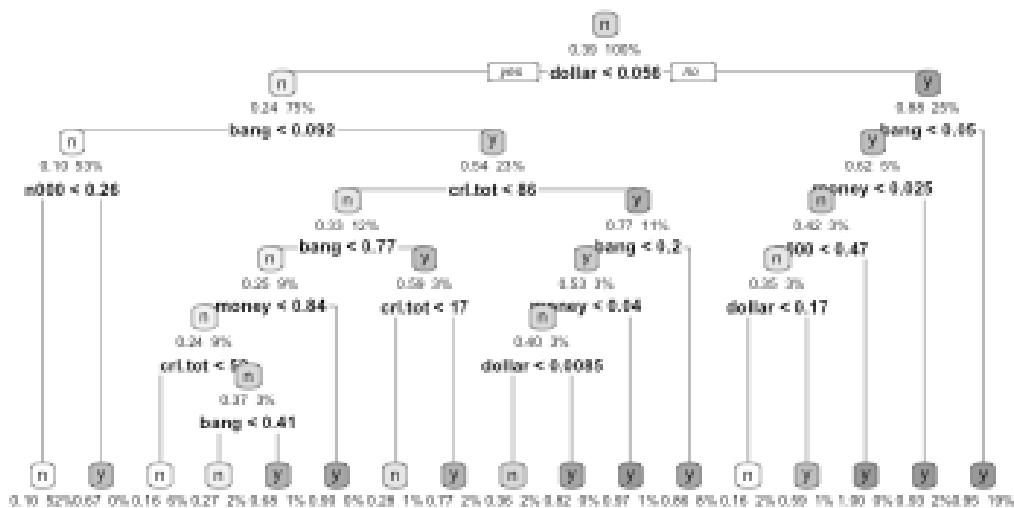
	CP	nsplit	rel	error	xerror	xstd
1	0.47656	0	1.000	1.000	0.0183	
2	0.07557	1	0.523	0.550	0.0154	
3	0.01158	3	0.372	0.390	0.0135	
4	0.01048	4	0.361	0.386	0.0134	

5	0.00634	5	0.350	0.374	0.0133
6	0.00552	10	0.317	0.360	0.0130
7	0.00441	11	0.311	0.357	0.0130
8	0.00386	12	0.307	0.352	0.0129
9	0.00276	16	0.291	0.339	0.0127
10	0.00221	17	0.288	0.339	0.0127
11	0.00200	18	0.286	0.336	0.0127

```

cpdf <- signif(as.data.frame(spam7a.rpart$cptable),3)
minRow <- which.min(cpdf[, "xerror"])
upr <- sum(cpdf[minRow, c("xerror", "xstd")])
takeRow <- min((1:minRow)[cpdf[1:minRow, "xerror"] < upr])
newNsplit <- cpdf[takeRow, 'nsplit']
cpval <- mean(cpdf[c(takeRow-1, takeRow), "CP"])

```



```

spam7b.rpart <- prune(spam7a.rpart, cp=cpval)
rpart.plot::rpart.plot(spam7b.rpart, under=TRUE, box.palette="Grays", tweak=1.65)

```

How does the one standard error rule affect accuracy of estimates?

```
requireNamespace('randomForest', quietly=TRUE)
DAAG::compareTreecalcs(data=DAAG::spam7, fun="rpart")
```

rpSEcvI	rpcvI	rpSEtest	rptest	nSErule	nREmin
0.1396	0.1387	0.1269	0.1217	7.0000	8.0000

```
acctree.mat <- matrix(0, nrow=100, ncol=6)
spam7 <- DAAG::spam7
for(i in 1:100)
  acctree.mat[i,] <- DAAG::compareTreecalcs(data=spam7, fun="rpart")
```

**How is the standard error calculated?**

**When are tree-based methods appropriate?**

## Section 8.4: From one tree to a forest – a more global optimality

```
suppressPackageStartupMessages(library(randomForest))
spam7.rf <- randomForest(yesno ~ ., data=spam7, importance=TRUE)
spam7.rf
```

Call:

```
randomForest(formula = yesno ~ ., data = spam7, importance = TRUE)
      Type of random forest: classification
      Number of trees: 500
```

No. of variables tried at each split: 2

OOB estimate of error rate: 11.56%

Confusion matrix:

	n	y	class.error
n	2651	137	0.04914
y	395	1418	0.21787

```
z <- tuneRF(x=spam7[, -7], y=spam7$yesno, plot=FALSE)
```

```

mtry = 2  OOB error = 11.8%
Searching left ...
mtry = 1  OOB error = 12.67%
-0.07366 0.05
Searching right ...
mtry = 4  OOB error = 11.76%
0.003683 0.05

```

```

zdash <- t(z[,2,drop=F])
colnames(zdash) <- paste0(c("mtry=",rep("",2)), z[,1])
round(zdash,3)

```

```

      mtry=1      2      4
OOBError 0.127 0.118 0.118

```

```
importance(spam7.rf)
```

	n	y	MeanDecreaseAccuracy	MeanDecreaseGini
crl.tot	46.73	54.19	70.57	248.10
dollar	56.21	55.35	76.13	431.75
bang	91.66	100.46	115.95	588.53
money	33.09	51.87	53.49	206.51
n000	58.25	15.74	62.29	115.46
make	13.67	21.76	26.72	41.13

### Subsection 8.4.1: Prior probabilities

```

Pima.tr <- MASS::Pima.tr
table(Pima.tr$type)

```

```

No Yes
132 68

```

```

set.seed(41)      # This seed should reproduce the result given here
Pima.rf <- randomForest(type ~ ., data=Pima.tr)
## The above is equivalent to:
## Pima.rf <- randomForest(type ~ ., data=Pima.tr, sampsize=200)
round(Pima.rf$confusion,3)

```

	No	Yes	class.error
No	110	22	0.167
Yes	32	36	0.471

```
tab <- prop.table(table(Pima.tr$type))
```

```
Pima.rf <- randomForest(type ~ ., data=Pima.tr, sampsize=c(68,68))
```

```
Pima.rf <- randomForest(type ~ ., data=Pima.tr, sampsize=c(132,68))
```

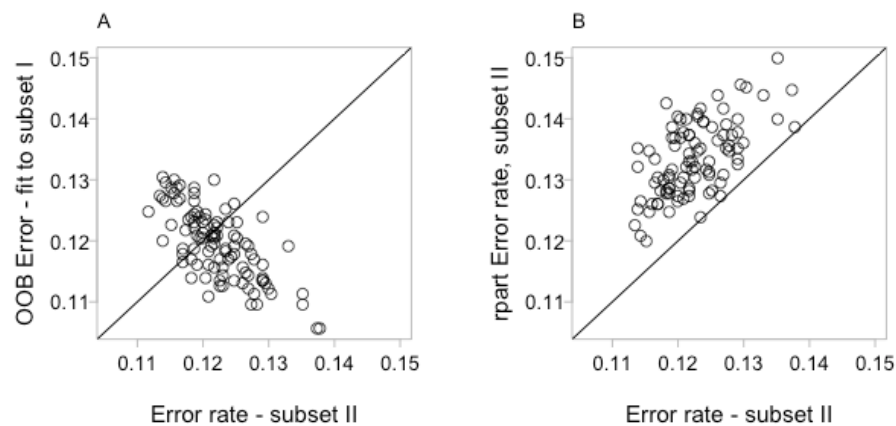
#### Subsection 8.4.2: A low-dimensional representation of observations

#### Subsection 8.4.3: Models with a complex error structure

### Section 8.5: Additional notes – one tree, or many trees?

#### Subsection 8.5.1: Differences between rpart() and randomForest()

#### Error rates – rpart() versus randomForest()



```
## Accuracy comparisons
acctree.mat <- matrix(0, nrow=100, ncol=8)
colnames(acctree.mat) <- c("rpSEcvI", "rpcvI", "rpSEtest", "rpctest",
                           "n.SERule", "nre.min.12", "rfOOBI", "rfctest")
for(i in 1:100) acctree.mat[i,] <- DAAG::compareTreecalcs(data=spam7, cp=0.0004,
                                                           fun=c("rpart", "randomForest"))
```

```

acctree.df <- data.frame(acctree.mat)
lims <- range(acctree.mat[, c(4,7,8)], na.rm=TRUE)
cthrublack <- adjustcolor("black", alpha.f=0.75)
# Panel A
plot(rfOOBI ~ rfctest, data=acctree.df, xlab="Error rate - subset II", xlim=lims,
     ylim=lims, ylab="OOB Error - fit to subset I", col=cthrublack, fg="gray")
abline(0,1)
mtext(side=3, line=0.5, "A", adj=0)
# Panel B
plot(rpctest ~ rfctest, data=acctree.df, xlab="Error rate - subset II",
     ylab="rpart Error rate, subset II", xlim=lims, ylim=lims,
     col=cthrublack, fg="gray")
abline(0,1)
mtext(side=3, line=0.5, "B", adj=0)

```

```

acctree.mat <- matrix(0, nrow=100, ncol=8)
colnames(acctree.mat) <- c("rpSEcvI", "rpcvI", "rpSEtest", "rpctest",
                          "n.SErule", "nre.min.12", "rfcvI", "rfctest")
for(i in 1:100)acctree.mat[i,] <- DAAG::compareTreecalcs(data=spam7,
                                                         fun=c("rpart", "randomForest"))

## Panel A: Plot `rfOOBI` against `rfctest`
## Panel B: Plot `rpctest` against `rfctest`

```

## Times required for computation

### Subsection 8.5.2: Tree-based methods, versus other approaches

Tree-based methods may usefully complement other approaches

### Subsection 8.5.3: Further notes

Pruning as variable selection

## Section 8.6: Further reading and extensions

### Exercises (8.7)

8.5

```
sapply(MASS::biopsy, function(x)sum(is.na(x))) ## Will omit rows with NAs
```

ID	V1	V2	V3	V4	V5	V6	V7	V8	V9	class
0	0	0	0	0	0	16	0	0	0	0

```
biops <- na.omit(MASS::biopsy)[,-1] ## Omit also column 1 (IDs)
## Examine list element names in randomForest object
names(randomForest(class ~ ., data=biops))
```

```
[1] "call"          "type"          "predicted"
[4] "err.rate"      "confusion"     "votes"
[7] "oob.times"     "classes"       "importance"
[10] "importanceSD"  "localImportance" "proximity"
[13] "ntree"         "mtry"          "forest"
[16] "y"            "test"          "inbag"
[19] "terms"
```

8.5a

```
## Repeated runs, note variation in OOB accuracy.
for(i in 1:10) {
  biops.rf <- randomForest(class ~ ., data=biops)
  OOBerr <- mean(biops.rf$err.rate[, "OOB"])
  print(paste(i, ": ", round(OOBerr, 4), sep=""))
  print(round(biops.rf$confusion, 4))
}
```

```
[1] "1: 0.0288"
      benign malignant class.error
benign    432         12    0.0270
malignant    7        232    0.0293
[1] "2: 0.0344"
      benign malignant class.error
benign    431         13    0.0293
malignant    9        230    0.0377
[1] "3: 0.0308"
      benign malignant class.error
benign    433         11    0.0248
malignant    9        230    0.0377
[1] "4: 0.0307"
```

```

      benign malignant class.error
benign      431         13      0.0293
malignant     6        233      0.0251
[1] "5: 0.0311"

      benign malignant class.error
benign      433         11      0.0248
malignant     8        231      0.0335
[1] "6: 0.0301"

      benign malignant class.error
benign      431         13      0.0293
malignant     6        233      0.0251
[1] "7: 0.0312"

      benign malignant class.error
benign      433         11      0.0248
malignant     7        232      0.0293
[1] "8: 0.0301"

      benign malignant class.error
benign      433         11      0.0248
malignant     8        231      0.0335
[1] "9: 0.0285"

      benign malignant class.error
benign      433         11      0.0248
malignant     6        233      0.0251
[1] "10: 0.0301"

      benign malignant class.error
benign      432         12      0.0270
malignant     7        232      0.0293

```

8.5b

```

## Repeated train/test splits: OOB accuracy vs test set accuracy.
for(i in 1:10){
  trRows <- sample(1:dim(biops)[1], size=round(dim(biops)[1]/2))
  biops.rf <- randomForest(class ~ ., data=biops[trRows, ],
    xtest=biops[-trRows,-10], ytest=biops[-trRows,10])
  oobErr <- mean(biops.rf$err.rate[, "OOB"])
  testErr <- mean(biops.rf$test$err.rate[, "Test"])
  print(round(c(oobErr, testErr), 4))
}

```

```

[1] 0.0282 0.0340
[1] 0.0366 0.0327

```



```
[1] 0.0360 0.0226
[1] 0.0330 0.0311
[1] 0.0360 0.0384
[1] 0.0357 0.0353
[1] 0.0248 0.0399
[1] 0.0442 0.0270
[1] 0.0407 0.0268
[1] 0.0347 0.0242
```

8.5c

```
randomForest(class ~ ., data=biops, xtest=biops[,-10], ytest=biops[,10])
```

Call:

```
randomForest(formula = class ~ ., data = biops, xtest = biops[,      -10], ytest = biops[,
              Type of random forest: classification
              Number of trees: 500
```

No. of variables tried at each split: 3

OOB estimate of error rate: 2.93%

Confusion matrix:

	benign	malignant	class.error
benign	433	11	0.02477
malignant	9	230	0.03766

Test set error rate: 0%

Confusion matrix:

	benign	malignant	class.error
benign	444	0	0
malignant	0	239	0

8.7

```
## Run model on total data
randomForest(as.factor(type) ~ ., data=Pima.tr)
```

Call:

```
randomForest(formula = as.factor(type) ~ ., data = Pima.tr)
              Type of random forest: classification
              Number of trees: 500
```

No. of variables tried at each split: 2

OOB estimate of error rate: 28%

Confusion matrix:

	No	Yes	class.error
No	109	23	0.1742
Yes	33	35	0.4853

```
rowsamp <- sample(dim(Pima.tr)[1], replace=TRUE)
randomForest(as.factor(type) ~ ., data=Pima.tr[rowsamp, ])
```

Call:

```
randomForest(formula = as.factor(type) ~ ., data = Pima.tr[rowsamp, ])
```

Type of random forest: classification

Number of trees: 500

No. of variables tried at each split: 2

OOB estimate of error rate: 8.5%

Confusion matrix:

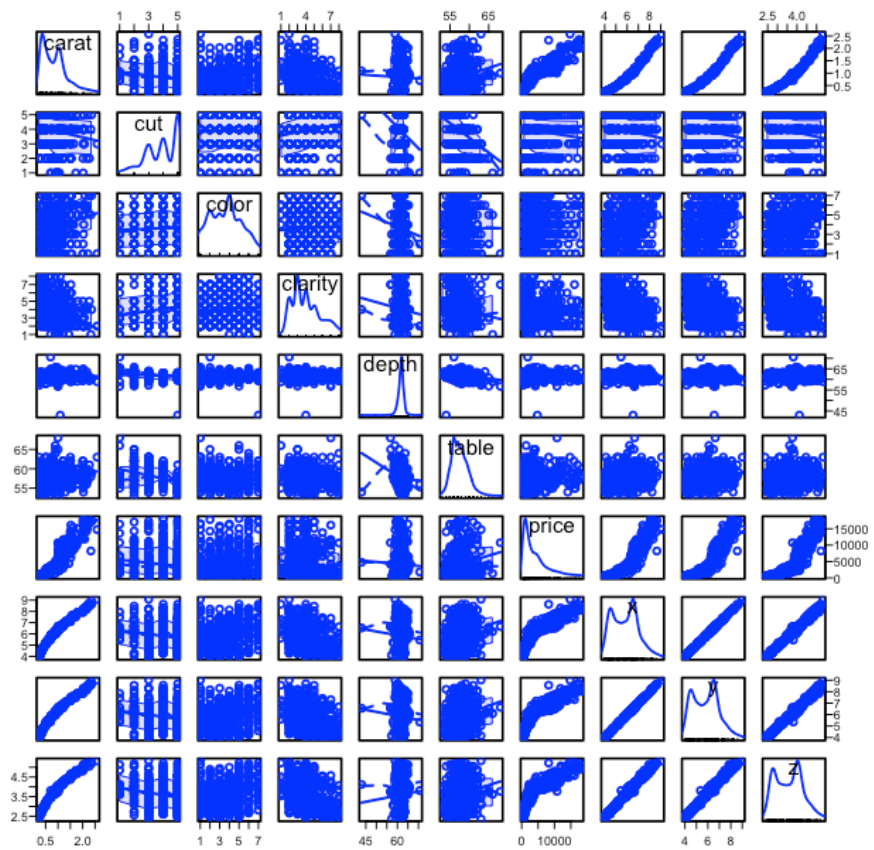
	No	Yes	class.error
No	129	3	0.02273
Yes	14	54	0.20588

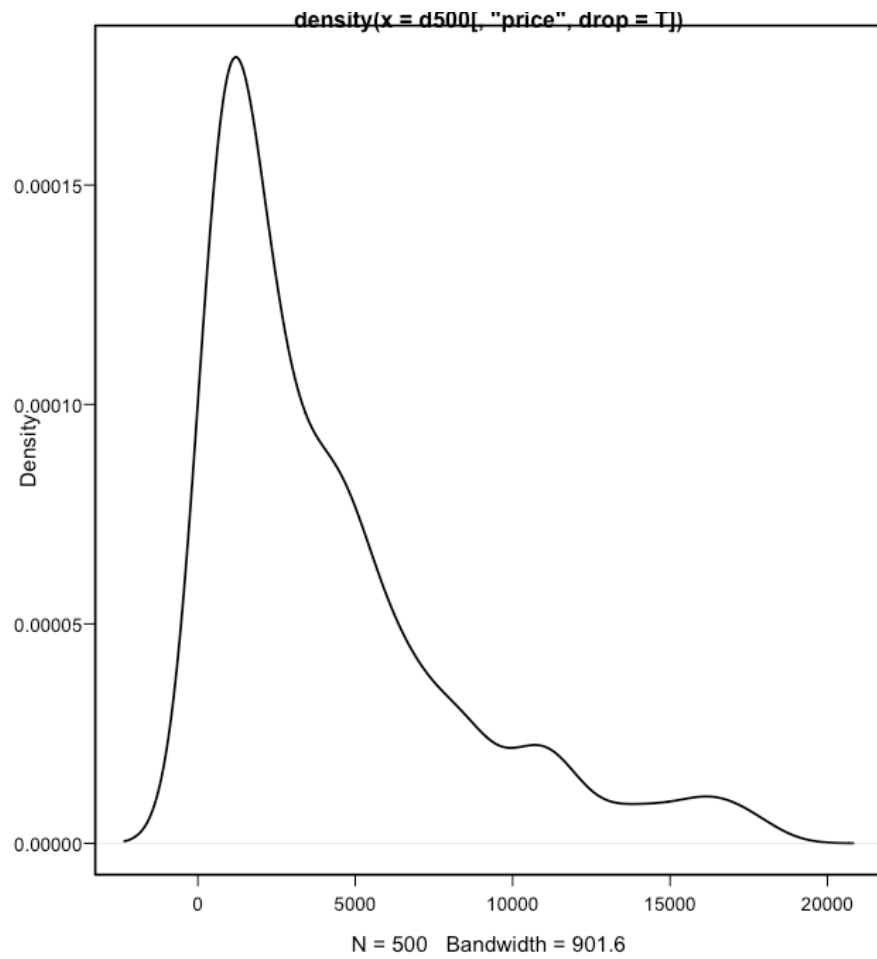
8.8a

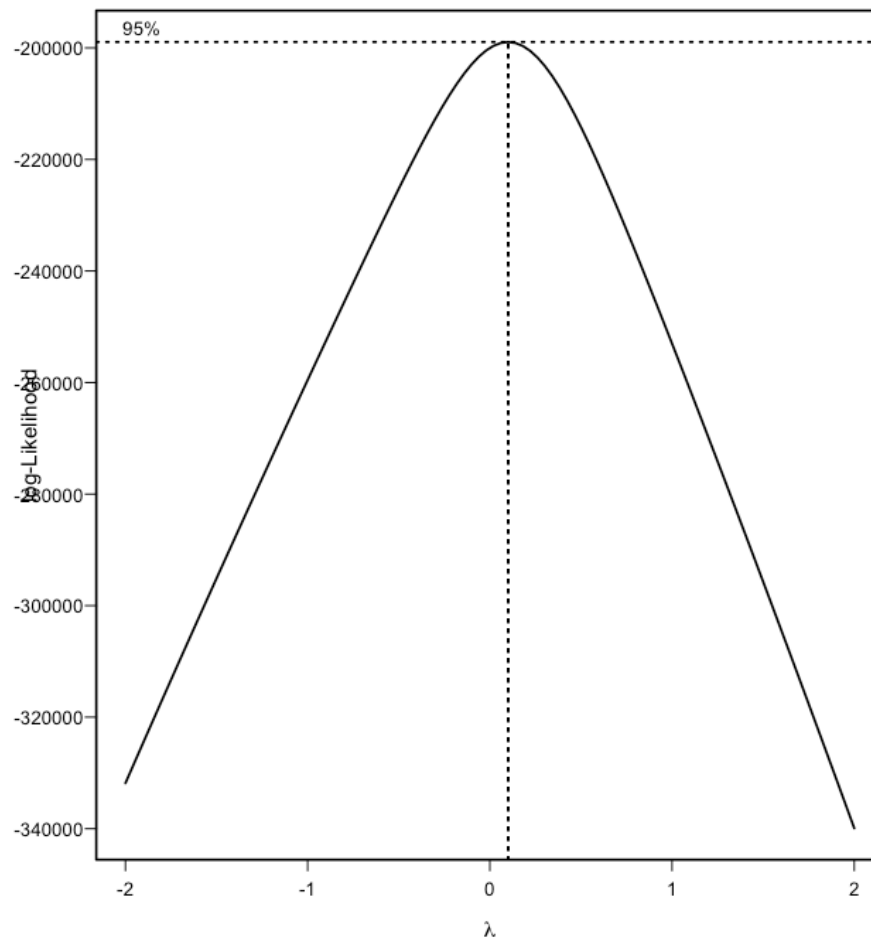
```
d500 <- ggplot2::diamonds[sample(1:nrow(ggplot2::diamonds), 500),]
unlist(sapply(d500, class)) # Check the class of the 10 columns
```

carat	cut1	cut2	color1	color2	clarity1	clarity2
"numeric"	"ordered"	"factor"	"ordered"	"factor"	"ordered"	"factor"
depth	table	price	x	y	z	
"numeric"	"numeric"	"integer"	"numeric"	"numeric"	"numeric"	"numeric"

```
car::spm(d500) # If screen space is limited do two plots, thus:
# 1) variables 1 to 5 and 7 (`price`); 2) variables 6 to 10
plot(density(d500[, "price", drop = T])) # Distribution is highly skew
MASS::boxcox(price~., data=ggplot2::diamonds) # Suggests log transformation
```







8.8b

```
diamonds <- ggplot2::diamonds; Y <- diamonds[, "price", drop=T]
library(rpart)
d7.rpart <- rpart(log(Y) ~ ., data=diamonds[, -7], cp=5e-7) # Complex tree
d.rpart <- prune(d7.rpart, cp=0.0025)
printcp(d.rpart) # Relative to `d7.rpart`, simpler and less accurate
```

Regression tree:

```
rpart(formula = log(Y) ~ ., data = diamonds[, -7], cp = 0.0000005)
```

Variables actually used in tree construction:

```
[1] carat   clarity color    x        y
```

Root node error: 55531/53940 = 1

n= 53940

	CP	nsplit	rel	error	xerror	xstd
1	0.7244	0		1.000	1.000	0.00409
2	0.0885	1		0.276	0.276	0.00141
3	0.0661	2		0.187	0.188	0.00104
4	0.0290	3		0.121	0.121	0.00075
5	0.0105	4		0.092	0.092	0.00059
6	0.0072	5		0.082	0.082	0.00054
7	0.0071	6		0.074	0.072	0.00049
8	0.0030	7		0.067	0.068	0.00047
9	0.0026	8		0.064	0.065	0.00046
10	0.0026	9		0.062	0.061	0.00044
11	0.0025	10		0.059	0.059	0.00044

```
nmin <- which.min(d7.rpart$cptable[, 'xerror'])
d0pt.rpart <- prune(d7.rpart, cp=d7.rpart$cptable[nmin, 'CP'])
print(d0pt.rpart$cptable[nmin])
```

```
[1] 0.0000008743
```

```
(xerror12 <- d0pt.rpart$cptable[c(nrow(d.rpart$cptable), nmin), "xerror"])
```

```
11      1931
0.05934 0.01074
```

```
## Subtract from 1.0 to obtain R-squared statistics
```

```
rbind("d.rpart"=d.rpart[['variable.importance']],
      "d0pt.rpart"=d0pt.rpart[['variable.importance']]) |>
  (\(x)100*apply(x,1,function(x)x/sum(x)))() |> round(1) |> t()
```

	y	x	carat	z	clarity	table	color	depth	cut
d.rpart	23.8	23.1	22.9	22.4	4.8	2.8	0.2	0.1	0.1
d0pt.rpart	23.5	22.9	22.7	22.2	5.2	2.7	0.5	0.2	0.1

8.9

```
Y <- ggplot2::diamonds[,"price", drop=T]
samp5K <- sample(1:nrow(diamonds), size=5000)
(diamond5K.rf <- randomForest(x=diamonds[samp5K,-7], y=log(Y[samp5K]),
                             xtest=diamonds[-samp5K,-7], ytest=log(Y[-samp5K])))
```

Call:

```
randomForest(x = diamonds[samp5K, -7], y = log(Y[samp5K]), xtest = diamonds[-samp5K,
                                     Type of random forest: regression
                                     Number of trees: 500
```

No. of variables tried at each split: 3

```
Mean of squared residuals: 0.01434
% Var explained: 98.59
Test set MSE: 0.01
% Var explained: 98.65
```

```
## Omit arguments `xtest` and `ytest` if calculations take too long
```

```
sort(importance(diamond5K.rf)[,1], decreasing=T) |>
  (\(x)100*x/sum(x))() |> round(1) |> t()
```

```
      y carat  x      z clarity color depth table cut
[1,] 33.5  23.6 21 17.2      2.6   1.2   0.4   0.3 0.2
```

8.9a

```
(diamond5KU.rf <- randomForest(x=diamonds[samp5K,-7], y=Y[samp5K],
                               xtest=diamonds[-samp5K,-7], ytest=Y[-samp5K]))
```

Call:

```
randomForest(x = diamonds[samp5K, -7], y = Y[samp5K], xtest = diamonds[-samp5K,
                                     Type of random forest: regression
                                     Number of trees: 500
```

No. of variables tried at each split: 3

```
Mean of squared residuals: 450742
```

```
% Var explained: 97.11
      Test set MSE: 493916
% Var explained: 96.9
```

```
if(file.exists("/Users/johnm1/pkg/PGRcode/inst/doc/")){
code <- knitr::knit_code$get()
txt <- paste0("\n## ", names(code), "\n", sapply(code, paste, collapse='\n'))
writeLines(txt, con="/Users/johnm1/pkg/PGRcode/inst/doc/ch8.R")
}
```



## 9 Chapter 9: Multivariate data exploration and discrimination

### Packages required (plus any dependencies)

Packages used are: DAAG MASS RColorBrewer teigen BiocManager DAAGbio hddplot lmttest splines cobalt mice datasets car micemd oz randomForest ggplot2 latticeExtra mvtnorm teigen limma hddplot mgev MatchIt sandwich gridExtra DAAGbio mlbench (exercise).

Additionally, knitr and Hmisc are required in order to process the Rmd source file.

```
Hmisc::knitrSet(basename="mva", lang='markdown', fig.path="figs/g", w=7, h=7)
oldopt <- options(digits=4, formatR.arrow=FALSE, width=70, scipen=999)
library(knitr)
opts_chunk[['set']](cache.path='cache-', out.width="80%", fig.align="center",
                    fig.show='hold', ps=10, strip.white = TRUE,
                    comment=NA, width=70, tidy.opts = list(replace.assign=FALSE))
```

### Section 9.1: Multivariate exploratory data analysis

```
## Make the lattice package and the possum dataset available
library(latticeExtra)
possum <- DAAG::possum
```

#### Subsection 9.1.1: Scatterplot matrices

```
## Colors distinguish sexes; symbols distinguish sites
sitenames <- row.names(DAAG::possumsites)[c(1,2,4:6,3,7)]
key <- list(points = list(pch=0:6), text=list(sitenames),
            columns=4, between=1, between.columns=2)
colr <- c("red","blue")
vnames <- c("tail\nlength","foot\nlength", "conch\nlength")
gphA <- with(possum, splom(~ possum[, 9:11], pch=(0:6)[site], col=colr[sex],
```

```

      xlab="", varnames=vnames, key=key, axis.line.tck=0.6))
gphB <- with(possum, cloud(earconch~taill+footlgth, data=possum,
  col=colr[sex], key=key, pch = (0:6)[site],
  zlab=list("earconch", rot=90), zoom=0.925))
update(c("A: Scatterplot matrix"=gphA, "B: Cloud plot"=gphB),
  between=list(x=1))

```

## Subsection 9.1.2: Principal components analysis

### Preliminary data scrutiny

```

## Ratios of largest to smallest values: possum[, 6:14] (DAAG)
possum <- DAAG::possum
sapply(na.omit(possum[, 6:14]), function(x)round(max(x)/min(x),2))

```

```

## Principal components calculations: possum[, 6:14] (DAAG)
here <- complete.cases(possum[, 6:14])
possum.prc <- prcomp(log(possum[here, 6:14]))
scores <- cbind(predict(possum.prc), possum[here, c('sex', 'site')])

```

```

## For parset, key and colr; see code for Fig 9.1
pchr <- c(3,4,0,8,2,10,1)
parset <- list(fontsize=list(text=10, points=6), cex=0.75, pch=pchr, alpha=0.8)
key <- modifyList(key, list(columns=1, space="right"))
gph <- with(scores, xyplot(PC2 ~ PC1, aspect="iso", key = key,
  col = colr[sex], pch = (0:6)[site]))
update(gph, scales=list(tck=0.5), par.settings=parset,
  xlab="1st Principal Component", ylab="2nd Principal Component")

```

```

print(summary(possum.prc),digits=2)
cat("\nRotations (otherwise called Loadings)\n")
print(possum.prc$rotation, digits=2)
## By default, blanks are shown for loadings < 0.1 in magnitude

```

### The stability of the principal components plot

```

suppressPackageStartupMessages(library(ggplot2))
theme_set(theme_gray(base_size=8))
## Bootstrap principal components calculations: possum (DAAG)

```

```

## Sample from rows where there are no missing values
rowsfrom <- (1:nrow(possum))[complete.cases(possum[, 6:14])]
logpossum6to14 <- log(possum[rowsfrom, 6:14])
sexPop <- possum[rowsfrom, c("sex","Pop")]
n <- length(rowsfrom); ntimes <- 3
bootscores <- data.frame(scores1=numeric(ntimes*n), scores2=numeric(ntimes*n))
for (i in 1:ntimes){
  samprows <- sample(1:n, n, replace=TRUE)
  bootcores[n*(i-1)+(1:n), 1:2] <-
    prcomp(logpossum6to14[samprows, ])$x[, 1:2]
}
bootcores[, c("sex","Pop")] <- sexPop[samprows, ]
bootcores$sampleID <- factor(rep(1:ntimes, rep(n,ntimes)))
gph <- quickplot(x=scores1, y=scores2, colour=sex, size=I(1.0),
  asp=1, shape=Pop, facets=~sampleID, data=bootcores) +
  scale_shape_discrete(solid=F)
gph + scale_colour_manual(values=c("m"="blue","f"="red")) +
  xlab("First Principal Component") + ylab("Second Principal Component")

```

### Subsection 9.1.3: Multi-dimensional scaling

#### Distance measures

#### Ordination

```

## Code that will display individual graphs
d.possum <- dist(possum[,6:14]) # Euclidean distance matrix
MASS::sammon(d.possum, k=2, trace=FALSE)$points |> as.data.frame() |>
  setNames(paste0("ord",1:2)) |> cbind(Pop=DAAG::possum$Pop) -> sammon.possum
MASS::isoMDS(d.possum, k=2, trace=FALSE) |> as.data.frame() |>
  setNames(paste0("ord",1:2)) |> cbind(Pop=DAAG::possum$Pop) -> mds.possum
gph1 <- xyplot(ord2~ord1, groups=Pop, aspect="iso", data=sammon.possum)
gph2 <- xyplot(ord2~ord1, groups=Pop, aspect="iso", data=mds.possum)
update(c(gph1, gph2, layout=c(2,1)),
  par.settings=simpleTheme(pch=c(1,3)),
  between=list(x=0.5), auto.key=list(columns=2),
  strip=strip.custom(factor.levels=c("A: Sammon","B: IS0mds"))))

```

#### Binary data

## Section 9.2: Principal component scores in regression

```
## Principal components: data frame socsupport (DAAG)
socsupport <- DAAG::socsupport
ss.pr1 <- prcomp(as.matrix(na.omit(socsupport[, 9:19])), retx=TRUE, scale=TRUE)
```

```
oldpar <- par(fg='gray40',col.axis='gray20',lwd=0.5,col.lab='gray20')
pairs(ss.pr1$x[, 1:3], col='gray40', gap=0.2)
par(oldpar)
```

```
summary(sort(ss.pr1$rotation[,1]))
## Note the very large maximum value
which.max(ss.pr1$x[,1])
## Try also boxplot(ss.pr1$x[,1])
## ss.pr1$x["36",1] ## Check that this returns 42
```

```
use <- complete.cases(socsupport[, 9:19])
use[36] <- FALSE
ss.pr <- prcomp(as.matrix(socsupport[use, 9:19]))
```

```
## Output from summary()
print(summary(ss.pr), digits=1) # Compare contributions
```

```
comp <- as.data.frame(ss.pr$x[,1:6])
ss.lm <- lm(socsupport[use, "BDI"] ~ ., data=comp)
signif(round(coef(summary(ss.lm)),5), digits=3)
```

```
print(ss.pr$rotation[, 1], digits=2)
```

```
## Plot BDI against first principal component score
gph <- xyplot(BDI ~ ss.pr$x[,1], groups=gender, data=socsupport[use,],
par.settings=simpleTheme(pch=1:2), auto.key=list(columns=2))
bw9 <- list(pch=c(1,3), list(text=9, points=5))
update(gph, scales=list(tck=0.5), par.settings=bw9,
xlab ="1st principal component")
```

## Section 9.3: Cluster analysis

### Subsection 9.3.1: Hierarchical Clustering

```
library(mvtnorm)
makeClust <- function(n=6, d1=4, d2=4, sigs=c(1, 1, 1, 1), seed=NULL){
  if(!is.null(seed))set.seed(seed)
  g1 <- rmvnorm(n, mean = c(-d1,d2), sigma=sigs[1]*diag(2))
  g2 <- rmvnorm(n, mean = c(d1,d2), sigma=sigs[2]*diag(2))
  g3 <- rmvnorm(n, mean = c(-d1,-d2), sigma=sigs[3]*diag(2))
  g4 <- rmvnorm(n, mean = c(d1,-d2), sigma=sigs[4]*diag(2))
  rbind(g1,g2,g3,g4)
}
```

```
## Code for the plots
datA <- makeClust(seed=35151)
datB <- makeClust(d2=16, seed=35151)
datC <- makeClust(d1=2,d2=2, seed=35151)
plot(datA, xlab="X1", ylab="X2", fg="gray")
title(main="A: 4blobsA", adj=0, line=0.5, font.main=1)
## Repeat previous two lines for datB and datC
plot(datB, xlab="X1", ylab="X2", fg="gray")
title(main="B: 4blobsB", adj=0, line=0.5, font.main=1)
plot(datC, xlab="X1", ylab="X2", fg="gray")
title(main="C: 4blobsC", adj=0, line=0.5, font.main=1)
```

```
## Possible alternative
config <- c('Equidistant blobs', 'Pulled vertically', 'Closer centers')
dat123 <- cbind(as.data.frame(rbind(datA, datB, datC)),
                gp=factor(rep(1:3, rep(6*4,3)), labels=config))
xyplot(V2 ~ V1 | gp, data=dat123, scales=list(relation='free'),
       strip=strip.custom(factor.levels=config), between=list(x=0.5),
       par.settings=DAAG::DAAGtheme(color=F))
```

```
## Code for single linkage plots: `?plot.hclust` gives help for the plot method
clusres_sing <- hclust(dist(datA), method="single")
par(fig=c(0,0.75,0,1))
plot(clusres_sing, sub="", xlab="", ylab="Distance joined", adj=0.5,
     main="", fg="gray")
mtext('A: Single linkage cluster dendrogram, for 4blobsA layout', side=3, adj=0,
     font=1, line=1, cex=1.15)
```

```

par(fig=c(0.72,1,0,1), new=TRUE)
membs <- cutree(clusres_sing, 4)
col4= RColorBrewer::brewer.pal(4,'Set1')
plot(datA, xlab="X1", ylab="X2", col=col4[membs], fg='gray', pch=membs+1)
mtext('B: 4blobsA, by color', side=3, adj=1.0, font=1, cex=1.15, line=1)
## To see plots from 'average' and 'complete' linkage methods, do:
# plot(hclust(dist(datB), method="average"))
# plot(hclust(dist(datC), method="complete"))

```

```

## Dendrograms from data where blobs were pulled vertically
## Follow each use of `hclust()` with a `plot()` command
sclusres_sing <- hclust(dist(datB), method="single")
plot(sclusres_sing, sub="", xlab="", ylab="Distance Joined", main="")
title(main='A: Single linkage, blobs pulled vertically (4blobsB)',
      adj=0, font.main=1)
sclusres_sing_s <- hclust(dist(scale(datA)), method="single")
plot(sclusres_sing_s, sub="", xlab="", ylab="Distance Joined", main="")
title(main='B: Single linkage, (4blobsB, rescaled to variance 1)',
      adj=0, font.main=1)
# sclusres_avg_s <- hclust(dist(scale(datB)), method="average")
# #plot(sclusres_avg_s, sub="", xlab="", ylab="")
# sclusres_comp_s <- hclust(dist(scale(datB)), method="complete")
# #plot(sclusres_comp_s, sub="", xlab="", ylab="")

```

```

## Code. Follow each use of `hclust()` with a `plot()` command
clusres_sing2 <- hclust(dist(datC), method="single")
plot(clusres_sing2, sub="", xlab="", ylab="", cex=1.25, cex.main=1.65,
     main="A: Single linkage, closer clusters (4blobsC)", adj=0, font.main=1)
clusres_avg2 <- hclust(dist(datC), method="average")
plot(clusres_avg2, sub="", xlab="", ylab="", cex=1.25, cex.main=1.65,
     main="B: Average linkage, closer clusters (4blobsC)", adj=0, font.main=1)
clusres_comp2 <- hclust(dist(datC), method="complete")
plot(clusres_comp2, sub="", xlab="", ylab="", cex=1.25, cex.main=1.65,
     main="C: Complete linkage, closer clusters (4blobsC)", adj=0, font.main=1)

```

### Subsection 9.3.2: *k*-Means Clustering

```

set.seed(35151)
kdat <- makeClust(n=100, d1=5, d2=5, sigs=c(.5, .5, 6, 6))

```

```
plot(kdat, xlab="X1", ylab="X2", fg="gray")
kmres <- kmeans(kdat, 4, nstart=30)
plot(kdat, col=rainbow(4)[kmres$cluster], pch=kmres$cluster+1,
     xlab="X1", ylab="X2", fg="gray")
```

## Comments on $k$ -means and hierarchical clustering

### Subsection 9.3.3: Mixture model-based clustering

```
## Code
plotMix2 <- function(taus=c(.5, .5), means=c(10,15), sds=c(3,1), xlims=c(0,20)){
  curve(taus[1]*dnorm(x, mean=means[1], sd=sds[1]) +
        taus[2]*dnorm(x, mean=means[2], sd=sds[2]),
        from=xlims[1], to=xlims[2], ylab="Density", fg="gray")
  curve(taus[1]*dnorm(x, mean=means[1], sd=sds[1]),
        from=xlims[1], to=xlims[2], col="red", lty=2, add=TRUE, fg="gray")
  curve(taus[2]*dnorm(x, mean=means[2], sd=sds[2]),
        from=xlims[1], to=xlims[2], col="blue", lty=3, add=TRUE, fg="gray")
}
plotMix2(taus=c(.2, .8))
plotMix2(taus=c(.5, .5))
plotMix2(taus=c(.9, .1))
```

```
library(teigen)
possm1 <- na.omit(DAAG::possum[,c(3,9:11)])
set.seed(513451)
gaus_fit <- teigen(possm1[,2:4], models="UUUU", gauss=TRUE, verbose=FALSE,
                  scale=FALSE)
```

```
## BIC values are plotted against number of groups
gaus_fit$allbic
plot(gaus_fit$allbic, type="b", ylab="", xlab="Number of Groups", fg="gray")
mtext(side=2, line=3.5, "BIC", las=0)
axis(1, at=1:9, fg="gray")
```

```
table(possm1$Pop, gaus_fit$classification)
```

```

par(fig=c(0, 0.5, 0.5, 1))
plot(gaus_fit, what="contour", xmargin=1, ymargin=2, draw.legend=FALSE, fg="gray")
## See ?teigen::plot.teigen for details of the plot command used here.
par(fig=c(0, 0.5, 0, 0.5), new=TRUE)
plot(gaus_fit, what="contour", xmargin=1, ymargin=3, draw.legend=FALSE, fg="gray")
par(fig=c(0.5, 1, 0, 0.5), new=TRUE)
plot(gaus_fit, what="contour", xmargin=2, ymargin=3, draw.legend=FALSE, fg="gray")
par(fig=c(0,1,0,1))

```

## Issues of high parametrization and scaling

### Subsection 9.3.4: Relationship between $k$ -means and mixture models

## Section 9.4: Discriminant analysis

### Subsection 9.4.1: Example – plant architecture

```

leafshape17 <- DAAG::leafshape17
plot(bladelen ~ bladewid, data=leafshape17, pch=c(1,3)[arch+1])
## For panel B, specify log="xy" in the call to plot()

```

## Logistic regression, versus linear discriminant analysis

### Subsection 9.4.2: Logistic regression

```

## Fit logistic regression model
leafshape17 <- DAAG::leafshape17
leaf17.glm <- glm(arch ~ logwid + loglen, family=binomial(link=logit),
data=leafshape17)
print(DAAG::sumry(leaf17.glm)$coef, digits=2)

```

## Predictive accuracy



```
set.seed(29)
leaf17.cv <- DAAG::CVbinary(leaf17.glm)
tCV <- table(DAAG::leafshape17[["arch"]], round(leaf17.cv$cvhat))
rownames(tCV) <- colnames(tCV) <- c("0=Plagiotropic", "1=Orthotropic")
cbind(tCV, "Proportion correct"=c(tCV[1,1], tCV[2,2])/(tCV[,1]+tCV[,2]))
```

```
round(unlist(leaf17.cv[c("acc.training", "acc.cv")]), 3)
```

### Subsection 9.4.3: Linear discriminant analysis

```
suppressPackageStartupMessages(library(MASS))
## Discriminant analysis; data frame leafshape17 (DAAG)
leaf17.lda <- lda(arch ~ logwid+loglen, data=DAAG::leafshape17)
print(leaf17.lda)
```

### Assessments of predictive accuracy

```
set.seed(29)
leaf17cv.lda <- lda(arch ~ logwid+loglen, data=leafshape17, CV=TRUE)
## the list element 'class' gives the predicted class
## The list element 'posterior' holds posterior probabilities
tab <- table(leafshape17$arch, leaf17cv.lda$class)
rownames(tab) <- colnames(tab) <- c("0=Plagiotropic", "1=Orthotropic")
cbind(tab, "Proportion correct"=c(tCV[1,1], tCV[2,2])/(tCV[,1]+tCV[,2]))
cbind(tab, c(tab[1,1], class.acc=tab[2,2])/(tab[,1]+tab[,2]))
cat("Overall proportion correct =", sum(tab[row(tab)==col(tab)]/sum(tab), "\n")
```

### The function qda(), and other alternatives to lda()

### Subsection 9.4.4: An example with more than two groups

```
## Linear discriminant calculations for possum data
possum <- DAAG::possum
possum.lda <- lda(site ~ hdlngth + skullw + totlngth + taill + footlngth +
                  earconch + eye + chest + belly, data=na.omit(possum))
# na.omit() omits any rows that have one or more missing values
```

```
plot(possum.lda, dimen=3, col=1:9)
# Scatterplot matrix - scores on 1st 3 canonical variates
# See `?plot.lda` for details of the generic lda plot function

## Linear discriminant calculations for possum data
print(possum.lda, digits=3)
```

## Section 9.5: \*High-dimensional data — RNA-Seq gene expression

### Setup for installing and using Bioconductor packages

```
## For latest details, see: https://www.bioconductor.org/install/
if (!require("BiocManager", quietly = TRUE))
  install.packages("BiocManager")
BiocManager::install()
BiocManager::install('limma','multtest')
```

### \*Brief note on mRNA technical issues

#### Subsection 9.5.1: Data and design matrix setup

```
counts <- DAAGbio::plantStressCounts
colSums(counts)
```

```
## Require at least 3 counts per million that are > 1
keep <- rowSums(counts)>=3
counts <- counts[keep,]
```

```
treatment <- factor(rep(c("CTL", "L", "D"), rep(3,3)))
design <- model.matrix(~0+treatment)
colnames(design) <- levels(treatment)
```

### A two-dimensional representation

```
library(limma)
v <- voom(counts, design, plot=TRUE)
```

```

par(oma=c(0,0,1,0))
library(limma)
v <- voom(counts, design, plot=TRUE)
firstchar <- substring(colnames(counts),1,1)
plotMDS(counts, labels=paste0(firstchar, rep(1:3,3)), cex=0.8)
box(col="gray")
mtext(side=3, line=0.4, adj=0, "MDS summary plot")
mtext(side=3, line=-0.25, adj=0.105, "A", outer=TRUE)
mtext(side=3, line=-0.25, adj=0.605, "B", outer=TRUE)

```

## Fitting the model

```
fit <- lmFit(v, design)
```

```

contrs <- c("D-CTL", "L-CTL", "L-D")
contr.matrix <- makeContrasts(contrasts=contrs,
levels=levels(treatment))
fit2 <- contrasts.fit(fit, contr.matrix)
efit2 <- eBayes(fit2)

```

## Subsection 9.5.2: From $p$ -values to false discovery rate (FDR)

```

## First contrast only; Drought-CTL
print(round(topTable(efit2, coef=1, number=4),15), digits=3)

```

```
round(sort(p.adjust(p=efit2$p.value[,1], method="BH"))[1:4], 15) # Not run
```

```
round(topTable(efit2, number=4), 15)
```

```
head(decideTests(fit2),5)
```

```

summary(decideTests(fit2))
## Try also
## summary(decideTests(fit2, p.value=0.001))

```

## Section 9.6: High dimensional data from expression arrays

### Subsection 9.6.1: Molecular classification of cancer — an older technology

#### Breakdown of ALL B-type data, with one observation excluded

```
library(hddplot)
data(golubInfo)
with(golubInfo, table(cancer, tissue.mf))

## Identify all B samples that are BM:f or BM:m or PB:m
subsetB <- with(golubInfo,
cancer=="allB" & tissue.mf%in%c("BM:f","BM:m","PB:m"))
## Separate off the relevant columns of the matrix Golub
## NB: variables (rows) by cases (columns)
GolubB <- with(golubInfo, Golub[, subsetB])
## Form vector that identifies these as BM:f or BM:m or PB:m
tissue.mfB <- with(golubInfo, tissue.mf[subsetB, drop=TRUE])
## Change the level names to leave out the colons
levels(tissue.mfB) <- list("b_f"="BM:f", "b_m"="BM:m", "PBm"="PB:m")
```

### Subsection 9.6.2: Classifications and associated graphs

#### Preliminary data manipulation

```
## Display distributions for the first 20 observations
boxplot(data.frame(GolubB[, 1:20])) # First 20 columns (observations)
## Random selection of 20 rows (features)
boxplot(data.frame(GolubB[sample(1:7129, 20), ]))
```

#### Flawed graphs

```
colr <- c("red","blue","gray40", "magenta")
tissue.mf <- golubInfo[, "tissue.mf"]
cancer <- golubInfo[, "cancer"]
G.PBf <- Golub[, tissue.mf=="PB:f" & cancer=="allB", drop=FALSE]
set.seed(41)
rGolubB <- matrix(rnorm(prod(dim(GolubB))), nrow=dim(GolubB)[1])
rownames(rGolubB) <- rownames(Golub)
rG.PBf <- matrix(rnorm(prod(dim(G.PBf))), nrow=dim(G.PBf)[1])
```

```

plot2 <- function(x = GolubB, cl=tissue.mfB, x.omit=Golub.PBf, cl.omit="PBf",
                 ncol = length(cl), nfeatures=12, device = "", seed = 37,
                 pretext="", colr=1:3, levnames = NULL,
                 ylab="2nd discriminant function"){
  cl <- factor(cl)
  if(!is.null(levnames))levels(cl) <- levnames
  ord15 <- orderFeatures(x, cl=cl)[1:nfeatures]
  dfB <- t(x[ord15, ])
  dfB.lda <- lda(dfB, grouping=cl)
  scores <- predict(dfB.lda, dimen=2)$x
  df.PBf <- data.frame(t(x.omit[ord15, drop=FALSE]))
  colnames(df.PBf) <- colnames(dfB)
  scores.other <- predict(dfB.lda, newdata=df.PBf)$x
  scoreplot(list(scores=scores, cl=cl, other=scores.other, cl.other=cl.omit,
                 fg="gray", params=list(other=list(pch=4, cex=1.5)),
                 xlab="1st discriminant function", ylab=ylab)
  }
plot2(x = GolubB, cl = tissue.mfB, x.omit=G.PBf, cl.omit="PBf",
      nfeatures=15, device = "", seed = 37, ylab="2nd discriminant function",
      colr=colr, pretext="A: ALL B-cell:")
plot2(x = rGolubB, cl = tissue.mfB, x.omit=rG.PBf, cl.omit="Gp 4",
      device = "", seed = 37, colr=colr, levnames = c("Gp 1", "Gp 2", "Gp 3"),
      pretext="B: Random data:", ylab="")

```

```

## Uses orderFeatures() (hddplot); see below
ord15 <- orderFeatures(GolubB, cl=tissue.mfB)[1:15]

```

```

## Panel A: Take 1st 15 features & transpose to observations by features
dfB15 <- data.frame(t(GolubB[ord15, ]))
dfB15.lda <- MASS::lda(dfB15, grouping=tissue.mfB)
scores <- predict(dfB15.lda, dimen=2)$x
## Scores for the single PB:f observation
chooseCols <- with(golubInfo, tissue.mf=="PB:f"& cancer=="allB")
df.PBf <- data.frame(t(GolubB[ord15, chooseCols, drop=FALSE]))
scores.PBf <- predict(dfB15.lda, newdata=df.PBf, dimen=2)$x
## Warning! The plot that now follows may be misleading!
## Use hddplot::scoreplot()
scoreplot(list(scores=scores, cl=tissue.mfB, other=scores.PBf, cl.other="PB:f"),
           fg="gray")

```

```
## Panel B: Repeat plot, now with random normal data
simscores <- simulateScores(nrow=7129, cl=rep(1:3, c(19,10,2)),
cl.other=4, nfeatures=15, seed=41)
# Returns list elements: scores, cl, scores.other & cl.other
scoreplot(simscores)
```

### Subsection 9.6.3: The mean-variance relationship

```
par(oma=c(0,0,1,0))
designG <- model.matrix(~0+tissue.mfB)
colnames(designG) <- levels(tissue.mfB)
vG <- vooma(GolubB, designG, plot=TRUE) # Panel A
plotMDS(vG, pch=unclass(tissue.mfB), cex=0.8) # Panel B
leglabs <- c("BM:female", "BM:male", "PB:female")
legend(x="bottomright", bty="n", legend=leglabs, pch=1:3)
mtext(side=3, line=0.4, adj=0, "MDS summary plot")
mtext(side=3, line=-0.275, adj=0.085, "A", outer=TRUE)
mtext(side=3, line=-0.275, adj=0.585, "B", outer=TRUE)
```

### Cross-validation to determine the optimum number of features

#### Cross-validation for a range of choices of number of features

```
## Cross-validation to determine the optimum number of features
## 10-fold (x4). Warning messages are omitted.
## Accuracy measure will be: tissue.mfB.cv$acc.cv
tissue.mfB.cv <- cvdisc(GolubB, cl=tissue.mfB, nfeatures=1:23,
nfold=c(10,4), print.progress=FALSE)
## Defective measures will be in acc.resub (resubstitution)
## and acc.sel1 (select features prior to cross-validation)
tissue.mfB.badcv <- defectiveCVDisc(GolubB, cl=tissue.mfB,
foldids=tissue.mfB.cv$folds, nfeatures=1:23)
##
## Calculations for random normal data:
set.seed(43)
rGolubB <- matrix(rnorm(prod(dim(GolubB))), nrow=nrow(GolubB))
rtissue.mfB.cv <- cvdisc(rGolubB, cl=tissue.mfB, nfeatures=1:23,
nfold=c(10,4), print.progress=FALSE)
rtissue.mfB.badcv <- defectiveCVDisc(rGolubB, cl=tissue.mfB,
```

```
nfeatures=1:23,
foldids=rtissue.mfB.cv$folds)
```

### Which features?

```
genelist <- matrix(tissue.mfB.cv$genelist[1:3, ], nrow=3)
tab <- table(genelist, row(genelist))
ord <- order(tab[,1], tab[,2], tab[,3], decreasing=TRUE)
tab[ord,]
```

### Subsection 9.6.4: Graphs derived from the cross-validation process

```
## Uses tissue.mfB.acc from above
tissue.mfB.scores <-
cvscores(cvlist = tissue.mfB.cv, nfeatures = 3, cl.other = NULL)
scoreplot(scorelist = tissue.mfB.scores, cl.circle=NULL,
prefix="B-cell subset -", fg='gray')
```

### The key role of cross-validation

### Subsection 9.6.5: Estimating contrasts, and calculating False Discovery Rates

```
fitG <- lmFit(vG, designG)
contrs <- c("b_f-b_m", "b_f-PBm", "b_m-PBm")
contr.matrix <- makeContrasts(contrasts=contrs,
levels=levels(tissue.mfB))
fit2 <- contrasts.fit(fitG, contr.matrix)
fit2 <- eBayes(fit2)
```

### From $p$ -values to false discovery rate (FDR)

```
print(topTable(fit2, number=5), digits=2)
```

```
summary(decideTests(fit2))
## Try also
## summary(decideTests(fit2, p.value=0.001))
```

## Distributional extremes

### Section 9.7: Causal inference from observational data — balance and matching

#### Subsection 9.7.1: Tools for the task

```
library(DAAG)
## Columns 4:7 are factors; columns 9:10 (re75 & re78) are continuous
propmat <- matrix(0, ncol=6, nrow=8)
dimnames(propmat) <- list(c("psid1", "psid2", "psid3", "cps1", "cps2", "cps3",
"nsw-ctl", "nsw-trt"), names(nswdemo)[c(4:7, 9:10)])
for(k in 1:8){
  dframe <- switch(k, psid1, psid2, psid3, cps1, cps2, cps3,
    subset(nswdemo, trt==0), subset(nswdemo, trt==1))
  propmat[k,] <- c(sapply(dframe[,4:7], function(x){
    z <- table(x); z[2]/sum(z)}),
    sapply(dframe[,9:10], function(x)sum(x>0)/sum(!is.na(x))))
}
```

```
PGtheme <- DAAG::DAAGtheme(color=TRUE)
library(DAAG)
if(!require(grid))return("Package 'grid' is not installed -- cannot proceed")
dsetnames <- c("nsw-ctl", "nsw-trt", "psid1", "psid2", "psid3",
  "cps1", "cps2", "cps3")
colrs <- c("gray","black", PGtheme$superpose.line$col[1:3])
lty <- c(1,2,1,1,1)
lwd <- c(1,0.75,0.75,0.75,0.75)
denplot <-
  function(sel=c(1:2,6:8), yvar="re75", offset=30, ylim=c(0,1.75),
    from=NULL, at=c(.5,1,1.5), labels=paste(at), bw="nrd0",
    ylab="Density", takelog=TRUE, col.axis="black"){
    nzre <- unlist(lapply(list(subset(nswdemo, trt==0),
      subset(nswdemo, trt==1),
      psid1, psid2, psid3, cps1, cps2, cps3)[sel],
      function(x){z <- x[,yvar]; z[z>0]}))
    num <- unlist(lapply(list(subset(nswdemo, trt==0), subset(nswdemo, trt==1),
      psid1, psid2, psid3, cps1, cps2, cps3),
      function(x){z <- x[,yvar]; sum(z>0)}))
    xy <- data.frame(nzre=nzre, fac = factor(rep(dsetnames[sel], num[sel]),
      levels=dsetnames[sel]))
    if(takelog) {
```



```

y <- log(xy$nzre+offset)
xlab <- paste("log(", yvar, "+", offset, ")", sep="") } else
{
  y <- xy$nzre
  xlab <- yvar
}
densityplot(~ y, groups=fac, data=xy, bw=bw, from=from,
  scales=list(y=list(at=at, labels=labels, col=col.axis), tck=0.25),
  plot.points=FALSE, col=colrs[1:5], lwd=lwd, lty=lty,
  key=list(x=0.01, y=0.99, text=list(dsetnames[sel[3:5]]), col=colrs[3:5],
    cex=0.75, lines=list(lwd=rep(1.5,3)), between=1),
  par.settings=list(col=colrs, lty=lty, cex=0.75, lwd=lwd,
    fontsize=list(text=9, points=5)),
  fg="gray", ylim=ylim, ylab=ylab, xlab=xlab)
}
## Plot base graph; overlay with lattice graphs on same page
par(fig=c(0,1,0,1), mar=c(0,0,0,0))
plot(0:1,0:1, axes=FALSE, type="n", bty="n", xlab="", ylab="")
legend(x="top", legend=dsetnames[1:2], lty=1:2, lwd=c(1,0.75),
  col=colrs[1:2], bty="n", ncol=2, yjust=0.75)
print(denplot(), position=c(0, 0, 0.32, 0.505), newpage=FALSE)
print(denplot(1:5, ylab=" ", col.axis="white"),
  position=c(0.21, 0, .53, 0.505), newpage=FALSE)
print(denplot(ylab=" ", yvar="re78", col.axis="white"),
  position=c(0.47, 0, 0.79, 0.505), newpage=FALSE)
print(denplot(1:5, ylab=" ", yvar="re78", col.axis="white"),
  position=c(0.68, 0, 1, 0.505), newpage=FALSE)
## Age
print(denplot(yvar="age", takelog=FALSE, ylim=c(0,0.1), from=16,
  at=c(.02,.04,.06,.08), labels=c(".02",".04",".06",".08")),
  position=c(0, 0.475, 0.32, .98), newpage=FALSE)
print(denplot(1:5, yvar="age", takelog=FALSE, ylim=c(0,0.1), from=16,
  at=c(.02,.04,.06,.08), labels=c(".02",".04",".06",".08"),
  ylab=" ", col.axis="white"),
  position=c(0.21, 0.475, .53, .98), newpage=FALSE)
## educ
print(denplot(1:5, yvar="educ", takelog=FALSE, ylim=c(0,0.5), bw=0.5,
  at=c(.1,.2,.3,.4), ylab=" "),
  position=c(0.47, 0.475, .79, .98), newpage=FALSE)
print(denplot(yvar="educ", takelog=FALSE, ylim=c(0,0.75), bw=0.5,
  at=c(.1,.2,.3,.4), ylab=" ", col.axis="white"),
  position=c(0.68, 0.475, 1, .98), newpage=FALSE)

```

```

addControl <-
function(control, offset=30){
  nam <- deparse(substitute(control))
  if(nam=="nswdemo")nsw0 <- nswdemo else
    nsw0 <- rbind(control, subset(DAAG::nswdemo, trt==1))
  nsw0$z75 <- factor(nsw0$re75==0, labels=c("0", ">0"))
  nsw0$ethnicid <- factor(with(nsw0, ifelse(black==1, "black",
    ifelse(hisp==1, "hisp", "other"))), levels=c("other", "black", "hisp"))
  nsw0 <- nsw0[, -match(c("black", "hisp"), names(nsw0))]
  nsw0
}

```

```

## Create dataset that will be used for later analyses
nsw <- addControl(psid1)
nsw <- within(nsw, {re75log <- log(re75+30);
                  re78log <- log(re78+30);
                  trt <- factor(trt, labels=c("Control", "Treat"))})
## A treated values only dataset will be required below
trtdat <- subset(nsw, trt=="Treat")
trtdat$pres74 <- factor(!is.na(trtdat$re74), labels=c("<NA>", "pres"))
table(trtdat$pres74)

```

```

with(trtdat, table(pres74, z75))

```

## Subsection 9.7.2: Regression comparisons

### Regression calculations

```

nsw.gam <- gam(log(re78+30)~ trt + ethnicid + z75 + nodeg + s(age) +
              s(educ) + log(re75+30), data=nsw)

```

## Subsection 9.7.3: The use of scores to replace covariates

## Subsection 9.7.4: Two-dimensional representation using randomForest proximities

```

suppressPackageStartupMessages(library(randomForest))
form <- trt ~ age + educ + ethnicid + marr + nodeg + z75 + re75log
nsw.rf <- randomForest(form, data=nsw, sampsize=c(297, 297))

```

```
p.rf <- predict(nsw.rf,type="prob")[,2]
sc.rf <- log((p.rf+0.001)/(1-p.rf+0.001))
```

```
omitn <- match(c("PropScore","weights","subclass"), names(dat2RF), nomatch=0)
matchISO.rf <- matchit(trt ~ age + educ + ethnicid + marr + nodeg + z75 +
                      re75log, ratio=1, data=dat2RF[, -omitn], distance=isoScores[,1])
## summary(match.rf,un=F,improvement=F)
## summary(match.rf, un=F, interactions=T, improvement=F)$sum.matched[,1:4]
## In the first place, look only at the first 4 columns
```

```
dat1RF <- match.data(matchISO.rf, distance="PropScore")
dat1RF.lm <- lm(re78log ~ trt, data = dat1RF, weights = weights)
library(sandwich) # Allows use of `vcovCL()` from the `sandwich` package
lmtest::coeftest(dat1RF.lm, vcov. = vcovCL, cluster = ~subclass)
## Check for increase in number with non-zero earnings
dat1RF.glm <- glm(I(re78>0) ~ trt, data = dat1RF, weights = weights,
                 family=binomial)
lmtest::coeftest(dat1RF.glm, vcov. = vcovCL, cluster = ~subclass)
```

## Derivation and investigation of scores

```
library(mgcv)
formG <- trt ~ ethnicid + marr+ z75 + s(age) + s(educ) + s(re75log)
nsw.gam <- gam(formG, family=binomial(link="logit"), data=nsw)
pred <- predict(nsw.gam, type='response')
table(nsw$trt, round(pred))
## Alternative
library(splines) ## Fit normal cubic splines using splines::ns()
formNS <- trt ~ ethnicid + marr+ z75 + ns(age,2) +
ns(educ) + ns(re75log,3)
nsw.glm <- glm(formNS, family=binomial(link="logit"), data=nsw)
pred <- predict(nsw.glm, type='response')
table(nsw$trt, round(pred))
cbind(AIC(nsw.glm,nsw.gam), BIC(nsw.glm, nsw.gam))
```

```
## Include factor by factor and variable interactions with ethnicid
## and marr (Result not shown)
formGx <- trt ~ (ethnicid+marr+z75)^2 + s(age, by=ethnicid)+
s(educ, by=ethnicid) + s(re75log,by=ethnicid)+
s(age, by=marr)+ s(educ, by=marr) + s(re75log,by=marr)
```

```
nswx.gam <- gam(formula = formGx, data = nsw, family=binomial(link = "logit"))
predx <- predict(nswx.gam, type='response')
table(nsw$trt, round(predx))
AIC(nsw.glm,nsw.gam,nswx.gam)
```

```
library(MatchIt)
## Use data frame that omits re74. Otherwise matchit() will generate NAs
## where they occur in re74, even though re74 is not in the model formula.
nswG <- nsw[, c("trt","age","educ","ethnicid", "marr","nodeg","z75",
               "re75log","re78log","re78")]
formG <- trt ~ ethnicid + marr+ z75 + s(age) + s(educ) + s(re75log)
match.gam <- matchit(formula = formG, data = nswG, method = "nearest",
                    distance = "gam", link = "logit", reestimate=TRUE)
datG <- match.data(match.gam, distance="PropScore")
## Summary information
match.gam
## summary(match.gam,un=F,improvement=F)
## summary(match.gam, un=F, interactions=T, improvement=F)$sum.matched[,1:4]
## In the first place, look only at the first 4 columns
```

```
suppressPackageStartupMessages(library(gridExtra))
suppressPackageStartupMessages(library(ggplot2))
suppressPackageStartupMessages(library(cobalt))
gg1<- cobalt::love.plot(match.gam, position="bottom", grid=TRUE,
                       star.char="",stars='raw') +
  ggtitle("A: Differences from balance") +
  theme(plot.title = element_text(hjust=0, vjust=0.5, size=11),
        plot.margin=unit(c(9,15,0,9), 'pt'))
sub <- match(with(subset(datG, trt=="Control"),subclass),
             with(subset(datG, trt=="Treat"),subclass))
datGpaired <- cbind(subset(datG, trt=="Treat"),
                   with(subset(datG, trt=="Control")[sub,],
                         cbind("Cre78log"=re78log,"CPropScore"=PropScore)))
gg2 <- ggplot(datGpaired)+
  geom_point(aes(PropScore,I(re78log-Cre78log)), size=1)+
  geom_smooth(aes(PropScore,I(re78log-Cre78log)), method = "gam",
             formula = y ~ s(x, bs = "cs")) +
  xlab("Propensity score for treated")+
  ylab("Treatment vs control differences") +
  ggtitle("B: Treatment vs control differences") +
  theme(plot.title = element_text(hjust=0, vjust=0.5, size=11),
        plot.margin=unit(c(9,9,0,15), 'pt'))
```

```
grid.arrange(gg1, gg2, ncol=2)
```

```
library(sandwich)
datG.lm <- lm(re78log ~ trt, data = datG, weights = weights)
## With 1:1 matching, the weights argument is not really needed
## Print first two coefficients only.
lmtest::coeftest(datG.lm, vcov. = vcovCL, cluster = ~subclass)[1:2,]
## Check number whose income was greater than 0
datG.glm <- glm(I(re78>0) ~ trt, data = datG, weights = weights, family=binomial)
lmtest::coeftest(datG.glm, vcov. = vcovCL, cluster = ~subclass)[1:2,]
```

## Alternative matching approaches

### Subsection 9.7.5: Coarsened exact matching

```
form <- trt ~ age + educ + ethnicid + marr + nodeg + z75 + re75log
match5.cem <- matchit(formula=form, data=nsWG, method="cem", cutpoints=5)
datcem5 <- match.data(match5.cem)
match6.cem <- matchit(formula=form, data=nsWG, method="cem", cutpoints=6)
datcem6 <- match.data(match6.cem)
## Show the effect of adding another cutpoint
match5.cem
match6.cem
```

```
library(sandwich)
datcem5.lm <- lm(re78log ~ trt, data = datcem5, weights = weights)
## The function vcovHC() provides cluster robust standard errors
lmtest::coeftest(datcem5.lm, vcov. = vcovHC)
## Estimate treatment effect on number with some earnings:
datcem6.glm <- glm(I(re78>0) ~ trt, data = datcem6, weights = weights,
family=binomial)
lmtest::coeftest(datcem6.glm, vcov. = vcovHC)
```

## Section 9.8: Multiple imputation

```
suppressPackageStartupMessages(library(mice))
Boys <- with(subset(mice::boys, age>=9),
             data.frame(age=age, loghgt=log(hgt), logbmi=log(bmi), loghc=log(hc)))
(Pattern <- md.pattern(Boys, plot=F))
```

```
set.seed(31) # Set to reproduce result shown
PatternB <- rbind(Pattern[-c(1,nrow(Pattern)), -ncol(Pattern)],
                 c(0,1,1,1), c(0,1,0,0), c(0,0,1,0))
boys <- rbind(ic(Boys),
             ampute(cc(Boys), pattern=PatternB, freq=c(.3,.15,.15,.2,.1,.1),
                   prop=0.75)$amp)
md.pattern(boys, plot=FALSE)
```

```
set.seed(17) # Set to reproduce result shown
out <- capture.output( # Evaluate; send screen output to text string
  boys.mids <- mice(boys, method='pmm', m=8) )
impDFs <- complete(boys.mids, action='all') # Returns a list of m=8 dataframes
## Average over imputed dataframes (use for exploratory purposes only)
impArray <- sapply(impDFs, function(x)as.matrix(x), simplify='array')
boysAv <- as.data.frame(apply(impArray, 1:2, mean))
```

```
fits <- with(boys.mids, lm(logbmi~age+loghgt))
pool.coef <- summary(pool(fits)) # Include in table below
```

```
## 2) Regression that leaves out rows with NAs
omitNArows.coef <- coef(summary(lm(logbmi~age+loghgt, data=boys)))
## 3) Regression fit to average over data frames after imputation
boysAv.coef <- coef(summary(lm(logbmi~age+loghgt, data=boysAv)))
## 4) Fit to original data, with 36 rows had missing data
Orig.coef <- coef(summary(lm(logbmi ~ age+loghgt, data=Boys)))
```

```
ctab <- cbind(summary(pool(fits))[2:3], omitNArows.coef[,1:2], boysAv.coef[,1:2],
              Orig.coef[,1:2])
tab <- setNames(cbind(ctab[,c(1,3,5,7)], ctab[,c(2,4,6,8)]),
               paste0(rep(c('Est','SE'), c(4,4)), rep(1:4, 2)))
round(tab,3)
```

## Time series cross-sectional data – an example

```

airquality <- datasets::airquality
airq <- cbind(airquality[, 1:4], day=1:nrow(airquality))
# 'day' (starting May 1) replaces columns 'Month' & 'Day'
## Replace `Ozone` with `rt4ozone`:
airq <- cbind(rt4ozone=airq$Ozone^0.25, airq[,-1])

## Generate the scatterplot matrix, now with `rt4ozone` replacing `Ozone`
smoothPars <- list(col.smooth='red', lty.smooth=2, spread=0)
car::spm(airq, cex.labels=1.2, regLine=FALSE, col='blue',
         oma=c(1.95,3,4, 3), gap=.25, smooth=smoothPars)

airq.imp <- mice(airq, m=20, print=FALSE)
## 20 imputations shows up issues of concern very clearly

## Code for figure
out <- micemd::overimpute(airq.imp)

```

Some further points

## Section 9.9: Further reading

Data with more variables than observations

Causal inference

Multiple imputation

## Section 9.10: Exercises

9.3

```

library(DAAG)
oz::oz(sections=c(3:5, 11:16))
names(possumsites)[1:2] <- c("long", "lat")
with(possumsites, {
  points(long, lat);
  text(long, lat, row.names(possumsites), pos=c(2,4,2,2,4,2,2))
})

```

9.7

```
data(wine, package='gclus')
mat <- with(wine,
  round(1-cor(cbind(Alcohol, Malic, Magnesium, Phenols, Flavanoids)),2))
colnames(mat) <- rownames(mat) <- 1:5
print(mat)
```

9.9a

```
`confusion` <-
function(actual, predicted, digits=4){
  tab <- table(actual, predicted)
  confuse <- apply(tab, 1, function(x)x/sum(x))
  print(round(confuse, digits))
  acc <- sum(tab[row(tab)==col(tab)])/sum(tab)
  invisible(print(c("Overall accuracy" = round(acc,digits))))
}
data(Vehicle, package="mlbench")
lhat <- MASS::lda(Class ~ ., data=Vehicle, CV=TRUE)$class
qhat <- MASS::qda(Class ~ ., data=Vehicle, CV=TRUE)$class
DAAG::confusion(Vehicle$Class, lhat)
DAAG::confusion(Vehicle$Class, qhat)
randomForest::randomForest(Class ~ ., data=Vehicle, CV=TRUE)
```

9.9c

```
Vehicle.lda <- MASS::lda(Class ~ ., data=Vehicle)
twoD <- predict(Vehicle.lda)$x
ggplot2::quickplot(twoD[,1], twoD[,2], color=Vehicle$Class,
  geom=c("point","density2d"))
```

9.10

```
library(ape); library(MASS)
library(DAAGbio)
primates.dna <- as.DNAbin(primatesDNA)
primates.dist <- dist.dna(primates.dna, model="K80")
primates.cmd <- cmdscale(primates.dist)
eqscplot(primates.cmd)
rtleft <- c(4,2,4,2)[unclass(cut(primates.cmd[,1], breaks=4))]
text(primates.cmd, labels=row.names(primates.cmd), pos=rtleft)
```



```
d <- dist(primates.cmd)
sum((d-primates.dist)^2)/sum(primates.dist^2)
```

9.11

```
library(DAAG)
pacific.dist <- dist(x = as.matrix(rockArt[-c(47,54,60,63,92),28:641]),
                     method = "binary")
sum(pacific.dist==1)/length(pacific.dist)
plot(density(pacific.dist, to = 1))
## Check that all columns have at least one distance < 1
symmat <- as.matrix(pacific.dist)
table(apply(symmat, 2, function(x) sum(x<1)))
pacific.cmd <- cmdscale(pacific.dist)
pacific.sam <- sammon(pacific.dist)
```

9.15

```
Wine <- setNames(cbind(stack(wine, select=2:14), rep(wine[,-1], 13)),
                c("value", "measure", "Class"))
bwplot(measure ~ value, data=Wine)
```

```
wine.pr <- prcomp(wine[,-1], scale=TRUE)
round(wine.pr$sdev,2)
t(round(wine.pr$rotation[,1:2],2))
scores <- as.data.frame(cbind(predict(wine.pr), Class=wine[,1]))
xyplot(PC2 ~ PC1, groups=Class, data=scores, aspect='iso',
       par.settings=simpleTheme(pch=16), auto.key=list(columns=3))
```

```
library(MASS)
wine.lda <- lda(Class ~ ., data=wine)
wineCV.lda <- lda(Class ~ ., data=wine, CV=T)
t(round(wine.lda$scaling,2))
tab <- table(wine$Class, wineCV.lda$class,
             dnn=c('Actual', 'Predicted'))
tab
setNames(round(1-sum(diag(tab))/sum(tab),4), "CV error rate")
scores <- as.data.frame(cbind(predict(wine.lda)$x, Class=wine[,1]))
xyplot(LD2 ~ LD1, groups=Class, data=scores, aspect='iso',
       par.settings=simpleTheme(pch=16), auto.key=list(columns=3))
```

```
wine$Class <- factor(Wine$Class)
wine.rf <- randomForest(x=wine[,-1], y=wine$Class)

if(file.exists("/Users/johnm1/pkgs/PGRcode/inst/doc/")){
code <- knitr::knit_code$get()
txt <- paste0("\n## ", names(code), "\n", sapply(code, paste, collapse='\n'))
writeLines(txt, con="/Users/johnm1/pkgs/PGRcode/inst/doc/ch9.R")
}
```

# 10 Appendix A: The R System – A Brief Overview

On options for working with the code see the vignettes [Ch1-Learning](#) and [UsingCode](#).

\* Packages required (plus any dependencies)

DAAG dplyr tidyr tibble MASS gplots plotrix latticeExtra RColorBrewer

Additionally, knitr and Hmisc are required in order to process the qmd source file.

## The R System – A Brief Overview

### Section 10.1: Getting started with R

#### Subsection 10.1.1: Learn by typing code at the command line

```
> ## Arithmetic calculations. See the help page `?Arithmetic` {-}  
> 2*3+10          # The symbol `*` denotes 'multiply'
```

```
> ## Use the `c()` function to join results into a numeric vector  
> c(sqrt(10), 2*3+10, sqrt(10), 2^3) # 2^3 is 2 to the power of 3  
> ## R knows about pi  
> 2*pi*6378       # Approximate circumference of earth at equator (km)
```

```
?help           # Get information on the use of `help()`  
?sqrt           # Or, type help(sqrt)  
?Arithmetic     # See, in similar vein ?Syntax  
?'<'           # `?Comparison` finds the same help page
```

```
## Two commands on one line; Use ';' as separator  
2*3*4+10; sqrt(10)      ## Try also `cat(2*3*4+10, sqrt(10), sep='n')`  
## Convert CO2 carbon emissions from tonnes of carbon to tonnes of CO2  
3.664*c(.53, 2.56, 9.62) ## Data are for 1900, 1960 & 2020
```

```
## Use `cat()` to print several items, with control of formatting
cat(2*3*4+10, sqrt(10), '\n')
```

## Assignment

```
## Convert from amounts of carbon to amounts of CO2 (billions of tonnes)
## and assign result to a named object
fossilCO2vals <- c(.53, 2.56, 9.62)*3.664 # Amounts in 1900, 1960, and 2020
# Equivalently `fossilCO2vals <- c(.53, 2.56, 9.62)*rep(3.664,3)`
## To assign and print, enclose in round brackets
(fossilCO2vals <- c(.53, 2.56, 9.62)*3.664)
```

```
3.664*c(.53,2.56, 9.62) -> fossilCO2vals
```

## Entry of data at the command line, a graphics formula, and a graph

```
Year <- c(1900, 1920, 1940, 1960, 1980, 2000, 2020)
CO2 <- c(.53,.96,1.32,2.56,5.32,6.95,9.62)*3.664
## Now plot Carbon Dioxide emissions as a function of Year
plot(CO2 ~ Year, pch=16, fg="gray")
```

## Grouping vectors together into data frames

```
CO2byYear <- data.frame(year=Year, co2gas=CO2)
CO2byYear      # Display the contents of the data frame.
rm(Year, CO2)  # Optionally, remove `Year` and `Carbon` from the workspace
plot(co2gas ~ year, data=CO2byYear, pch=16)
```

```
sqrt(10)      # Number of digits is determined by current setting
options(digits=2) # Change until further notice,
sqrt(10)
```

## Wide-ranging information access and searches

## Section 10.2: R data structures

### Subsection 10.2.1: Vectors, dates, and arrays

```
vehicles <- c("Compact", "Large", "Midsize", "Small", "Sporty", "Van")
c(T, F, F, F, T, T, F)  # A logical vector, assuming F=FALSE and T=TRUE
```

```
## Character vector
mammals <- c("Rat", "Pig", "Rat", "Mouse", "Pig")
## Logical vector
rodent <- c("TRUE", "FALSE", "TRUE", "FALSE", "TRUE", "FALSE")
## From character vector `mammals`, create factor
mfac <- factor(mammals)
levels(mfac)
table(mfac)
```

#### Dates

```
day1 <- as.Date(c("2022-01-01", "2022-02-01", "2022-03-01"))
as.numeric(day1)  # Days since 1 January 1970
day1[3] - day1[2]
```

#### The use of square brackets to extract subsets of vectors

```
## Specify the indices of the elements that are to be extracted
x <- c(3, 11, 8, 15, 12, 18)
x[c(1, 4:6)]  # Elements in positions 1, 4, 5, and 6
## Use negative indices to identify elements for omission
x[-c(2, 3)]  # Positive and negative indices cannot be mixed
## Specify a vector of logical values.
x > 10  # This generates a vector of logical values
x[x > 10]
```

```
bodywt <- c(Cow=465, Goat=28, Donkey=187, Pig=192)
bodywt[c("Goat", "Pig")]
```

#### Matrices and arrays

```
arr123 <- array(1:24, dim=c(2,4,3))
## This prints as three 2 by 4 matrices. Print just the first of the three.
arr123[, 2, 1]      # Column 2 and index 1 of 3rd dimension
attributes(arr123)
```

### Subsection 10.2.2: Factors

```
gender <- c(rep("male",691), rep("female",692))
gender <- factor(gender) # From character vector, create factor
levels(gender)           # Notice that `female` comes first
```

```
Gender <- factor(gender, levels=c("male", "female"))
```

```
mf1 <- factor(rep(c('male','female'),c(2,3)), labels=c("f", "m"))
## The following has the same result
mf2 <- factor(rep(c('male','female'), c(2,3)))
levels(mf2) <- c("f","m") # Assign new levels
if(all(mf1==mf2))print(mf1)
```

```
sum(gender=="male")
```

```
table(chickwts$feed) # feed is a factor
source <- chickwts$feed
levels(source) <- c("milk","plant","plant","meat","plant","plant")
table(source)
```

### Ordered factors

```
stress <- rep(c("low","medium","high"), 2)
ord.stress <- ordered(stress, levels=c("low", "medium", "high"))
ord.stress
ord.stress >= "medium"
```

### Missing values in values of factors

### Subsection 10.2.3: Operations with data frames

```
Cars93sum <- DAAG::Cars93.summary # Create copy in workspace
Cars93sum
```

```
Cars93sum[4:6, 2:3] # Extract rows 4 to 6 and columns 2 and 3
Cars93sum[6:4, ]    # Extract rows in the order 6, 5, 4
Cars93sum[, 2:3]    # Extract columns 2 and 3
## Or, use negative integers to specify rows and/or columns to be omitted
Cars93sum[-(1:3), -c(1,4)] # In each case, numbers must be all +ve or all -ve
## Specify row and/or column names
Cars93sum[c("Small", "Sporty", "Van"), c("Max.passengers", "No.of.cars")]
```

### Data frames vs matrices

```
names(Cars93sum)[3] <- "numCars"
names(Cars9sum) <- c("minPass", "maxPass", "numCars", "code")
```

### Using a data frame as a database – with() and within()

```
## trees (datasets) has data on Black Cherry Trees
with(trees, round(c(mean(Girth), median(Girth), sd(Girth)), 1))
```

```
with(DAAG::pair65, # stretch of rubber bands
     {lenchange = heated-ambient
       c(mean(lenchange), median(lenchange))
     })
```

```
## Add variables `mph` and `gradient` to `DAAG::nihills`
nihr <- within(DAAG::nihills, {mph <- dist/time; gradient <- climb/dist})
```

### Extracting rows from data frames

```
unlist(Cars93sum[1, ])
```

### Subsection 10.2.4: Data manipulation functions used in earlier chapters

```
## For columns of `DAAG::jobs`, show the range of values
sapply(DAAG::jobs, range)
## Split egg lengths by species, calculate mean, sd, and number for each
with(DAAG::cuckoos, sapply(split(length,species),
                             function(x)c(av=mean(x), sd=sd(x), nobs=length(x))))
```

```
apply(UCBAdmissions, 3, function(x)(x[1,2]/(x[1,2]+x[2,2]))*100) # Females
apply(UCBAdmissions, 3, function(x)(x[1,1]/(x[1,1]+x[2,1]))*100) # Males
```

```
UCBAdmissions[, , 1]
```

```
DAAG::cricketer |> dplyr::count(year, left, name="Freq") -> handYear
names(handYear)[2] <- "hand"
byYear <- tidyr::pivot_wider(handYear, names_from='hand', values_from="Freq")
```

### Subsection 10.2.5: Writing data to a file, and reading data from a file

```
C02byYear <- data.frame(year = seq(from=1900, to=2020, by=20),
                        co2gas = c(1.94, 3.52, 4.84, 9.38, 19.49, 25.46, 35.25))
write.table(C02byYear, file='gas.txt') # Write data frame to file
C02byYear <- read.table(file="gas.txt") # Read data back in
write.csv(C02byYear, file='gas.csv') # Write data frame
C02byYear <- read.csv(file="gas.csv", row.names=1) # Read data back in
```

### Data input from the RStudio menu — data frames vs tibbles

#### Subsection 10.2.6: Issues for working with data frames and tibbles

##### Extraction of columns from data frames and tibbles

```
sites <- DAAG::possumsites # sites is then a data frame
sites[,3] # returns a vector
sites[,3, drop=FALSE] # returns a 1-column data frame
```

```
dplyr::as_tibble(sites)[,3] # returns a 1-column tibble
dplyr::as_tibble(sites)[[3]] # returns a vector
sites[[3]] # returns a vector
```



## Conversion between data frames and tibbles

```
attributes(DAAG::possumsites)[['row.names']]
```

```
possumSites <- tibble::as_tibble(DAAG::possumsites, rownames="Site")
possumSites
```

### Subsection 10.2.7: Lists

```
## Summary statistics for 31 felled black cherry tree
## Median (middle value), range, number, units
htstats <- list(med=76, range=c(low=63,high=87), n=31, units="ft")
htstats[1:2]      # Show first two list elements only
```

```
## The following are alternative ways to extract the second list element
htstats[2]        # First list element (Can replace `2` by 'range')
htstats[2][1]     # A subset of a list is a list
```

```
htstats[[2]]; htstats$range; htstats[["range"]]
```

```
unlist(htstats[2]) # Contents of second list element, with composite names
unlist(htstats[2], use.names=F) # Elements have no names
```

```
tstats <- with(MASS::shoes, t.test(B, A, paired=TRUE))
names(tstats)      ## Names of list elements. See `?t.test` for details.
tstats[1]          ## Type tstats[1] to see the first list element
## Compact listing of contents list elements 1 to 5, which are all numeric
unlist(tstats[1:5]) ## With `unlist(tstats)` all elements become character
```

## Section 10.3: Functions and operators

### Subsection 10.3.1: Common useful built-in functions

```
## Data indices
length()          # number of elements in a vector or a list
order()           # x[order(x)] sorts x (by default, NAs are last)
which()           # which indices of a logical vector are `TRUE`
which.max()       # locates (first) maximum (NB, also: `which.min()`)
```

```
## Data manipulation
c()          # join together (`concatenate`) elements or vectors or lists
diff()       # vector of first differences
sort()       # sort elements into order, by default omitting NAs
rev()        # reverse the order of vector elements
t()          # transpose matrix or data frame
             # (a data frame is first coerced to a matrix with())
with()       # do computation using columns of specified data frame
```

```
## Data summary
mean()       # mean of the elements of a vector
median()     # median of the elements of a vector
range()      # minimum and maximum value elements of vector
unique()     # form the vector of distinct values
## List function arguments
args()       # information on the arguments to a function
```

```
## Obtain details
head()       # display first few rows (by default 6) of object
ls()         # list names of objects in the workspace
```

```
## Print multiple objects
cat()        # prints multiple objects, one after the other
```

```
## Functions that return TRUE or FALSE?
all()        # returns TRUE if all values are TRUE
any()        # returns TRUE if any values are TRUE
is.factor()  # returns TRUE if the argument is a factor
is.na()      # returns TRUE if the argument is an NA
             # NB also is.logical(), etc.
```

```
seq(from =1, by=2, length.out=3) # Unabbreviated arguments
seq(from =1, by=2, length=3)     # Abbreviate `length.out` to `length`
```

### Subsection 10.3.2: User-written functions

```
distance <- c(148,182,173,166,109,141,166)
mean.and.sd(distance)
```

```
## Execute the function with the default argument:  
mean.and.sd()
```

```
## Thus, to return the mean, SD and name of the input vector  
## replace c(mean=av, SD=sdev) by  
list(mean=av, SD=sdev, dataset = deparse(substitute(x)))
```

### Subsection 10.3.3: Generic functions, and the class of an object

### Subsection 10.3.4: Pipes — a “Do this, then this, . . .” syntax

```
mean(rnorm(20, sd=2))  
20 |> rnorm(sd=2) |> mean()
```

```
logmammals <- MASS::mammals |> log() |> setNames(c("logbody","logbrain"))  
## Alternatively, use the ability to reverse the assignment operator.  
MASS::mammals |> log() |> setNames(c("logbody","logbrain")) -> logmammals  
## This last is more in the spirit of pipes.
```

```
MASS::mammals |>  
  log() |>  
  setNames(c("logbody","logbrain")) |>  
  (\(d)lm(logbrain ~ logbody, data=d))() |>  
  coef()
```

### Subsection 10.3.5: Operators

```
## Multiple of divisor that leaves smallest non-negative remainder  
c("Multiple of divisor" = 24 %/% 9, "Remainder after division" = 6)
```

## Section 10.4: Calculations with matrices, arrays, lists, and data frames

### Calculations in parallel across all elements of a vector

```
x <- 1:6
log(x)           # Natural logarithm of 1, 2, ... 6
log(x, base=10)  # Common logarithm (base 10)
log(64, base=c(2,10)) # Apply different bases to one number
log(matrix(1:6, nrow=2), base=2) # Take logarithms of all matrix elements
```

## Patterned data

```
seq(from=5, to=22, by=3) # The first value is 5.
rep(c(2,3,5), 4)         # Repeat the sequence (2, 3, 5) four times over
rep(c("female", "male"), c(2,3)) # Use syntax with a character vector
```

## Subsection 10.4.1: Missing values

```
nbranch <- subset(DAAG::rainforest, species=="Acacia mabellae")$branch
nbranch           # Number of small branches (2cm or less)
```

```
mean(nbranch, na.rm=TRUE)
```

```
nbranch == NA      # This always equals `NA`
```

```
is.na(nbranch)     # Use to check for NAs
```

```
nbranch[is.na(nbranch)] <- -999
# `mean(nbranch)` will then be a nonsense value
```

## NAs in modeling functions

```
options()$na.action # Version 3.2.2, following startup
```

## Counting and identifying NAs – the use of table()

```
with(DAAG::nswdemo, table(trt, re74>0, useNA="ifany"))
```

## Infinities and NaNs

```
summary(DAAG::primates)
```

```
primates <- DAAG::primates
```

```
gplots::plotCI()    # `plotCI()` function in package `gplots`  
plotrix::plotCI()   # `plotCI()` function in package `plotrix`
```

```
sessionInfo()[['basePkgs']]
```

```
## List just the workspace and the first eight packages on the search list:  
search()[1:9]
```

```
data(package="datasets")
```

## Section 10.5: Brief notes on R graphics packages and functions

### Subsection 10.5.1: Lattice graphics — a step beyond base graphics

```
grog <- DAAG::grog  
chr <- with(grog, match(Country, c('Australia', 'NewZealand')))  
# Australia: 1; matches 1st element of c('Australia', 'NewZealand')  
# NewZealand: 2; matches 2nd element  
plot(Beer ~ Year, data=grog, ylim=c(0, max(Beer)*1.1), pch = chr)  
with(grog, points(Wine ~ Year, pch=chr, col='red'))  
legend("bottomright", legend=c("Australia", "New Zealand"), pch=1:2)  
title(main="Beer consumption (l, pure alcohol)", line=1)
```

```
library(latticeExtra)    ## Loads both lattice and the add-on latticeExtra  
gph <- xyplot(Beer+Wine ~ Year, groups=Country, data=grog)  
update(gph, par.settings=simpleTheme(pch=19), auto.key=list(columns=2))
```

```
## Or, condition on `Country`  
xyplot(Beer+Wine+Spirit ~ Year | Country, data=grog,  
       par.settings=simpleTheme(pch=19), auto.key=list(columns=3))
```

```
tinting <- DAAG::tinting  
xyplot(csoa~it | tint*target, groups=agegp, data=tinting, auto.key=list(columns=2))
```

```
cuckoos <- DAAG::cuckoos
av <- with(cuckoos, aggregate(length, list(species=species), FUN=mean))
gph <- dotplot(species ~ length, data=cuckoos, alpha=0.4) +
  as.layer(dotplot(species ~ x, pch=3, cex=1.4, col="black", data=av))
update(gph, xlab="Length of egg (mm)")
```

```
## Alternatives, using `layer()` or `as.layer()`
avg <- with(cuckoos, data.frame(nspec=1:nlevels(species),
                               av=apply(split(length,species),mean)))
dotplot(species ~ length, data=cuckoos) +
  layer(lpoints(nspec~av, pch=3, cex=1.25, col="black"), data=avg)
```

```
dotplot(species ~ length, data=cuckoos) +
  as.layer(dotplot(nspec~av, data=avg, pch=3, cex=1.25, col="black"))
```

```
## Specify panel function
dotplot(species ~ length, data=cuckoos,
  panel=function(x,y,...){panel.dotplot(x, y, pch=1, col="gray40")
    avg <- data.frame(nspec=1:nlevels(y), av=apply(split(x,y),mean))
    with(avg, lpoints(nspec~av, pch=3, cex=1.25, col="black")) })
```

## Combining separately created graphics objects

```
cuckoos <- DAAG::cuckoos
## Panel A: Dotplot without species means added
gphA <- dotplot(species ~ length, data=cuckoos)
## Panel B: Box and whisker plot
gphB <- bwplot(species ~ length, data=cuckoos)
update(c("A: Dotplot"=gphA, "B: Boxplot"=gphB), between=list(x=0.4),
  xlab="Length of egg (mm)", layout=c(2,1))
## `latticeExtra::c()` joins compatible plots together.
## `layout=c(2,1)` : join horizontally; `layout=c(1,2)` : join vertically
```

## Subsection 10.5.2: Dynamic graphics – the rgl package

```
vignette('plot3D', package='plot3D')
```

## Section 10.6: Plotting characters, symbols, line types and colors

```
ycol <- -2.1 - (0:9) * 2.1
ftype <- c("plain", "bold", "italic", "bold italic", "symbol")
yline <- 4.2
ypmax <- 20
farleft <- -7.8
plot(c(-4, 31), c(4.25, ypmax), type = "n", xlab = "", ylab = "",
axes = F)
chh <- par()$cxy[2]
text(0:25, rep(ypmax + 0.8 * chh, 26), paste(0:25), srt = 90,
cex = 0.75, xpd = T)
text(-1.5, ypmax + 0.8 * chh, "pch = ", cex = 0.75, xpd = T)
points(0:25, rep(ypmax, 26), pch = 0:25, lwd=0.8)
letterfont <- function(ypos = ypmax, font = 2) {
  par(font = font)
  text(-1.35, ypos, "64-76", cex = 0.75, adj = 1, xpd = TRUE)
  text(19 - 1.35, ypos, "96-108", cex = 0.75, adj = 1)
  points(c(0:12), rep(ypos, 13), pch = 64:76)
  points(19:31, rep(ypos, 13), pch = 96:108)
  text(farleft, ypos, paste(font), xpd = T)
  text(farleft, ypos - 0.5, ftype[font], cex = 0.75)
}
plotfont <- function(xpos = 0:31, ypos = ypmax, font = 1,
sel32 = 2:4, showfont = TRUE) {
  par(font = font)
  i <- 0
  for (j in sel32) {
    i <- i + 1
    maxval <- j * 32 - 1
    if(j==4)maxval <- maxval-1
    text(-1.35, ypos - i + 1, paste((j - 1) * 32, "-",
maxval, sep = ""), cex = 0.75, adj = 1, xpd = TRUE)
    if(j!=4)
      points(xpos, rep(ypos - i + 1, 32), pch = (j - 1) *
32 + (0:31))
    else
      points(xpos[-32], rep(ypos - i + 1, 31), pch = (j - 1) *
32 + (0:30))
  }
  lines(rep(-1.05, 2), c(ypos - length(sel32) + 1, ypos) +
c(-0.4, 0.4), xpd = T, col = "grey40")
}
```

```

if (showfont) {
text(farleft, ypos, paste("font =", font, " "), xpd = T)
text(farleft, ypos - 0.5, ftype[font], cex = 0.75,
xpd = T)
}
}
plotfont(ypos = ymax - 1.5, font = 1, sel32 = 2:4)
for (j in 2:4) letterfont(ypos = ymax - 2.1 - 1.4 * j, font = j)
plotfont(ypos = ymax - 9.1, font = 5, sel32 = 3)
plotfont(xpos = c(-0.5, 1:31), ypos = ymax - 10.1, font = 5,
sel32 = 4, showfont = FALSE)
par(font = 1)
ltyes <- c("blank", "solid", "dashed", "dotted", "dotdash",
"longdash", "twodash")
lcode <- c("", "", "44", "13", "1343", "73", "2262")
for (i in 0:6) {
lines(c(4, 31), c(ylines + 4.5 - 0.8 * i, ylines + 4.5 -
0.8 * i), lty = i, lwd = 2, xpd = T)
if (i == 0)
numchar <- paste("lty =", i, " ")
else numchar <- i
text(farleft, ylines + 4.5 - 0.8 * i, numchar, xpd = TRUE)
text(farleft + 3.5, ylines + 4.5 - 0.8 * i, ltyes[i +
1], cex = 0.85, xpd = TRUE)
text(farleft + 7.5, ylines + 4.5 - 0.8 * i, lcode[i +
1], cex = 0.85, xpd = TRUE)
}

```

```

ycol <- -2.1 - (0:9) * 2.1
ftype <- c("plain", "bold", "italic", "bold italic", "symbol")
ylines <- 4.2
ymax <- 20
farleft <- -7.8
plot(c(-4, 31), c(4.25, ymax), type = "n", xlab = "", ylab = "",
axes = F)
chh <- par()$cxy[2]
text(0:25, rep(ymax + 0.8 * chh, 26), paste(0:25), srt = 90,
cex = 0.75, xpd = T)
text(-1.5, ymax + 0.8 * chh, "pch = ", cex = 0.75, xpd = T)
points(0:25, rep(ymax, 26), pch = 0:25, lwd=0.8)
letterfont <- function(ypos = ymax, font = 2) {
par(font = font)

```



```

text(-1.35, ypos, "64-76", cex = 0.75, adj = 1, xpd = TRUE)
text(19 - 1.35, ypos, "96-108", cex = 0.75, adj = 1)
points(c(0:12), rep(ypos, 13), pch = 64:76)
points(19:31, rep(ypos, 13), pch = 96:108)
text(farleft, ypos, paste(font), xpd = T)
text(farleft, ypos - 0.5, ftype[font], cex = 0.75)
}
plotfont <- function(xpos = 0:31, ypos = ymax, font = 1,
sel32 = 2:4, showfont = TRUE) {
par(font = font)
i <- 0
for (j in sel32) {
i <- i + 1
maxval <- j * 32 - 1
if(j==4)maxval <- maxval-1
text(-1.35, ypos - i + 1, paste((j - 1) * 32, "-",
maxval, sep = ""), cex = 0.75, adj = 1, xpd = TRUE)
if(j!=4)
points(xpos, rep(ypos - i + 1, 32), pch = (j - 1) *
32 + (0:31))
else
points(xpos[-32], rep(ypos - i + 1, 31), pch = (j - 1) *
32 + (0:30))
}
lines(rep(-1.05, 2), c(ypos - length(sel32) + 1, ypos) +
c(-0.4, 0.4), xpd = T, col = "grey40")
if (showfont) {
text(farleft, ypos, paste("font =", font, " "), xpd = T)
text(farleft, ypos - 0.5, ftype[font], cex = 0.75,
xpd = T)
}
}
plotfont(ypos = ymax - 1.5, font = 1, sel32 = 2:4)
for (j in 2:4) letterfont(ypos = ymax - 2.1 - 1.4 * j, font = j)
plotfont(ypos = ymax - 9.1, font = 5, sel32 = 3)
plotfont(xpos = c(-0.5, 1:31), ypos = ymax - 10.1, font = 5,
sel32 = 4, showfont = FALSE)
par(font = 1)
ltypes <- c("blank", "solid", "dashed", "dotted", "dotdash",
"longdash", "twodash")
lcode <- c("", "", "44", "13", "1343", "73", "2262")
for (i in 0:6) {

```

```

lines(c(4, 31), c(yline + 4.5 - 0.8 * i, yline + 4.5 -
0.8 * i), lty = i, lwd = 2, xpd = T)
if (i == 0)
numchar <- paste("lty =", i, " ")
else numchar <- i
text(farleft, yline + 4.5 - 0.8 * i, numchar, xpd = TRUE)
text(farleft + 3.5, yline + 4.5 - 0.8 * i, ltypes[i +
1], cex = 0.85, xpd = TRUE)
text(farleft + 7.5, yline + 4.5 - 0.8 * i, lcode[i +
1], cex = 0.85, xpd = TRUE)
}

```

## Font families

## Colors

```

library(RColorBrewer)
palette(brewer.pal(12, "Set3"))

```

```

if(file.exists("/Users/johnm1/pkgsg/PGRcode/inst/doc/")){
code <- knitr::knit_code$get()
txt <- paste0("\n## ", names(code), "\n", sapply(code, paste, collapse='\n'))
writeLines(txt, con="/Users/johnm1/pkgsg/PGRcode/inst/doc/Appendix.R")
}

```