LORENTZ NON INVARIANCE OF SPHERICAL LIGHTWAVES IN SPECIAL RELATIVITY

J. H. MARTEL

Contents

1. Admitted Apparent Incompatibilities at the Basis of Special Relativity

The subject of this brief note is Einstein's alleged proof of the Lorentz invariance of spherical light waves in his special relativity theory. Our purpose here is to describe a positive gap in above mentioned proof, and to further demonstrate the untenability of its conclusions. This is a controversial subject developed by other critical authors, e.g. [Bry], [Cro19]. The present note arises from the author's own study of the controversy, and his attempt to identify the error in plain mathematical terms.

We recall that the purpose of Einstein's alleged proof [Ein19] is to reconcile the fundamental assumptions of special relativity, namely

(A1) that the laws of physics are the same in all nonaccelerated reference frames, i.e. if K' is a coordinate system moving uniformly (and devoid of rotation) with respect to a coordinate system K, then natural phenomena run their course with respect to K' according to exactly the same laws as with respect to K.

(A2) that light in vacuum propagates along straight lines with constant velocity $c \approx 300,000$ kilometres per second.

In Einstein's own words [Ein19, Ch.7, 11]:

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> "*There is hardly a simpler law in physics than that according to which light is propagated in empty space. Every child at school knows, or believes he knows, that this propagation takes place in straight lines with velocity* $c = 300,000 km/sec$*.... Who would imagine that this simple law has plunged the conscientiously thoughtful physicist into the greatest intellectual difficulties?*"

Notice the mention of "law" in the above quote. Einstein presents (A2) as the *law* of propagation of light. To be simultaneously consistent with (A1) requires (A2) be satisfied in every nonaccelerated reference frame. However the conjunction of (A1) and (A2), abbreviated (A12), appears contradictory according to classical mechanics, Galilean transformations, and Fizeau's law of addition of velocities, etc.. However Einstein claims – and this popularly lauded as among his greatest intellectual achievements – that this is only an *apparent* incompatibility. The incompatibility is reconciled, according to Einstein's proposal, by postulating Lorentz-Fitzgerald's length contractions and time dilations, c.f. [Mic95, Ch.XIV]. What needs be shown is that Lorentz transformation preserves the form of the law (A2) in every inertial frame K. This requires the Lorentz invariance of luminal spherical waves, as we now discuss.

2. Lorentz Invariant Tensors: Existence and Nonexistence

The Lorentz transformations arose as a possible solution to the null effect of the Michelson-Morley experiment. The transformations relate the space- and time-coordinates x, y, z, t and ξ, η, ζ, τ of two inertial observers K, K', respectively. The key assumption is that Minkowski's line element $dx^2 + dy^2 + dz^2 - c^2dt^2$ is invariant under Lorentz transformations. Of course c is the constant luminal velocity from (A2). Here we pause to warn against the mathematician's habit of casually setting $c = 1$ and treating t as a space-variable immediately comparable to x, y, z. The constant c is required to transition from units of time to units of space.

Now we turn to our critical analysis. We claim the positive gap in Einstein's attempted proof has a twofold source.

Firstly, an error arises when quadratic expressions like

$$\xi^2 + \eta^2 + \zeta^2 = c^2\tau^2 \tag{1}$$

are *misidentified* with "the equation of a sphere".

Actually (1) is a three-dimensional cone in the four variables ξ, η, ζ, τ. Of course the cone contains numerous spherical two-dimensional subsets, but a further equation is required. For instance the standard geometric sphere S centred at the origin simultaneously satisfies (1) and the equation

$$\frac{1}{2}d(\xi^2+\eta^2+\zeta^2) = \xi d\xi + \eta d\eta + \zeta d\zeta = 0.$$

In short, the standard geometric sphere requires that *two*(!) quadratic forms $\xi^2 + \eta^2 + \zeta^2$ and $c^2\tau^2$ be *simultaneously constant*. This leads us to Einstein's second error, which is the failure to realize that **the Lorentz invariance of the quadratic form** $h = x^2 + y^2 + z^2 - c^2t^2$ **in no way implies the Lorentz invariance of the forms** $h_1 = x^2 + y^2 + z^2$ **and** $h_2 = c^2t^2$. Indeed the forms $h_1.h_2$ are positive definite, and degenerate. But if h_1 and h_2 were G-invariant, then G would be reducible representation on $\mathbb{R}^4$.

If we fix a reference frame K, then the set of Lorentz transformations is the Lie group $G := O(h)$ of isometries of the Minkowski form $h = x^2 + y^2 + z^2 - c^2t^2$. Invariance says $\xi^2 + \eta^2 + \zeta^2 - c^2\tau^2$ is numerically equal to $x^2 + y^2 + z^2 - c^2t^2$ for every Lorentz transform ϕ satisfying $(\xi, \eta, \zeta, \tau) = \phi.(x, y, z, t)$. But as we argue below, Minkowski's form is essentially the *only* Lorentz invariant quadratic form on $\mathbb{R}^4$. Now returning to (A12), we see Einstein's argument concerns the geometry of a wave front generated by a light pulse. That light satifies (A2) implies light trajectories are restricted to the so-called null cone $N = \{h = 0\}$. Obviously N is G-invariant and defined by the equation $x^2 + y^2 + z^2 = c^2t^2$. Let $\mathscr{C}$ be the vector space of (possibly degenerate) quadratic forms q on $\mathbb{R}^4$, and let $h \in \mathscr{C}$ be Minkowski's form.

Theorem. *If $q \in \mathscr{C}$ is a quadratic form on $\mathbb{R}^4$ such that the restriction $q|_N$ is G-invariant, then q is proportional to h.*

Proof. The G-action on N is transitive on nonzero vectors. Therefore if $q|_N$ is G-invariant, then $q|_N$ is constant. By continuity it follows that $q|_N$ is identically zero since $q(0) = 0$. So the zero locus of q contains the zero locus of h, i.e.

$$N \subset \{q = 0\}. \tag{2}$$

Now we use a theorem of J. H. Elton to conclude q, h are proportional, c.f. [Elt10]. We outline his argument. Let $q \otimes_{\mathbb{R}} \mathbb{C}$ and $h \otimes_{\mathbb{R}} \mathbb{C}$ be the complexifications of the real quadratic forms q, h. Thus $q \otimes_{\mathbb{R}} \mathbb{C} : \mathbb{R}^4 \otimes_{\mathbb{R}} \mathbb{C} \to \mathbb{C}$ is a complex-valued quadratic form. Elton's proof establishes the following inclusion

$$\{h \otimes \mathbb{C} = 0\} \subset \{q \otimes \mathbb{C} = 0\}. \tag{3}$$

According to the tensor construction we have $h \otimes \mathbb{C}(x + iy) = h(x) - h(y) + 2ih(x, y)$. Therefore $h \otimes \mathbb{C}(x + iy) = 0$ if and only if $h(x) = h(y)$ and $h(x, y) = 0$, where $h(\cdot, \cdot)$ is the bilinear form canonically defined by h. Elton's proof reduces to establishing the implication: if (2) is satisfied, then $h(x) = h(y)$ and $h(x, y) = 0$ implies $q(x) = q(y)$ and $q(x, y) = 0$ for all $x, y \in \mathbb{R}^4$. That q vanishes on the null cone N implies q is indefinite if it is not identically zero. Once the inclusion (3) is established, Hilbert's Nullstellensatz [Eis13] implies $q \otimes \mathbb{C} = \lambda \cdot h \otimes \mathbb{C}$ for some $\lambda \in \mathbb{C}$. But then obviously $\lambda \in \mathbb{R}$ and the theorem follows. □

That is, the Minkowski form is the unique Lorentz invariant quadratic form (modulo scalars) on $\mathbb{R}^4$ which vanishes on the null cone. Another argument is possible [Arm+18] which establishes nonexistence of any other Lorentz invariant $(0,2)$-tensors (the proof is long algebraic computation). Naturally the question arises whether there exist any other invariant (p,q)-tensors which arise independantly of the Minkowski form h.

Now we return to the subject at hand, namely Einstein's alleged proof that (A12) are compatible with respect to Lorentz transforms. The argument is very general, and to illustrate we consider the two-dimensional case in the variables x, t. Here we find $h = x^2 - c^2t^2$ is invariant with respect to the group $G = SO(1,1)$ generated by $a_\theta := \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix}$, where $\theta \in \mathbb{R}$. We see $SO(1,1)$ is isomorphic to the split multiplicative torus $\mathbb{R}^\times_{>0}$ using the logarithm. The unit sphere includes two vectors $\langle 1,1\rangle$, $\langle -1,1\rangle$, and which are mapped by $a_\theta \in SO(1,1) \simeq \mathbb{R}^\times_{>0}$ to

$$\langle \xi, \tau\rangle = \langle \cosh\theta + \sinh\theta, \cosh\theta + \sinh\theta\rangle,\ \langle -\cosh\theta + \sinh\theta, \cosh\theta - \sinh\theta\rangle.$$

But evidently $\xi^2 \neq x^2 = 1$ and $\tau^2 \neq t^2 = 1$ when $\theta \neq 0$. Thus the quadratic forms $h_1 = x^2$ and $h_2 = t^2$ are not invariant. Likewise we find the image of the unit sphere $x^2 = 1$ does not correspond to a sphere in $\xi\tau$ coordinates.

3. Radius is Not a Lorentz Invariant Variable

A dual formulation of the above observation can be phrased as follows. Consider the solutions of the homogeneous wave equation

$$\phi_{xx} + \phi_{yy} + \phi_{zz} - \frac{1}{c^2}\phi_{tt} = 0. \tag{4}$$

If ϕ is expressed as a function of ξ, η, ζ, τ using a Lorentz transformation, then evidently $\phi = \phi(\xi, \eta, \zeta, \tau)$ is again a solution of the homogeneous wave equation

$$\phi_{\xi\xi} + \phi_{\eta\eta} + \phi_{\zeta\zeta} - \frac{1}{c^2}\phi_{\tau\tau} = 0.$$

This is obvious. **However what is perhaps not obvious is that the class of *radial solutions* of (4) is *not* Lorentz invariant, because indeed there is no Lorentz invariant definition of "radius".**

In group theoretic terms, a solution $\phi = \phi(x,t)$ of (4) is said to be *radial by an observer* K iff $\phi(Ax, t) = \phi(x, t)$ for every rigid motion $A \in SO(3)$ in the space variables x, y, z. But implicitly this requires a Lie group representation ρ of $SO(3)$

into the Lorentz group $G \simeq O(3,1)$, and this representation of the maximal compact subgroup is noncanonical. Recall the quotient $G/SO(3)$ is contractible modulo a finite subgroup and a model of hyperbolic space $\mathbb{H}^3$.

Different inertial observers K, K' generally choose different orthogonal symmetry groups, for instance as defined by their own "sum of squares" formula (applied to physical squares ξ^2, η^2, ζ^2 in their local variables ξ, η, ζ, τ). Of course the Minkowski element (1) is invariant and canonical, but any attempted decomposition into "spatial" and "time" parts is arbitrary choice of the observer and not Lorentz invariant.

Furthermore, for the motion of light pulses according to (A12) the Minkowski element vanishes identically, and the only canonical tensor element becomes the constant zero element. After a Lorentz change of variables, we find a new solution $\phi' = \phi(\xi, \eta, \zeta, \tau)$, but this solution needs not be radial in the above sense. Indeed the rigid K-space motion A does not often preserve the space coordinates ξ, η, ζ of K', and does not act on the subspace generated by ξ, η, ζ. That is, A-motions nontrivially depend on the K'-time variable τ. As such, we conclude the nonexistence of a Lorentz invariant radius.

4. Objections and Response

To conclude we briefly recall some popular objections to the above line of reasoning. The author submits this article in good faith to all honest persons, and we attempt to stimulate some critical discourse with a reasonable reader.

First one might object that our argument simply reduces to the observation that spheres in the frame K are transformed to ellipsoids in K', as well-known [Ein+05, §4]. But we remind the reader that the Lorentz contraction is assumed to affect *material* objects, even independantly of the nature of the material, and such that material spheres in K are seen as material ellipsoids in K', where again the eccentricity of the K'-ellipsoid is nontrivial and independant of the material nature of the sphere. However we respond that light spheres are not material, and not themselves subject to Lorentz contraction if (A2) holds. At least not without further evidence and hypotheses.

Secondly critics might object that (A12) only requires the consistent *measurement* of c in arbitrary reference frames K, K'. This would replace the formal "law" (A2) with a more pragmatic rule of thumb for measurements. And indeed this article welcomes such an approach, and quickly we are led to an important experimental difficulty at the core of special relativity. For we remind the reader that space- and time-measurements are always dependant on material objects, and often non local, i.e. the source and receiver are possibly separated by large distances. The impossibiliy of synchronizing non local clocks leads to the apparent impossibility of measuring the "one-way" velocity of light. It strikes the author that the incompatibility of (A12) is not merely "apparent" but actually *essential*, and even natural evidence that (A2) is not a proper natural law. That all measurements of c only succeed in measuring the "two-way" or "round-trip" velocities of light where source and receiver coincide, is discussed in [Zha97], [Pér11]. See also [Ver] for entertaining introduction. That is to say, we argue (A2) has never been and cannot be subject to measurement.

Thirdly, the book [Rin89, pp.8-10, 21–22] attempts

> *"in spite of its historical and heuristic importance, . . . to de-emphasize the logical role of the law of light propagation (A2) as a pillar of special relativity."*

Rindler further claims

> *"a second axiom is needed* only *to determine the value of a constant c of the dimensions of a velocity that occurs naturally in the theory. But this could come from any number of branches of physics – we need only think of the energy formula $E = mc^2$, or de Broglie's velocity relation $uv = c^2$."*

The above objection is interesting, and to which we have a simple response, namely that the above quoted formulas are *equivalent* to (A2), and not independant in any physical sense. Even the Weber-Kohlrausch formula [Ass03], for example, is another form of (A2). So another independant law involving c could potentially serve as a logical substitute for (A2), but this is hypothetical since no such formula appears to exist – thus far all formulas involving c are based essentially on some form of (A2). So the pillar is unmoved, logically speaking.

Moreover the incompatibility of (A2) with the hypothesis of "wave–particle" duality is analyzed by A.K.T. Assis [Ass99, §7.2.4, pp.133]. This is subject of future investigations.

5. R.Sansbury's Experiment: Does Light Travel Through Space?

Finally we think it possible that (A2) is incorrect for another reason, namely that ***light* is not something that *travels through space***. The daring idea was introduced and developed by R. Sansbury, for example in his interesting book [San12], which begins with the following experiment which we quote in full:

> *(Case 1) A 15 nanosecond light pulse from a laser was sent to a light detector, 30 feet away. When the light pulse was blocked at the photodiode during the time of emission, but unblocked at the expected time of arrival, 31.2 nanoseconds after the beginning of the time of emission, for 15 nanosecond duration, little light was received. (A*

little more than the 4mV noise on the oscilloscope). This process was repeated thousands of times per second.

(Case 2) When the light was unblocked at the photodiode during the time of emission (15 nanosecond) but blocked after the beginning of the time of emission, during the expected time of arrival for 15 nanosecond, twice as much light was received (8mV). This process was repeated thousands of times per second.

This indicated that light is not a moving wave or photon, but rather the cumulative effect of instantaneous forces at a distance. That is, undetectable oscillations of charge can occur in the atomic nuclei of the photodiode that spill over as detectable oscillations of electrons after a delay.

This important experiment has apparently not been repeated, despite it's simplicity. We refer the reader to R. Sansbury's book [sic] for further details.

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Email address: jhmartel@protonmail.com