ON EINSTEIN-LORENTZ' ALLEGED PROOF OF SPHERICAL INVARIANCE OF LIGHT PULSES IN SPECIAL RELATIVITY

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The subject of this brief note is controversial. We observe the Lorentz $SO(3,1)$-invariance of the quadratic form $h = x^2 + y^2 + z^2 - c^2t^2$ in no way implies the Lorentz invariance of the forms $h_1 = x^2 + y^2 + z^2$ and $h_2 = c^2t^2$. As corollary we find Einstein's proposed resolution of the apparent incompatibility of the principle of restricted relativity and the purported law of the constancy of light invalidated by basic computations, c.f. [Ein19, Ch.7, 11]. The argument is general and applies to split orthogonal groups of arbitrary rank $SO(1,1)$, $SO(2,1)$, etc..

As far as mathematics is concerned, here is the issue in simplest terms. Minkowski's form $h = x^2 + y^2 + z^2 - c^2t^2$ is invariant with respect to Lorentz transformations, i.e. the Lie group $G = SO(h) = SO(3,1)$ where invariance means $\xi^2 + \eta^2 + \zeta^2 - c^2\tau^2$ is equal to $x^2 + y^2 + z^2 - c^2t^2$ for every Lorentz transform (ξ, η, ζ, τ) of (x, y, z, t). That is, the Minkowski form is a real valued function, and is invariant under Lorentz transformations. Now as Einstein's argument concerns the geometry of a wave front generated by a light pulse, i.e. the action of the Lorentz group on (subsets of) the so-called null cone $N = \{h = 0\}$, where obviously N is G-invariant and defined by the equation $x^2 + y^2 + z^2 = c^2t^2$. Next comes the key claim, that N has no nonempty proper G-invariant subsets. More specifically the Lorentz invariance of h, N in no way implies the Lorentz invariance of the forms $h' = x^2 + y^2 + z^2$ and $h'' = c^2t^2$, even despite their difference $h = h' - h''$ being invariant.

The two-dimensional case in xt-variables is illustrative, where $h = x^2 - c^2t^2$ is invariant with respect to the group $G = SO(1,1)$ generated by $a_\theta := \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix}$, where $\theta \in \mathbb{R}$. We see $SO(1,1)$ is isomorphic to the split multiplicative torus $\mathbb{R}^\times_{>0}$ using the logarithm. The unit sphere includes two vectors

$$\langle 1, 1\rangle,\ \langle -1, 1\rangle,$$

and which are mapped by $a_\theta \in SO(1,1) \simeq \mathbb{R}^\times_{>0}$ to

$$\langle \xi, \tau\rangle = \langle \cosh\theta + \sinh\theta, \cosh\theta + \sinh\theta\rangle,\ \langle -\cosh\theta + \sinh\theta, \cosh\theta - \sinh\theta\rangle.$$

But evidently $\xi^2 \neq x^2 = 1$ and $\tau^2 \neq t^2 = 1$ when $\theta \neq 0$. That is, the image of the unit sphere does not correspond to a sphere in $\xi\tau$ coordinates. In other words the

Date: December 5, 2020.

equation $x^2 = c^2t^2$ is not the equation of sphere when x, t are variable, and likewise $\xi^2 = c^2\tau^2$ is not the equation of sphere when ξ, τ are both variable and nonconstant.

The author submits that the above argument positively demonstrates a gap in Einstein's purported proof, c.f. [Ein19][Ch.11, pp.39] where the reader can verify that Einstein does not treat the general case but restricts himself to a velocity parallel to x-axis. Nearly one hundred years expired before the gap was (re)discoverd by more critical authors [Bry], [Cro19]. What are the consequences of such a gap? For Einstein, the purpose of his proof was to demonstrate that his so-called law of the propogation of light was indeed compatible with the principle of restricted relativity. The fundamental assumptions of Einstein's special relativity are (i) that the Laws of physics are the same in all nonaccelerated reference frames, where if K' is a coordinate system moving uniformly (and devoid of rotation) with respect to a coordinate system K, then natural phenomena run their course with respect to K' according to exactly the same laws as with respect to K. Next Einstein claims "*There is hardly a simpler law in physics than that according to which light is propograted in empty space. Every child at school knows, or believes he knows, that this propogation takes place in straight lines with velocity* $c = 300000km/sec.$ *... Who would imagine that this simple law has plunged the conscientiously thoughtful physicist into the greatest intellectual difficulties?*" Thus in view of Fizeau's Addition of Velocities in classical mechanics, apparently (i) and (ii) are incompatible. Einstein claims this apparent incompatibility is resolved by Lorentz transformations – however it does not and a definite gap remains. For material motions, the null result of Michelson-Morley is compatible with the postulates of length contraction and time dilations, as consistent with the Lorentz group. But for light pulses – which is the subject of (ii) – the Lorentz group is not consistent with (ii).

References

[Bry] Steven Bryant. "The Failure of the Einstein-Lorentz Spherical Wave Proof". In: *Proceedings of the NPA* 8 (), p. 64.

[Cro19] S.J. Crothers. "Special Relativity and the Lorentz Sphere". In: (2019). URL: https://vixra.org/abs/1911.0013 (visited on 12/04/2020).

[Ein19] Albert Einstein. *Relativity: The Special and the General Theory-100th Anniversary Edition*. Princeton University Press, 2019.