LORENTZ NON INVARIANCE OF SPHERICAL LIGHTWAVES IN SPECIAL RELATIVITY

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Abstract. This note examines whether spherical light waves are Lorentz invariant or non invariant in Einstein's special relativity. This note argues that the spatial quantity called "radius" in the literature is not a Lorentz invariant quantity. Equivalently we argue that the homogeneous wave equation does not have a nontrivial class of Lorentz invariant radial solutions. Thus we find the hypothesis that wave fronts generated by light pulses are round spheres in every inertial frame is not consistent with the principle of relativity. This leads to a reexamination of the assumptions underlying Einstein's special relativity theory. This controversial subject has already been developed by [Bry], [Cro19]. The present article aims to present the controversy in elementary mathematical terms.

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1. Admitted Incompatibilities and Attempted Resolutions

1.1. **Einstein's Assumptions.** We begin with Einstein's presentation of the basic principles of special relativity [Ein19], [Ein+05], where appears the alleged proof of the Lorentz invariance of spherical lightwaves. In Einstein's words [Ein19, Ch.7, 11]:

> "*There is hardly a simpler law in physics than that according to which light is propagated in empty space. Every child at school knows, or believes he knows, that this propagation takes place in straight lines with velocity* $c = 300,000$ *km/sec [one foot per nanosecond]. . . . Who would imagine that this simple law has plunged the conscientiously thoughtful physicist into the greatest intellectual difficulties?*"

The primary difficulty is to reconcile the fundamental axioms of special relativity, namely:

(A1) that the laws of physics are the same in all nonaccelerated reference frames, i.e. if K' is a coordinate system moving uniformly (and devoid of rotation) with respect to a coordinate system K, then natural phenomena run their course with respect to K' according to exactly the same laws as with respect to K.

(A2) that light in vacuum propagates along straight lines with constant velocity $c \approx 300,000$ *km/sec* [one foot per nanosecond].

Observe how "law" and "propagation" is repeated in the above quotes. The assumption (A2) is coined the *law* of propagation of light. If (A2) is to be consistent with (A1), then the law (A2) must hold true in every inertial reference frame K'. Except the conjunction of (A1) and (A2) – abbreviated (A12) – appears contradictory to the laws of classical mechanics, e.g. Fizeau's law of addition of velocities, and this is the "great intellectual difficulty" alluded to by the thoughtful physicist. Yet Einstein claims the difficulty is only an *apparent incompatibility* which is reconciled - according to Einstein's reasoning - by postulating Lorentz-Fitzgerald's length contractions and time dilations, c.f. [Mic95, Ch.XIV]. Thus what needs be demonstrated is that the formulae of the Lorentz transformations preserves the form of the law (A2) in every inertial frame. We examine the action of Lorentz transformations on the law (A2) and the relation to luminal spherical waves in the next section. We remark that (A2) is not strictly a well-formed sentence, for we must clearly distinguish between velocity and speed. As is well known, velocity is an "arrow" which has a magnitude ("speed") and a direction. With this distinction in mind, we read (A2) as declaring that light propagates in vacuum in fixed directions (along straight lines) and with constant magnitude (the "speed" c).

2. Lorentz Invariant and Non Invariant Tensors

2.1. **Lorentz Transformations.** The linear group of Lorentz transformations was hypothesized as an attempt to explain the observed null effect of Michelson-Morley's famous interferometer experiment. This experiment was supposed to measure the speed of light relative to the *ether*. The Lorentz transformations relate the space and time coordinates (x, y, z, t) and (ξ, η, ζ, τ) of two inertial observers K and K', respectively. They are defined as the isometry group of the quadratic form

$$h := ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2. \tag{1}$$

It is assumed that h is a scalar invariant for all inertial observers. Here c is the constant luminal velocity posited by (A2) in vacuum. The Lorentz invariance of h says $\xi^2 + \eta^2 + \zeta^2 - c^2\tau^2$ is numerically equal to $x^2 + y^2 + z^2 - c^2t^2$ for every Lorentz transform λ satisfying $(\xi, \eta, \zeta, \tau) = \lambda \cdot (x, y, z, t)$. It can be shown that Minkowski's form is the *only* Lorentz invariant quadratic form on $\mathbb{R}^4$ modulo homothety, c.f. [Elt10], [Arm+18].

2.2. **Photons.** Now we say something about the photon model of light, as treated in [Lev, III.XI.6, pp.301]. If light satisfies (A2), then in a reference frame K, light is *something* $\gamma(t) = (x(t), y(t), z(t))$ that travels through space with time, and whose velocity γ', if it could be materially measured as a function of t, would satisfy

$$||\gamma'||^2 = (dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2 = c^2. \tag{2}$$

Thus it is argued that light trajectories are constrained to the null cone $N := \{h = 0\}$ of Minkowski's metric h. Obviously N is Lorentz invariant and defined by the equation $x^2 + y^2 + z^2 = c^2t^2$.

What does the identity (2) say? The common interpretation is (2) demonstrates that the *speed* of γ, as measured by K, is identically equal to c. The Lorentz invariance of the cone N implies that differentiable curves $\gamma(t)$ contained in N will represent light trajectories exhibiting a constant speed c *independant of the trajectory* of γ on N. The *speed* is c, yet we can say nothing about the directions of the curve γ along its path neither it's parameterization. Einstein's (A2) postulates the propagation is along straight lines, i.e. with constant direction in vacuum. This straight line propagation is preserved by the *linearity* of the Lorentz group. Thus the hypothesis (A2) that light propagates in *straight lines* with constant speed c in every inertial reference frame K' *is* invariant with respect to Lorentz transformations. Consequently the conjunction (A12) is consistent up to this point.

However the propagation of light still remains underdetermined by (A2). For straight lines on N have *no* canonical parameterization, even affine. This reveals a clear distinction between Riemannian straight lines (which have a canonical arclength parameter ds) and the null lines $\ell \subset N$ (which do not admit canonical ds

arclength parameter except the vanishing $ds = 0$). And at this point we must ask the question again: what does (A2) *really* say about the propagation of light? In this article we argue that there is an implicit hidden assumption about (A2), namely that the straight lines in N can be canonically parameterized, and that light propagates according to a definite rule along a straight line as a function of t. This hidden assumption becomes more prominent in the wave or undulatory model of light.

2.3. **Critique of Einstein's Proof.** Now we turn to our critical analysis. We claim the positive gap in Einstein's attempted proof has a twofold source. **First an error arises when quadratic expressions like**

$$\xi^2 + \eta^2 + \zeta^2 = c^2\tau^2 \tag{3}$$

are *misidentified* with "the equation of a sphere". Strictly speaking, (3) is a three-dimensional cone in the four variables ξ, η, ζ, τ. Of course the cone contains many spherical two-dimensional subsets – but a second independant equation is necessary to specify these metric spheres.

The Penrose approach is to projectivize the equation (3) and obtain a projective sphere with a well defined conformal structure [PR84, Ch 1.]. But again there is no canonical metric invariant with respect to the Lorentz group on the projectivization. In otherwords, the null sphere has a Lorentz invariant *conformal* structure, but it does not have a Lorentz invariant *metric* structure.

For instance the standard round sphere S centred at the origin simultaneously satisfies (3) and additionally the equation

$$\frac{1}{2}d(\xi^2 + \eta^2 + \zeta^2) = \xi d\xi + \eta d\eta + \zeta d\zeta = 0.$$

In short, the round sphere requires that *two* quadratic forms $\xi^2 + \eta^2 + \zeta^2$ and $c^2\tau^2$ be *simultaneously constant*. This leads us to Einstein's second error, which is the failure to observe that **the Lorentz invariance of the quadratic form $h = x^2+y^2+z^2-c^2t^2$ in no way implies the Lorentz invariance of $h_1 := x^2+y^2+z^2$ and $h_2 := c^2t^2$.** . Indeed the quadratic forms h_1, h_2 are degenerate, with nontrivial radicals $rad(h_1) = \{x = y = z = 0\}$ and $rad(h_2) = \{t = 0\}$. The radicals are linear subspaces of $\mathbb{R}^4$. But if h_1, h_2 are invariant, then $rad(h_1)$ and $rad(h_2)$ are also nontrivial invariant subspaces. This contradicts the fact that the standard linear representation of the Lorentz group acts irreducibly on $\mathbb{R}^4$.

2.4. **Elementary Computations in $SO(1,1)$.** Now we present a simple computation to illustrate the numerics involved in (A12). The computation is essentially two-dimensional in the variables (x,t) and (ξ,τ). For numerical convenience we set $c := 1$. Then $h = x^2 - t^2$ is a quadratic form on $\mathbb{R}^2$ invariant with respect to the one-dimensional Lorentz group $G = SO(1,1)_0$ generated by $a_\theta := \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix}$, for $\theta \in \mathbb{R}$. In two dimensions the null cone $x^2 - t^2 = 0$ is projectively a 0-dimensional sphere, hence consisting of two projective points represented by the affine lines $x - t = 0$ and $x + t = 0$. The round sphere $\{x^2 = 1\}$ consists of two vectors in the null cone, namely $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$. Multiplying by a_θ on the left of these vectors, we find the images $\begin{pmatrix} \xi \\ \tau \end{pmatrix}$ equal to $\begin{pmatrix} \cosh\theta + \sinh\theta \\ \cosh\theta + \sinh\theta \end{pmatrix}$ and $\begin{pmatrix} -\cosh\theta + \sinh\theta \\ \cosh\theta - \sinh\theta \end{pmatrix}$. But evidently $\xi^2 \neq x^2 = 1$ and $\tau^2 \neq t^2 = 1$ when $\theta \neq 0$. Thus the quadratic forms $h_1 = x^2$ and $h_2 = t^2$ are not a_θ-invariant. Likewise we find the image of the unit sphere $x^2 = 1$ does not correspond to a spatial sphere in (ξ, τ) coordinates. But these trivial computations have the effect of falsifying the alleged Lorentz invariance of spherical lightwaves. However the "slope" of $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $a_\theta . \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \cosh\theta + \sinh\theta \\ \cosh\theta + \sinh\theta \end{pmatrix}$ *is* identically equal to $c = 1$ in accordance with (A2).

To further illustrate our earlier comments on the nonexistence of canonical parameters on N, consider that for any C^1 function $f : \mathbb{R} \to \mathbb{R}$ we obtain a curve $\epsilon(t) = \epsilon_f(t) = \begin{pmatrix} f(t) \\ f(t) \end{pmatrix} = f(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ for $t \in \mathbb{R}$. Then $\epsilon_f(t)$ is supported on a straight line in N, and the "photon" ϵ_f can be said to propagate in a straight line with constant speed $c = 1$. For example the curves $\begin{pmatrix} \cos(t) \\ \cos(t) \end{pmatrix}$, and $\begin{pmatrix} e^t \\ e^t \end{pmatrix}$ appear to equally satisfy Einstein's "law" of propagation (A2) as the more conventional curve with $f(t) = t$ and $\begin{pmatrix} t \\ t \end{pmatrix}$. In this sense we argue that the propagation of light, and specifically it's velocity, is underdetermined by (A2).

In [Lev, p. III.XI.16] the laws of geometric optics are characterized by two equations:

$$\delta \int ds = 0 \quad \text{and} \quad ds^2 = 0. \tag{4}$$

The first equation says the variational derivative of the functional $\gamma \mapsto \int ds$ vanishes on the light trajectories, and the second equation says the trajectory is constrained to the null cone. In the Riemannian setting, where ds is positive definite, the first equation $\delta \int ds = 0$ is essentially equivalent to the geodesic equation. However in the Lorentzian setting this equation reduces to $0 = 0$ and has no meaning on the null cone N. This is acknowledged in [Lev, XI.XI.14, pp.332]. But Levi-Civita argues that zero length geodesics are limits of Riemannian geodesics ($ds > 0$) and that "there is a process of passing to the limit (in conditions of complete analytical regularity) from ordinary geodesics", [Lev, XI.XI.14, pp.332]. However Levi-Civita claims that the equation $\delta \int ds = 0$ somehow implies the standard geodesic equation $\nabla_{\gamma'}\gamma' = 0$. But this correspondance between geodesic equations on N and $\delta \int ds = 0$ is informal and nonrigorous.

Levi-Civita modifies (A2) somewhat by asserting that "the propagation of light is rectilinear, uniform, and with velocity c". The term "uniform" does not feature in Einstein's formulation of (A2), although it speaks to the hidden assumption that the light rays have a canonical parameter (describing the *uniform* motion of the light ray).

3. Radius is Not a Lorentz Invariant Variable

3.1. **Homogeneous Wave Equation and Radial Solutions.** In this section we provide another view based on the homogeneous wave equation. Here we are considering smooth real-valued functions $\phi = \phi(x, y, z, t)$ in $3 + 1$ independant variables.

$$\phi_{xx} + \phi_{yy} + \phi_{zz} - \frac{1}{c^2}\phi_{tt} = 0. \tag{5}$$

If λ is a Lorentz transformation with $(\xi, \eta, \zeta, \tau) = \lambda \cdot (x, y, z, t)$, then

$$\phi' := \lambda^* \phi = \phi \circ \lambda^{-1}$$

is again a solution of the homogeneous wave equation

$$\phi_{\xi\xi} + \phi_{\eta\eta} + \phi_{\zeta\zeta} - \frac{1}{c^2}\phi_{\tau\tau} = 0.$$

This is verified by elementary computation, substituting the formulae for the Lorentz transform. **But the set of *radial solutions* of (5) is *not* Lorentz invariant**.

There are different ways to see the Lorentz noninvariance of radius r. In group theoretic terms, a solution $\phi = \phi(x, t)$ of (5) is said to be *radial by an observer K* iff $\phi(Ax, t) = \phi(x, t)$ for every rigid motion $A \in SO(3)$ in the space variables x, y, z of K. Implicitly this requires a Lie group representation ρ of $SO(3)$ into the Lorentz group $G \simeq O(3, 1)$. But this representation of the maximal compact subgroup is noncanonical and cannot be invariantly chosen. Different inertial observers K, K' generally choose different orthogonal symmetry groups, for instance as defined by their own "physical sum of squares" formulae, applied to physical squares ξ^2, η^2, ζ^2 in their local variables ξ, η, ζ, τ. Of course the Minkowski element (3) is invariant and canonical, but any decomposition into "spatial" and "time" requires arbitrary choices by the observer, and again is not Lorentz invariant. For example, while the open set of timelike vectors $\{h(v) < 0\}$ is invariantly defined, there is no Lorentz invariant choice of timelike vector. Likewise among the spacelike set $\{h(v) > 0\}$ there is no Lorentz invariant choice of orthogonal three-dimensional frame.

Furthermore, for the motion of light pulses according to (A12) the Minkowski element vanishes identically, and the only canonical tensor element becomes the constant zero element. After a Lorentz change of variables we find a new solution ϕ' as above, but this solution need not be radial in the inertial frame K' – the rigid K-space motion A does not often preserve the space coordinates ξ, η, ζ of K'. Thus we find A-motions nontrivially depend on the K'-time variable τ. This again reflects the nonexistence of a Lorentz invariant radius.

4. Some Objections and Responses

Given the controversial nature of this article, we here respond to some potential objections. First, one might object that our argument reduces to the observation that spheres in the frame K are transformed to ellipsoids in K', as well-known [Ein+05, §4]. But we remind the reader that the Lorentz contraction is assumed to affect *material* objects, even independantly of the nature of the material, and material spheres in K are seen as material ellipsoids in K', where again the eccentricity of the K'-ellipsoid is nontrivial and independant of the material nature of the sphere. But we respond that light spheres are *not* material, and not themselves subject to Lorentz contraction if (A2) holds.

Second, critics may object that (A12) only requires the consistent *measurement* of c in arbitrary reference frames K, K'. This would replace the formal "law" (A2) with some rule of thumb for measurements. But this immediately leads to a well-known experimental difficulty at the core of special relativity, namely the impossibility of measuring the *one-way* speed of light. For space and time measurements are always dependant on material objects and often non local, having sources and receivers separated by large distances. The impossibility of synchronizing non local clocks leads to the impossibility of measuring the one-way velocity of light. That all measurements of c only succeed in measuring the "two-way" or "round-trip" velocities of light where source and receiver coincide, is discussed in [Zha97], [Pér11]. See also [Ver]. Moreover in studying the two-way velocity of light, one needs further postulate that the velocity c is constant (uniform) throughout its journey, as Einstein himself supposed, [Ein19, Ch.8]. But this assumption is unverifiable. This article argues that the incompatibility of (A12) is not merely apparent, but *essential* evidence that (A2) is not the correct natural law for (A2) and is not subject to measurement.

Third, the interesting textbook [Rin89, pp.8-10, 21–22] attempts

> *"in spite of its historical and heuristic importance, . . . to de-emphasize the logical role of the law of light propagation [(A2)] as a pillar of special relativity."*

Rindler claims that

> *"a second axiom [(A2)] is needed* only *to determine the value of a constant c of the dimensions of a velocity that occurs naturally in the theory. But this could come from any number of branches of physics – we need only think of the energy formula $E = mc^2$, or de Broglie's velocity relation $uv = c^2$."*

Rindler's objection is very interesting. Our response is simply that the above quoted formulas are *equivalent* to (A2), and not independant in any logical or physical sense. The constant c is of course central to physics. But it appears not widely known that c was first formulated and estimated by Wilhelm Weber circa 1846, and even before J.C. Maxwell's famous treatise. Weber further studied c with G. Kirchoff in the telegraphy equations. C.f. [Ass99b] and [Ass03]. Here c is the velocity of an electric signal propagated through a wire of arbitrarily small resistance. But observe that the Weber-Kirchoff definition of c is not equivalent to the c of Einstein's special relativity: Einstein defines c as the velocity in vacuum, and Weber-Kirchoff define c as velocity of signal propagation in a *material* wire! So all formulas involving c are based essentially on some form of (A2), and the logical pillar remains unmoved.

A fourth objection might criticize our argument for not properly accounting for the so-called wave-particle duality of light, or Bohr's complementarity. Our article treats both cases – corpuscular and undulatory – showing that (A12) is underdetermined in both cases. In section 3 we observed that "radial solutions" of the homogeneous wave equation do not constitute a Lorentz invariant set. That is, there exists no solutions ϕ which are radial in every inertial frame. The photon theory is addressed in §2.

The incompatibility of (A12) with *both* the wave and particle model has been highlighted by A.K.T. Assis [Ass99a, §7.2.4, pp.133]:

> *"we can only conclude that for Einstein the velocity of light is constant not only whatever the state of motion of the emitting body [source], but also whatever the state of motion of the receiving body (detector) and of the observer."*

For waves in physical medium, the velocity of emission is independant of the velocity of the source. For waves are transmitted *by* the medium and their velocities become properties *of* the medium. Furthermore for both particles and waves, it is known that the velocity of the wave is dependant on the velocity of the receiver. According to (A2), light then exhibits properties quite unlike both waves and particles. In this sense we argue that (A2) itself contradicts the supposed complementarity and wave-particle duality, i.e. that (A2) requires light to behave contrary to both the wave and particle hypothesis. We refer the reader to Assis' work for further details [Ibid].

5. Ralph Sansbury's Experiment

Is it possibly that **light is not *something* that travels through space?**. This was proposed by Ralph Sansbury [San], and the following experiment is quoted in full from R. Sansbury's book [San12]. Recall that c is well approximated at 1 foot per nanosecond.

> *(Case 1) A* 15 *nanosecond light pulse from a laser was sent to a light detector,* 30 *feet away. When the light pulse was blocked at the photodiode during the time of emission, but unblocked at the expected time of arrival,* 31.2 *nanoseconds after the beginning of the time of emission, for 15 nanosecond duration, little light was received. (A little more than the* $4mV$ *noise on the oscilloscope). This process was repeated thousands of times per second.*
>
> *(Case 2) When the light was unblocked at the photodiode during the time of emission (*15 *nanoseconds) but blocked after the beginning of the time of emission, during the expected time of arrival for* 15 *nanoseconds, twice as much light was received (*$8mV$*). This process was repeated thousands of times per second.*
>
> *This indicated that light is not a moving wave or photon, but rather the cumulative effect of instantaneous forces at a distance. That is, undetectable oscillations of charge can occur in the atomic nuclei of the photodiode that spill over as detectable oscillations of electrons after a delay.*

This important experiment has apparently not been repeated, despite it's simplicity. We refer the reader to R. Sansbury's book [Ibid] for further details and explanation via his *cumulative instantaneous action at a distance* theory of light.

6. Conclusion

This article lays out in plain mathematical terms a persistent error in the first principles of special relativity, namely that radius r is not Lorentz invariant. The error is subtle, and is easily overlooked. It is possible that the error stems from a deeper error, namely that light needs be *something that travels* through space in the eyes of men. This is the subject of future investigation.

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