LORENTZ NON INVARIANCE OF SPHERICAL LIGHTWAVES IN SPECIAL RELATIVITY

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ABSTRACT. The subject of this note is to consider whether spherical light waves are Lorentz invariant or non invariant in Einstein's special relativity. In this article we reason that the spatial quantity called "radius" in the literature is not a Lorentz invariant quantity. Equivalently we argue that the homogeneous wave equation does not have a nontrivial class of Lorentz invariant radial solutions. Thus we find the hypothesis that wave fronts generated by light pulses are round spheres in every inertial frame is not consistent with the principle of relativity. This is controversial subject already developed by other authors, e.g. [Bry], [Cro19]. The present article aims to present the controversy in elementary mathematical terms.

CONTENTS

1. Admitted Incompatibilities and Attempted Resolutions

We begin with Einstein's presentation of the basic principles of special relativity [Ein19], [Ein+05], where appears the alleged proof of the Lorentz invariance of spherical lightwaves. In Einstein's words [Ein19, Ch.7, 11]:

> "*There is hardly a simpler law in physics than that according to which light is propagated in empty space. Every child at school knows, or believes he knows, that this propagation takes place in straight lines with velocity* $c = 300,000 km/sec$*.... Who would imagine that this simple*

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law has plunged the conscientiously thoughtful physicist into the greatest intellectual difficulties?"

The primary intellectual difficulty is to reconcile the fundamental axioms of special relativity, namely:

(A1) that the laws of physics are the same in all nonaccelerated reference frames, i.e. if K' is a coordinate system moving uniformly (and devoid of rotation) with respect to a coordinate system K, then natural phenomena run their course with respect to K' according to exactly the same laws as with respect to K.

(A2) that light in vacuum propagates along straight lines with constant velocity $c \approx 300,000$ kilometres per second [one foot per nanosecond].

Let the reader observe the reference to "law" in the above quote. Einstein presents (A2) as the *law* of propagation of light. If (A2) is to be consistent with (A1), then (A2) must necessarily hold true in *every* inertial reference frame. But the conjunction of (A1) and (A2), abbreviated (A12), appears contradictory to the laws of classical mechanics, e.g. Fizeau's law of addition of velocities, etc.. And this is the difficulty which faced by the thoughtful physicist.

Yet Einstein claims that all this *is only an apparent incompatibility*. The incompatibility is reconciled, according to Einstein's reasoning, by postulating Lorentz-Fitzgerald's length contractions and time dilations, c.f. [Mic95, Ch.XIV]. Thus what needs be demonstrated is that the formulae of the Lorentz transformations preserves the form of the law (A2) in every inertial frame. This requires the Lorentz invariance of luminal spherical waves, as we now discuss.

2. Lorentz Invariant and Non Invariant Tensors

Here we review the basic linear algebra involved in special relativity, namely a linear group of transformations called the Lorentz transformations. These transformations were hypothesized as an attempt to explain the observed null effect of Michelson-Morley's famous interferometer experiment. We recall that this experiment was supposed to measure the speed of light relative to the *aether*. The transformations relate the space and time coordinates (x, y, z, t) and (ξ, η, ζ, τ) of two inertial observers K and K', respectively. They are defined as the isometry group of the quadratic form $h := ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$. It is assumed that h is a scalar invariant for all inertial observers. Here c is the constant luminal velocity posited by (A2) in vacuum. We caution the reader against casually setting $c := 1$ and treating t as a space variable immediately comparable to x, y, z. The constant c, whether its numerical value is 1 or not (and with respect to which units?) is necessary to transition from time units to space units.

Next we describe the *photon* model of light. This is similarly treated in [Lev]. *If light satifies (A2), then in a reference frame K, light is something $\gamma(t) = (x(t), y(t), z(t))$ that travels through space with time, and whose velocity, if it could be materially measured, would satisfy*

$$(dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2 = c^2.$$

And in this sense it is argued that light trajectories are constrained to the null cone $N = \{h = 0\}$ of Minkowski's metric h. Obviously N is Lorentz invariant and is defined by the equation $x^2 + y^2 + z^2 = c^2t^2$. Invariance says $\xi^2 + \eta^2 + \zeta^2 - c^2\tau^2$ is numerically equal to $x^2 + y^2 + z^2 - c^2t^2$ for every Lorentz transform λ satisfying $(\xi, \eta, \zeta, \tau) = \lambda \cdot (x, y, z, t)$. It can be shown that Minkowski's form is the *only* Lorentz invariant quadratic form on $\mathbb{R}^4$ modulo rescaling, c.f. [Elt10], [Arm+18].

Now we turn to our critical analysis. We claim the positive gap in Einstein's attempted proof has a twofold source. **First an error arises when quadratic expressions like**

$$\xi^2 + \eta^2 + \zeta^2 = c^2\tau^2 \tag{1}$$

are *misidentified* with "the equation of a sphere". Strictly speaking, (1) is a three-dimensional cone in the four variables ξ, η, ζ, τ. Of course the cone contains many spherical two-dimensional subsets – but a second independant equation is necessary to specify the metric sphere. We can projectivize the equation (1) and obtain a topological sphere, but again there is no canonical metric invariant with respect to the Lorentz group on the projectivization.

For instance the standard round sphere S centred at the origin simultaneously satisfies (1) and additionally the equation

$$\frac{1}{2}d(\xi^2 + \eta^2 + \zeta^2) = \xi d\xi + \eta d\eta + \zeta d\zeta = 0.$$

In short, the round sphere requires that *two* quadratic forms $\xi^2 + \eta^2 + \zeta^2$ and $c^2\tau^2$ be *simultaneously constant*. This leads us to Einstein's second error, which is the failure to observe that **the Lorentz invariance of the quadratic form** $h = x^2+y^2+z^2-c^2t^2$ **in no way implies the Lorentz invariance of** $h_1 := x^2+y^2+z^2$ **and** $h_2 := c^2t^2$. . Indeed the quadratic forms $h_1.h_2$ are degenerate, with nontrivial radicals satisfying $rad(h_1) = \{x = y = z = 0\}$ and $rad(h_2) = \{t = 0\}$. The radicals are linear subspaces of $\mathbb{R}^4$. But if h_1, h_2 are invariant, then $rad(h_1)$ and $rad(h_2)$ are also nontrivial invariant subspaces. This contradicts the fact that the standard linear representation of the Lorentz group acts irreducibly on $\mathbb{R}^4$.

Now we present a simple computation to illustrate the numerical incompatibility of (A12). The computation is essentially two-dimensional in the variables (x, t) and (ξ, τ). For numerical convenience we set $c := 1$. Then $h = x^2 - t^2$ is a quadratic form on $\mathbb{R}^2$ invariant with respect to the one-dimensional Lorentz group $G = SO(1, 1)_0$

generated by $a_\theta := \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix}$, for $\theta \in \mathbb{R}$. In two dimensions the null cone $x^2 - t^2 = 0$ is projectively a 0-dimensional sphere, hence consisting of two projective points represented by the affine lines $x - t = 0$ and $x + t = 0$. The round sphere $\{x^2 = 1\}$ consists of two vectors in the null cone, namely $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$. Multiplying by a_θ on the left of these vectors, we find the images $\begin{pmatrix} \xi \\ \tau \end{pmatrix}$ equal to $\begin{pmatrix} \cosh\theta + \sinh\theta \\ \cosh\theta + \sinh\theta \end{pmatrix}$ and $\begin{pmatrix} -\cosh\theta + \sinh\theta \\ \cosh\theta - \sinh\theta \end{pmatrix}$. But evidently $\xi^2 \neq x^2 = 1$ and $\tau^2 \neq t^2 = 1$ when $\theta \neq 0$. Thus the quadratic forms $h_1 = x^2$ and $h_2 = t^2$ are not a_θ-invariant. Likewise we find the image of the unit sphere $x^2 = 1$ does not correspond to a spatial sphere in (ξ, τ) coordinates. But these trivial computations have the effect of falsifying the alleged Lorentz invariance of spherical lightwaves.

3. Radius is Not a Lorentz Invariant Variable

In this section we provide another view based on the homogeneous wave equation. Here we are considering smooth real-valued functions $\phi = \phi(x, y, z, t)$ in $3 + 1$ independant variables.

$$\phi_{xx} + \phi_{yy} + \phi_{zz} - \frac{1}{c^2}\phi_{tt} = 0. \tag{2}$$

If λ is a Lorentz transformation with $(\xi, \eta, \zeta, \tau) = \lambda \cdot (x, y, z, t)$, then

$$\phi' := \lambda^* \phi = \phi \circ \lambda^{-1}$$

is again a solution of the homogeneous wave equation

$$\phi_{\xi\xi} + \phi_{\eta\eta} + \phi_{\zeta\zeta} - \frac{1}{c^2}\phi_{\tau\tau} = 0.$$

This is verified by elementary computation, substituting the formulae for the Lorentz transform. **But the set of *radial solutions* of (2) is *not* Lorentz invariant.**

There are different ways to see the Lorentz noninvariance of radius r. In group theoretic terms, a solution $\phi = \phi(x, t)$ of (2) is said to be *radial by an observer K* iff $\phi(Ax, t) = \phi(x, t)$ for every rigid motion $A \in SO(3)$ in the space variables x, y, z of K. Implicitly this requires a Lie group representation ρ of $SO(3)$ into the Lorentz group $G \simeq O(3, 1)$. But this representation of the maximal compact subgroup is noncanonical and cannot be invariantly chosen. Different inertial observers K, K' generally choose different orthogonal symmetry groups, for instance as defined by their own "physical sum of squares" formulae, applied to physical squares ξ^2, η^2, ζ^2 in their local variables ξ, η, ζ, τ. Of course the Minkowski element (1) is invariant and canonical, but any decomposition into "spatial" and "time" requires arbitrary choices by the observer, and again is not Lorentz invariant. For example, while the open set of timelike vectors $\{h(v) < 0\}$ is invariantly defined, there is no Lorentz invariant choice of timelike vector. Likewise among the spacelike set $\{h(v) > 0\}$ there is no Lorentz invariant choice of orthogonal three-dimensional frame.

Furthermore, for the motion of light pulses according to (A12) the Minkowski element vanishes identically, and the only canonical tensor element becomes the constant zero element. After a Lorentz change of variables , we find a new solution ϕ' as above, but this solution need not be radial in the inertial frame K' – the rigid K-space motion A does not often preserve the space coordinates ξ, η, ζ of K'. Thus we find A-motions nontrivially depend on the K'-time variable τ. This again reflects the nonexistence of a Lorentz invariant radius.

4. Some Objections and Responses

Given the controversial nature of this article, we here respond to some potential objections. First, one might object that our argument reduces to the observation that spheres in the frame K are transformed to ellipsoids in K', as well-known [Ein+05, §4]. But we remind the reader that the Lorentz contraction is assumed to affect *material* objects, even independantly of the nature of the material, and material spheres in K are seen as material ellipsoids in K', where again the eccentricity of the K'-ellipsoid is nontrivial and independant of the material nature of the sphere. But we respond that light spheres are *not* material, and not themselves subject to Lorentz contraction if (A2) holds.

Second, critics may object that (A12) only requires the consistent *measurement* of c in arbitrary reference frames K, K'. This would replace the formal "law" (A2) with some rule of thumb for measurements. But this immediately leads to a well-known experimental difficulty at the core of special relativity, namely the impossibility of measuring the *one-way* speed of light. For space and time measurements are always dependant on material objects and often non local, having sources and receivers separated by large distances. The impossibility of synchronizing non local clocks leads to the impossibility of measuring the one-way velocity of light. That all measurements of c only succeed in measuring the "two-way" or "round-trip" velocities of light where source and receiver coincide, is discussed in [Zha97], [Pér11]. See also [Ver]. Moreover in studying the two-way velocity of light, one needs further postulate that the velocity c is constant throughout its journey, as Einstein himself supposed, [Ein19, Ch.8]. But this assumption is unverifiable. This article argues that the incompatibility of (A12) is not merely apparent, but *essential* evidence that (A2) is not the correct natural law for (A2) has never been and cannot be subject to measurement.

Third, the interesting textbook [Rin89, pp.8-10, 21–22] attempts

> *"in spite of its historical and heuristic importance, ... to de-emphasize the logical role of the law of light propagation [(A2)] as a pillar of special relativity."*

Rindler claims that

> *"a second axiom [(A2)] is needed* only *to determine the value of a constant c of the dimensions of a velocity that occurs naturally in the theory. But this could come from any number of branches of physics – we need only think of the energy formula $E = mc^2$, or de Broglie's velocity relation $uv = c^2$."*

Rindler's objection is very interesting, and we have a simple response: the above quoted formulas are *equivalent* to (A2), and not independant in any logical or physical sense. The constant c is of course central to physics. But it appears not widely known that c was first formulated and estimated by Wilhelm Weber circa 1846, and even before J.C. Maxwell's famous treatise. Weber further studied c with G. Kirchoff in the telegraphy equations. C.f. [Ass99b] and [Ass03]. Here c is the velocity of an electric signal propagated through a wire of arbitrarily small resistance. Let the reader remark that the Weber-Kirchoff definition of c is not equivalent to the c of Einstein's special relativity: Einstein defines c as the velocity in vacuum, and Weber-Kirchoff define c as velocity of propagation in a material wire! Thus far all formulas involving c are based essentially on some form of (A2). And so the logical pillar remains unmoved.

A fourth objection might criticize our argument for not properly accounting for the so-called wave-particle duality of light, or Bohr's complementarity. Our article treats both cases (corpuscular and undulatory), showing that (A12) is false in either case. In section 3 we observe that "radial solutions" of the homogeneous wave equation is not a Lorentz invariant set. That is, there exists no solutions ϕ which are radial in every inertial frame. The photon theory is treated in section 2, see the second paragraph for example.

The incompatibility of (A12) with *both* the wave and particle model has been highlighted by A.K.T. Assis [Ass99a, §7.2.4, pp.133]:

> *"we can only conclude that for Einstein the velocity of light is constant not only whatever the state of motion of the emitting body [source], but also whatever the state of motion of the receiving body (detector) and of the observer."*

For waves in physical medium, the velocity of emission is independant of the velocity of the source. For waves are transmitted *by* the medium and their velocities become properties of the medium. Furthermore for both particles and waves, it is also known that the velocity of the wave is dependant on the velocity of the receiver. According to (A2), light then exhibits properties quite unlike both waves and particles. In this sense we argue that (A2) itself contradicts the supposed complementarity and wave-particle duality. See [Ibid] for further references.

5. Ralph Sansbury's Experiment

Is (A12) incoherent and contradictory because **light is not *something* that travels through space?**. This was proposed by Ralph Sansbury [San], and the following experiment is quoted in full from R. Sansbury's book [San12]. Recall that c is well approximated at 1 foot per nanosecond.

> *(Case 1) A* 15 *nanosecond light pulse from a laser was sent to a light detector,* 30 *feet away. When the light pulse was blocked at the photodiode during the time of emission, but unblocked at the expected time of arrival,* 31.2 *nanoseconds after the beginning of the time of emission, for 15 nanosecond duration, little light was received. (A little more than the* $4mV$ *noise on the oscilloscope). This process was repeated thousands of times per second.*
>
> *(Case 2) When the light was unblocked at the photodiode during the time of emission (*15 *nanoseconds) but blocked after the beginning of the time of emission, during the expected time of arrival for* 15 *nanoseconds, twice as much light was received (*$8mV$*). This process was repeated thousands of times per second.*
>
> *This indicated that light is not a moving wave or photon, but rather the cumulative effect of instantaneous forces at a distance. That is, undetectable oscillations of charge can occur in the atomic nuclei of the photodiode that spill over as detectable oscillations of electrons after a delay.*

This important experiment has apparently not been repeated, despite it's simplicity. We refer the reader to R. Sansbury's book [Ibid] for further details and explanation via *cumulative instantaneous action at a distance*.

6. Conclusion

This article lays out in plain mathematical terms a persistent error in the first principles of special relativity, namely that radius r is not Lorentz invariant. The error is subtle, and is easily overlooked. It is possible that the error stems from a deeper error, namely that light needs be *something that travels* through space in the eyes of men. This is the subject of future investigation.

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