Foundations of Special Relativity and Light Propagation in Vacuum : A Critical Review.

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Abstract

This essay is a critical review of the mathematical foundations of Einstein's special relativity (SR) and especially the propagation of light in vacuum. Constructive comments are welcome at the author's address jhmlabs at gmail dot com.

Introduction

This essay is a critical review of the mathematical foundations of special relativity (SR) and the propagation of light *in vacuum*. The present work developed out of the author reviewing articles [Bry], [Cro19] which claimed there were errors in Einstein's original proof of the Lorentz invariance of spherical light waves (c.f. §0.3). The goal of this essay is to critically review these mathematical controversies in the simplest possible terms.

The essay is structured as follows. In §0.1 we present Einstein's assumptions, labelled (A1) and (A2), which are the axiomatic foundations of SR. The conjunction of these assumptions, labelled (A12), leads to apparent incompatibilities which are reconciled, according to Einstein by the Lorentz-Fitzgerald formulae. In §0.2 we review the Lorentz-Fitzgerald formulae, and examine consequences of (A12) in the photon model of light on the Minkowski null cone. Here we find the law of propagation of the photon particle is *strongly underdetermined* by (A12). This is contrary to Riemannian intuition and implies that the motion of a photon along a straight line with speed c in vacuum is not uniqly prescribed. In §0.3 we examine Einstein's proof of the compatibility of (A12) and his appeal to spherical light waves. Here we consider two critical errors in Einstein's classical argument. These errors are summarized in the fact that *radius r is not a Lorentz invariant variable*. In §0.4 we illustrate with some elementary computations in the $(1+1)$-spacetime $\mathbb{R}^{1,1}$. The wave model is considered in §0.5 where we examine the homogeneous wave equation (HWE). The Lorentz non-invariance of spherical lightwaves is expressed as the impossibility of defining Lorentz invariant *radius* and *Lorentz invariant radial solutions to HWE*. In §0.6 we look to anticipate and address possible objections to our arguments. In §0.7 we discuss an experiment proposed by Ralph Sansbury's experiment, and which we present as a modified Fizeau sawtooth experiment.

1 Assumptions, Difficulties, Lorentz Formulae

We begin with Einstein's introduction to his theory of special relativity [Ein+05], [Ein19, Ch7, 11]:

> "*There is hardly a simpler law in physics than that according to which light is propagated in empty space. Every child at school knows, or believes he knows, that this propagation takes place in straight lines with velocity $c = 300,000$ km/sec [one foot per nanosecond].... Who would imagine that this simple law has plunged the conscientiously thoughtful physicist into the greatest intellectual difficulties?*"

Elaborating the "intellectual difficulties" of this supposedly "simple law" is the primary subject of this essay. The difficulty is to reconcile the basic assumptions of SR, namely:

(A1) that the laws of physics are the same in all nonaccelerated reference frames, i.e. if K' is a coordinate system moving uniformly (and devoid of rotation) with respect to a coordinate system K, then natural phenomena run their course with respect to K' according to exactly the same laws as with respect to K.

(A2) that light in vacuum propagates along straight lines with constant velocity $c \approx 300,000$ *km/sec* [one foot per nanosecond].

Assumption (A1) is generally known as the *principle of restricted relativity*, and represents a meta principle which posits that the *form* of the laws of physics need be the same for all observers in uniform relative motion. This assumption introduces the class of inertial frames and raises the question *how to change variables between different inertial frames K, K'?* Einstein does not precisely define what a "law" is, but we might assume law includes logical mathematical law. More specifically (A2) is assumed to represent such a law, and this is clear from the "schoolboy" quoted above. The assumption (A2) is called the *law of propagation of light in vacuum*.

What are the intellectual difficulties arising from the conjunction of (A1) and (A2), abbreviated (A12)? Evidently (A2) posits a law of nature and (A1) requires this law to hold true in every inertial frame K'. But (A2) is not invariant with respect to Galilean change of inertial frames and is indeed contradictory to classical mechanics, e.g. the additivity of energy and Fizeau's law of addition of velocities. Yet Einstein asserts that these difficulties are only *apparent incompatibilities*. The incompatibilities are reconciled – according to Einstein's reasoning – by postulating Lorentz-Fitzgerald's formulae for length contractions and time dilations, e.g.[Mic95, Ch.XIV]. The assertion that Lorentz transformations reconcile (A12) in fact contains two claims:

Claim (i): that inertial frames K, K' are related by Lorentz transformations;

Claim (ii): that the law of propagation of light (A2) is Lorentz invariant, i.e. if (A2) is satisfied in K, then (A2) is satisfied in every Lorentz translate $K' = \lambda.K$.

We remark that the assumption (A2) does not explicitly assume either a particle or wave model of light. As we describe below, Einstein attempts to prove assertion (ii) from a wave-theoretic viewpoint, arguing in terms of light pulses and spherical wavefronts. However Minkowski's linear algebra and Lorentz transformations represent the photon corpuscular model. Our critical analysis applies to both models, and we argue that (A12) is incompatible with *both* the particle and wave models of light.

The linear algebra of Minkowski and Lorentz transformations plays a definitive role in SR. Lorentz transformations in the setting of SR can be defined as the group of linear transformations $\lambda : \mathbb{R}^4 \to \mathbb{R}^4$ which satisfy $\lambda^*(h) = h$ where $h = ds^2$ is the Minkowski-Lorentz quadratic form

$$h := ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2. \tag{1}$$

Here c is the constant luminal velocity in vacuum posited by (A2). The Lorentz invariance of h says $\xi^2 + \eta^2 + \zeta^2 - c^2\tau^2$ is numerically equal to $x^2 + y^2 + z^2 - c^2t^2$ for every Lorentz transform λ satisfying $(\xi, \eta, \zeta, \tau) = \lambda \cdot (x, y, z, t)$. It is assumed that h is a scalar invariant for all inertial observers. It is interesting result of Elton and Arminjon that Minkowski's form is the *only* Lorentz invariant quadratic form on $\mathbb{R}^4$ modulo homothety, c.f. [Elt10], [Arm+18]. This has useful consequence for the homogeneous wave equation as we discuss below.

The linear group of Lorentz transformations was hypothesized as an attempt to explain the observed null result of Michelson-Morley's interferometer experiment. The experiment was intended to measure variations of the speed of light relative to the *aether*. No such variations were discovered, and it was *postulated* that the usual space and time coordinates (x, y, z, t) and (ξ, η, ζ, τ) of two inertial observers K and K', respectively, were not related by Galilean transformations, but related by the Lorentz-Fitzgerald formulae.

The postulate asserts that incredibly and contrary to all observation, the material arm of the interferometer contracts in the direction of motion and simultaneously the time parameter contracts by the inverse ratio, namely the so-called beta factor $\beta = 1/\sqrt{1 - v^2/c^2}$, and moreover this contraction is *independant* of the materials involved.

2 Photons, Triviality of Variational Equation on Null Cone

For the purposes of our discussion, it is critical that we remind the reader of the distinction between "velocity" and "speed". As is well known, velocity is a vector in a tangent space, an "arrow" which has a magnitude ("speed") and a direction. We read (A2) as declaring that light propagates in vacuum in fixed directions (along straight lines) and with constant magnitude ("speed" c). This is the standard Riemannian interpretation of velocity. Yet in Lorentzian geometry, the magnitude or "speed" loses its Riemannian meaning, and as we argue below, the law of propagation (A2) is strongly underdetermined from the Riemannian perspective.

Now we say something about the photon model of light as treated in [Lev77, III.XI.6, pp.301]. If light satisfies (A2), then in a reference frame K light is *something* $\gamma(t) = (x(t), y(t), z(t))$ that travels through space with time, and whose velocity γ' *if it could be materially measured as a function of time* t would satisfy

$$||\gamma'||^2 = (dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2 = c^2. \tag{2}$$

Thus it is argued that light trajectories are constrained to the null cone $N := \{h = 0\}$ of Minkowski's metric h. Obviously the null cone N is a Lorentz invariant subspace of $\mathbf{R}^{1,3}$. and defined by the equation $x^2 + y^2 + z^2 = c^2t^2$ in any reference frame K with coordinates (x, y, z, t).

What does the identity (2) *say about the propagation of light?* If light is something γ that travels through vacuum, then (2) implies the *speed* of γ is identically equal to c as measured in K. But the speed of γ only partially prescribes the motion. Indeed the Lorentz invariance of the cone N implies that *any* differentiable curve satisfying $\gamma(t) \in N$ for all t represents a trajectory with constant "speed" c. Curves γ maintain a large degree of freedom on the null cone N.

Einstein's assumption (A2) attempts to prescribe the light propagation by assuming the motion is along straight lines with constant speed. The straight line propagation is a convenient hypothesis since it's preserved by the linearity of the Lorentz action. Thus the hypothesis (A2) that light propagates along straight lines with constant *speed* c in every inertial reference frame K' *is* invariant with respect to Lorentz transformations. We admit that the conjunction (A12) is consistent up to this point. But if γ has constant direction and constant speed, the motion of γ is still underdetermined on N! See §0.4 for illustration. This is critical distinction between Riemannian and Lorentzian metrics. Therefore we argue that Claim (ii) is not correct and that (A2) is *not* Lorentz invariant.

The problem is that the schoolboy's image of the propagation of light in vacuum is a Riemannian perspective. But this Riemannian perspective is *not* Lorentz invariant. Indeed in Riemannian geometry, if a particle is travelling in a straight line and with constant velocity, then the motion of the particle (it's position as a function of time) *is* uniquely determined. However in Lorentzian geometry, a particle which is travelling in a straight line along the null cone will always have a constant speed, regardless of its trajectory. Supposing that the trajectory is confined to a straight line, there still remains the question of the position of the particle as a function of time. The problem is that the *uniformity* of the straight line propagation is meaningless on the null cone of Lorentzian geometry.

Here it's convenient to introduce Levi-Civita's approach as represented in his excellent text [Lev77]. Levi-Civita modifies (A2) somewhat by asserting that "the propagation of light is rectilinear, uniform, and with velocity c". The term "uniform" does not feature in Einstein's formulation of (A2), although it speaks to the hidden assumption that the light rays have a canonical parameter (describing the *uniform* motion of the light ray). The above remarks are directly related to Levi-Civita's characterization of geometric optics in the following two equations (see [Lev77, p. III.XI.16]):

$$\delta \int ds = 0 \tag{3}$$

and

$$ds^2 = 0. \tag{4}$$

The first equation (3) says the variational derivative of the functional $\gamma \mapsto \int_\gamma ds$ vanishes on the light trajectories, and the second equation (4) says the trajectory is constrained to the null cone. In the Riemannian setting where ds is positive definite, the equation (3) is variationally equivalent to the geodesic equation $\nabla_{\gamma'}\gamma' = 0$. However in the Lorentzian setting we find (3) reduces to $0 = 0$ on the null cone N. Thus the usual Riemannian $ds > 0$ argument does *not* establish the corresponding “geodesic” equation on N. This is acknowledged in [Lev77, p. III.XI.14] but Levi-Civita argues that zero length geodesics are limits of Riemannian geodesics ($ds > 0$) and that “there is a process of passing to the limit (in conditions of complete analytical regularity) from ordinary geodesics”. Levi-Civita maintains that the variational equation (3) somehow “implies” the geodesic-type equation $\nabla_{\gamma'}\gamma' = 0$ for light rays, c.f. [Lev77, p. III.XI.18].

Our viewpoint is that $\nabla_{\gamma'}\gamma' = 0$ is an *independant* hypothesis, and by no means a formal consequence of (3). We reason that contrary to Levi-Civita's claims, the variational equation (3) on $N = \{ds = 0\}$ is *trivial*. In Riemannian geometric terms, we find straight lines on N have *no* canonical parameterizations, even affine. This reveals a clear distinction between Riemannian straight lines which *do* have a canonical arclength parameter ds, and the null lines $\ell \subset N$ which *do not* admit canonical ds arclength parameters *except the trivial* $ds = 0$.

3 Critique of Einstein's Spherical Wave Proof and (A12)

Now we turn to our critical analysis of Einstein's spherical wave “proof” of the compatibility of (A12). We were much influenced by the works of [Bry], [Cro19], and here present our approach to Einstein's subtle error. We recall that Einstein's spherical wave proof looks to derive Claim (ii) from §0.1 as a consequence of Claim (i). The starting point is the fundamental property of Lorentz transforms, that the quadratic expression

$$x^2 + y^2 + z^2 = c^2 t^2$$

is invariant with respect to Lorentz transforms. In other words the null cone N is Lorentz *covariant*. Thus if

$$(\xi, \eta, \zeta, \tau) = \lambda.(x, y, z, t)$$

then the equation

$$\xi^2 + \eta^2 + \zeta^2 = c^2 \tau^2 \tag{5}$$

holds. But what does the equation (5) signify?

We claim the errors in Einstein's argument are twofold. **First an error arises when quadratic expressions like (5) are *misidentified* with “the equation of a sphere”.** Strictly speaking (5) is a three-dimensional cone in the four independant variables ξ, η, ζ, τ. Of course the cone contains many spherical two-dimensional subsets, but our point is that a second independant equation is necessary to specify these metric spheres.

The Penrose approach is to projectivize the equation (5) and obtain a projective sphere with a well defined conformal structure [PR84, Ch 1.]. But again there is no canonical metric invariant with respect to the Lorentz group on the projectivization. In otherwords, the null sphere has a Lorentz invariant *conformal* structure, but it does not have a Lorentz invariant *Riemannian metric* structure. For instance the standard round sphere S centred at the origin simultaneously satisfies (5) and additionally the equation

$$\frac{1}{2}d(\xi^2+\eta^2+\zeta^2)=\xi d\xi+\eta d\eta+\zeta d\zeta=0.$$

In short, the round sphere requires that *two* quadratic forms $\xi^2+\eta^2+\zeta^2$ and $c^2\tau^2$ be *simultaneously constant*.

This leads us to Einstein's second error, the failure to observe that **the Lorentz invariance of the quadratic form $h=x^2+y^2+z^2-c^2t^2$ does not imply the Lorentz invariance of $h_1:=x^2+y^2+z^2$ and $h_2:=c^2t^2$**. Indeed the quadratic forms h_1, h_2 are degenerate, with nontrivial radicals $rad(h_1)=\{x=y=z=0\}$ and $rad(h_2)=\{t=0\}$. The radicals are linear subspaces of $\mathbb{R}^4$. But if h_1, h_2 are invariant, then $rad(h_1)$ and $rad(h_2)$ are also nontrivial invariant subspaces. This contradicts the fact that the standard linear representation of the Lorentz group acts irreducibly on $\mathbb{R}^4$.

To summarize the two errors in Einstein's spherical wave argument: the cone (5) is misidentified as the equation of a sphere; and the Lorentz invariance of $\xi^2+\eta^2+\zeta^2-c^2\tau^2$ does not imply the Lorentz invariance of the forms $\xi^2+\eta^2+\zeta^2$ and $c^2\tau^2$. That is, the numerical equality between $\xi^2+\eta^2+\zeta^2$ and $c^2\tau^2$ is always maintained, but the numerical values obtained by these forms are not Lorentz invariant. We risk repeating ourselves here because this is subtle point which is easily overlooked.

4 Elementary Computations in $\mathbb{R}^{1,1}$

Now we present a simple computation to illustrate the numerics involved in (A12), and to illustrate the basic ideas of the previous sections. We restrict ourselves to two variables (x,t) and (ξ,τ). For numerical convenience we set $c:=1$. Thus $h=x^2-t^2$ is a quadratic form on $\mathbb{R}^2$ invariant with respect to the one-dimensional Lorentz group $G=SO(1,1)_0$ generated by

$$a_\theta := \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix}$$

for $\theta \in \mathbb{R}$. In two dimensions the null cone

$$N = \{x^2 - t^2 = 0\}$$

projectivizes to a 0-dimensional sphere consisting of two projective points represented by the affine lines $x - t = 0$ and $x + t = 0$. The round 0-dimensional sphere $\{x^2 = 1\}$ consists of two vectors in the null cone, namely $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$. Left translating these vectors by a_θ we find the translates

$$\begin{pmatrix} \xi \\ \tau \end{pmatrix} = \begin{pmatrix} \cosh\theta + \sinh\theta \\ \cosh\theta + \sinh\theta \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -\cosh\theta + \sinh\theta \\ \cosh\theta - \sinh\theta \end{pmatrix}.$$

But evidently

$$\xi^2 \neq x^2 = 1^2 = 1 \quad \text{and} \quad \tau^2 \neq t^2 = 1$$

when $\theta \neq 0$. Thus the quadratic forms $h_1 = x^2$ and $h_2 = t^2$ are not a_θ-invariant. Likewise we find the image of the unit sphere $x^2 = 1$ does not correspond to a spatial sphere in (ξ, τ) coordinates.

These trivial computations have the effect of falsifying the alleged Lorentz invariance of spherical lightwaves. However the "slope" of $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and

$$a_\theta . \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \cosh\theta + \sinh\theta \\ \cosh\theta + \sinh\theta \end{pmatrix}$$

is identically equal to $c = 1$ in accordance with (A2).

To further illustrate our earlier comments on the nonexistence of canonical parameters on N, consider that for any C^1 monotonic function $f : \mathbb{R} \to \mathbb{R}$ we obtain a curve $\epsilon(t) = \epsilon_f(t) = \begin{pmatrix} f(t) \\ f(t) \end{pmatrix} = f(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ for $t \in \mathbb{R}$. Then $\epsilon_f(t)$ is supported on a straight line in N, and the "photon" ϵ_f can be said to propagate in a straight line with constant speed $c = 1$. It might be said that ϵ_f is *not uniform* in the parameter t, but we argue that there exists no *Lorentz invariant* definition of "uniform parameter", especially on the null cone.

5 Homogeneous Wave Equation, Radius is Not Lorentz Invariant

At this stage in our essay we now consider the wave interpretation in more detail. Our first step is to demonstrate a simple relationship between the Lorentz invariance of Minkowski's ds^2 (1) and d'Alembert's operator $\Box$. Informally we view $\Box$ as "dual" to ds^2 in the following sense. In §0.2 we referred to the results of [Elt10], [Arm+18] on the uniqueness of Lorentz invariant quadratic forms modulo homothety. Their same proof implies the following:

Lemma. *Let C be the algebra of polynomial functions on $\mathbb{R}^{1,3}$ and the contragredient representation ρ^* of the Lorentz group. Then d'Alembert's wave operator $\Box$ is the unique Lorentz invariant second order linear operator on C modulo homothety.*

Therefore Einstein's assertion (i) that Lorentz transformations define the change-of-variables formulae for inertial observers K, K' also implies that d'Alembert's operator $\Box$ is essentially the unique second order operator defined simultaneously for all inertial observers.

The Lorentz invariance of $\Box$ implies the solutions of the homogeneous wave equation (HWE) are Lorentz covariant. So if $\phi = \phi(x, y, z, t)$ is a regular function satisfying (HWE)

$$\phi_{xx} + \phi_{yy} + \phi_{zz} - \frac{1}{c^2}\phi_{tt} = 0. \tag{6}$$

and if λ is a Lorentz transform with $(\xi, \eta, \zeta, \tau) = \lambda \cdot (x, y, z, t)$, then

$$\phi' := \lambda^*(\phi) = \phi \circ \lambda^{-1}$$

is a solution of the homogeneous wave equation

$$\phi_{\xi\xi} + \phi_{\eta\eta} + \phi_{\zeta\zeta} - \frac{1}{c^2}\phi_{\tau\tau} = 0.$$

This is verified by elementary computation, substituting the formulae for the Lorentz transform.

The main point we want to emphasize is this: **the set of *radial solutions* of (6) is *not* Lorentz invariant**. Indeed the spatial radius variable $r^2 = x^2 + y^2 + z^2$ is not a Lorentz invariant variable. This was discussed in §0.3, but we give another explanation in terms of Lie groups below. In group theoretic terms, a solution $\phi = \phi(x, t)$ of (6) is said to be *radial by an observer K* iff $\phi(Ax, t) = \phi(x, t)$ for every rigid motion $A \in SO(3)$ in the space variables x, y, z of K. Implicitly this requires a Lie group representation ρ of $SO(3)$ into the Lorentz group $G \simeq O(3, 1)$. But this choice of maximal compact subgroup is noncanonical. Different inertial observers K, K' generally choose different orthogonal symmetry groups, e.g. by their own "physical sum of squares" formula $\xi^2 + \eta^2 + \zeta^2$. Of course the Minkowski element (5) is invariant and canonical, but any decomposition into "spatial" and "time" requires arbitrary choices by the observer, and these choices are not Lorentz invariant. For example, while the open set of timelike vectors $\{h(v) < 0\}$ is invariantly defined, there is no Lorentz invariant choice of timelike vector. Likewise among the spacelike set $\{h(v) > 0\}$ there is no Lorentz invariant choice of orthogonal three-dimensional frame.

Furthermore, for the motion of light pulses according to (A12) the Minkowski element vanishes identically, and the only canonical tensor element becomes the constant zero element. After a Lorentz change of variables we find a new solution ϕ' as above, but this solution need not be radial in the inertial frame K'. Indeed the rigid K-space motion A will not generally preserve the space coordinates ξ, η, ζ of K'. Thus we find A-motions nontrivially depend on the K'-time variable τ. This again reflects the nonexistence of a Lorentz invariant radius.

6 Some Objections and Responses

Given the critical nature of this essay, we here respond to some potential objections. First, one might object that all our arguments reduce to the observation that spheres in the frame K are transformed to ellipsoids in K' as is well-known [Ein+05, §4]. But we remind the reader that Lorentz contraction is assumed to affect *material* objects, even independantly of the nature of the material. So material spheres in K become material ellipsoids in K', where the eccentricity of the K'-ellipsoid is nontrivial and independant of the material nature of the sphere. But we respond that light spheres are *immaterial* and not themselves subject to Lorentz contraction. In fact if light spheres were subject to the same effects as material spheres, then (A2) would definitely be false.

Second, critics may object that (A12) only requires the consistent *measurement* of c in arbitrary reference frames K, K'. This would replace the formal "law" (A2) with some rule of thumb for measurements. But this immediately leads to a well-known experimental difficulty at the core of special relativity, namely the impossibility of measuring the *one-way* speed of light. For space and time measurements are always dependant on material objects and often non local, having sources and receivers separated by large distances. The impossibility of synchronizing non local clocks leads to the impossibility of measuring the one-way velocity of light. That all measurements of c only succeed in measuring the "two-way" or "round-trip" velocities of light where source and receiver coincide is discussed in [Zha97], [Pér11]. See also [Ver]. Moreover in studying the two-way velocity of light, one needs further postulate that the velocity c is constant (uniform) throughout its two-way journey, as Einstein argued [Ein19, Ch.8]. But this assumption is arbitrary and unverifiable.

Third, the interesting textbook [Rin89, pp.8-10, 21–22] admittedly attempts

> *"in spite of its historical and heuristic importance, ... to de-emphasize the logical role of the law of light propagation [(A2)] as a pillar of special relativity."*

Rindler claims that

> *"a second axiom [(A2)] is needed* only *to determine the value of a constant c of the dimensions of a velocity that occurs naturally in the theory. But this could come from any number of branches of physics – we need only think of the energy formula* $E = mc^2$*, or de Broglie's velocity relation* $uv = c^2$*."*

Rindler's objection is interesting, and our response is simply that the above quoted formulas are *equivalent* to (A2), and not independant in any logical or physical sense. The constant c is of course central to physics, and c was first formulated and estimated by Wilhelm Weber circa 1846, and even before J.C. Maxwell's famous treatise. Weber further studied c with G. Kirchoff in the telegraphy equations. C.f. [Ass99b] and [Ass03]. Here c is the velocity of an electric signal propagated through a wire of arbitrarily small resistance. But observe that the Weber-Kirchoff definition of c is not equivalent to the c of Einstein's special relativity: Einstein defines c as the velocity in vacuum, and Weber-Kirchoff define c as velocity of signal propagation in a *material* wire! So all formulas involving c are based essentially on some form of (A2), and the logical pillar remains unmoved. In otherwords, there does not appear any independant relation involving c *en vacuo* apart from Einstein's (A2).

A fourth objection might criticize our argument for not properly accounting for the so-called wave-particle duality of light, e.g. "Bohr's complementarity". Our presentation has addressed both corpuscular and undulatory models, showing that (A12) is underdetermined in *both* cases. In section 0.5 we observed that "radial solutions" of the homogeneous wave equation do not constitute a Lorentz invariant set: there does not exist solutions ϕ of the wave equation which are radial in every inertial frame. The photon theory is addressed in §0.2. The incompatibility of (A12) with *both* the wave and particle model has been highlighted by A.K.T. Assis [Ass99a, §7.2.4, pp.133]:

> *"we can only conclude that for Einstein the velocity of light is constant not only whatever the state of motion of the emitting body [source], but also whatever the state of motion of the receiving body (detector) and of the observer."*

For waves in physical medium, the velocity of emission is independant of the velocity of the source, since waves are transmitted *by* the medium and their velocities a property *of* the medium. Furthermore for both particles and waves, it is known that velocity is dependant on the velocity of the receiver. According to (A2), light is postulated to exhibit properties unlike both waves and particles. Indeed we argue that (A2) *contradicts* the supposed complementarity and wave-particle duality, i.e. (A2) requires light to behave contrary to *both* the wave and particle interpretations. We refer the reader to Assis' work for further details [Ibid].

7 Ralph Sansbury's Experiment

Is it possible that **light is not *something* that travels through space?**. This was proposed by Ralph Sansbury [San], and the following experiment is quoted in full from R. Sansbury's book [San12]. Recall that c is well approximated at 1 foot per nanosecond.

> *(Case 1) A* 15 *nanosecond light pulse from a laser was sent to a light detector,* 30 *feet away. When the light pulse was blocked at the photodiode during the time of emission, but unblocked at the expected time of arrival,* 31.2 *nanoseconds after the beginning of the time of emission, for* 15 *nanosecond duration, little light was received. (A little more than the* $4mV$ *noise on the oscilloscope). This process was repeated thousands of times per second.*
>
> *(Case 2) When the light was unblocked at the photodiode during the time of emission (*15 *nanoseconds) but blocked after the beginning of the time of emission, during the expected time of arrival for* 15 *nanoseconds, twice as much light was received (*$8mV$*). This process was repeated thousands of times per second.*
>
> *This indicated that light is not a moving wave or photon, but rather the cumulative effect of instantaneous forces at a distance. That is, undetectable oscillations of charge can occur in the atomic nuclei of the photodiode that spill over as detectable oscillations of electrons after a delay.*

To the author's knowledge, Sansbury's experiment has never been sufficiently investigated nor reviewed. Apparently Sansbury found the equipment necessary for his setup too expensive to rent for an extended period of time, and it's possible that Sansbury was not sufficiently experienced in calibrating the equipment. We refer the reader to R. Sansbury's book [Ibid] for further details and his explanation via *cumulative instantaneous action at a distance* theory of light. Sansbury's book and papers are very interesting, but they are not completely persuasive on this point.

In this section we propose a modification of Hippolyte Fizeau's 1849 spinning sawtooth experiment to test Sansbury's proposal. We remark that Fizeau was historically looking to estimate the luminal velocity c. Sansbury's experiment assumes c as given, estimated at 1 foot per nanosecond. Sansbury's goal is to distinguish light propagation from particle model, and his setup is meant to test whether light is even something that travels at all.

In Fizeau's original setup, light travelled a total path of $\approx 16km$. With an expected speed of light $c = 3 \times 10^8 km$, then the expected travel time is

$$\frac{16 \times 10^3 [m]}{3 \times 10^8 [m]/[sec]} = 5.333 \ldots \times 10^{-5} [sec]. \tag{7}$$

Now consider the wheel with angular velocity ω having units of $[degrees]/[sec]$ and having a toothlength equal to 1/720 degrees. The time required to turn one toothlength is therefore $\frac{1/720}{\omega} = \frac{1}{720\omega}$ (units of [sec]). Thus we find that the expected travel time is equal to the time to rotate one toothlength if the following equality holds

$$\frac{16 \times 10^3}{3 \times 10^8} = \frac{1}{720\omega},$$

which implies

$$\omega \approx 26 \quad \frac{[rotations]}{[sec]}.$$

Fizeau's involves several reflecting mirrors (beam splitters). Therefore there is more interaction involved in Fizeau's setup than with Sansbury's. For Fizeau's is a type of two-way trip of light, where the source and receiver are space-coincident. But Sansbury's is a one-way trip, requiring some electronics at the receiver namely a photodiode, to measure the amount of electrons released by the light emission.

We could test some of Sansbury's ideas if we could increase the angular velocity of the Fizeau wheel by factor of 4, i.e. we need a wheel of roughly 100 revolutions per second instead of 20 revolutions per second. Given such a revolution speed, then we could change the sawtooth pattern of the wheels, having some that are 1/4 closed, 1/2 closed, and 3/4 closed wheels. For example, we could have the alternating sawtooth

$$\ldots 0101010101 \ldots$$

or we could have

$$\ldots 001100110011 \ldots$$

both of which are 1/2 closed but having different patterns. And these patterns would have different predictions depending on the photon model or Sansbury's cumulative action-at-a-distance. Likewise it would be interesting to compare the predictions given a wheel having a 1/4-closed sawtooth pattern

$$\ldots 0001000100010001 \ldots$$

versus a 3/4-closed pattern

$$\ldots 0111011101110111 \ldots.$$

If we could get the Fizeau wheel to spin 200 revolutions per second, then we could test the theories according to 8-periodic patterns, i.e. with sawtooth patterns being $1/8, 2/8, \ldots, 7/8$ths closed, etc.. If we could build a larger wheel with more teeth, say, 1440 teeth, then 2880 teeth, then basic gear ratio would increase the speed of the initial pinion wheel by factor of $2, 4, \ldots$. Thus a Fizeau saw-wheel with regular but variable sawtooth patterns could provide surprising outcomes, and an apparatus to validate or invalidate R. Sansbury's proposal.

References

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