ON CRITICAL FOUNDATIONS OF SPECIAL RELATIVITY AND LIGHT PROPAGATION IN VACUUM

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Abstract.

Contents

1. Introduction

This article is a critical essay on the mathematical foundations of special relativity (SR) and the propagation of light in vacuum. The present article arose from the author reviewing several articles [Bry], [Cro19] which claimed there were subtle errors in Einstein's original proof of the Lorentz invariance of spherical light waves, see §4 below. The purpose of this article is to present some controversial issues in SR in plain mathematical terms. Thus we hope this article is interesting to both adherents and skeptics of the SR theory. For something of the extremely interesting history of skepticism and Einstein, the reader might be interested in [Ken16].

The sections of this essay are structured as follows. In §2 we present Einstein's assumptions (A1), (A2) which logically underpin SR, and the admitted apparent incompatibility in their conjunction (A12). In §3 we introduce the Lorentz formulae and the null cone and consider consequences of (A12) in the photon model of light, but here find the law of propagation of the photon particle is *underdetermined* by

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(A12). This is counter to the Riemannian intuition, and says one cannot determine the time-position or motion of a photon moving along a straight line with speed c in vacuum. Indeed the photon's motion with time still has degrees of freedom even though bound to a line and having speed c. In §4 we examine Einstein's proof of the compatibility of (A12) and his appeal to spherical light waves. We here identify two critical errors in Einstein's argument, but summarized in the fact that r is not a Lorentz invariant variable. In §5 we illustrate with some elementary computations in the $1+1$ spacetime $\mathbb{R}^{1,1}$. Thus we argue that (A2) is an incomplete description of light, and additional assumptions are necessary. These further assumptions were provided by Levi-Civita in the formulas of geometric optics. The wave model is considered in §6 where we examine the homogeneous wave equation (HWE). The Lorentz non invariance of spherical lightwaves is phrased in terms of the impossibility of defining a Lorentz invariant *radius* and a Lorentz invariant class of radial solutions to HWE. In §7 we try to anticipate and address some counter-objections to our arguments. In §8 we enter into controversial issue of Ralph Sansbury's experiment, and an intriguing alternative to the assumptions (A2).

2. Assumptions and Difficulties in SR

We begin our study of the foundations of SR with Einstein's presentation of the theory [Ein19], [Ein+05]. In Einstein's words [Ein19, Ch.7, 11]:

> "*There is hardly a simpler law in physics than that according to which light is propagated in empty space. Every child at school knows, or believes he knows, that this propagation takes place in straight lines with velocity* $c = 300,000$ *km/sec [one foot per nanosecond]. . . . Who would imagine that this simple law has plunged the conscientiously thoughtful physicist into the greatest intellectual difficulties?*"

The elaboration of the "intellectual difficulties" of this supposedly "simple law" is the main subject of this essay. The primary difficulty is to reconcile the following axiomatic assumptions of SR, namely:

(A1) that the laws of physics are the same in all nonaccelerated reference frames, i.e. if K' is a coordinate system moving uniformly (and devoid of rotation) with respect to a coordinate system K, then natural phenomena run their course with respect to K' according to exactly the same laws as with respect to K.

(A2) that light in vacuum propagates along straight lines with constant velocity $c \approx 300,000$ *km/sec* [one foot per nanosecond].

Assumption (A1) is generally known as the *principle of restricted relativity*, and represents a meta principle that the *form* of the laws of physics need be the same for all observers in uniform relative motion. This assumption introduces the class of inertial frames and raises the question of how to "change variables" between different inertial frames K, K'.

It's left somewhat undefined what a "law" is, but we might assume law includes mathematical laws. More specifically (A2) is assumed to represent such a law, and this is clear from the "schoolboy" quoted above. The assumption (A2) is coined the *law* of *propagation* of light in vacuum.

The great intellectual difficulty arises from the conjunction of (A1) and (A2), abbreviated (A12). If (A2) is to be consistent with (A1), then the law (A2) must hold true in every inertial reference frame K'. But (A12) appears contradictory to classical mechanics, e.g. Fizeau's law of addition of velocities and the additivity of energy. Yet Einstein claims the difficulty is only an *apparent incompatibility.* The incompatibility is reconciled according to Einstein's reasoning by postulating Lorentz-Fitzgerald's length contractions and time dilations, c.f. [Mic95, Ch.XIV]. This claim that Lorentz transformations reconcile (A12) in fact contains two assertions:

(i) that inertial frames K, K' are related by Lorentz transformations;
(ii) that the law of propagation of light (A2) is Lorentz covariant, i.e. if (A2) is satisfied in K, then (A2) is satisfied in every Lorentz translate $K' = \lambda.K$.

The assumption (A2) does not explicitly assume either a particle or wave model of light. As we describe below, Einstein attempts to prove assertion (ii) from a wave-theoretic viewpoint, arguing in terms of light pulses and spherical wavefronts. However the Minkowski linear algebra which arises from the Lorentz transformations represents the photon corpuscular model. Our critical analysis applies to both models, and in fact we argue that ultimately (A12) is incompatible with *both* the particle and wave interpretations of light. We develop our argument and line of reasoning in the following sections.

3. Photons and Null Cone

For a proper analysis of (A2) we must remind the reader of the distinction between "velocity" and "speed". As is well known, velocity is a vector in a tangent space, an "arrow" which has a magnitude ("speed") and a direction. We read (A2) as declaring that light propagates in vacuum in fixed directions (along straight lines) and with constant magnitude (the "speed" c). This is the standard Riemannian conception of vector. Yet in Lorentzian geometry, the magnitude or "speed" loses its Riemannian meaning, and as we argue below, the law of propagation (A2) is strongly underdetermined from the Riemannian perspective.

The linear algebra of Minkowski and Lorentz transformations plays a definitive role in SR. The linear group of Lorentz transformations was hypothesized as an attempt to explain the observed null result of Michelson-Morley's interferometer experiment. The experiment was intended to measure variations of the speed of light relative to the *aether*. No such variations were discovered, and it was *postulated* that the usual space and time coordinates (x, y, z, t) and (ξ, η, ζ, τ) of two inertial observers K and K', respectively, were not related by Galilean transformations, but related by the Lorentz-Fitzgerald formulae. The heuristic was that incredibly and contrary to all expectations, the material arm of the interferometer contracted in the direction of motion and simultaneously the time parameter was contracted by the same ratio, namely the so-called gamma factor $\gamma = 1/\sqrt{1 - v^2/c^2}$. A review of the history of mechanics [Dug] shows that Voigt studied a similar set of transformations of solutions to the homogeneous wave equations, but which solutions did not form a group under composition. Lorentz transformations in the setting of SR can be defined as the group of linear transformations $\lambda : \mathbb{R}^4 \to \mathbb{R}^4$ which satisfy $\lambda^*(h) = h$ where $h = ds^2$ is the Minkowski-Lorentz quadratic form

$$h := ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2. \tag{1}$$

Here c is the constant luminal velocity in vacuum posited by (A2). The Lorentz invariance of h says $\xi^2 + \eta^2 + \zeta^2 - c^2\tau^2$ is numerically equal to $x^2 + y^2 + z^2 - c^2t^2$ for every Lorentz transform λ satisfying $(\xi, \eta, \zeta, \tau) = \lambda \cdot (x, y, z, t)$. It is assumed that h is a scalar invariant for all inertial observers. We remark that Minkowski's form is the *only* Lorentz invariant quadratic form on $\mathbb{R}^4$ modulo homothety, c.f. [Elt10], [Arm+18].

Now we say something about the photon model of light as treated in [Lev, III.XI.6, pp.301]. If light satisfies (A2), then in a reference frame K light is *something* $\gamma(t) = (x(t), y(t), z(t))$ that travels through space with time, and whose velocity γ' *if it could be materially measured as a function of* t would satisfy

$$||\gamma'||^2 = (dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2 = c^2. \tag{2}$$

Thus it is argued that light trajectories are constrained to the null cone

$$N := \{h = 0\}$$

of Minkowski's metric h. Obviously the null cone N is Lorentz invariant subspace of $\mathbf{R}^{1,3}$. and defined by the equation

$$x^2 + y^2 + z^2 = c^2t^2$$

in any reference frame K with coordinates (x, y, z, t).

What does the identity (2) say about the propagation of light? Typically it's argued that (2) demonstrates the *speed* of γ is identically equal to c as measured by K. The Lorentz invariance of the cone N implies that differentiable curves $\gamma(t)$ contained in N will represent light trajectories exhibiting a constant speed c *independant of the trajectory of* γ *on* N. Einstein's (A2) postulates that the propagation is along straight lines, i.e. with constant *direction* in vacuum. This straight line propagation is a convenient hypothesis since it's preserved by the linearity of the Lorentz action.

Thus the hypothesis (A2) that light propagates along straight lines with constant *speed* c in every inertial reference frame K' *is* invariant with respect to Lorentz transformations. Thus we admit that the conjunction (A12) is consistent up to this point.

However the above facts notwithstanding, we maintain that the propagation of light remains *underdetermined* by (A2) from the point of view of Riemannian geometry. Indeed in Riemannian geometry, if a particle is travelling in a straight line and with constant velocity, then the propagation of the particle, namely it's position as a function of time, *is* uniquely determined. However, in Lorentzian geometry, a particle which is travelling in a straight line along the null cone will always have a constant speed, regardless of its trajectory. Supposing that the trajectory is confined to a straight line, there still remains the question of the position of the particle as a function of time. The problem is that the *uniformity* of the straight line propagation is meaningless on the null cone of Lorentzian geometry.

Here we introduce Levi-Civita's approach as represented in his excellent text [Lev]. Levi-Civita modifies (A2) somewhat by asserting that "the propagation of light is rectilinear, uniform, and with velocity c". The term "uniform" does not feature in Einstein's formulation of (A2), although it speaks to the hidden assumption that the light rays have a canonical parameter (describing the *uniform* motion of the light ray). The above remarks are directly related to Levi-Civita's characterization of geometric optics in the following two equations (see [Lev, p. III.XI.16]):

$$\delta \int ds = 0 \tag{3}$$

and

$$ds^2 = 0. \tag{4}$$

The first equation (3) says the variational derivative of the functional $\gamma \mapsto \int_\gamma ds$ vanishes on the light trajectories, and the second equation says the trajectory is constrained to the null cone. In the Riemannian setting where ds is positive definite, the equation (3) is essentially equivalent to the geodesic equation $\nabla_{\gamma'}\gamma' = 0$. However in the Lorentzian setting we find (3) reduces to $0 = 0$ on the null cone N. Thus the usual Riemannian $ds > 0$ argument does *not* establish the corresponding "geodesic" equation on N. This is acknowledged in [Lev, p. III.XI.14] but Levi-Civita argues that zero length geodesics are limits of Riemannian geodesics ($ds > 0$) and that "there is a process of passing to the limit (in conditions of complete analytical regularity) from ordinary geodesics". Levi-Civita maintains that the variational equation (3) somehow "implies" the geodesic-type equation $\nabla_{\gamma'}\gamma' = 0$ for light rays, c.f. [Lev, p. III.XI.18].

Our viewpoint is that $\nabla_{\gamma'}\gamma' = 0$ is an *independant* hypothesis, and by no means a formal consequence of (3). This is related to our contention that contrary to Levi-Civita's claims, the variational equation

$$\delta \int ds = 0$$

on $N = \{ds = 0\}$. In Riemannian geometric terms, we find straight lines on N have *no* canonical parameterizations, even affine. This reveals a clear distinction between Riemannian straight lines which *do* have a canonical arclength parameter ds, and the null lines $\ell \subset N$ which do *not* admit canonical ds arclength parameter *except the trivial* $ds = 0$.

4. Critique of Einstein's Proof of Compatibility of (A12)

Now we turn to our critical analysis of Einstein's "proof" of the compatibility of (A12). We were much influenced by the main results of [Bry], [Cro19].

Einstein's proof looks to derive the assertion (ii) from §2 as a consequence of assertion (i). We claim that the errors in Einstein's attempted proof are twofold. **First an error arises when quadratic expressions like**

$$\xi^2 + \eta^2 + \zeta^2 = c^2\tau^2 \tag{5}$$

are *misidentified* with "the equation of a sphere". Strictly speaking (5) is a three-dimensional cone in the four independant variables ξ, η, ζ, τ. Of course the cone contains many spherical two-dimensional subsets, but our point is that a second independant equation is necessary to specify these metric spheres.

The Penrose approach is to projectivize the equation (5) and obtain a projective sphere with a well defined conformal structure [PR84, Ch 1.]. But again there is no canonical metric invariant with respect to the Lorentz group on the projectivization. In otherwords, the null sphere has a Lorentz invariant *conformal* structure, but it does not have a Lorentz invariant *Riemannian metric* structure. For instance the standard round sphere S centred at the origin simultaneously satisfies (5) and additionally the equation

$$\frac{1}{2}d(\xi^2 + \eta^2 + \zeta^2) = \xi d\xi + \eta d\eta + \zeta d\zeta = 0.$$

In short, the round sphere requires that *two* quadratic forms $\xi^2 + \eta^2 + \zeta^2$ and $c^2\tau^2$ be *simultaneously constant*.

This leads us to Einstein's second error, which is the failure to observe that **the Lorentz invariance of the quadratic form $h = x^2 + y^2 + z^2 - c^2t^2$ in no way implies the Lorentz invariance of $h_1 := x^2+y^2+z^2$ and $h_2 := c^2t^2$.** . Indeed the quadratic forms h_1, h_2 are degenerate, with nontrivial radicals $rad(h_1) = \{x = y = z = 0\}$ and $rad(h_2) = \{t = 0\}$. The radicals are linear subspaces of $\mathbb{R}^4$. But if h_1, h_2

are invariant, then $rad(h_1)$ and $rad(h_2)$ are also nontrivial invariant subspaces. This contradicts the fact that the standard linear representation of the Lorentz group acts irreducibly on $\mathbb{R}^4$.

5. Elementary Computations in $\mathbb{R}^{1,1}$

Now we present a simple computation to illustrate the numerics involved in (A12), and to illustrate the basic ideas of the previous sections. We restrict ourselves to two variables (x,t) and (ξ,τ). For numerical convenience we set $c := 1$. Thus $h = x^2 - t^2$ is a quadratic form on $\mathbb{R}^2$ invariant with respect to the one-dimensional Lorentz group $G = SO(1,1)_0$ generated by

$$a_\theta := \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix}$$

for $\theta \in \mathbb{R}$. In two dimensions the null cone

$$N = \{x^2 - t^2 = 0\}$$

projectivizes to a 0-dimensional sphere consisting of two projective points represented by the affine lines $x - t = 0$ and $x + t = 0$. The round 0-dimensional sphere $\{x^2 = 1\}$ consists of two vectors in the null cone, namely $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$. Left translating these vectors by a_θ we find the translates

$$\begin{pmatrix} \xi \\ \tau \end{pmatrix} = \begin{pmatrix} \cosh\theta + \sinh\theta \\ \cosh\theta + \sinh\theta \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -\cosh\theta + \sinh\theta \\ \cosh\theta - \sinh\theta \end{pmatrix}.$$

But evidently

$$\xi^2 \neq x^2 = 1^2 = 1 \quad \text{and} \quad \tau^2 \neq t^2 = 1$$

when $\theta \neq 0$. Thus the quadratic forms $h_1 = x^2$ and $h_2 = t^2$ are not a_θ-invariant. Likewise we find the image of the unit sphere $x^2 = 1$ does not correspond to a spatial sphere in (ξ,τ) coordinates.

These trivial computations have the effect of falsifying the alleged Lorentz invariance of spherical lightwaves. However the “slope” of $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and

$$a_\theta . \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \cosh\theta + \sinh\theta \\ \cosh\theta + \sinh\theta \end{pmatrix}$$

is identically equal to $c = 1$ in accordance with (A2).

To further illustrate our earlier comments on the nonexistence of canonical parameters on N, consider that for any C^1 monotonic function $f : \mathbb{R} \to \mathbb{R}$ we obtain a curve $\epsilon(t) = \epsilon_f(t) = \begin{pmatrix} f(t) \\ f(t) \end{pmatrix} = f(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ for $t \in \mathbb{R}$. Then $\epsilon_f(t)$ is supported on a straight line in N, and the "photon" ϵ_f can be said to propagate in a straight line with constant speed $c = 1$. It might be said that ϵ_f is *not uniform* in the parameter t, but we argue that there exists no *Lorentz invariant* definition of "uniform parameter", especially on the null cone.

6. Radius is Not a Lorentz Invariant Variable

At this stage in our essay we now consider the wave interpretation in more detail. Our first step is to demonstrate a simple relationship between the Lorentz invariance of Minkowski's ds^2 (1) and d'Alembert's operator $\Box$. Informally we view $\Box$ as "dual" to ds^2 in the following sense. In §3 we referred to the results of [Elt10], [Arm+18] on the uniqueness of Lorentz invariant quadratic forms modulo homothety. Their same proof implies the following:

Lemma. *Let C be the algebra of polynomial functions on $\mathbb{R}^{1,3}$ and the contragredient representation ρ^* of the Lorentz group. Then d'Alembert's wave operator $\Box$ is the unique Lorentz invariant second order linear operator on C modulo homothety.*

Therefore Einstein's assertion (i) that Lorentz transformations define the change-of-variables formulae for inertial observers K, K' also implies that d'Alembert's operator $\Box$ is essentially the unique second order operator defined simultaneously for all inertial observers.

The Lorentz invariance of $\Box$ implies the solutions of the homogeneous wave equation (HWE) are Lorentz covariant. So if $\phi = \phi(x, y, z, t)$ is a regular function satisfying (HWE)

$$\phi_{xx} + \phi_{yy} + \phi_{zz} - \frac{1}{c^2}\phi_{tt} = 0. \tag{6}$$

and if λ is a Lorentz transform with $(\xi, \eta, \zeta, \tau) = \lambda \cdot (x, y, z, t)$, then

$$\phi' := \lambda^*(\phi) = \phi \circ \lambda^{-1}$$

is a solution of the homogeneous wave equation

$$\phi_{\xi\xi} + \phi_{\eta\eta} + \phi_{\zeta\zeta} - \frac{1}{c^2}\phi_{\tau\tau} = 0.$$

This is verified by elementary computation, substituting the formulae for the Lorentz transform.

The main point we want to emphasize is this: **the set of *radial solutions* of (6) is *not* Lorentz invariant**. Indeed the spatial radius variable $r^2 = x^2 + y^2 + z^2$ is not a Lorentz invariant variable. This was discussed in §4, but we give another explanation in terms of Lie groups below. In group theoretic terms, a solution $\phi = \phi(x, t)$ of (6) is said to be *radial by an observer* K iff $\phi(Ax, t) = \phi(x, t)$ for every rigid motion $A \in SO(3)$ in the space variables x, y, z of K. Implicitly this requires a Lie group representation ρ of $SO(3)$ into the Lorentz group $G \simeq O(3,1)$. But this choice of maximal compact subgroup is noncanonical. Different inertial observers K, K' generally choose different orthogonal symmetry groups, e.g. by their own "physical sum of squares" formula $\xi^2+\eta^2+\zeta^2$. Of course the Minkowski element (5) is invariant and canonical, but any decomposition into "spatial" and "time" requires arbitrary choices by the observer, and these choices are not Lorentz invariant. For example, while the open set of timelike vectors $\{h(v) < 0\}$ is invariantly defined, there is no Lorentz invariant choice of timelike vector. Likewise among the spacelike set $\{h(v) > 0\}$ there is no Lorentz invariant choice of orthogonal three-dimensional frame.

Furthermore, for the motion of light pulses according to (A12) the Minkowski element vanishes identically, and the only canonical tensor element becomes the constant zero element. After a Lorentz change of variables we find a new solution ϕ' as above, but this solution need not be radial in the inertial frame K'. Indeed the rigid K-space motion A will not generally preserve the space coordinates ξ, η, ζ of K'. Thus we find A-motions nontrivially depend on the K'-time variable τ. This again reflects the nonexistence of a Lorentz invariant radius.

7. Some Objections and Responses

Given the critical nature of this article, we here respond to some potential objections. First, one might object that all our arguments reduce to the observation that spheres in the frame K are transformed to ellipsoids in K' as is well-known [Ein+05, §4]. But we remind the reader that Lorentz contraction is assumed to affect *material* objects, even independantly of the nature of the material. So material spheres in K become material ellipsoids in K', where the eccentricity of the K'-ellipsoid is nontrivial and independant of the material nature of the sphere. But we respond that light spheres are *immaterial* and not themselves subject to Lorentz contraction. In fact if light spheres were subject to the same effects as material spheres, then (A2) would definitely be false.

Second, critics may object that (A12) only requires the consistent *measurement* of c in arbitrary reference frames K, K'. This would replace the formal "law" (A2) with some rule of thumb for measurements. But this immediately leads to a well-known experimental difficulty at the core of special relativity, namely the impossibility of measuring the *one-way* speed of light. For space and time measurements are always dependant on material objects and often non local, having sources and receivers separated by large distances. The impossibility of synchronizing non local clocks leads to the impossibility of measuring the one-way velocity of light. That all measurements of c only succeed in measuring the "two-way" or "round-trip" velocities of light where source and receiver coincide is discussed in [Zha97], [Pér11]. See also

[Ver]. Moreover in studying the two-way velocity of light, one needs further postulate that the velocity c is constant (uniform) throughout its two-way journey, as Einstein argued [Ein19, Ch.8]. But this assumption is arbitrary and unverifiable.

Third, the interesting textbook [Rin89, pp.8-10, 21–22] admittedly attempts

> *"in spite of its historical and heuristic importance, ... to de-emphasize the logical role of the law of light propagation [(A2)] as a pillar of special relativity."*

Rindler claims that

> *"a second axiom [(A2)] is needed* only *to determine the value of a constant c of the dimensions of a velocity that occurs naturally in the theory. But this could come from any number of branches of physics – we need only think of the energy formula $E = mc^2$, or de Broglie's velocity relation $uv = c^2$."*

Rindler's objection is interesting, and our response is simply that the above quoted formulas are *equivalent* to (A2), and not independant in any logical or physical sense. The constant c is of course central to physics, and c was first formulated and estimated by Wilhelm Weber circa 1846, and even before J.C. Maxwell's famous treatise. Weber further studied c with G. Kirchoff in the telegraphy equations. C.f. [Ass99b] and [Ass03]. Here c is the velocity of an electric signal propagated through a wire of arbitrarily small resistance. But observe that the Weber-Kirchoff definition of c is not equivalent to the c of Einstein's special relativity: Einstein defines c as the velocity in vacuum, and Weber-Kirchoff define c as velocity of signal propagation in a *material* wire! So all formulas involving c are based essentially on some form of (A2), and the logical pillar remains unmoved. In otherwords, there does not appear any independant relation involving c *en vacuo* apart from Einstein's (A2).

A fourth objection might criticize our argument for not properly accounting for the so-called wave-particle duality of light, e.g. "Bohr's complementarity". Our article treats both cases (corpuscular and undulatory) showing that (A12) is underdetermined in *both* cases. In section 6 we observed that "radial solutions" of the homogeneous wave equation do not constitute a Lorentz invariant set: there does not exist solutions ϕ of the wave equation which are radial in every inertial frame. The photon theory is addressed in §3. The incompatibility of (A12) with *both* the wave and particle model has been highlighted by A.K.T. Assis [Ass99a, §7.2.4, pp.133]:

> *"we can only conclude that for Einstein the velocity of light is constant not only whatever the state of motion of the emitting body [source], but also whatever the state of motion of the receiving body (detector) and of the observer."*

For waves in physical medium, the velocity of emission is independant of the velocity of the source, since waves are transmitted *by* the medium and their velocities a property *of* the medium. Furthermore for both particles and waves, it is known that velocity is dependant on the velocity of the receiver. According to (A2), light is postulated to exhibit properties unlike both waves and particles. Indeed we argue that (A2) *contradicts* the supposed complementarity and wave-particle duality, i.e. (A2) requires light to behave contrary to *both* the wave and particle interpretations. We refer the reader to Assis' work for further details [Ibid].

8. Ralph Sansbury's Experiment

Is it possible that **light is not *something* that travels through space?**. This was proposed by Ralph Sansbury [San], and the following experiment is quoted in full from R. Sansbury's book [San12]. Recall that c is well approximated at 1 foot per nanosecond.

> *(Case 1) A* 15 *nanosecond light pulse from a laser was sent to a light detector,* 30 *feet away. When the light pulse was blocked at the photodiode during the time of emission, but unblocked at the expected time of arrival,* 31.2 *nanoseconds after the beginning of the time of emission, for* 15 *nanosecond duration, little light was received. (A little more than the* $4mV$ *noise on the oscilloscope). This process was repeated thousands of times per second.*
>
> *(Case 2) When the light was unblocked at the photodiode during the time of emission (*15 *nanoseconds) but blocked after the beginning of the time of emission, during the expected time of arrival for* 15 *nanoseconds, twice as much light was received (*$8mV$*). This process was repeated thousands of times per second.*
>
> *This indicated that light is not a moving wave or photon, but rather the cumulative effect of instantaneous forces at a distance. That is, undetectable oscillations of charge can occur in the atomic nuclei of the photodiode that spill over as detectable oscillations of electrons after a delay.*

This important experiment has apparently not been repeated, despite it's simplicity. We refer the reader to R. Sansbury's book [Ibid] for further details and explanation via his *cumulative instantaneous action at a distance* theory of light. Discussions on Sansbury's experiment typically argue that one needs the ability to block and unblock the photodiode at 15 nanosecond intervals, and that this is below the speed of so-called "Pockel cells." A modification of Fizeau's tooth wheel experiment might render an experimental apparatus similar to R. Sansbury's setup.

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