ON EINSTEIN-LORENTZ' ALLEGED PROOF OF SPHERICAL INVARIANCE OF LIGHT PULSES IN SPECIAL RELATIVITY

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The subject of this brief note is controversial. We observe the Lorentz $SO(3,1)$-invariance of the quadratic form $h = x^2 + y^2 + z^2 - c^2t^2$ in no way implies the Lorentz invariance of the forms $h_1 = x^2 + y^2 + z^2$ and $h_2 = c^2t^2$. As corollary we find Einstein's proposed resolution of the apparent incompatibility of the principle of restricted relativity and the purported law of the constancy of light invalidated by basic computations. The argument is general and applies to $SO(1,1)$, $SO(2,1)$, etc.

Our goal is to describe a controversy in the foundations of the Einstein's special relativity theory, and especially the apparent incompatibility of the law of propogation of light with the principle of relativity, see [Ein19, Ch.7, 11].

As far as mathematics is concerned, here is the issue in simplest terms. Evidently Minkowski's form $h = x^2 + y^2 + z^2 - c^2t^2$ is invariant with respect to Lorentz transformations, i.e. the Lie group $G = SO(h) = SO(3,1)$ where invariance means $\xi^2 + \eta^2 + \zeta^2 = c^2\tau^2$ for every Lorentz transform ξ, η, ζ, τ of x, y, z, t.

Einstein's argument concerns the geometry of a wave front generated by a light pulse, i.e. the action of the Lorentz group on (subsets of) the so-called null cone $N = \{h = 0\}$, i.e. on subsets where the equation $x^2 + y^2 + z^2 = c^2t^2$ is satisfied. Obviously N is invariant under G. Now comes the key claim: the action on the null cone is nontrivial. More specifically: the Lorentz invariance of h, N in no way implies the Lorentz invariance of the forms $h' = x^2 + y^2 + z^2$ and $h'' = c^2t^2$, and even despite their difference $h = h' - h''$ being so invariant.

The two-dimensional case in xt-variables is illustrative, where $h = x^2 - c^2t^2$ is invariant with respect to the group $G = SO(1,1)$ generated by $a_\theta := \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix}$, where $\theta \in \mathbb{R}$. We see $SO(1,1)$ is isomorphic to the split multiplicative torus $\mathbb{R}^\times_{>0}$ using the logarithm. The unit sphere includes two vectors

$$\langle 1, 1\rangle,\ \langle -1, 1\rangle,$$

and which are mapped by $a_\theta \in SO(1,1) \simeq \mathbb{R}^\times_{>0}$ to

$$\langle \xi, \tau\rangle = \langle \cosh\theta + \sinh\theta, \cosh\theta + \sinh\theta\rangle,\ \langle -\cosh\theta + \sinh\theta, \cosh\theta - \sinh\theta\rangle.$$

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But evidently $\xi^2 \neq x^2 = 1$ and $\tau^2 \neq t^2 = 1$ when $\theta \neq 0$. That is, the image of the unit sphere does not correspond to a sphere in $\xi\tau$ coordinates. In other words the equation $x^2 = c^2t^2$ is not the equation of sphere when x, t are variable, and likewise $\xi^2 = c^2\tau^2$ is not the equation of sphere when ξ, τ are both variable and nonconstant.

The author claims that the above argument shows Einstein's purported proof is invalid in $SO(1,1)$ and similarly in $SO(3,1)$. But what are the consequences of such an argument? According to Einstein, the principles of special relativity begin with Galilean relativity, namely the invariance of the laws of kinematics under affine transformations. It is apparently an experimental fact, that we can all discover, that the Laws of physics are the same in all nonaccelerated reference frames. Einstein calls this the Principle of Restricted Relativity: If K' is a coordinate system moving uniformly (and devoid of rotation) with respect to a coordinate system K, then natural phenomena run their course with respect to K' according to exactly the same laws as with respect to K.

Next Einstein claims "There is hardly a simpler law in physics than that according to which light is propograted in empty space. Every child at school knows, or believes he knows, that this propogation takes place in straight lines with velocity $c = 300000km/sec$. ... Who would imagine that this simple law has plunged the conscientiously thoughtful physicist into the greatest intellectual difficulties?"

Einstein claimed to have resolved this apparent contradiction by Lorentz transformations. I.e. by postulating length contraction and time dilation in the direction of uniform velocity. Einstein attempts to resolve the contradiction in [Ein19][Ch.11, pp.39]. The reader can verify that Einstein does not treat the general case, but restricts himself to a velocity parallel to x-axis. Unfortunately a more detailed demonstration was left to future more critical authors, e.g. [Cro19], [Bry], and nearly one hundred years expired before the error began to be noticed.

References

[Bry] Steven Bryant. "The Failure of the Einstein-Lorentz Spherical Wave Proof". In: *Proceedings of the NPA* 8 (), p. 64.

[Cro19] S.J. Crothers. "Special Relativity and the Lorentz Sphere". In: (2019). URL: https://vixra.org/abs/1911.0013 (visited on 12/04/2020).

[Ein19] Albert Einstein. *Relativity: The Special and the General Theory-100th Anniversary Edition*. Princeton University Press, 2019.