Towards a unified model

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1 Notation

$L_\lambda(\theta)$ denotes a monochromatic luminosity observed at a given viewing angle (erg s^{-1} sr^{-1} Å^{-1}). $L'(\theta)$ denotes the same quantity integrated across a range of wavelengths (erg s^{-1} sr^{-1}) and $\bar{L}$ represents something integrated across wavelength and over the whole sphere (erg s^{-1}). $\hat{L}_\lambda$ represents something integrated over viewing angle but not wavelength (erg s^{-1} Å^{-1}). Possibly it would be better to do this in terms of 'intensities'.

We assume full azimuthal symmetry throughout, symmetry above and below the disc plane. We ignore limb darkening for the moment, even though the code doesn't. We assume isotropic line emission.

2 Calculation

We want to produce spectra that resemble quasars in terms of their emission line equivalent widths at a range of inclinations, especially low ones.

Let us begin by defining the equivalent width, EW, at a given angle, θ, in terms of luminosities

$$\mathrm{EW}(\theta) = -\int 1 - \frac{L_\lambda(\theta)}{L_{\lambda,C}(\theta)} d\lambda \tag{1}$$

where the integral is across the line profile and the subscript C denotes continuum and the minus sign goes against convention by ensuring positivity for emission lines. If the continuum is constant across the line profile we can then write

$$\mathrm{EW}(\theta) = 1/L_{\lambda_0,C}(\theta) \int L_\lambda(\theta) - L_{\lambda,C}(\theta) d\lambda \tag{2}$$

Since the integral is now just the escaping line luminosity at that angle, we have

$$\mathrm{EW}(\theta) = \frac{L'_{ul}(\theta)}{L_{\lambda_0,C}(\theta)} \tag{3}$$

where $L'_{ul}(\theta)$ is the escaping line luminosity at that angle for transition $u \to l$. Note that the total line luminosity integrated across all viewing angles – which is the line luminosity we would get from a "log spec tot" file – is, for an isotropic line, i.e. $L'_{ul}(\theta) =$ constant:

$$\bar{L}_{ul} = \int_0^{2\pi} d\phi \int_0^{\pi} L'_{ul}(\theta) \sin(\theta) d\theta = 4\pi L'_{ul}(\theta) \tag{4}$$

Now let us illuminate some plasma that subtends angles $\theta_{\rm min} \to \theta_{\rm max}$ to the source, defined from the polar axis. The source has total luminosity integrated across all angles and wavelengths of $\bar{L}_{\rm bol}$. The plasma intercepts a fraction of this radiation $f_{\rm illum}$ given by

$$f_{\rm illum}(\theta_{\rm min}, \theta_{\rm max}, \epsilon) = \frac{1}{2\pi} \int_0^{2\pi} d\phi \int_{\theta_{\rm min}}^{\theta_{\rm max}} \epsilon(\theta) \sin(\theta) d\theta. \tag{5}$$

where $\epsilon(\theta)$ is the "angular emissivity function". This function is a constant in the case of an isotropic source, and $\cos\theta$ for a foreshortened disc. The 2π rather than 4π is because we integrate assume symmetry above and below the disc plane. We can also define a line reprocessing efficiency, η_{ul}, which governs how much of the illuminating luminosity is converted into (isotropic) line emission, i.e.

$$\eta = \frac{\bar{L}_{ul}}{f_{\rm illum}\bar{L}_{\rm bol}} = \frac{4\pi L_{ul}(\theta)}{f_{\rm illum}\bar{L}_{\rm bol}} \tag{6}$$

η can be calculated from my simulation grids and tends to be about $0.1 - 0.14$ for Lyman alpha. I adopt 0.1 below. The final thing to worry about is the monochromatic continuum luminosity, $L_{\lambda_0,C}(\theta)$. This must integrate over viewing angle to give the monochromatic continuum luminosity in the log spec tot file, $\hat{L}_{\lambda_0,C}$, i.e.

$$\hat{L}_{\lambda_0,C} = 2\int_0^{2\pi} d\phi \int_0^{\pi/2} L_{\lambda_0,C}(\theta)\sin(\theta)d\theta = 4\pi L_{\lambda_0,C}(\theta=0)\int_0^{\pi/2} \cos(\theta)\sin(\theta)d\theta = 2\pi L_{\lambda_0,C}(\theta=0) \tag{7}$$

which says that if the monochromatic disc luminosity at angle θ is $L_{\lambda_0,C}(\theta = 0)\cos\theta$, then the angle-integrated monochromatic disc luminosity is 2π times the face on value. We can now substitute into our expression for EW to give

$$\mathrm{EW}(\theta) = \eta\hat{L}_{\rm bol}\frac{1}{\hat{L}_{C,\lambda_0}\epsilon(\theta)}\frac{1}{4\pi}\int_0^{2\pi} d\phi \int_{\theta_{\rm min}}^{\theta_{\rm max}} \epsilon(\theta)\sin(\theta)d\theta. \tag{8}$$

Adopting $\theta_{\rm max} = \pi/2$ and integrating gives:

$$\mathrm{EW}(\theta,\theta_{\rm min},\eta) = \eta\hat{L}_{\rm bol}\frac{1}{\hat{L}_{C,\lambda_0}\cos(\theta)}\frac{1}{4}\left[1-\sin^2(\theta_{\rm min})\right] \tag{9}$$

Now we can predict what wind geometry we need. Let's look at Lyman alpha. Our target is EW= 75Å. Let's adopt $\hat{L}_{\rm bol} = 1/2(GM\dot{m})/R_* \approx 2\times 10^{46}$, $\eta = 0.1$, and read the angle-integrated monochromatic luminosity L_{C,λ_0} from our log spec tot file. The results are shown in Fig. 1, which shows EW as a function of viewing angle for different wind geometries.

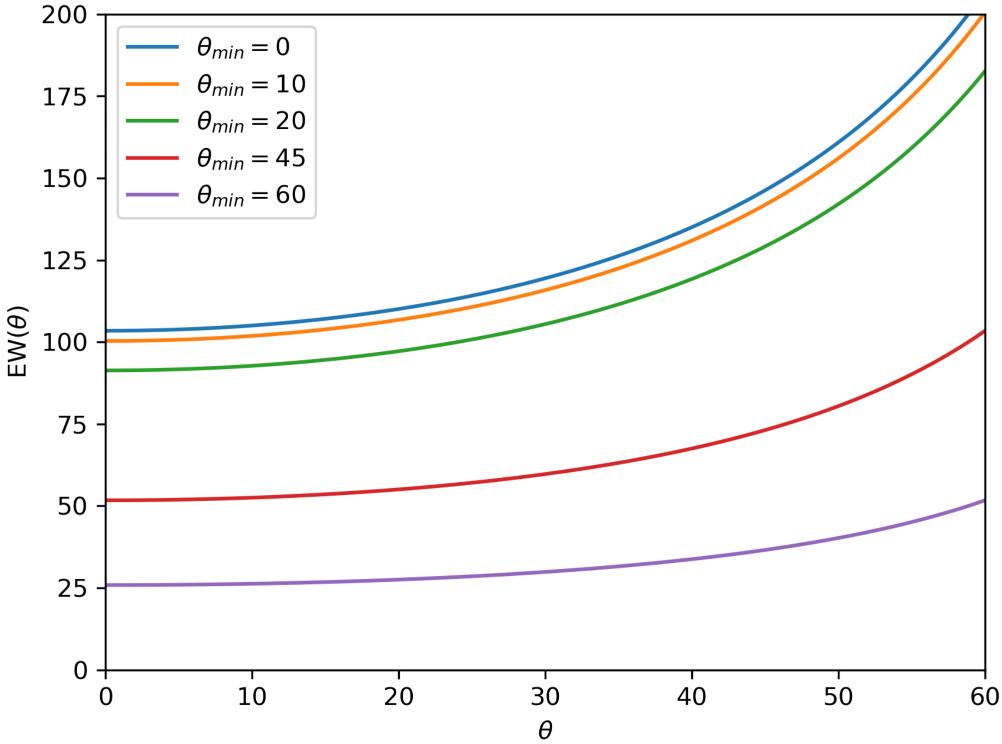

Figure 1: EW as a function of viewing angle for different wind geometries. **I think these should all be lowered by a factor of about 2**

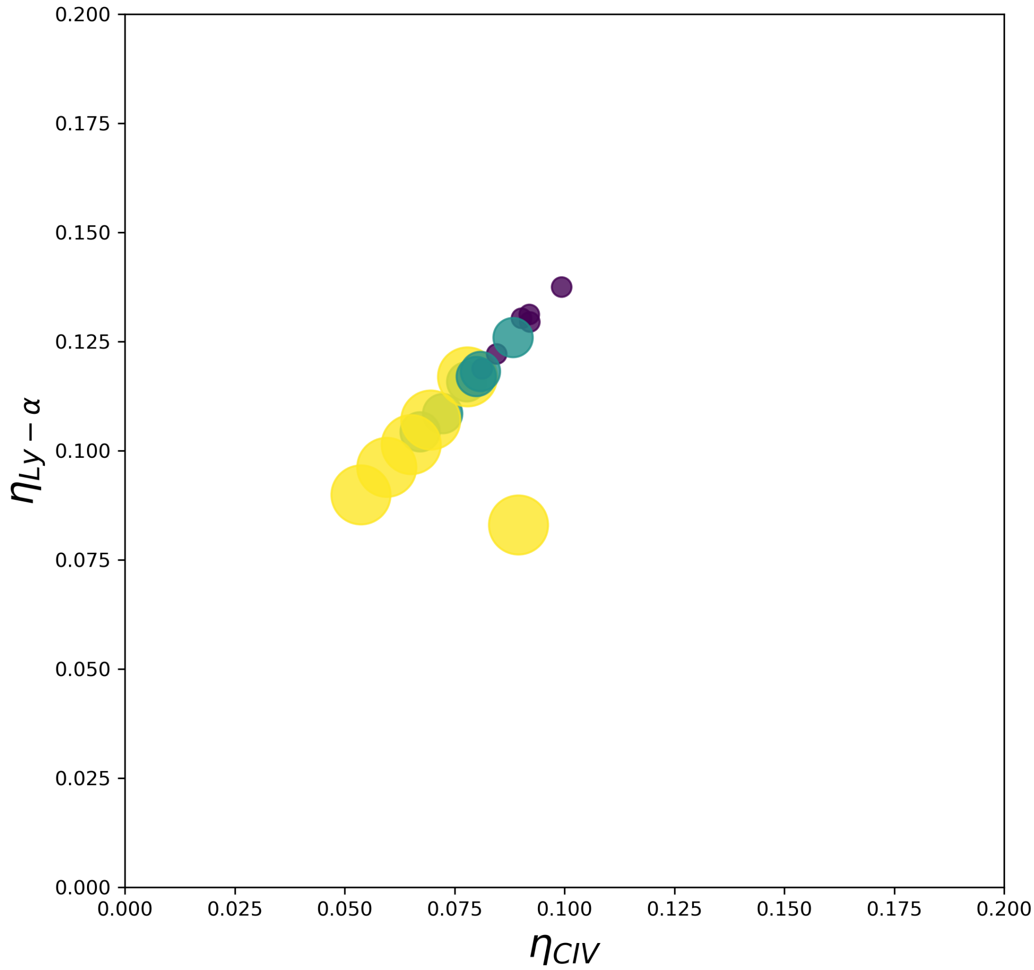

Figure 2: η for CIV and Lyman alpha for a sample wind grid. Larger, darker points mean larger values of $\theta_{\min}$. There is a trend for higher values of η for more collimated winds.