Derivation of Python's expression for adiabatic cooling

James Matthews

February 5, 2014

We can obtain an expression for the adiabatic cooling rate of a parcel of gas by starting from the first law of themodynamics and then considering the continuity equation. In adiabatic expansion, there is no heat input and so the cooling rate is given by

$$\text{cooling} = \frac{dE}{dt} = P\frac{dV}{dt} \tag{1}$$

Which just comes from the 1st law of TD (work done along an adiabat is proportional to PdV). Now consider the continuity equation which arises from conservation of mass:

$$\rho(\nabla \cdot \vec{v}) + \frac{d\rho}{dt} = 0 \tag{2}$$

This equation simply states that if your velocity field is diverging, then your parcel of gas better have a change in density else it requires a net input of mass.

Since $\rho = M/V$ and mass is conserved

$$\frac{d\rho}{dt} = \frac{d}{dt}(M/V) = -M\frac{1}{V^2}\frac{dV}{dt} \tag{3}$$

Substituting into (2)

$$\rho(\nabla \cdot \vec{v}) = \frac{M}{V^2}\frac{dV}{dt} \tag{4}$$

$$\frac{M}{V}(\nabla \cdot \vec{v}) = \frac{M}{V^2}\frac{dV}{dt} \tag{5}$$

$$\frac{dV}{dt} = V(\nabla \cdot \vec{v}) \tag{6}$$

and we now have our expression for $\frac{dV}{dt}$ to plug into equation (1). The only thing that remains is to obtain the expression for pressure, P. We can simply use kinetic theory of an ideal gas to obtain

$$P = n_e k_B T \tag{7}$$

where here we are making the assumption that the electrons dominate the contribution to the pressure. Note that this is clearly not valid in a non-ionized regime. Through this we obtain

$$\text{cooling} = n_e k_B T V (\nabla \cdot \vec{v}) \tag{8}$$

which is identical to Python's expression except for a factor of 1.5 which Python includes, and I don't know why!