Chapter 1

A 'by hand' test of the odd neutral hydrogen cells

Here I'm going to show the results of some bny hand calculations (well actually using a toy code written in python, plus results from interrogativng bython directly) to see if the results seen in some cells of the Proga model are reasonable.

This is all for a single cell, to summarise, in this cell we have the following abundances

$$\frac{n(H\textsc{ii})}{n(H\textsc{i})} = \frac{0.618}{0.382} = 0.618$$

i.e., slightly more Hydrogen in neutral than ioinzed, whereas for oxygen - which has an almost identical ioinization potential

$$\frac{n(O\textsc{ii})}{n(O\textsc{i})} = \frac{3.26e-16}{2.37e-14} = 72.5$$

i.e. much more Oxygen in the singly ionized state than neutral. NB, these number densities are relative to the total number density of that element, so we can see that actually both the neutral and singly ionized states of Oxygen are in the minority - the dominant stage is agcutlly the full ionized one Oix

We can test if this is reasonble by calculating the ratio from slightly more first principles. Implicit in our approximateionization scheme is the assumption that the photo ionization rate should equal the radiative recombination rate or

$$n_i \gamma^p_{i\to i+1} = n_{i+1} n_e \alpha^r_{i+1\to i} \tag{1.1}$$

Where $\gamma^p_{i\to i+1}$ is the photoioinsation rate coefficient and $\alpha^r_{i+1\to i}$ is the radiative recombination rate coefficient.Rearranging we obtain

$$\frac{n_{i+1} n_e}{n_i} = \frac{\gamma^p_{i\to i+1}}{\alpha^r_{i+1\to i}}$$

We can calculate each of these. Firstly, the recombination rate coefficient can be compurted from the Milne relation at the electron temperature of the cell, $T_e = 16200$. This is

$$\alpha^r_{i+1\rightarrow i} = \left(\frac{2\pi m_e k}{h^2}\right)\frac{8\pi}{c^2}\frac{g_i}{g_{i+1}g_e}T^{-3/2}\int_{\nu_0}^{\infty}\nu^2\sigma_{i\rightarrow i+1}(\nu)\exp\left(\frac{-h(\nu-\nu_0)}{kT}\right)d\nu$$

where $\sigma_{i\rightarrow i+1}(\nu)$ is the photoionization cross section Carrying out this integral we obtain

$$\alpha^r_{i+1\rightarrow i} = 1.26\times10^{-13}cm^3s^{-1}$$

for Hydrogen and

$$\alpha^r_{i+1\rightarrow i} = 1.86\times10^{-13}cm^3s^{-1}$$

for Oxygen. We can also calculate the photoinoization rate coefficient from a simple integration over the specific intensity modelled in the cell.

$$\gamma^p_{i\rightarrow i+1} = \frac{4\pi}{h}\int_{\nu_0}^{\infty}J_\nu\alpha_i(\nu)\nu^{-1}d\nu$$

This is where we take account of the actual J_ν in the cell, and the figure below shows that along with the cross sections for HI and OI carrying out the integrals gives us

$$\gamma^p_{i\rightarrow i+1} = 1.86\times10^{-8}s^{-}1$$

for hydrogen and

$$\gamma^p_{i\rightarrow i+1} = 1.66\times10^{-6}s^{-1}$$

for Oxygen.

So, we can write

$$\frac{n(H\textsc{ii})}{n(H\textsc{i})} = \frac{1.86\times10^{-8}}{n_e\times1.26\times10^{-13}} = \frac{1.48\times10^{-5}}{n_e}$$

and

$$\frac{n(O\textsc{ii})}{nO\textsc{i})} = \frac{1.66\times10^{-6}}{n_e\times1.86\times10^{-13}} = \frac{8.92\times10^{-6}}{n_e}$$

So we can see that as expected, the ratio for the oxygen ions is nearly 100 times that of the hydrogen ions, as we saw in the cell. We can cheat a bit, and ask what n_e actually was in the simulation - it was $n_e = 1.23\times10^5$. Clearly, plugging this into the equations above gicves us about the same abundances as seen. So, I think Python is doing it right.

So we come to the question is it physical. Well, possibly not, since a more complete version of equation 1.2 would be

$$n_{i,j}\gamma^{p}_{i\rightarrow i+1} + n_{i,j}\gamma^{c}_{i\rightarrow i+1} = n_{i+1,j}n_e\alpha^{r}_{i+1\rightarrow i} + n_{i+1,j}n_e\alpha^{dr}_{i+1\rightarrow i} + n_{i+1,j}n_e\alpha^{tbr}_{i+1\rightarrow i} \quad (1.2)$$

Where we have added a couple more rates

$\gamma^{c}_{i\rightarrow i+1}$ - the collisional ionisation rate coefficient

$\alpha^{dr}_{i+1\rightarrow i}$ - the dielectronic recombination rate coefficient

and $\alpha^{tar}_{i+1\rightarrow i}$ - the three body recombination rate coefficient.

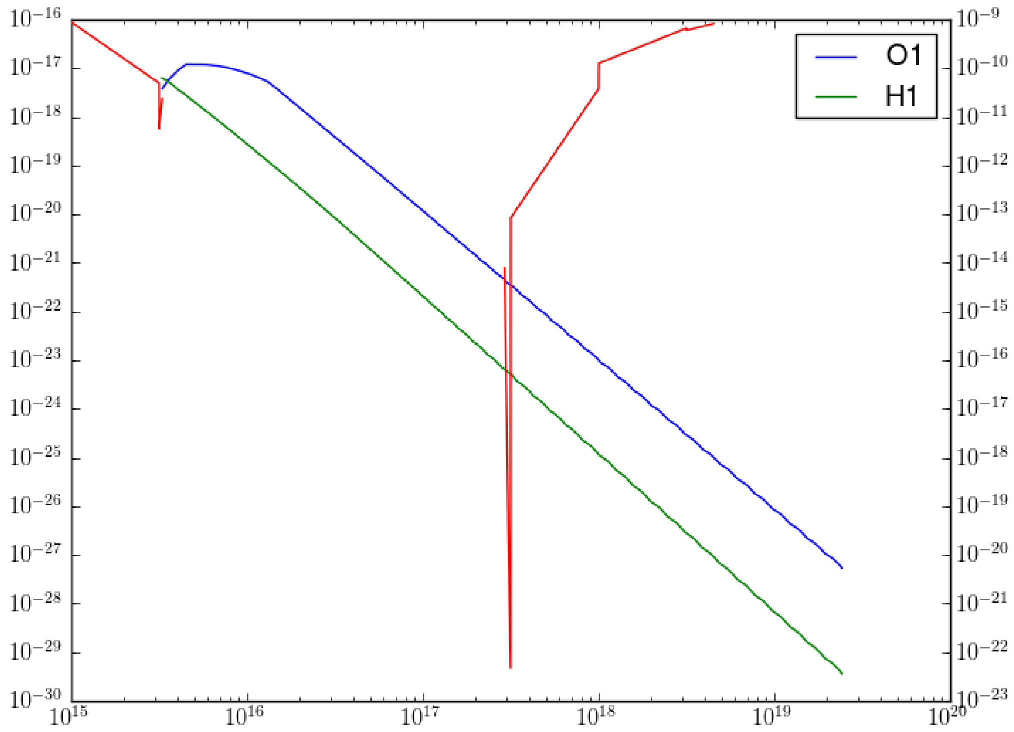
It is a distinct possibility that the collisional rate could dominate over the photoionization rate here - but because it is particularly large, but because the PI rate is so small. This may be the next thing to try and calculate to see if it is important. How to incorporate it into the treatment is up for discussion, however we include dielectronic recombination as an extra additive term in ζ, the recombination rate correction factor:

$$\zeta = \frac{\alpha^{r*}_{i+1,0\rightarrow i,0}}{\sum_l \alpha^{r*}_{i+1,0\rightarrow i,l} + \alpha^{dr}_{i+1\rightarrow i}}$$

so, couled we similarly include collisional ionization in s, our photoinoization rate correction factor.?

$$S = \frac{4\pi/h\int_{\nu_0}^{\infty} J_\nu \alpha_i(\nu)\nu^{-1}d\nu + \gamma^{c}_{i\rightarrow i+1}}{4\pi/h\int_{\nu_0}^{\infty} B_\nu(T_e)\alpha_i(\nu)\nu^{-1}d\nu}$$

If this were reasonable, there are lots of places where these rates are tabulated or functional forms are given....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