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Python

Knox S. Long and Christian Knigge

Space Telescope Science Institute, 3700 San Martin Drive, Baltimore, MD 21218

1.   Introduction

These are notes associated with the MC code Python which has been developed to model CV disk winds. They are intended to elucidate some of the physics behind Python, especially those portions of the physics which evoked some discussion between us as it was written, as well as to write down in one place some of the basic formulae and definitions used in Python. The notes are not systematic.

2.   Basics

2.1.   Definition of thermal equilibrium and local thermodynamic equilibrium

A plasma and radiation field are in **thermal equilibrium** if occupation numbers of photons, ions, free electrons, excited states etc. can be described in terms of a single temperature and statistical weights. In order for a plasma and its associated radiation field to be in thermal balance, the radiation field must be that of a BB and therefore completely isotropic.

If a plasma is in thermal equilibrium, every process within the plasma will be in **detailed balance**. Detailed balance means simply that each excitation process will exactly balanced by an inverse process. The utility of detailed balance is that it implies that each process and inverse can be treated independently of other processes, and so one if one knows with high accuracy the rate association with the excitation processes one can infer the rate for the inverse process with the same accuracy. This is the basis of the Milne relation between photoionization and recombination processes for example.

In any "interesting" radiative transfer equation, the plasma will never be in thermal equilibrium, because there can be no flow of radiation in such a situation.

If a plasma or some processes in a plasma are said to be in **local thermal equilibrium**, it is an assertion that the occupation numbers of these processes are describable in terms of a temperature and the statistical weights associated with each state, even though the radiation field is not necessarily a BB.

In Python (and most other radiative transfer codes), it is assumed that the electron distribution is in LTE, describable in terms of the Boltzmann equation and a temperature. This is a good assumption usually because the timescale for the electron population to readjust itself is small compared to most of the other relevant processes. (An example of a situation where this assumption fails in some calculations of ionization equilibrium where "supra-thermal" electrons created say by X-rays ionize the atoms within the plasma multiple times before "joining" the electron population".)

The statement that the ion populations are in LTE then is a statement that the Saha equation holds (and, at least in Python, that $T = T_e$).

The statement that lines (in a two level atom) are in LTE is a statement that (either as an assumption or as a result of a non-LTE calculation) that

$$n_2 = n_1 \frac{g_2}{g_1} e^{-\chi/kT_e} \tag{1}$$

Note that Mihalas uses a notation for in which * indicates LTE. The previous equation in Mihalas is written:

$$\left(\frac{n_{ijk}}{n_{ojk}} \right)^* = \frac{g_{ijk}}{g_{ojk}} e^{-\chi_{ijk}/kT_e} \tag{2}$$

Furthermore, in Mihalas, the indices ijk refer to level, ionization state, and atom.

(An alternative LTE expression for the number in an excited state is given by:

$$\left(\frac{n_{ijk}}{N_{jk}}\right)^* = \frac{g_{ijk}e^{-\chi_{ijk}/kT}}{U_{jk}(T)} \tag{3}$$

where $U_{jk}(T)$ is the partition function. Note that this is NOT the Saha equation since if does not tell you what N_{jk} is.)

By in large, Python is designed to be a non-LTE code since we are concerned with a wide range of densities etc.

If a process, and its inverse are treated in LTE, that is the occupation numbers are assumed to be the LTE occupation numbers, it simplifies the calculation of both the opacity and the emissivity associated with that process.

Specifically, if a process is in LTE, then the source function S will always be B the blackbody source function, and $\epsilon = \kappa B$. The absorption coefficient κ will also generally be simpler, of a form something like

$$\kappa = \kappa_o \left(1 - e^{h\nu/kT}\right) \tag{4}$$

where the second term (which may have some fudge factors for statistical weights) accounts for stimulated emission.

2.2. Ionization basics

The Saha equation defines the relationship between ions in LTE. If one was only concerned with ground states one would write

$$\left(\frac{n_{o,j+1,k}n_e}{n_{o,j,k}}\right)^* = \frac{2g_{o,j+1,k}}{g_{o,j,k}}(2\pi mkT/h^2)^{3/2}e^{-\chi_{Ijk}/kT} \tag{5}$$

(This was actually the formula being used for calculating LTE up until Python_35 and it would be fine if nearly all of the ions were in the ground state.)

But the problem with this is that is that this equation does not allow for excited state ions. For this one needs to use the Saha equation (with the partition function U)

$$\left(\frac{N_{j+1,k}n_e}{N_{j,k}}\right)^* = \frac{2U_{j+1,k}}{U_{j,k}}(2\pi mkT/h^2)^{3/2}e^{-\chi_{Ijk}/kT} \tag{6}$$

or

$$\left(\frac{N_{j+1,k}n_e}{N_{j,k}}\right)^* = 4.83 \times 10^{15}\frac{2U_{j+1,k}}{U_{j,k}}(T)^{3/2}e^{-\chi_{Ijk}/kT}|endequation$$

The ionization equilibrium equation that we use is a variant of one that is in Mihalas and is appropriate in radiatively dominated situations. The Mihalas expression (5-46) is:

$$\left(\frac{n_{o,j+1,k}n_e}{n_{o,j,k}}\right)^* = W\frac{2g_{o,j+1,k}}{g_{o,j,k}}(2\pi mkT/h^2)^{3/2}\left(\frac{T_e}{T_r}\right)^{1/2}e^{-\chi_{Ijk}/kT_r} \tag{8}$$

The expression above is identical to the one used by Lucy & Abbot 1993 in their paper on Wolf-Rayet winds.

ML93 suggested several improvements to this ionization formula. They began by

referring to Abbott and Lucy who apparently were the first to use the expression

$$\frac{n_{j+1}n_e}{n_j} = \xi W \left(\frac{T_e}{T_R}\right)^{1/2} \left(\frac{n_{j+1}n_e}{n_j}\right)^*_{T_R},$$
(9)

where * refers to Saha equation abundances and ξ is the fraction of recombinations going directly to the ground state. They state that this follows directly from the nebular approximation and the assumption that all ionizations occur directly from the ground state and radiative recombinations to all levels. So using this equation the plasma will be a little less ionized than the simpler Mihalas expression.

They then argue that one can include ionizations from the excited levels with the following expression:

$$\frac{n_{j+1}n_e}{n_j} = W\left[\xi + W(1 - \xi)\right] \left(\frac{T_e}{T_R}\right)^{1/2} \left(\frac{n_{j+1}n_e}{n_j}\right)^*_{T_R},$$
(10)

This has the positive attribute that one recovers the Saha equation when W is 1. THIS IS THE FORMULA WE CURRENT IMPLEMENT IN PYTHON (UP TO AT LEAST VERSION 35)

ML93 go on from here however to suggest a further improvement to this formula, which depends on the frequency at which the continuum becomes optically thick, namely:

$$\frac{n_{j+1}n_e}{n_j} = W[\delta\xi + W(1 - \xi)] \left(\frac{T_e}{T_R}\right)^{1/2} \left(\frac{n_{j+1}n_e}{n_j}\right)^*_{T_R},$$
(11)

In this equation, W is an effective dilution factor, ξ is the fraction of recombinations going directly to the ground state, T_R is the radiation temperature, and δ is a correction factor that can be used to account for the effect of an optically thick continuum.

More specifically, δ takes on two values depending on whether a species ground state ionization threshold $\chi_T > \chi_o$. It is not immediately obvious that this approach would help in the situations we are trying to address. To first order it appears that the modification

would reduce the ionization which has not been the main problem. It is more likely, or at least that is the way I feel at present that a completely different ionization approach is required for ions that one "cares" about!

2.3. Collision basics

Detailed balance requires that in LTE

$$n_2 = n1 \frac{g_2}{g_1} e^{-h\nu/kT} \tag{12}$$

and therefore for collision processes we must have

$$c_{12} = c_{21} \frac{g_2}{g_1} e^{-h\nu/kT} \tag{13}$$

so

$$c_{21} = c_{12} \frac{g_1}{g_2} e^{h\nu/kT} \tag{14}$$

2.4. Radiation Basics

The intensity I is the flux in given direction across a surface normal to that direction per unit solid angle. It generally has units of $ergs\ cm^{-2}\ s^{-1}\ sr^{-1}$.

The surface flux F or flux density is given by

$$F = \int_{4\pi} I\ cos(\theta)d\Omega \qquad (15)$$

The emittance is similar but it is integrated over the forward hemisphere only, i.e.

$$F = \int_{2\pi} I\ cos(\theta)d\Omega\ =\ \pi I \qquad (16)$$

for isotropic radiation.

The radiation density u is defined by

$$u\ =\ \frac{1}{c}\int_{4\pi} I\ d\Omega \qquad (17)$$

2.5. Blackbody formulae

$$B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \; ergs \; cm^{-2} \; s^{-1} sr^{-1} \; Hz^{-1} \qquad (18)$$

The relationship between the specific energy density u_ν, specific intensity B_ν, and specific emittance F_ν for blackbody radiation is as follows:

$$\frac{c}{4}u_\nu = \pi B_\nu = F_\nu = \frac{2\pi h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \qquad (19)$$

The associated units are $ergs \; cm^{-3} \; Hz^{-1}$, $ergs \; cm^{-2} \; s^{-1} sr^{-1} \; Hz^{-1}$, and $ergs \; cm^{-2} \; s^{-1} \; Hz^{-1}$

The total emittance:

$$F = \int B_\nu d\nu = \sigma T^4 \qquad (20)$$

and the units are $ergs \; cm^{-2} \; s^{-1}$.

The BB intensity is

$$I = \frac{\sigma}{\pi} T^4 \qquad (21)$$

and the radiation density is

$$U = aT^4 \qquad (22)$$

Note that $a = 4\sigma/c$

2.6. Radiative Transfer Basics

The simplest version of the radiative transfer equation (in the notation that ksl normally uses is:

$$\frac{dI}{ds} = -\kappa I + \epsilon \tag{23}$$

Mihalas (2-28) uses somewhat different notation:

$$\mu \frac{dI}{dz} = \eta - \chi I \tag{24}$$

It is often useful to rewrite this equation in terms of an optical depth τ and a source function S where

$$\tau = \int_z^{zmax} \chi dz' \tag{25}$$

and

in which case the radiative transfer equation takes the form

2.7. Line radiation and the Einstein coefficients

The Einstein coefficients were developed to reflect a very specific model for the way in which photons and matter interact. The processes involved are direct absorption, spontaneous emission and induced emission. There Einstein relations arise from the specific requirement that the rates associated with the 3 processes must yield the LTE level populations in a BB radiation field.

Consider a two level atom with lower level 1 and upper level 2.

The total cross section for a line is given by

$$\sigma_{tot} = \sigma = \frac{\pi e^2}{mc} f_{12} = B_{12} \frac{h\nu}{4\pi} \tag{26}$$

where f_{12} is the absorption(!) oscillator strength.

The Einstein relations are

$$A_{21} = \frac{2h\nu^3}{c^2} B_{21} = \frac{8\pi^2 e^2}{mc^3} \nu^2 \frac{g_1}{g_2} f_{12} = 7.428 \times 10^{-22} \nu^2 \frac{g_1}{g_2} f_{12} \tag{27}$$

and

$$g_1 B_{12} = g_2 B_{21} \tag{28}$$

The derivation of the Einstein relations is straightforward, see e.g. Mihalas Section 4-1.

Often in Python, B_{12} and B_{21} are calculated in terms of A_{21} and f_{12} and therefore we list some additional relations for completeness:

$$B_{21} = \frac{c^2}{2h\nu^3} A_{21} = \frac{4\pi}{h\nu} \frac{\pi e^2}{mc} \frac{g_1}{g_2} f_{12} \tag{29}$$

$$B_{12} = \frac{g_2}{g_1} \frac{c^2}{2h\nu^3} A_{21} = \frac{4\pi}{h\nu} \frac{\pi e^2}{mc} f_{12} \tag{30}$$

The associated radiative transfer equation is

$$\mu \frac{I_\nu}{dz} = [n_j A_{ji} \psi_{nu} - (n_i B_{ij} \psi_\nu) I_\nu] \, (h\nu_{ij}/4\pi) \tag{31}$$

In the context of the simplest expression of the radiative transfer equation, e.g.

$$\frac{dI}{ds} = -\kappa I + \epsilon \tag{32}$$

$$\epsilon = \frac{h\nu}{4\pi} n_2 A_{21} \phi = \frac{2h\nu^3}{c^2} \frac{\pi e^2}{mc} \frac{g_1}{g_2} f_{12} n_2 \tag{33}$$

and

$$\kappa = \frac{h\nu}{4\pi} (n_1 B_{12} - n_2 B_{21}) \phi \tag{34}$$

and ϕ the emission profile function integrates to 1.

This can be rewritten

$$\kappa = \frac{c^2}{8\pi\nu^2} (\frac{g_2}{g_1} n_1 - n_2) A_{21} \phi \tag{35}$$

or

$$\kappa = (n_1 - \frac{g_1}{g_2} n_2) \frac{\pi e^2}{mc} f_{12} \phi \tag{36}$$

As noted above, the Einstein coefficients were defined to "produce" equilibrium in when a plasma is in thermal equilibrium. In this limit,

$$dI/ds = 0 \qquad (37)$$

To see that this is indeed true, note that in TE (as well as LTE)

$$n_2 = n_1 \frac{g_2}{g_1} e^{-h\nu/kT} \qquad (38)$$

and therefore the emissivity

$$\epsilon = n_1 \frac{g_2}{g_1} e^{-h\nu/kT} \frac{h\nu}{4\pi} A_{21}\phi = \frac{2h\nu^3}{c^2} \frac{\pi e^2}{mc} f_{12} n_1 e^{-h\nu/kT} \phi \qquad (39)$$

which is certainly not 0. Therefore one has to get the right absorption.

But in TE (and LTE)

$$\kappa = \frac{h\nu}{4\pi} n_1 \frac{g_2}{g_1} (1 - e^{-h\nu/kT}) \frac{c^2}{2h\nu^3} A_{21}\phi \qquad (40)$$

or

$$\kappa = n_1 (1 - e^{-h\nu/kT}) \frac{\pi e^2}{mc} f_{12}\phi \qquad (41)$$

And this does imply that $dI/ds = 0$ since

$$0 = n_1 (1 - e^{-h\nu/kT}) \frac{\pi e^2}{mc} f_{12}\phi I_\nu + \frac{2h\nu^3}{c^2} \frac{\pi e^2}{mc} f_{12} n_1 e^{-h\nu/kT} \qquad (42)$$

will be correct if $I_\nu = B_\nu$.

Note: Here κ and ϵ include both absorption and scattering processes.

3.   Radiative transfer

In the context of Python, the issue of what is scattered and what is absorbed radiation is a serious one because while we treat the radiative transfer in essentially a microscopic manner we treat ionization equilibrium much more statistically. Therefore one has to be careful about energy flow between the radiation field and the gas and vice versus. The problem manifests itself primarily if not entirely in resonance line scattering, because other process, e.g. bremsstrahlung and electron scattering are treated as pure absorption or emission processes.

3.1.   Intuition

The simplest radiative transfer equation **??** does not split scattering and absorption. A more generalized equation would

$$\frac{dI}{ds} = -(\kappa_{abs} + \kappa_{sc})I + \epsilon_{scat} + \epsilon_{thermal} \tag{43}$$

To make use of an equation like this one needs to know how to split κ into its scattering and absorptive parts. This requires an understanding (in the case of the two-level atom) of what we mean by scattering and absorption on a microscopic level. Whatever the splitting is then we expect

$$\epsilon_{thermal} = \kappa_{abs}B_\nu(T_e) = q\kappa_{tot}B_\nu(T_e) \tag{44}$$

and for isotropic scattering,

$$\epsilon_{scat} = \kappa_{sc}J = (1-q)\kappa_{tot}J \tag{45}$$

One needs to relate this to the total emissivity of a two level atom which is

$$\epsilon_{tot} = n_2 A_{21} \frac{h\nu}{4\pi} \qquad (46)$$

but this includes both a scattering and a thermal fraction.

If stimulated emission can be ignored, then we take (by pure thought) in the single scattering approximation

$$\kappa_{scat} = \kappa_{tot} \frac{A_{21}}{c_{21} + A_{21}} = \kappa_{tot}(1 - q) \qquad (47)$$

and

$$\kappa_{abs} = \kappa_{tot} \frac{c_{21}}{c_{21} + A_{21}} = \kappa_{tot} q \qquad (48)$$

This "guess" is based on our "microscopic definition" that scattering corresponds to radiative excitation followed by spontaneous emission while absorption is a radiative excitation (which leaves the photon field with the same energy as previously) followed by collisional deexcitation (which robs the photon field of energy). Our "guess" clearly leaves open the question of how to account for stimulated emission.

Our intuitive ideas for single scattering can be carried over into multiple scattering if we argue that the fraction of the energy which absorbed in each scattering is $c_{21}/(c_{21} + a_{21}$, and therefore if N is the number of scatters that a photon will undergo before it escapes the medium, then

$$\kappa_{scat} = \kappa_{tot} \left(\frac{A_{21}}{c_{21} + A_{21}}\right)^N \qquad (49)$$

and

$$\kappa_{abs} = \kappa_{tot} \left(1 - \left(\frac{A_{21}}{c_{21} + A_{21}}\right)^N\right)$$

(50)

3.2.   Radiative transfer revisited for the two-level atom

Although the above "pure thought" approach produces reasonable results it is not really rigorous. A better approach is to note that in all scattering plus absorption problems the source function needs to be cast in the following form:

$$S = \frac{\epsilon}{\kappa} = (1 - q)\, J + q\, B(T_e) \tag{51}$$

but it is also true for line transitions (whether or not one is treating it in the two-level approximation) that

$$S = \frac{\epsilon}{\kappa} = \frac{n_2 A_{21}}{n_1 B_{12} - n_2 B_{21}} = \frac{2h\nu^3}{c^2} \frac{1}{\frac{g_2 n_1}{g_1 n_2} - 1} \tag{52}$$

The "trick" is to decompose S so that it has the form of **??**.

For a two level atom explicitly, the rate equations require

$$n_1(c_{12} + B_{12}J) = n_2(c_{21} + A_{21} + B_{21}J) \tag{53}$$

but this implies

$$\frac{n_1}{n_2} = \frac{A_{21} + B_{21}J + c_{21}}{B_{12}J + c_{12}} = \frac{g_1}{g_2}\frac{A_{21} + B_{21}J + c_{21}}{B_{21}J + c_{21}e^{-h\nu/kT_e}} \tag{54}$$

and therefore it is possible to replace $g_2 n_1/g_1 n_2$ in equation **??** This results in an equation for S of the following form:

$$S = \frac{A_{21}J + c_{21}\frac{2h\nu^3}{c^2}e^{-h\nu/kT_e}}{A_{21} + c_{21}(1 - e^{-h\nu/kT_e})} \tag{55}$$

But since this must be true for all J, this implies that the scattering fraction

$$1 - q = \frac{A_{21}}{A_{21} + c_{21}(1 - e^{-h\nu/kT_e})} \tag{56}$$

and therefore the absorption fraction

$$q = \frac{c_{21}(1 - e^{-h\nu/kT_e})}{A_{21} + c_{21}(1 - e^{-h\nu/kT_e})} \tag{57}$$

The "magical" thing about all of this is that one does find that the second term in the equation ?? for S is indeed qB.

Note: In general with our approximations q rises as the temperature drops. For example at 1500 Å, $q = 2.1 \times 10^{-3}$ at $T_e = 64{,}000$K; 4.7×10^{-3} at 21,000 K; 6.9×10^{-3} at 10,000 K; 9.8×10^{-3} at 5,000 K. The absorption also increases with n_e and decreases with ν. In the low temperature limit, the absorbed and scattered fractions agree with "pure thought".

A reasonable question to ask at this point is what does $\epsilon_{thermal}$ turn out to be:

$$\epsilon_{thermal} = q\kappa_{tot}B(T_e) = \frac{h\nu}{4\pi} \left(\frac{g_2}{g_1}n_1 - n_2 \right) A_{21}\frac{e^{-h\nu/kT_e}}{1 - e^{-h\nu/kT_e}} \tag{58}$$

$$\epsilon_{thermal} = q\kappa_{tot}B(T_e) = \frac{h\nu}{4\pi} \left(\frac{c_{21}e^{-h\nu/kT_e}}{1 + \frac{c_{21}}{A_{21}} (1 - e^{-h\nu/kT_e})} \right) \left(\frac{g_2}{g_1}n_1 - n_2 \right) \tag{59}$$

$$\epsilon_{thermal} == \frac{h\nu}{4\pi} \left(\frac{c_{12}}{1 + \frac{c_{21}}{A_{21}} (1 - e^{-h\nu/kT_e})} \right) \left(n_1 - \frac{g_1}{g_2}n_2 \right) \tag{60}$$

[This is the formula now implemented for total line emission. It has the "desirable" property that $\epsilon_{thermal}$ decreases with temperature even if n_1 and n_2 are controlled by the radiative rates. The old formula

$$\epsilon_{thermal} = \frac{h\nu}{4\pi}qn_2A_{21} \tag{61}$$

is wrong and did not have this property!]

3.3. Radiative transfer in terms of MC methods

The simplest form of the radiative transfer equation is

$$\frac{dI}{ds} = -\kappa I + \epsilon \tag{62}$$

Suppose we were trying to construct a MC simulation of the emergent intensity of a radiating cloud. At each point in the cloud we would produce photons, according to rules determined by the specific physics embodied by the emissivity ϵ. These photons would traverse the medium until and unless they were absorbed by the medium. In a very loose sense to carry out a MC simulation, we would associate I with a certain number of photons traversing a medium. The distance that a photon would travel before being absorbed would be given

$$\tau_{scat} = \int_o^x \kappa(x')dx' \geq -ln(1-p) \tag{63}$$

The basic problem in any MC problem therefore is to determine the emissivity ϵ. Now instead of expressing the radiative transfer equation in physical dimensions, it is far more common to write the radiative transfer equation directly in terms of τ, e.g.

$$\frac{dI}{d\tau} = I - S \tag{64}$$

where the source function

$$S = \frac{\epsilon}{\kappa} \tag{65}$$

Typically κ consists of a scattering term and pure absorption term, and in this case

$$\epsilon = \kappa_{abs}B + \sigma J \tag{66}$$

It is with the addition of the scattering term that problems begin to arise. But if the scattering is isotropic, then it is clear that on a microscopic level at least we associated J with a scattering event, and that what the code must do in this case is simply redirect photons over 4π. Indeed for the pure scattering case, e.g. no absorption it is pretty obvious that all a scattering event can do is to redirect a photon (possibly changing its frequency in the process). This must be true even if there are multiple scatters on the microscopic scale that cause a complicated radiation pattern. The only issue, is what that radiation pattern is.

4. Sobolev Approximation

Scattering is treated in the Sobolev approximation. A photon travels until it encounters a resonant surface.

In the Sobolev approximation:

$$\phi = \delta(\nu - \nu_i(1 + v/c)) \tag{67}$$

The optical depth through the resonant surface is give by

$$\delta\tau = n_i \sigma_{tot} \frac{\lambda}{|dv/ds|} = \left(n_l - \frac{g_l}{g_u} n_u\right) \frac{\pi e^2}{mc} f_{lu} \frac{\lambda}{|dv/ds|} \tag{68}$$

Scattering is not a conservative process, because of the effects of stimulated emission and because of collisions, although it is an option in the code to ignore the nonconservative parts of the process.

Scattering (including electron and resonance line scattering) and continuous absorption are treated differently in the wind. Both need to satisfy the appropriate LTE constraints.

4.1. $\delta\tau$ derived

Equation ?? is not particularly obvious, and in particular its does not appear to be dimensionless. Therefore it is useful to actually derive it in detail. Now,

$$\tau = \int n(s)\sigma(\nu(s))ds = n\sigma_o \int \phi(\nu(s))ds \tag{69}$$

which through a change in variables can be rewritten

$$\tau = n\sigma_o \int \phi(\nu)\frac{\delta s}{\delta\nu}d\nu \tag{70}$$

But

$$\frac{\delta s}{\delta\nu} = \frac{c}{\nu_o|dv/ds|} \tag{71}$$

which implies that

$$\tau = n\sigma_o\frac{c}{\nu_o|dv/ds|} \int \phi(\nu d\nu = n\sigma_o\frac{c}{\nu_o|dv/ds|} \tag{72}$$

5. Escape Probabilities

Escape probabilities in the Sobolev approximation represent a way to try to take into account for the fact that the optical depth through a Sobolev surface can be very large and therefore the energy loss in a scattering is not that of a single scattering. In Python, currently, the escape probability P_{esc} is defined in terms of an optical depth, e.g.

$$P_{esc} = \frac{1 - e^{-\tau}}{\tau} \tag{73}$$

where

$$\delta\tau = n_i\sigma_{tot}\frac{\lambda}{<|dv/ds|>} \tag{74}$$

and $<|dv/ds|>$ is some suitably chosen average.

The escape probability affects the code in two distinct ways:

(a) It affects the effective line emissivity of the plasma. Specifically $\epsilon_{thermal}$ is reduced by the escape probability to be

$$\epsilon_{thermal} = q\kappa_{tot}B(T_e)P_{esc} \tag{75}$$

and so

$$L_{thermal} = q\kappa_{t}otB(T_e)P_{esc}Volume \tag{76}$$

(b) it modifies the source function because it changes the number of times that a photon will scatter in the medium.

With multiple scattering, one can show that the scattering fraction SF is given by

$$SF = (1 - q)\frac{P_{esc}}{q + P_{esc}(1 - q)} \tag{77}$$

where q is absorptive fraction of photons absorbed in a single scattering interaction. To see this let W_0 describe the initial intensity (or equivalently in Python the number of photons which are scattered). All W_0 photons interact in the first scattering event. Of these, $q\,W_0$ are absorbed and $W_1 = (1 - q)\,W_0$ remain. Of those that remain $P_{esc}W_1$ exit the interaction region and $(1 - P_{esc})\,W_1$ interact a second time. Of these, $q\,(1 - P_{esc})\,W_1$ are absorbed and $W_2 = (1 - q)\,(1 - P_{esc})\,W_1$ remain. This process continues. After the nth scatter the number of photons which remain is

$$W_n = (1 - P_{esc})(1 - q)W_{n-1} = (1 - P_{esc})^{n-1}(1 - q)^n W_0 \tag{78}$$

The total energy absorbed can now be written down:

$$Absorbed = qW_0 + \Sigma_{n=1}^{\infty}q(1 - P_{esc})W_n = q\Sigma_{n=0}^{\infty}(1 - P_{esc})^n(1 - q)^n W_0 \tag{79}$$

But this is a geometric series, and so the absorbed energy is just

$$Absorbed = \frac{q}{1 - (1 - P_{esc})(1 - q)}W_O = \frac{q}{q + P_{esc}(1 - q)}W_O \tag{80}$$

and therefore

$$Scattered = 1 - Absorbed = (1 - q)\frac{P_{esc}}{q + P_{esc}(1 - q)} \tag{81}$$

as advertised.

6. Other Details

6.1. Emission from surfaces

In general, Python uses black bodies, Kurucz or Hubeny models to create emission from a surface.

In the case of the Kurucz models, particularly, there is a problem in that in some cases we would like to generate temperatures that are not in the model grid. Suppose we have a model with temperature T_1, but want a spectrum for T_2. We use the following approximation:

$$F_\nu(T_2) = \frac{e^{h\nu/kT_1} - 1}{e^{h\nu/kT_2} - 1} F_\nu(T_1) \tag{82}$$

This approximation has the property that it makes the spectrum bluer when $T_2 > T_1$, and it increases the total surface flux as T_2^4/T_1^4, as required.

6.2. Effects of limb darkening and anisotropic scattering

In generating photons from the disk, we account not only for geometrical foreshortening, but also for limb darkening[1]. In the absence of limb darkening, the probability, $p(\Omega)$, that a primary photon is emitted in a direction $(\theta, \phi))$ *relative to the local outward normal* is proportional to

$$p(\Omega)d\Omega \propto \cos\theta d\Omega = \cos\theta \sin\theta d\theta d\phi \tag{83}$$

Linear limb-darkening can be accounted for multiplying the right hand side of this expression by a factor $\eta(\theta) = a(1 + b\cos\theta)$. Here, the parameter b defines the strength of the limb-darkening la, and the parameter a is a normalization constant that can be included either explicitly or implicitly – the only important point in the MC approach is that the total (integrated) probability must sum to unity. This equation can be integrated directly to find the cumulative distribution function for emission in the a direction defined by $cos\theta$.

In extracting photons for computing spectra one also has to take limb darkening into account. Basically, since in this case one has already determined the photon direction one must adjust the weight of the photon relative to a photon that is created isotropically in the wind. If the wind emits or scatters isotropically, then the probabilty that this photons scatter toward the observer is given by

$$P_w = \frac{d\Omega_{obs}}{4\pi} \tag{84}$$

By contrast, a disk photon has a probability of heading toward the observer if

$$P_c = \frac{1}{g}f(\theta_{obs}\phi_{obs})d\Omega_{obs} \tag{85}$$

where $f(\theta_{obs}\phi_{obs})$ is given by $cos(\theta\eta(\theta)$. Here the normalizing factor g is determined by the

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[1]Most if not all of this is in Knigge's thesis

fact that the photon has to be emitted, e.g

$$P_{tot} = 1 = \frac{1}{g} \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} f(\theta_{obs}\phi_{obs})d\Omega_{obs} \tag{86}$$

For the case of linear limb darkening everything can now be integrated to show:

$$\frac{P_c}{P_{wind}} = \frac{12cos(\theta)(1 + b\ cos(\theta))}{3 + 2b} \tag{87}$$

Similarly, if scattering is anisotropic this also has to be taken into account in the code. Again, one has to account for anisotropic distributions both in the live or die portions of the code, this is in where the primary photons go, and in the extract portion of the code. For the live-or-die photons, (as was in fact the case for the disk photons) there are two basic options for dealing with anisotropic scattering. One can either adjust the weights of the photons and continue with an isotropic distribution, or alternatively one can adjust the distribution itself. Adjusting the weight injects noise into the system, because some photons will have weights greater than the average and hence the effect of these photons on the systems will greater than those whose weights have been adjusted downward. On extract, since one has to bring the photon out in a fixed direction, there is no real alternative to adjusting the weights. There is obviously a close connection between photons.

We are currently considering and may implement an form of anisotropic scattering in which

$$p(\Omega) \propto \frac{1 - e^{-\tau/|cos\theta|}}{1 - e^{-\tau}}\ |cos(\theta)|d\phi dcos(\theta) \tag{88}$$

This is the form of the scattering function that would result from a slab with a constant source function, in which τ is the optical depth normal to the slab. The principles for placing this in the code are identical to those associated with Eddington limb darkening. The only problems are this cannot be solved in closed form, and in this case the integrals to calculate the normalization are over 4π.

6.3. Level populations

In Python, the level populations are established from the radiation fields calculated in a previous iteration of the program. In general the radiation field is assumed to be given by that of a diluted BB, e.g.

$$J = W B_\nu(T_r) \tag{89}$$

In this case, the rate equation for a two-level atoms).

$$\frac{n_2}{n_1} = \frac{c_{12} + B_{12}J}{c_{21} + A_{21} + B_{21}J} \tag{90}$$

becomes

$$\frac{n_2}{n_1} = \frac{c_{12} + B_{12}W B_\nu(T_r)}{c_{21} + A_{21} + B_{21}W B_\nu(T_r)} \tag{91}$$

Note that these are the same equations as in the discussion of the source function. See e.g. equation **??**.

In the large J limit, the usual situation near the disk, this becomes

$$\frac{n_2}{n_1} = \frac{g_2}{g_1} \frac{W}{e^{h\nu/kT} + W - 1} \tag{92}$$

At present, Python assumes that the vast majority of all ions are in the ground state, and hence transitions that involve excited lower states, e.g. the Balmer series, have historically not been included in the code. In fact the lines are all treated independently. (The level populations are calculated in the routine "two_level_atom".)

The limitation is a significant one, even in the FUV since there are various excited states of C III and N IV in FUSE and ORFEUS spectra of high state CVs. The difficulty is that unless one were very judicious the number of levels is going to be fairly large, and it can be computationally and memory quite expensive to carry all the necessary level populations. The most likely hope would be to carry some kind of value of a partition function along with the ion densities, and to calculate the population of the levels for the specific transitions of interest.

As a stop gap measure beginning with Python 32 I have implemented an extremely simple approach for transitions with an excited lower levels. Specifically I have assumed that both lower level and the upper level have a density which are determined directly by equation **??**.

Leon Lucy does not include corrections for collisions in calculating levels. So the equation he uses is identical to that of **??**. If collisions are not included then line transfer is a pure scattering process, and no energy is deposited by this process into the wind. In fact, for UV and EUV lines the only reason, one expects much energy deposition from lines is due to trapping. See the section on collision cross sections.

In the equation above, Leon sets W to one for meta-stable levels! Prior to Python_36, Python did not have a way to identify meta-stable levels. However, as of Python 36, an attempt is made to associate lines with electronic configurations. If a configuration can be identified with a line or lines, then the spontaneous decay rate is calculated for the level. We then use the spontaneous decay rate to identify a level as metastable (i.e with a rate of less than $10^4 s^{-1}$), and in that case we set W to 1.

6.4. Atomic Data

The line list for resonance scattering that I have been used most often is from Verner, Verner, & Ferland 1996. The entire list contains 5971 lines (and 1877 multuplets). Some of the less astrophysically abundant elements are not generally included in the calculations. More specifically we have 3213 lines from various levels of H, He, C, N, O, Ne, Na, Mg, Al, Si, S, Ca, Fe, and Ni in the current line list.

There is a program permitted.c which converts one of VVF's tables to something appropriate for python. When I searched for this program in November 01, I could not find the latest version of permitted.c; I found an old version which had been used apparently in the 1-d version (Monte) of the program, but there must have been a newer version because the version I found does not produce the same number of output lines as the input lines. The current version of the file lines.data was produced prior to May 1998, when I was working with python_20. Pike needs to be checked.

6.5. Electron scattering

We use the Thompson cross section for electron scattering. Note that heating due to electron scattering is not included. In principle, a few percent of the energy is lost at each scatter.

6.6. Collision Cross Sections

In general, we followed Gaetz & Salpeter when we started thinking about electron collisions, who begin with

$$c_{12} = n_e q_{12} \tag{93}$$

As was introduced by Bely and Van Regemorter (1970), we assume the rate coefficient can be written:

$$q_{12} = \frac{h^2}{(2\pi m)^{3/2}} \frac{1}{(kT_e)^{1/2}} \frac{\Omega}{g_1} e^{-E_{12}/kT_e} \tag{94}$$

or

$$q_{12} = \frac{8.629 \times 10^{-6}}{g_1 T_e^{0.5}} e^{-E_{12}/kT_e} \Omega \, cm^3 s^{-1} \tag{95}$$

and we take (following Seaton 1962)

$$\Omega = \frac{8\pi}{\sqrt{3}} \frac{g_1 f_{12}}{E_{12}} g_{eff}(T_e) = \frac{16\pi^3}{\sqrt{3}} \frac{m \, e^4}{h^3 \nu} g_1 f_{12} g_{eff}(T_e) \tag{96}$$

where E_{12} is the excitation energy in Rydbergs

It is possible to write q_{12} in terms of A_{21}, e.g.

$$q_{12} = \left(\frac{m}{6\pi kT_e} \right)^{1/2} \frac{g_2}{g_1} \frac{e^2 c^3}{h\nu^3} A_{21} \, g_{eff}(T_e) \, e^{-E_{12}/kT_e} \tag{97}$$

$$q_{21} = \left(\frac{m}{6\pi kT_e} \right)^{1/2} \frac{e^2 c^3}{h\nu^3} A_{21} \, g_{eff}(T_e) \tag{98}$$

In practice, the program right now assumes the gaunt factors are 1.

In this approximation, term $n_e q_{21}/A_{21}$

becomes

$$n_e q_{21}/A_{21} = 1.16141 \times 10^{16} \frac{n_e \Omega}{\sqrt{T} \nu^2 g_1 f_{12}} \tag{99}$$

and therefore using our approximation for Ω, we obtain

$$n_e q_{21}/A_{21} = 5.54 \times 10^{32} \frac{n_e g_1}{g_2 T^{0.5} \nu^3} \tag{100}$$

or

$$n_e = 4.86 \times 10^{13} \left(\frac{\lambda}{1000\mathring{A}}\right)^{-3} \frac{g_2}{g_1} \sqrt{T} \tag{101}$$

In this approximation, collisional deexcitation and spontaneous emission will be equal at densities of 1.4×10^{15} cm^{-3} at 1500 Å and 1000 times this at 150 Å. Since the densities are lower than this, spontaneous emission is usually the dominant process.

One might also ask under what situations do collisional excitations compete with radiative excitations. Collisional excitation compete with photoionization if:

$$\frac{W}{n_e} < \left(\frac{m_e}{6\pi k T_e}\right)^{1/2} \frac{e^2 c^3}{h\nu^3} g_{eff} e^{-\chi/kT_e} \left(e^{h\nu/kT_r} - 1\right) \tag{102}$$

The right hand side of this equation is of order 10^{19} at 100 Å, 10^{16} at 1000 Å, 10^{14} at 5000 Å. The equation can be more usefully rewritten as follows:

$$n_e > 4.85 \times 10^{13} W \left(\frac{\lambda}{1000\mathring{A}}\right)^{-3} \frac{e^{\chi/kT_e}}{g_{eff}} \frac{1}{e^{h\nu/kT_r} - 1} \tag{103}$$

So most of the time radiation dominates in CV winds, except in the visible, and unless trapping is involved. Note that in O star winds, collisions become more important sooner.

Essentially the same arguments are made on pages 125-127 of Mihalas. Note that Mihalas has an interesting caution though in that he says that the fact that collisions seem to dominate at longer wavelengths by this argument might make one believe that LTE is pretty good at long wavelengths. In fact, he says the largest departures are often seen there. I wonder whether this is not due in part to the fact that recombinations may dominate excitations in many situations.

7. Continuous absorption and emission processes

7.1. Free Free absorption and emission

$$J_\nu = \frac{16}{3}(\frac{\pi}{6})^{1/2}\frac{e^2 Z^2}{m^2 c^3}(\frac{m}{kT})^{1/2}\, g\, e^{-h\nu/kT}\, n_e\, n_i\, ergs\, cm^{-3}\, s\, sr^{-1}\, Hz^{-1} \qquad (104)$$

We have set the gaunt factor g to be 1 at present.

The total free-free emission is

$$4\,\pi\int J_\nu d\nu = \frac{64\pi}{3}(\frac{\pi}{6})^{1/2}\frac{e^6 Z^2}{hc^3 m}(\frac{kT}{m})^{1/2}\, g\, n_e\, n_i\, ergs\, cm^{-3} \qquad (105)$$

At present, Python actually uses a an absorption coefficient which has been modified for stimulated emission. A more complete discussion of this needs to be added.

As is generally true, emissivities and opacities can be related through Kirchoff's law which in TE states:

$$B_\nu = S_\nu = \frac{\epsilon}{\kappa} \qquad (106)$$

or

$$\kappa = \epsilon_\nu\, \frac{c^2}{2h\nu^3}\frac{1 - e^{-h\nu/kT_e}}{e^{-h\nu/kT_e}} \qquad (107)$$

or, in this case,

$$\kappa_{ff} = \frac{8}{3}(\frac{\pi}{6})^{1/2}\frac{e^2 Z^2}{m^2 ch\nu^3}(\frac{m}{kT})^{1/2}(1 - e^{-h\nu/kT_e})n_e n_i \qquad (108)$$

7.2.   Photoionization and recombination

In versions of Python through Python 35, We relied solely currently using analytic fits for photoionization cross sections from Verner, et al. (1996). One weakness of this is that these are only ground state cross sections. In Python 36, possibility of using Topbase cross sections was added; these are not analytic fits, but are presented as cross sections as a function of energy or frequency. The Topbase data base contains cross sections for excited states.

Through at least Python 35, a simple formula for recombination emission was used

$$P(n, \nu) = n_e \, n_i \, \frac{Z^4 K}{kT^{3/2}} \, g_{fb} \, e^{-h(\nu - \nu_{threshold})/kT} \, d\nu \tag{109}$$

where

$$K = \frac{64\pi^{1/2}e^4 h}{3^{3/2}m^2 c^3} \left(\frac{2\pi^2 e^4}{mh^2}\right)^{3/2} = 3.51 \times 10^{-56} \tag{110}$$

and the total power between f_1 and f_2 is given by

$$\int_{f_1}^{f_2} P(n, \nu) d\nu = n_e \, n_i \frac{Z^4 K}{h(kT)^{1/2}} \frac{g_{fb}}{n^3} (e^{h(f_{min} - f_{threshold})/kT} - e^{h(f_{max} - f_{threshold})/kT}) \tag{111}$$

Thus bound-free and free-bound processes are not properly balanced, especially when one allows for recombination to the n=2 level. This is the underlying motivation for moving toward using Topbase data in Python 36 and beyond.

Do we really care? What is our problem here. It is really two-fold. We would like to have He1640 properly in our models and this line is produced primarily be recombination. In addition we would like to handle the recombination continuum for hydrogen at least

since there is significant problem with Balmer decrements in CVs. But at best we are not dealing self-consistently with this ion.

The proper way to handle photoionization and recombination is through use of the Milne relation and a determination of upper state populations. When I began to implement this into Python I had some problems with notation of the various sources I use for understanding radiative transfer, Mihalas, Cloudy and Osterbrock. A brief description of the argument each makes (and their reconciliation follows)

Mihalas (pp 130:131):

The number of photoionizations from a bound level i to the continuum level κ is given by

$$n_i R_{i\kappa} = n_i \, 4\pi \int_{\nu_o}^{\infty} \alpha_{i\kappa} \frac{J_\nu}{h\nu} d\nu \tag{112}$$

Note that i refers to a particular level of the κ-1 ion and κ refers to the next ionization state.

In thermodynamic equilibrium, the number of spontaneous recombinations must equal the number of photoinizations when (a) $J_\nu = B_\nu$ and (b) correcting for stimulated emission by a factor of $1 - e^{h\nu/kT}$. As a result if n_κ denotes the ion density, then:

$$(n_\kappa R'_{\kappa i})^* = n^* \, 4\pi \int_{\nu_o}^{\infty} \alpha_{i\kappa} \frac{B_\nu}{h\nu} 1 - e^{-h\nu/kT} \, d\nu \tag{113}$$

In these equations, the superscript * denotes the ratio that one would have in LTE. The total number of recombinations, allowing for stimulation emission, is

$$n_\kappa \left(R'_{\kappa i,spon} + R'_{\kappa i,stim} \right) == n_\kappa \left(\frac{n_i}{n_\kappa} \right)^* R_{\kappa i} = n_\kappa \left(\frac{n_i}{n_\kappa} \right)^* 4\pi \int_{\nu_o}^{\infty} \frac{\alpha_{i\kappa}}{h\nu} \left(\frac{2h\nu^3}{c^2} + J_\nu \right) e^{-h\nu/kT} d\nu \tag{114}$$

Note that one of the things that Mihalas is doing is defining $R\kappa i$. He is also trying to make the point that the notation for bound-free and bound-bound interactions can be made identical, specifically:

The upward radiative rate is

$$R_{ij} = 4\pi \int_{\nu_o}^{\infty} \alpha_{ij} \frac{J_\nu}{h\nu} d\nu \tag{115}$$

whether or not j is bound or free and the downward radiation rate is $n_j(n_i/n_j)^* R_{ji}$ where

$$R_{ji} == 4\pi \int_{\nu_o}^{\infty} \frac{\alpha_{ij}}{h\nu} \left(\frac{2h\nu^3}{c^2} + J_\nu \right) e^{-h\nu/kT} d\nu \tag{116}$$

We know that in LTE, the rates must balance, and the energy absorbed from the photon field in photoionization must equal to that emitted by recombination to the photon field. In addition, it must be true that energy absorbed into the electron thermal pool and the energy emitted from the thermal pool must be identical. But why does it work out this way, given that these various process involve different integrals? The reason is that in all cases the absorption integral has the factor

$$J_\nu \tag{117}$$

while the recombination integral has the factor

$$\left(\frac{2\nu^3}{c^2} + J_\nu \right) e^{-h\nu/kT} \tag{118}$$

and in the instance that $J_\nu = B_\nu$, the two factors become identical, and hence every integral balances.

How does the notation of Mihalas relate to that of Osterbrock?. In the Appendix to his book, Osterbrock states that

$$N_e N(X^{+i+1}) v\sigma(v) f(v) dv = \left(1 - e^{-h\nu/kT} \right) N(X^{+i}) \frac{4\pi B_\nu}{h\nu} a_\nu d\nu \tag{119}$$

where f(v) is Maxwell-Boltzmann distribution function,

$$f(v) = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2kT}\right)^{3/2} v^2 e^{-mv^2/kT} \tag{120}$$

Exactly what species or levels X^{+i+1} and X^{+i+1} are being referred to here is a little unclear, but the Osterbrock notation seems to be limited to ground state ionization. He then uses the Saha equation to derive the relationship between the photoionization and radiative recombination cross sections, viz.

$$\sigma(v) = \frac{w_i}{w_{i+1}} \frac{h^2 \nu^2}{m^2 c^2 v^2} a_\nu \tag{121}$$

By definition, the (spontaneous) recombination coefficient is given by

$$\alpha(X^{+i}) = \int_o^\infty v f(v) \sigma(v) dv \tag{122}$$

Since $h(\nu - \nu_T) = mv^2/2$, we can rewrite the equation above as follows:

$$\alpha(X^{+i}) = \int_{\nu_T}^\infty \frac{4}{\sqrt{\pi}} \left(\frac{m}{2kT}\right)^{3/2} e^{h(\nu-\nu_T)/kT} \frac{w_i}{w_{i+1}} \frac{h^2 \nu^2}{m^2 c^2} a_\nu d\nu \tag{123}$$

This has the same frequency dependence as Mihalas and it is pretty obvious that if you pull out $(N^{+i}/N^{+i+1})^*$ using the Saha equation, one will come very close to the Mihalas formula.

Perhaps more usefully for our purposes, Ferland (Hazy2, p 240) says explicitly that the emissivity is:

$$4\pi j_\nu = h\nu \left(\frac{2\pi mk}{h^2}\right)^{-3/2} \left(\frac{8\pi}{c^2}\right) \frac{g_n}{g_e g_{ion}} T^{-3/2} \nu^2 \alpha_\nu(n) e^{-h(\nu-\nu_T)/KT} \tag{124}$$

7.3. Non LTE ionization levels and radiative transfer

Mihalas seems to imply that if you are going to study an ion in a non-LTE situation, then you define the levels which you are going to include and calculate the population of the individual levels. You actually do not solve for an entire ion but the result for the entire ion is derived after the fact. The rates then involve individual levels.

The complete rate equation for level i of an ion is given by Mihalas (5-87) to be:

$$-\sum_{i''<i}(R_{i''i}+C_{i''i})+n_i\left[\sum_{i''<i}(n_{i''}/n_i)^*(R_{ii''}+C_{ii''}+\sum_{i'>i}^{\kappa}(R_{ii'}+C_{ii'})\right]-\sum_{i'>i}^{\kappa}n_{i'}(n_i/n_{i'})^*(R_{i'i}+C_{ii'})=0$$
(125)

There is obviously one such equation for each bound state. An additional equation is needed since in general one does not know the total for the next ionization state upward. One can either close the system by requiring the total number of atoms and ions of that species to reach the right value or by requiring charge conservation.

See equations 7-1 and 7-2 in Mihalas for a possible way to proceed with some of this. There he gives the "non-LTE opacity" of a material.

$$\chi_\nu=\sum_i\sum_{j>i}[n_i-(g_i/g_j)]\,\alpha_{ij}(\nu)+\sum_i\left(n_i-n_i^*e^{-h\nu/kt}\right)\alpha_{i\kappa}(\nu)+\sum_\kappa n_e n_\kappa\alpha_{\kappa\kappa}(\nu,T)\left(1-e^{-h\nu/kt}\right)+n_e\sigma_e$$
(126)

where the four terms represent bound-bound, bound-free, free-free and electron scattering, and * implies the LTE value, as usual with Mihalas. The corresponding non-LTE spontaneous thermal emission is

$$\eta_\nu=(2h\nu^3/c^2)\left[\sum_i\sum_{j>i}n_j(g_i/g_j)\alpha_{ij}(\nu)+\sum_i n_i^*\alpha_{i\kappa}e^{-h\nu/kt}+\sum_\kappa n_e n_\kappa\alpha_{\kappa\kappa}(\nu,T)e^{-h\nu/kT}\right]$$
(127)

In LTE, these equations are simpler:

$$\chi_\nu^* = \left[\sum_i \sum_{j>i} n_i^* \alpha_{ij}(\nu) + \sum_i n_i^* \alpha_{i\kappa}(\nu) + \sum_\kappa n_e n_\kappa \alpha_{\kappa\kappa}(\nu, T) \left(1 - e^{-h\nu/kt} \right) \right] (1 - e^{-h\nu/kT} + n_e \sigma_e$$

$$(128)$$

$$\eta_\nu = (2h\upsilon^3/c^2) e^{-h\nu/kT} \left[\sum_i \sum_{j>i} n_i^* \alpha_{ij}(\nu) + \sum_i n_i^* \alpha_{i\kappa} + \sum_\kappa n_e n_\kappa \alpha_{\kappa\kappa}(\nu, T) \right] \qquad (129)$$

8. Comparison to Shlosman & Vitello

Since much of the kinematic model for the wind derives from Shlosman & Vitello (1993), it is worthwhile to compare the results. The standard, or more properly generic, SV model assumes a 0.8 $M_\odot$ WD with R_{wd} of 7×10^8 cm with T_{wd} of 40,000 K and a negligible boundary layer temperature. The mass accretion rate $\dot{M}_{disk}$ is set to 1×10^{10} $M_\odot$ yr^{-1} and the wind mass loss rate $\dot{M}_{wind}$ is set to 1×10^{10} $M_\odot$ yr^{-1}. In this model the wind emanates from a region between 4 and 12 R_{wd} and the disk extends to 34 R_{wd} or 2.4×10^{10} cm.

Figures 1 and 2 illustrate the results of a calculation with Python_30 in November, 2000 for parameters equivalent to the SV standard model. In this model, we have included escape probabilities (as did SV) and the ionic abundances and temperatures have been recalculated so that the energy absorbed and emitted by the wind are equal. Broadly speaking the shapes of C IV resemble that of Shlosman and Vitello. In particular, inside the wind-cone one does see two broad emission peaks punctuated by a narrow absorption profile. Inside the wind-cone, one observes both absorption and emission. The strength of the emission profile grows with increasing inclination until it dominates outside the wind-cone.

The most significant difference between the SV profiles and the Python profiles are at 60°. The SV profiles are deepest in the middle of the absorption profile, while the Python profiles are deepest on the blue wing of the profile. The most likely explanation for this difference is a difference in the calculated ionization state for C. SV (figure 4) shows that CIV ionization fraction peaks a few WD radii above the disk and then declines. SV state that this is due to photoionization of the wind as it becomes "exposed to the central source". In Python, C IV continues to dominate far from the disk; it never has a significant fraction in C V. In the Python model in which $dotM_{wind}$ is reduced to 1×10^{-10} $M_\odot$ yr^{-1}, carbon does reach C V and the absorption profile is more symmetric.

9. Conclusions

Someday this code may eventually be accurate.

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Seaton, M. 1962 in Atomic and Molecular Processes, ed. D. R. Bates (New York: Academic), p 374

Verner, D. A., Ferland, G. J., Korista, K. T., & Yakovlev, D. G. 1996, ApJ, 465, 487

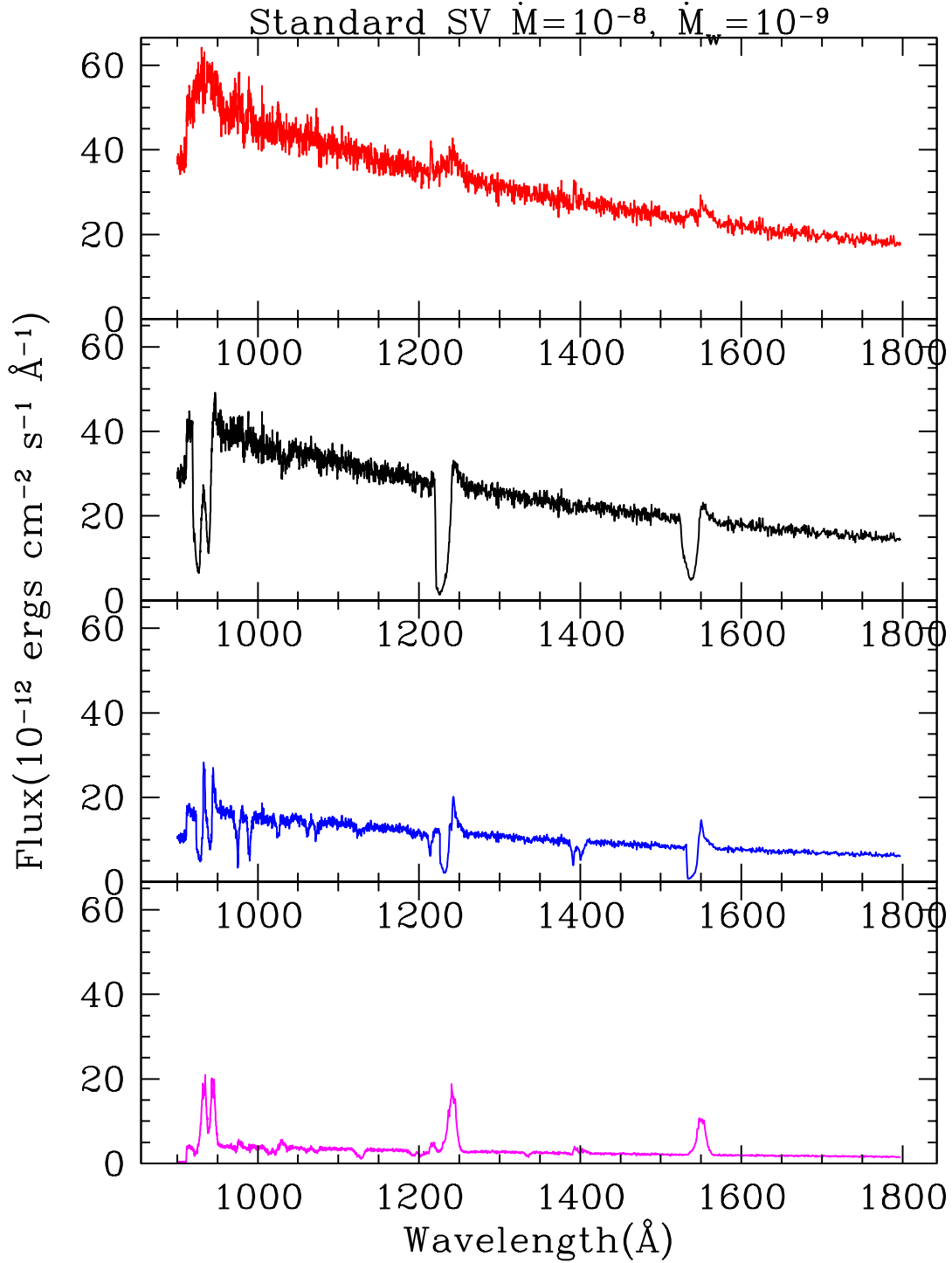Verner, D. A., Verner, E. M., & Ferland, G. J. 1996, Atomic Data Nucl. Data Tables, 64, 1

Fig. 1.— This a result of the standard Shlossman and Vitello geometry as calculated using Python 30. There were 10 ionization cycles and 200 spectral cycles in this calculation. Ionization of the wind was recalculated so that the absorbed and radiated energy are the same. Line radiation was calculated using escape probabilities. Blackbodies were used for the continuum spectra. An 0.8 $M_\odot$WD was utilized. The outer radius of the disk was taken to be 2.4×10^{10} cm. The angles calculated are 10, 30, 60, and 80 degrees. The wind emanates from 4-12 WD radii
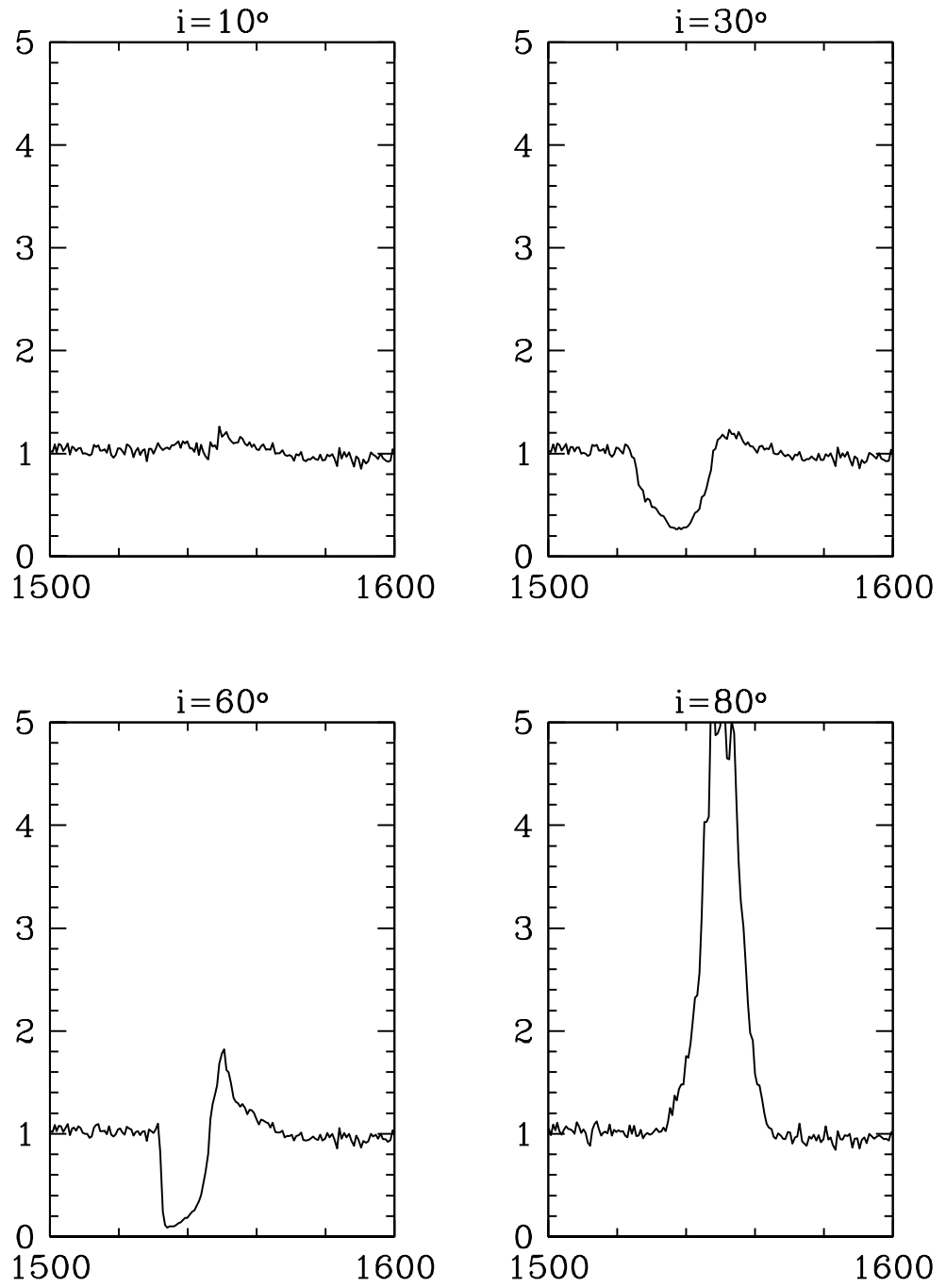
Fig. 2.— Normalized C IV profiles for the "standard" Shlossman & Vitello model.
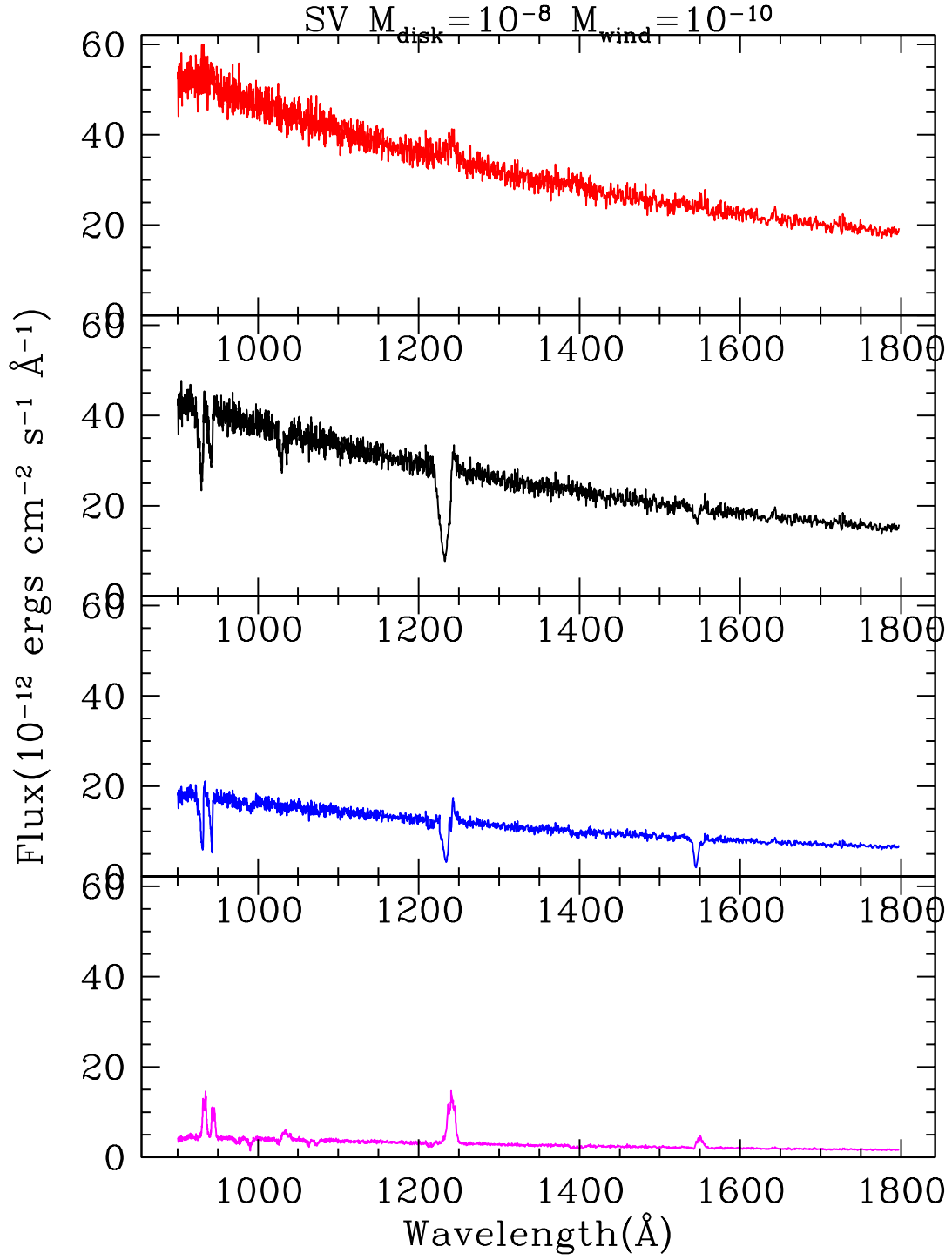
Fig. 3.— The geometry and other conditions is the same as in the "standard" SV model, except here the $\dot{M}_{wind}$ is 10^{-10} $M_\odot$ $year^{-1}$.
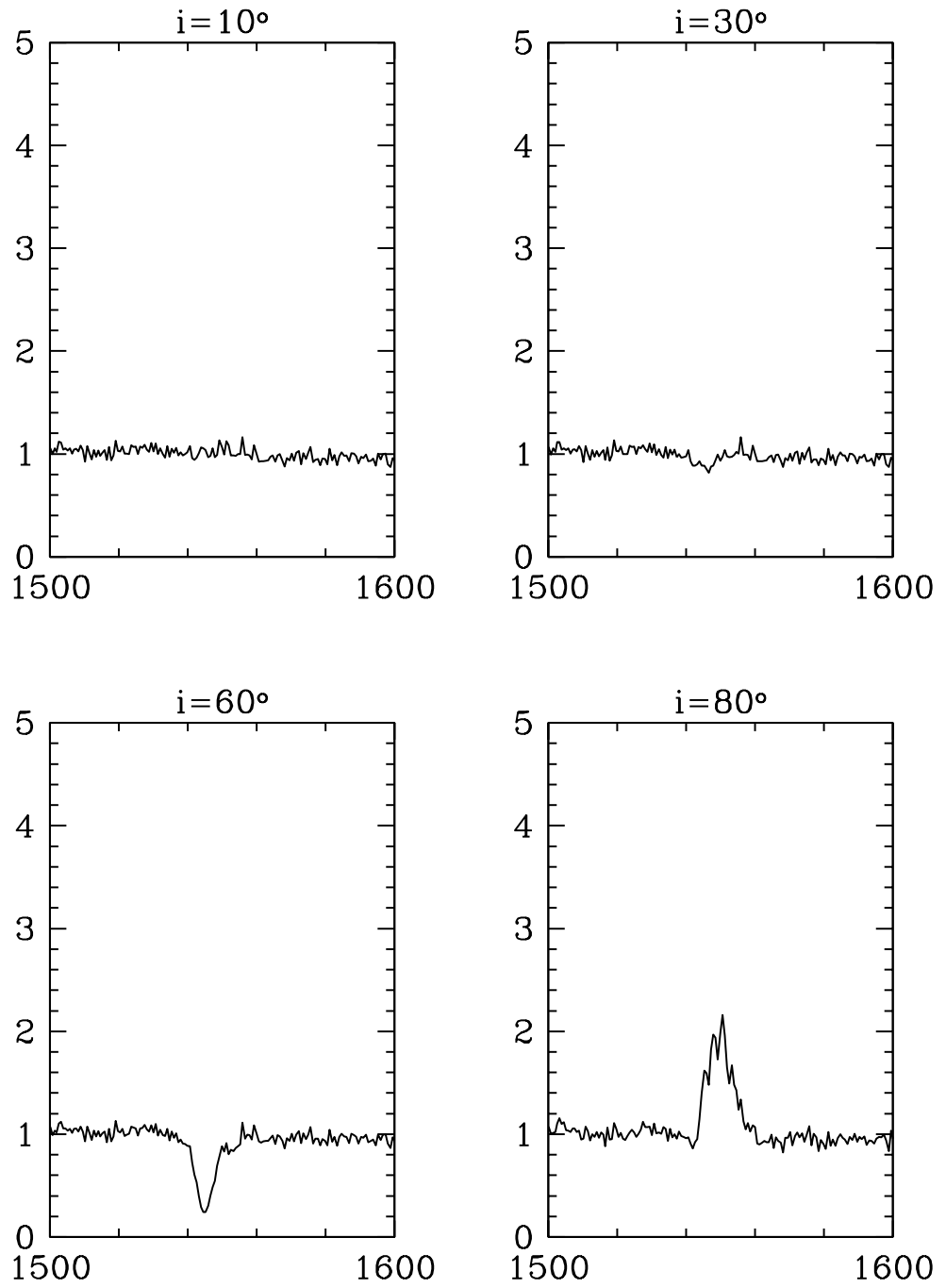
Fig. 4.— Normalized C IV profiles for the "standard" Shlossman & Vitello model, but with $\dot{M}_{wind}$ of 10^{-10} $M_\odot$ yr^{-1}.
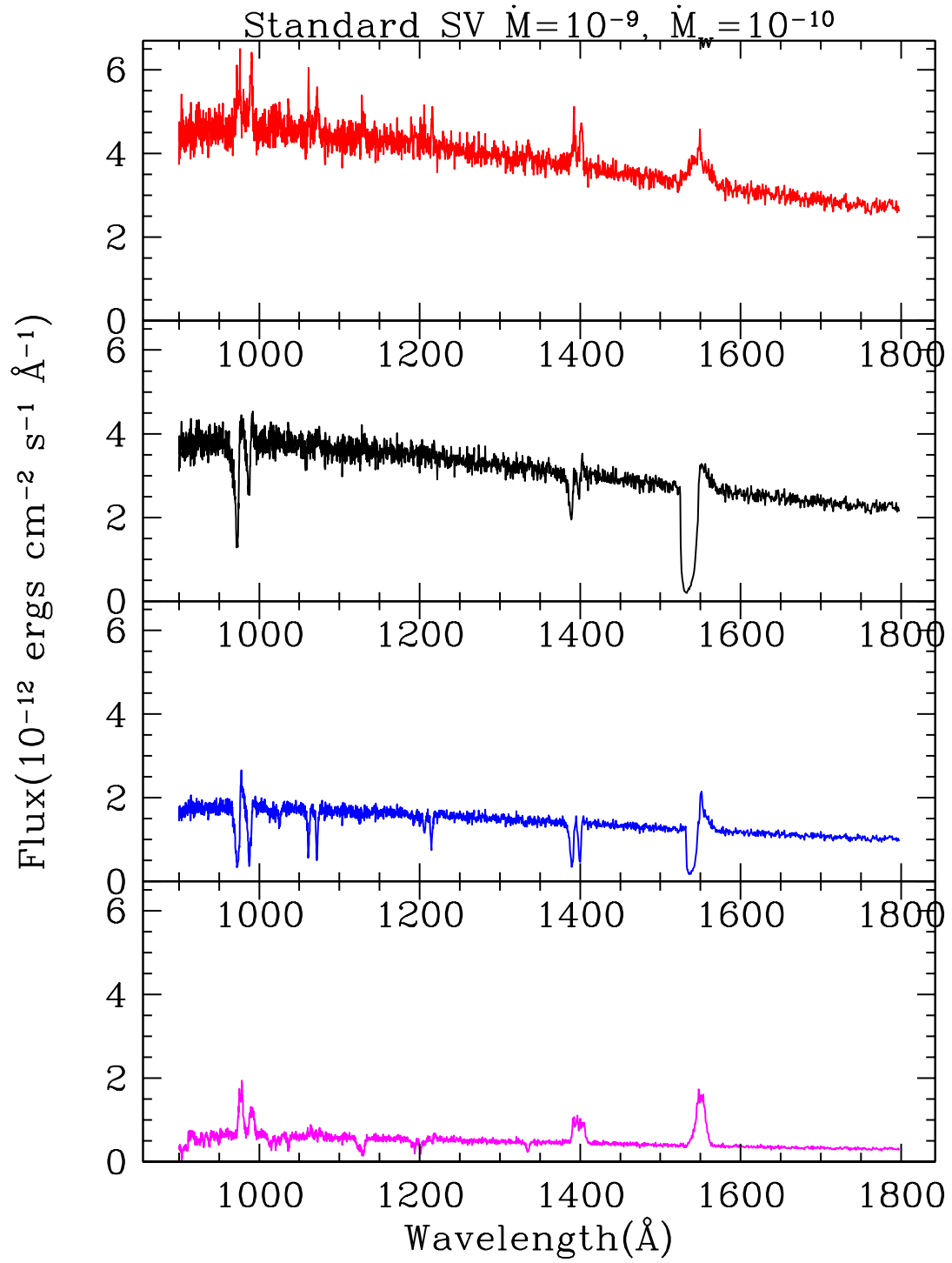
Fig. 5.— Similar to the "standard" SV model but with $\dot{M}_{disk} = 10^{-9}$ and $\dot{M}_{wind} = 10^{-10}$
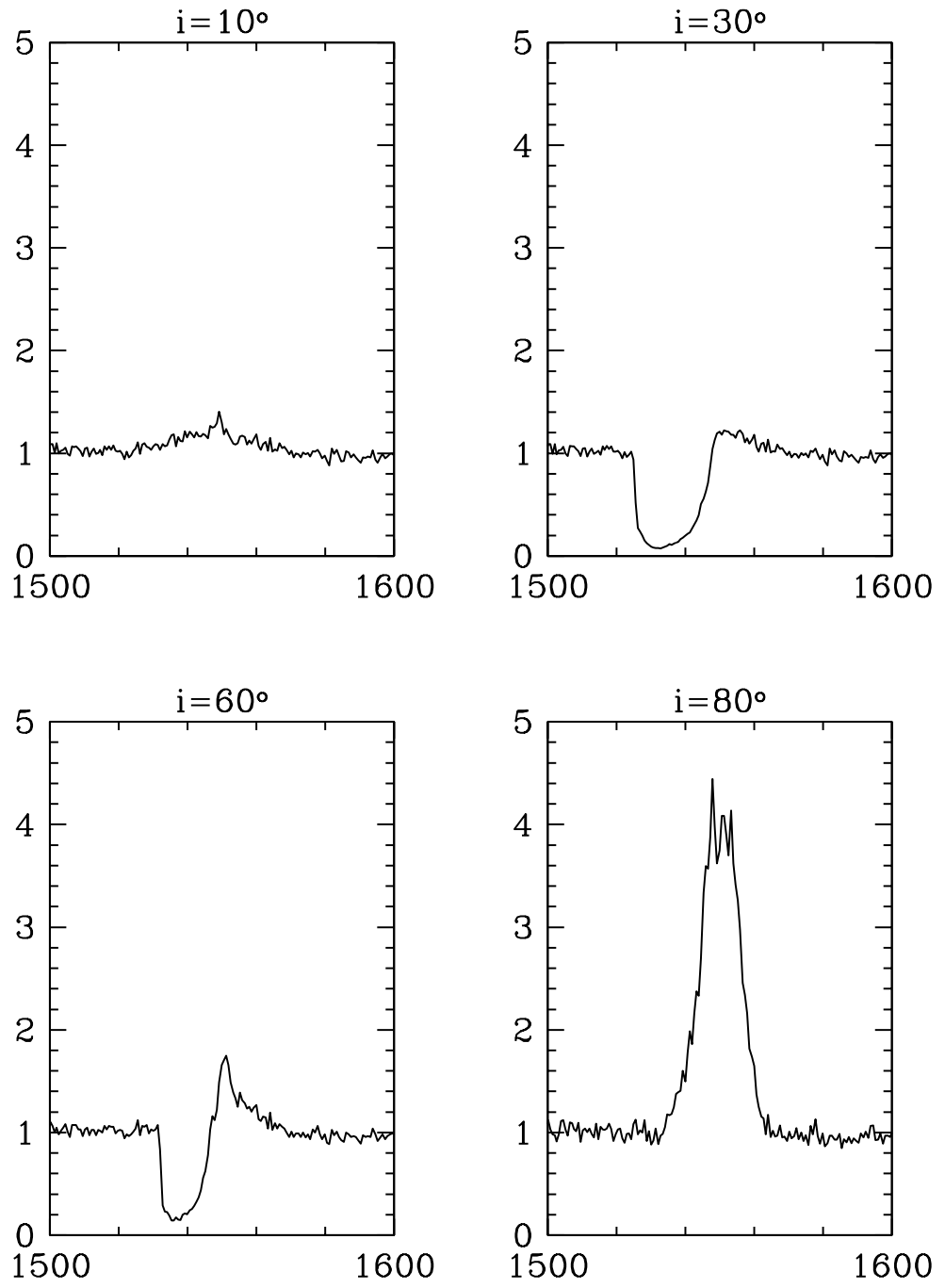
Fig. 6.— Normalized C IV profiles for the "standard" Shlossman & Vitello model, but with $\dot{M}_{disk}$ of 10^{-9} $M_\odot$ yr^{-1} and $\dot{M}_{wind}$ of 10^{-10} $M_\odot$ yr^{-1}.
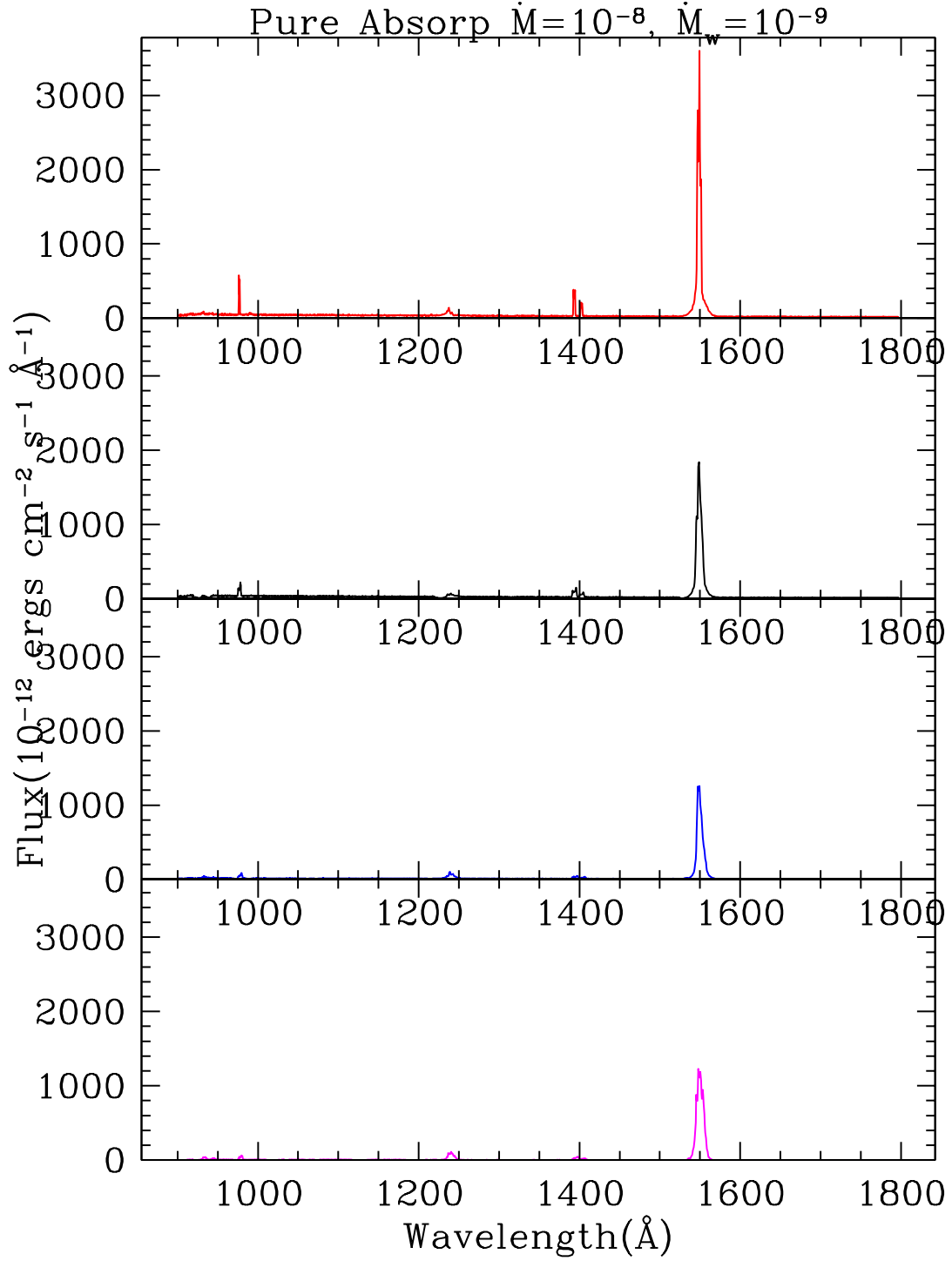
Fig. 7.— This is a model calculated for pure absorption. Ionization was recalculated, but the spectrum is still dominated by radiation from the wind. Presumably the reason that the wind lines are so strong in this case is that the wind has to radiate more.
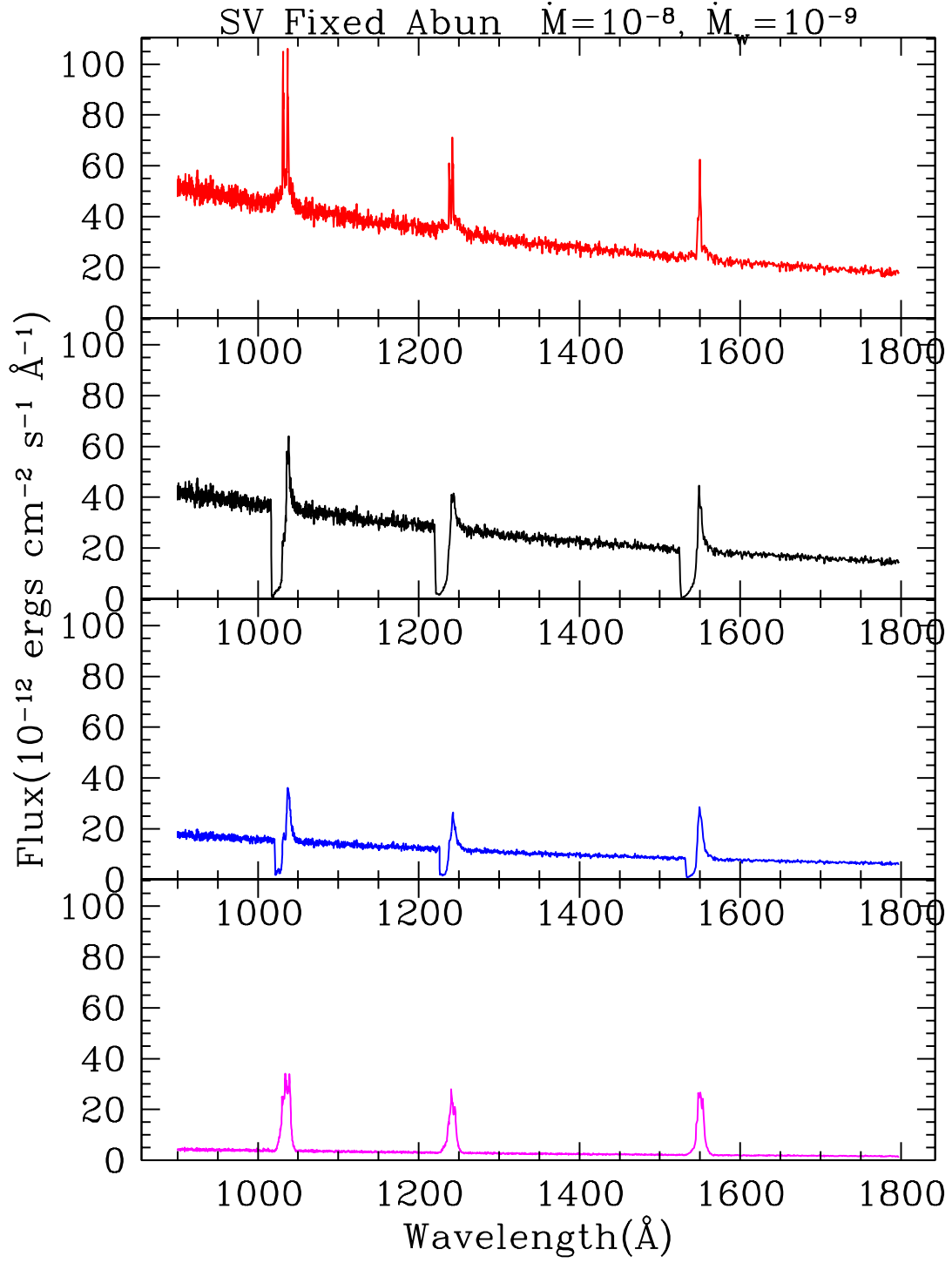
Fig. 8.— This is a "standard' SV model, but the ionization is set so that C, N, and O are completely in the Li-like ion.
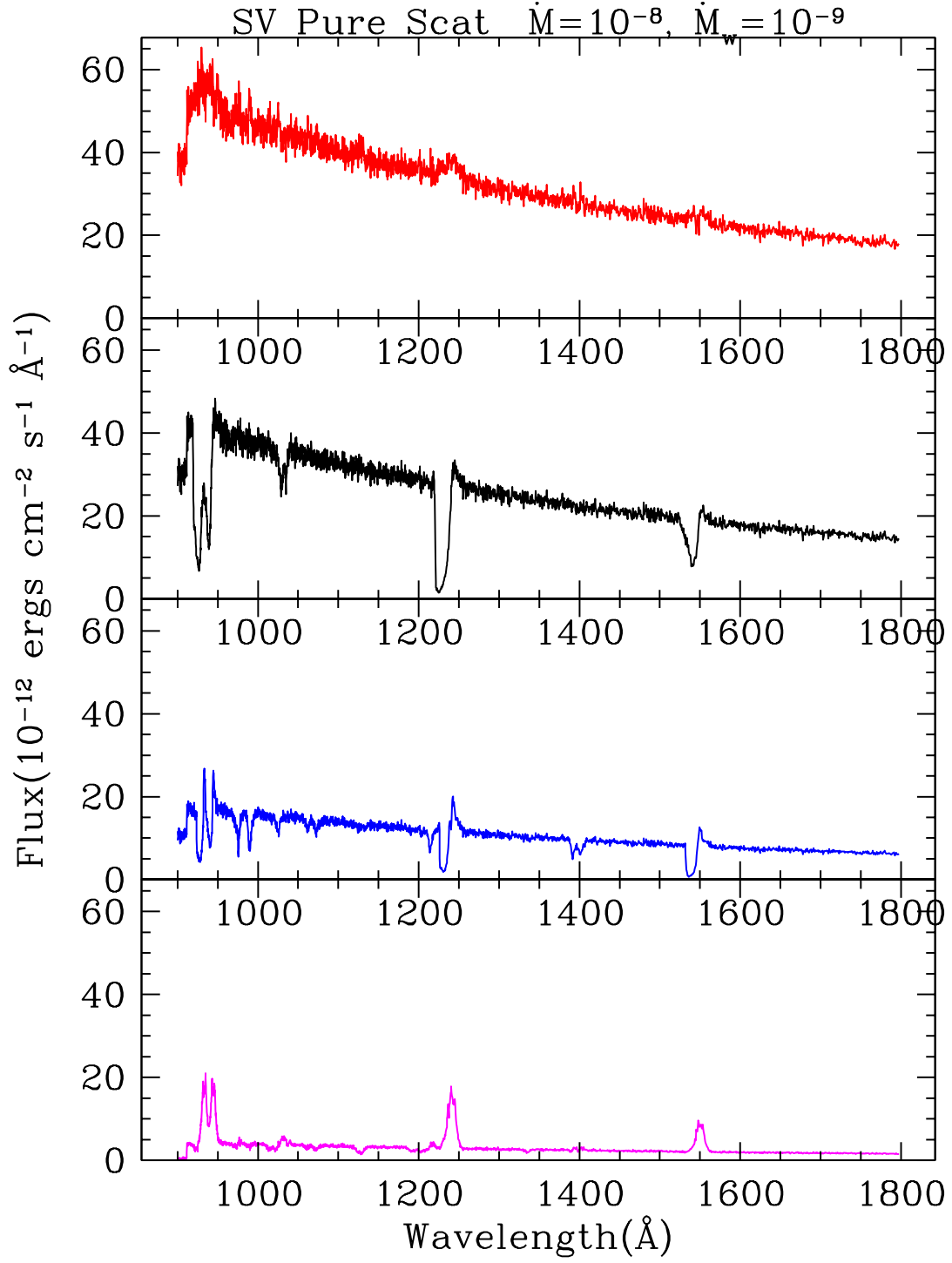
Fig. 9.— This is a "standard' SV model, but the radiative transfer is calculated in the pure scattering mode. Why this model doesn't result in a very cold wind is a mystery.
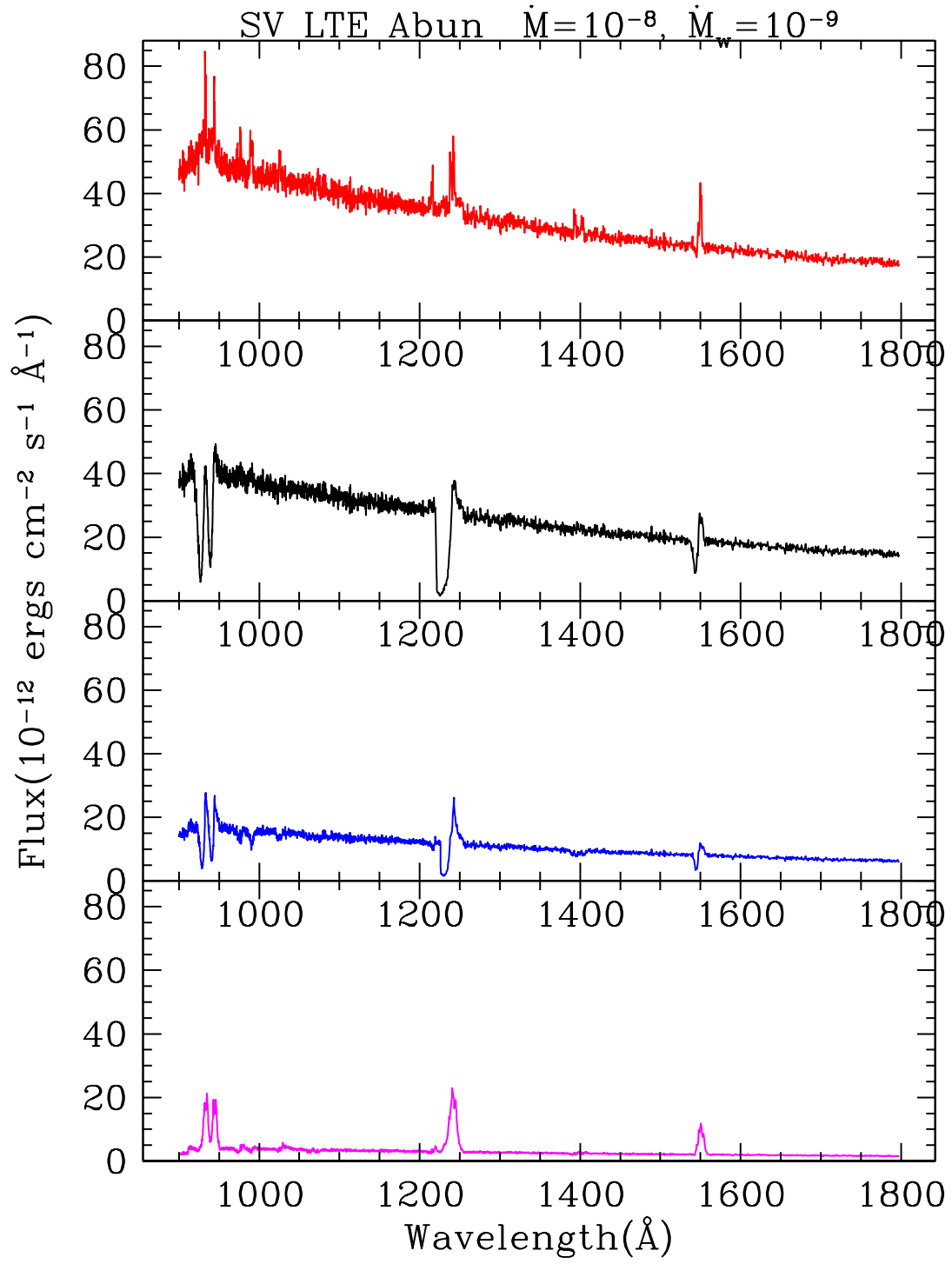
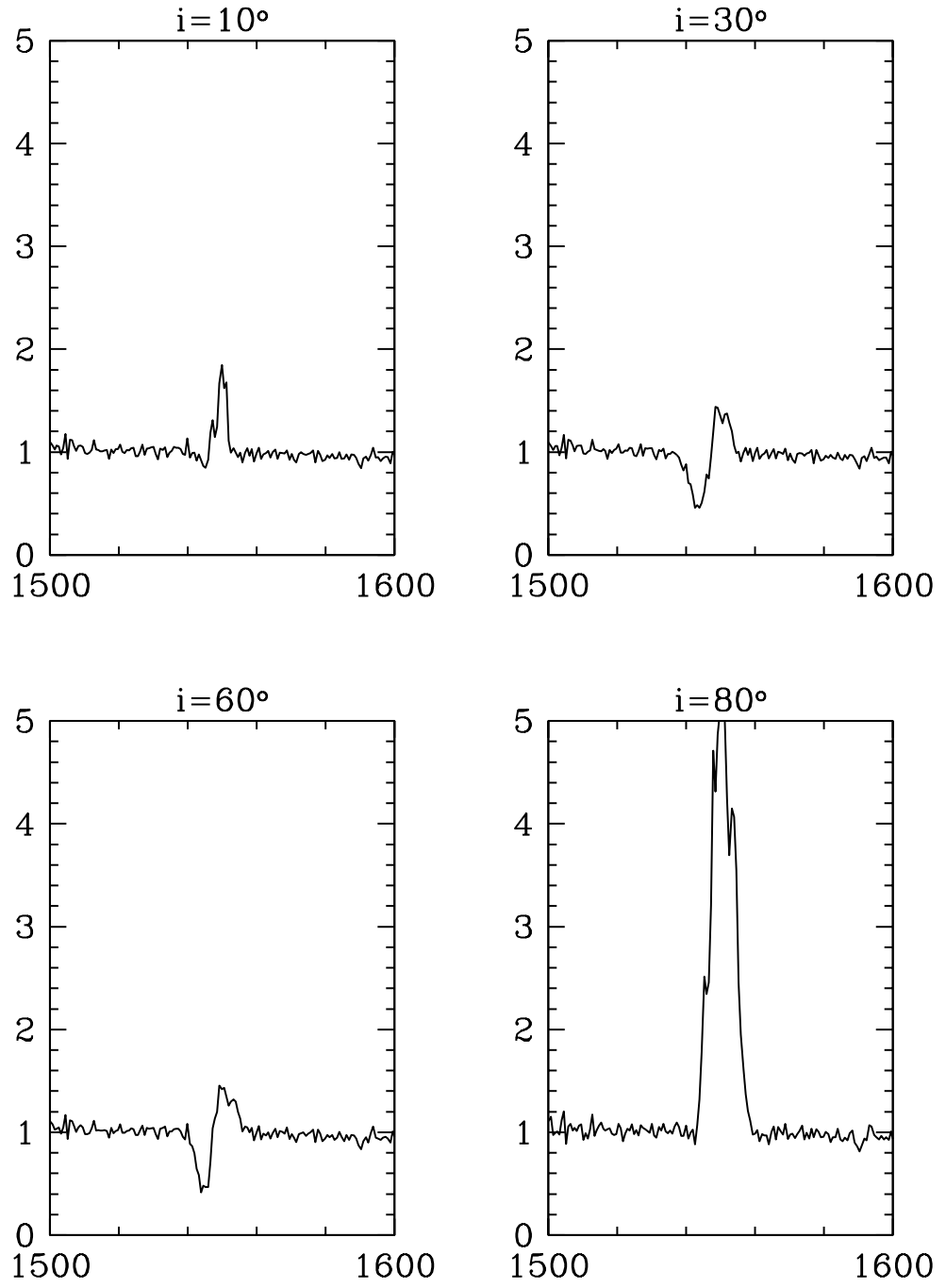Fig. 10.— Here ionization is in the LTE limit

Fig. 11.— Normalized C IV profiles for the "standard" Shlossman & Vitello model, but with LTE abundances.
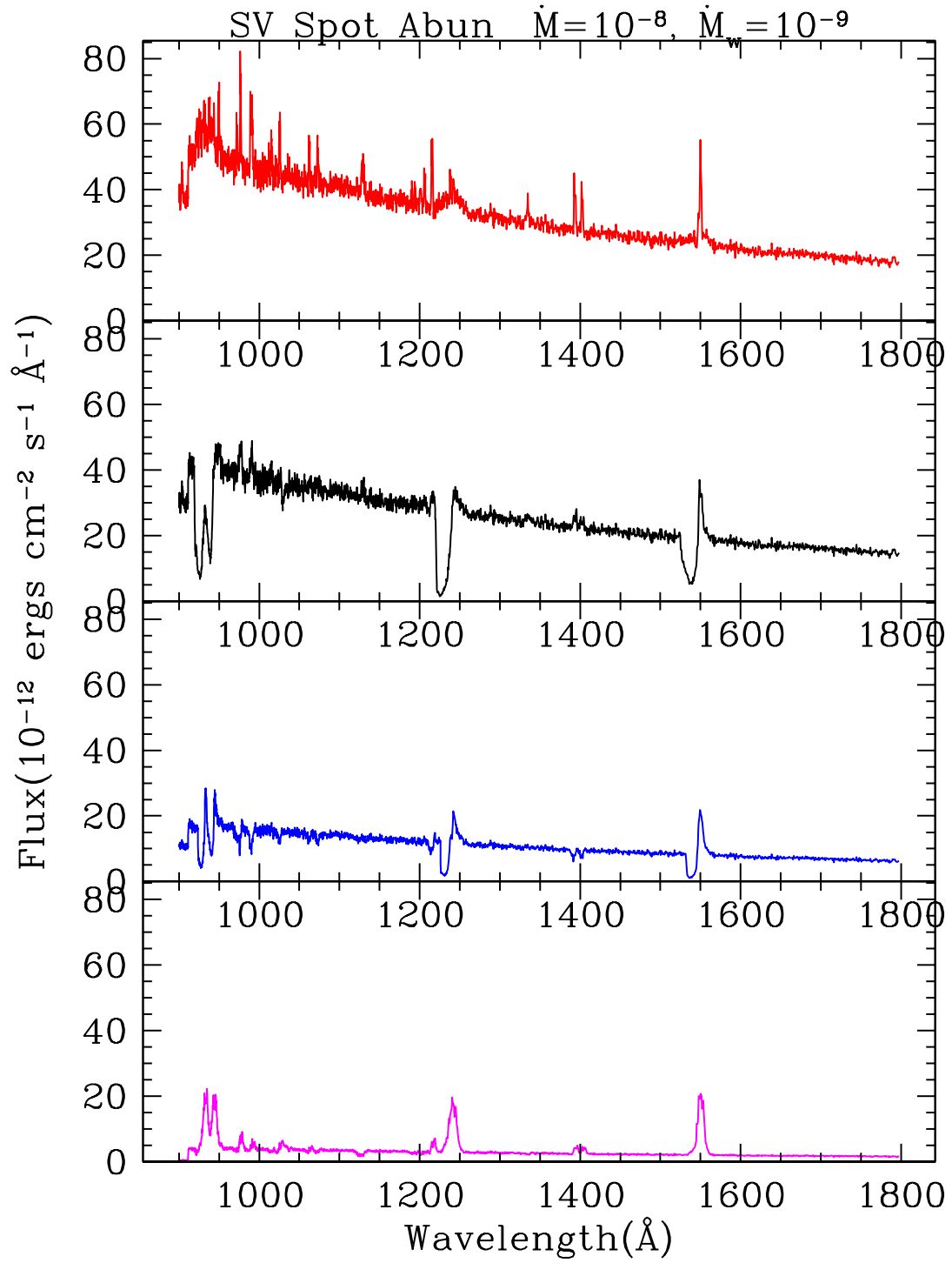
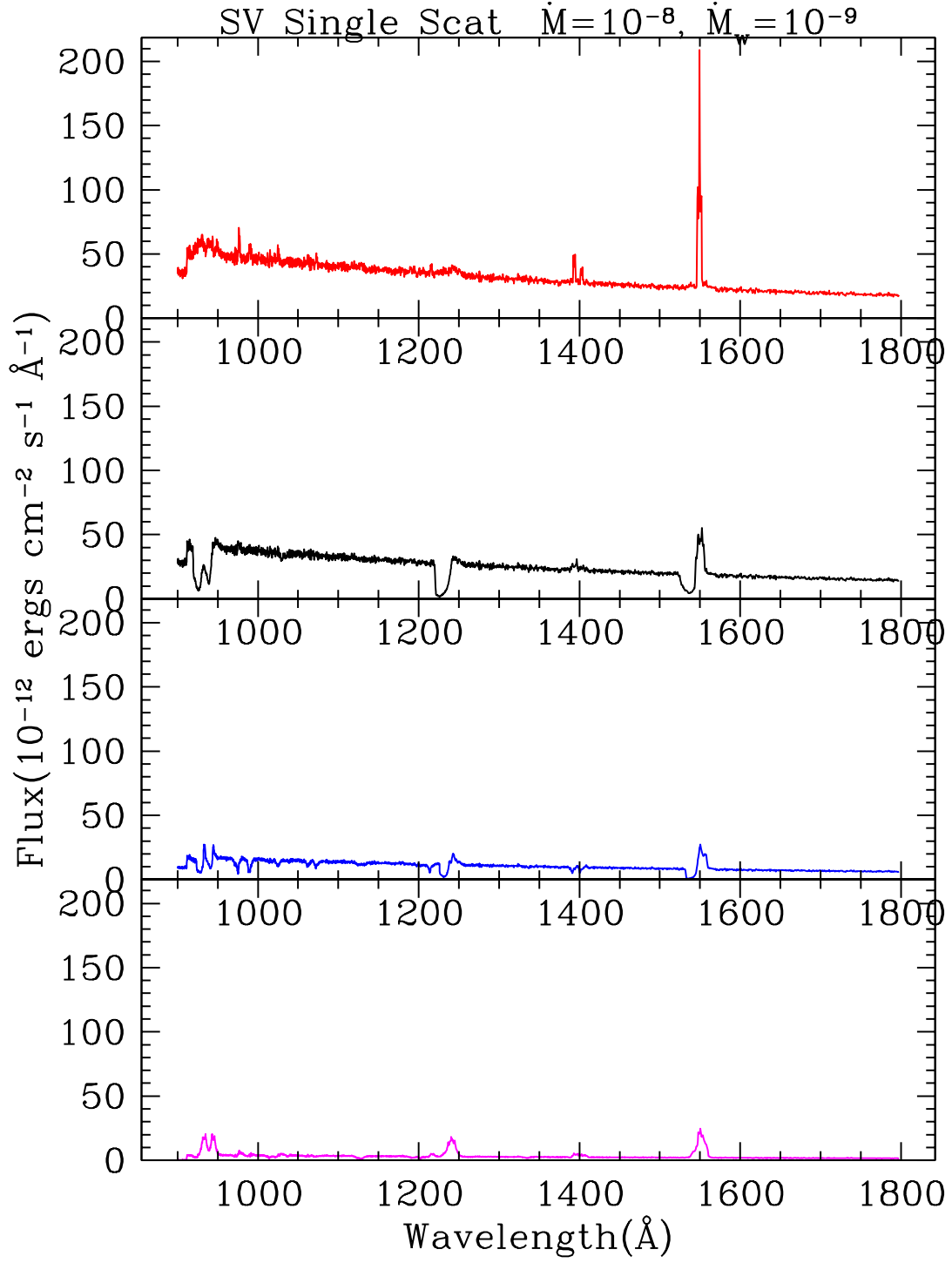Fig. 12.— Here ionization is calculated in the on the spot approximation.

Fig. 13.— s is a "standard' SV model, but the radiative transfer is in the single scattering limit. Escape probabilities have there fore been turned off.