

$l = 1, 2, \dots, L$	ORF
$m = 1, \dots, M_l$	Repeat
$n = 1, 2, \dots, N_{lm}$	Time Point

$$y_{lmn} \sim N(\hat{y}_{lmn}, (e^{\nu_l})^{-1})$$

$$\hat{y}_{lmn} = f(x_{lmn}; e^{K_{lm}}, e^{r_{lm}}, e^P)$$

Repeat level

$$K_{lm} \sim N(K_l^o, (e^{\tau_l^K})^{-1}) \quad \tau_l^K \sim N(\sigma^K, (\phi^K)^{-1})$$

$$r_{lm} \sim N(r_l^o, (e^{\tau_l^r})^{-1}) \quad \tau_l^r \sim N(\sigma^r, (\phi^r)^{-1})$$

ORF level

$$K_l^o \sim N(K^p, (e^{\sigma^{K,o}})^{-1}) \quad \sigma^{K,o} \sim N(\eta^{K,o}, (\psi^{K,o})^{-1})$$

$$r_l^o \sim N(r^p, (e^{\sigma^{r,o}})^{-1}) \quad \sigma^{r,o} \sim N(\eta^{r,o}, (\psi^{r,o})^{-1})$$

$$\nu_l \sim N(\nu^p, (e^{\sigma^\nu})^{-1}) \quad \sigma^\nu \sim N(\eta^\nu, (\psi^\nu)^{-1})$$

Population level

$$K^p \sim N(K^\mu, (\eta^{K,p})^{-1})$$

$$r^p \sim N(r^\mu, (\eta^{r,p})^{-1})$$

$$\nu^p \sim N(\nu^\mu, (\eta^{\nu,p})^{-1})$$

$$P \sim N(P^\mu, (\eta^P)^{-1})$$

$$\begin{aligned}
y_{lmn} &\sim N(\hat{y}_{lmn}, (e^{\nu_l})^{-1}) \\
\hat{y}_{lmn} &= f(x_{lmn}; e^{K_{lm}}, e^{r_{lm}}, e^P) \\
K_{lm} &\sim N(K_l^o, (e^{\tau_l^K})^{-1})
\end{aligned}$$

$$\pi(K_{lm}|\theta) = \sqrt{\frac{e^{\tau_l^K}}{2\pi}} e^{-\frac{1}{2}(K_{lm}-K_l^o)^2 e^{\tau_l^K}} \prod_{n=1}^{N_{lm}} \sqrt{\frac{e^{\nu_l}}{2\pi}} e^{-\frac{1}{2}(y_{lmn}-\hat{y}_{lmn})^2 e^{\nu_l}}$$

Taking logs

$$l\pi(K_{lm}|\theta) = -\frac{1}{2}(K_{lm} - K_l^o)^2 e^{\tau_l^K} - \sum_{n=1}^{N_{lm}} \frac{1}{2}(y_{lmn} - \hat{y}_{lmn})^2 e^{\nu_l} + C$$

Similarly for  $r_{lm}$

$$l\pi(r_{lm}|\theta) = -\frac{1}{2}(r_{lm} - r_l^o)^2 e^{\tau_l^r} - \sum_{n=1}^{N_{lm}} \frac{1}{2}(y_{lmn} - \hat{y}_{lmn})^2 e^{\nu_l} + C$$


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$$\begin{aligned}
K_{lm} &\sim N(K_l^o, (e^{\tau_l^K})^{-1}) \\
\tau_l^K &\sim N(\sigma^K, (\phi^K)^{-1})
\end{aligned}$$

$$\pi(\tau_l^K|\theta) = \sqrt{\frac{\phi^K}{2\pi}} e^{-\frac{1}{2}(\tau_l^K - \sigma^K)^2 \phi^K} \prod_{m=1}^{m_l} \sqrt{\frac{e^{\tau_l^K}}{2\pi}} e^{-\frac{1}{2}(K_{lm}-K_l^o)^2 e^{\tau_l^K}}$$

Taking logs

$$l\pi(\tau_l^K|\theta) = -\frac{1}{2}(\tau_l^K - \sigma^K)^2 \phi^K + \sum_{m=1}^{m_l} \frac{1}{2}(\tau_l^K - (K_{lm} - K_l^o)^2 e^{\tau_l^K}) + C$$

Similarly for  $\tau_l^r$

$$l\pi(\tau_l^r|\theta) = -\frac{1}{2}(\tau_l^r - \sigma^r)^2 \phi^r + \sum_{m=1}^{m_l} \frac{1}{2}(\tau_l^r - (r_{lm} - r_l^o)^2 e^{\tau_l^r}) + C$$

$$K_{lm} \sim N(K_l^o, (e^{\tau_l^K})^{-1})$$

$$K_l^o \sim N(K^p, (e^{\sigma^{K,o}})^{-1})$$

$$\pi(K_l|\theta) = \sqrt{\frac{e^{\sigma^{K,o}}}{2\pi}} e^{-\frac{1}{2}(K_l^o - K^p)^2 e^{\sigma^{K,o}}} \prod_{m=1}^{M_l} \sqrt{\frac{e^{\tau_l^K}}{2\pi}} e^{-\frac{1}{2}(K_{lm} - K_l^o)^2 e^{\tau_l^K}}$$

Taking logs

$$l\pi(K_l|\theta) = -\frac{1}{2}(K_l^o - K^p)^2 e^{\sigma^{K,o}} - \sum_{m=1}^{M_l} \frac{1}{2}(K_{lm} - K_l^o)^2 e^{\tau_l^K} + C$$

Similarly for  $r_{lm}$

$$l\pi(r_l|\theta) = -\frac{1}{2}(r_l^o - r^p)^2 e^{\sigma^{r,o}} - \sum_{m=1}^{M_l} \frac{1}{2}(r_{lm} - r_l^o)^2 e^{\tau_l^r} + C$$


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$$y_{lmn} \sim N(\hat{y}_{lmn}, (e^{\nu_l})^{-1})$$

$$\hat{y}_{lmn} = f(x_{lmn}; e^{K_{lm}}, e^{r_{lm}}, e^P)$$

$$\nu_l \sim N(\nu^p, (e^{\sigma^\nu})^{-1})$$

$$\pi(\nu_l|\theta) = \sqrt{\frac{e^{\sigma^\nu}}{2\pi}} e^{-\frac{1}{2}(\nu_l - \nu^p)^2 e^{\sigma^\nu}} \prod_{m=1}^{m_l} \prod_{n=1}^{n_{lm}} \sqrt{\frac{e^{\nu_l}}{2\pi}} e^{-\frac{1}{2}(y_{lmn} - \hat{y}_{lmn})^2 e^{\nu_l}}$$

Taking logs

$$l\pi(\nu_l|\theta) = -\frac{1}{2}(\nu_l - \nu^p)^2 e^{\sigma^\nu} + \sum_{m=1}^{m_l} \sum_{n=1}^{n_{lm}} \frac{1}{2}(\nu_l - (y_{lmn} - \hat{y}_{lmn})^2 e^{\nu_l}) + C$$


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$$K_l^o \sim N(K^p, (e^{\sigma^{K,o}})^{-1})$$

$$\sigma^{K,o} \sim N(\eta^{K,o}, (\psi^{K,o})^{-1})$$

$$\pi(\sigma^{K,o}|\theta) = \sqrt{\frac{\psi^{K,o}}{2\pi}} e^{-\frac{1}{2}(\sigma^{K,o} - \eta^{K,o})^2 \psi^{K,o}} \prod_{l=1}^L \sqrt{\frac{e^{\sigma^{K,o}}}{2\pi}} e^{-\frac{1}{2}(K_l^o - K^p)^2 e^{\sigma^{K,o}}}$$

Taking logs

$$l\pi(\sigma^{K,o}|\theta) = -\frac{1}{2}(\sigma^{K,o} - \eta^{K,o})^2 \psi^{K,o} + \sum_{l=1}^L \frac{1}{2}(\sigma^{K,o} - (K_l^o - K^p)^2 e^{\sigma^{K,o}}) + C$$

Similarly for  $\sigma^{r,o}$

$$l\pi(\sigma^{r,o}|\theta) = -\frac{1}{2}(\sigma^{r,o} - \eta^{r,o})^2 \psi^{r,o} + \sum_{l=1}^L \frac{1}{2}(\sigma^{r,o} - (r_l^o - r^p)^2 e^{\sigma^{r,o}}) + C$$

And also for  $\sigma^\nu$

$$l\pi(\sigma^\nu|\theta) = -\frac{1}{2}(\sigma^\nu - \eta^\nu)^2 \psi^\nu + \sum_{l=1}^L \frac{1}{2}(\sigma^\nu - (\nu_l - \nu^p)^2 e^{\sigma^\nu}) + C$$

$$K_l^o \sim N(K^p, (e^{\sigma^{K,o}})^{-1})$$

$$K^p \sim N(K^\mu, (\eta^{K,p})^{-1})$$

$$\pi(K_p|\theta) = \sqrt{\frac{\eta^{K,p}}{2\pi}} e^{-\frac{1}{2}(K^p - K^\mu)^2 \eta^{K,p}} \prod_{l=1}^L \sqrt{\frac{e^{\sigma^{K,o}}}{2\pi}} e^{-\frac{1}{2}((K_l^o - K^p) e^{\sigma^{K,o}})^2}$$

Taking logs

$$l\pi(K_p|\theta) = -\frac{1}{2}(K^p - K^\mu)^2 e^{\eta^{K,p}} - \sum_{l=1}^L \frac{1}{2}(K_l^o - K^p)^2 e^{\sigma^{K,o}} + C$$

Similarly for  $r_p$

$$l\pi(r_p|\theta) = -\frac{1}{2}(r^p - r^\mu)^2 e^{\eta^{r,p}} - \sum_{l=1}^L \frac{1}{2}(r_l^o - r^p)^2 e^{\sigma^{r,o}} + C \text{ and similarly for } \nu_p$$

$$l\pi(\nu^p|\theta) = -\frac{1}{2}(\nu^p - \nu^\mu)^2 \eta^{\nu,p} - \sum_{l=1}^L \frac{1}{2}(\nu_l - \nu^p)^2 e^{\sigma^\nu} + C$$

$$y_{lmn} \sim N(\hat{y}_{lmn}, (e^{\nu_l})^{-1})$$

$$\hat{y}_{lmn} = f(x_{lmn}; e^{K_{lm}}, e^{r_{lm}}, e^P)$$

$$P \sim N(P^\mu, (\eta^P)^{-1})$$

$$\pi(P|\theta) = \sqrt{\frac{\eta^P}{2\pi}} e^{-\frac{1}{2}(P^p - P^\mu)^2 \eta^P} \prod_{l=1}^L \prod_{m=1}^{m_l} \prod_{n=1}^{n_{lm}} \sqrt{\frac{e^{\nu_l}}{2\pi}} e^{-\frac{1}{2}(y_{lmn} - \hat{y}_{lmn})^2 e^{\nu_l}}$$

Taking logs

$$l\pi(P|\theta) = -\frac{1}{2}(P^p - P^\mu)^2 \eta^P - \sum_{l=1}^L \sum_{m=1}^{M_l} \sum_{n=1}^{N_{lm}} \frac{1}{2}(y_{lmn} - \hat{y}_{lmn})^2 e^{\nu_l} + C$$