CS-559: Assignment 1

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10 October 2020

I pledge my honor that I have abided by the Stevens Honor System. *Justin Ho*

1. $U(\text{no bet}) = U(\text{no bet, lose}) = U(\text{no bet, win}) = \$B - \$B = 0$
   $U(\text{bet, win}) = \$W - \$B > 0$
   $U(\text{bet, lose}) = \$L - \$B < 0$
   $$\begin{aligned} \therefore U(\text{bet}) &= p_w \cdot U(\text{bet, win}) + p_l \cdot U(\text{bet, lose}) \\ &= p_w \cdot (\$W - \$B) + p_l \cdot (\$L - \$B) \\ &= p_w \cdot (\$W - \$B) + (1 - p_w) \cdot (\$L - \$B) \\ &= p_w\$W - p_w\$B + \$L - \$B - p_w\$L + p_w\$B \\ &= p_w \cdot (\$W - \$L) + \$L - \$B \end{aligned}$$
   $p_w \cdot (\$W - \$L) + \$L - \$B = 0 \implies p_w = \frac{\$B - \$L}{\$W - \$L}$
   Therefore, to accept the bet with a positive expected value, $p_w > \frac{\$B - \$L}{\$W - \$L}$.

2. (a) W: wallet color, $W = \{b, g\}$ C: coin pulled from wallet, $C = \{p, d\}$
   $P(W = g) = 4 \cdot P(W = b) = 0.8 \implies P(W = b) = 0.2$
   $P(C = p \mid W = g) = 0.6 \implies P(C = d \mid W = g) = 0.4$
   $P(C = p \mid W = b) = 0.8 \implies P(C = d \mid W = b) = 0.2$
   $P(C = p) = 0.7 \implies P(C = d) = 0.3$
   $P(W = g \mid C = d, p, p) = \frac{P(d,p,p|g) \cdot P(g)}{P(d,p,p)} = \frac{0.4 \times 0.6^2 \times 0.8}{0.3 \times 0.7^2} \approx 0.784$
   $P(W = b \mid C = d, p, p) = \frac{P(d,p,p|b) \cdot P(b)}{P(d,p,p)} = \frac{0.2 \times 0.8^2 \times 0.2}{0.3 \times 0.7^2} \approx 0.174$
   The green wallet is more likely to have been picked in this situation.

   (b) $P(\text{error} \mid C = d, p, p) = P(W = b \mid C = d, p, p) \approx 0.174$

3. (b) $\hat{\mu} = \frac{N_1\mu_1 + N_2\mu_2}{N_1 + N_2} = \frac{2000(1) + 1000(4)}{2000 + 1000} = 2$
   $\hat{\sigma}^2 = \frac{N_1\sigma_1^2 + N_2\sigma_2^2}{N_1 + N_2} + \frac{N_1 N_2}{(N_1 + N_2)^2}(\mu_1 - \mu_2) = \frac{2000(4) + 1000(9)}{2000 + 1000} + \frac{2000(1000)}{(2000 + 1000)^2}(1 - 4)^2 \approx 7.667$
   Using the experimental data collected from the Python script, the empirical data fits the theoretical data.