

$$\eta_{Th} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{r^\alpha} \quad \alpha = \frac{k-1}{k}$$

$$W_{outmax} = C_p (\sqrt{T_3} - \sqrt{T_1})^2 \quad * \text{ use as check when } \eta = 1$$

$$W_{out} = h_3 - h_4$$

$$W_{in} = h_1 - h_2$$

$$\eta_{Th} = \frac{h_{4s} - h_3}{h_{4s} - h_3}$$

$$W_{out} = \eta (T_3 - T_{4,s})$$

$$= \frac{T_4}{T_{3,s}} = r_p^\alpha$$

$$\eta_c W_{act} = (T_{3,s} - T_1)$$

$$W_{act} = \frac{1}{\eta_c} (T_{3,s} - T_1)$$

$$\eta_{c, Th} = \frac{h_{3s} - h_1}{h_{3s} - h_1}$$

$$T_{4,s} = T_3 r_p^\alpha$$

$$\frac{T_2}{T_1} = r_p^\alpha$$

$$W_{out} = \eta T_3 (1 - r_p^\alpha)$$

$$W_{in} = \frac{T_1}{\eta_c} (r_p^\alpha - 1)$$

$$W_{net} = \frac{T_1}{\eta_c} (r_p^\alpha - 1) - \eta T_3 (1 - r_p^\alpha)$$

$$\eta_{II} = \frac{\eta_{Th, Brayton}}{\eta_{Th, Carnot}} = \frac{1 - \frac{q_{out}}{q_{in}}}{1 - \frac{T_L}{T_H}} = \frac{1 - r_p^{-\alpha}}{1 - \frac{T_0 \ln(T_2 T_3)}{T_2 + T_3}} = \frac{(T_2 + T_3)(1 - r_p^{-\alpha})}{(T_2 + T_3) - T_0 \ln(T_2 T_3)}$$

$$S_{gen} = \frac{q_{in}}{T_H} - \frac{q_{out}}{T_L}$$

$$\phi_x = T_0 S_{gen}$$

$$T_0 = 25^\circ\text{C}$$

1st Law turb

$$S_{gen,T} = -\frac{q_{in}}{T_3} + \frac{q_{out}}{T_{4,s}}$$

$$S_{gen,T} = -\frac{q_{in}}{T_3} + \frac{c_p(T_3 - T_{4,a}) - \gamma_{T_{TH}} T_3(1 - r_p^\alpha)}{T_{4,s}}$$

$$= -\frac{q_{in}}{T_3} + \frac{c_p(T_3 - T_3 + \gamma_{T_{TH}}(T_3 - T_{4,s})) - \gamma_{T_{TH}} T_3(1 - r_p^\alpha)}{T_3 - \gamma_{T_{TH}} T_3(1 - r_p^\alpha)}$$

$$= -\frac{q_{in}}{T_3} + \frac{c_p \gamma_{T_{TH}} T_3(1 - r_p^\alpha) - \gamma_{T_{TH}}(1 - r_p^\alpha)}{T_3 - \gamma_{T_{TH}} T_3(1 - r_p^\alpha)}$$

$$= -\frac{q_{in}}{T_3} + \frac{\gamma_{T_{TH}} T_3(1 - r_p^\alpha)(c_p - 1)}{T_3 - \gamma_{T_{TH}} T_3(1 - r_p^\alpha)}$$

$$S_{gen,T} = -\frac{q_{in}}{T_3} + \frac{\gamma_{T_{TH}}(1 - r_p^\alpha)(c_p - 1)}{1 - \gamma_{T_{TH}}(1 - r_p^\alpha)}$$

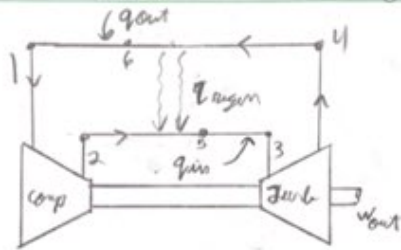
$$\dot{m} h_3 - \dot{m} h_4 - \dot{Q}_{out} - \dot{W}_{out} = 0$$

$$h_3 - h_4 - w_{out} = q_{out}$$

$$h_3 - h_4 - \gamma_T T_3(1 - r_p^\alpha) = q_{out}$$

$$\gamma_{T_{TH}} = \frac{c_p(T_3 - T_{4,a})}{c_p(T_3 - T_{4,s})}$$

$$T_3 - \gamma_{T_{TH}}(T_3 - T_{4,s}) = T_{4,a}$$



$$\frac{d}{dr_p} W_{\text{out,net}} = C_p (T_3 + T_1) - C_p T_3 (r_p^{-\alpha} + r_p^{-\alpha} \frac{T_1}{T_3})$$

$$0 = 0 + \alpha C_p T_3 r_p^{\alpha-1} - \alpha C_p T_3 \frac{T_1}{T_3} r_p^{\alpha-1}$$

$$0 = \alpha C_p (T_3 r_p^{\alpha-1} - T_1 r_p^{\alpha-1})$$

$$T_1 r_p^{\alpha-1} = T_3 r_p^{\alpha-1}$$

$$\frac{r_p^{\alpha-1}}{r_p^{\alpha-1}} = \frac{T_3}{T_1}$$

$$r_p^{2\alpha} = \frac{T_3}{T_1}$$

$$r_p^* = \left(\frac{T_3}{T_1} \right)^{\frac{1}{2\alpha}}$$

$$W_{\text{out,net,max}} = C_p (T_3 + T_1 - 2\sqrt{T_1 T_3})$$

$$W_{\text{out,net,max}} = C_p (\sqrt{T_3} + \sqrt{T_1})^2$$

$$W_{out,net} = 2 C_p [T_{max}(1 - r_{p,1}^{-\alpha}) - T_{min}(r_{p,1}^{\alpha} - 1)] \quad ; \quad r_{p,1} = \sqrt{r_p}$$

$$= 2 C_p [T_{max}(1 - r_p^{-\frac{1}{2}\alpha}) - T_{min}(r_p^{\frac{\alpha}{2}} - 1)]$$

$$\frac{dW}{dr_p} = 2 C_p T_{max} \left(-\frac{1}{2} \alpha r_p^{-\frac{1}{2}\alpha - 1} \right) - 2 C_p T_{min} \left(\frac{\alpha}{2} r_p^{\frac{\alpha}{2} - 1} \right) = 0$$

$$0 = T_{max} r_p^{-\frac{\alpha}{2} - 1} - T_{min} r_p^{\frac{\alpha}{2} - 1}$$

$$\frac{r_p^{\frac{\alpha}{2} - 1}}{r_p^{-\frac{\alpha}{2} - 1}} = \frac{T_{max}}{T_{min}}$$

$$r_p^{\alpha} = \frac{T_{max}}{T_{min}}$$

$$r_p^* = \left(\frac{T_{max}}{T_{min}} \right)^{\frac{1}{\alpha}} \quad r_{p,1} = \left(\frac{T_{max}}{T_{min}} \right)^{\frac{1}{2\alpha}}$$

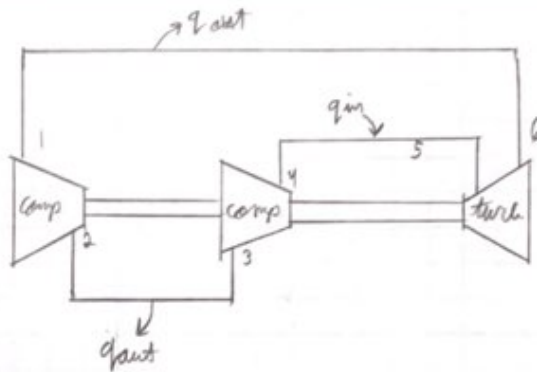
$$W_{out,net,max} = 2 C_p \left[T_{max} \left(1 - \left(\frac{T_{max}}{T_{min}} \right)^{-\frac{1}{2}} \right) - T_{min} \left(\left(\frac{T_{max}}{T_{min}} \right)^{\frac{1}{2}} - 1 \right) \right]$$

$$= 2 C_p [T_{max} - \sqrt{T_{min} T_{max}}] - 2 C_p [\sqrt{T_{min} T_{max}} - T_{min}]$$

$$= 2 C_p [T_{max} - \sqrt{T_{min} T_{max}} - \sqrt{T_{min} T_{max}} + T_{min}]$$

$$= 2 C_p [T_{max} - 2\sqrt{T_{min} T_{max}} + T_{min}]$$

$$W_{out,net,max} = 2 C_p [\sqrt{T_{max}} - \sqrt{T_{min}}]^2$$



$$\dot{m} h_1 + \dot{W}_{in} - \dot{m} h_2 = 0$$

$$\dot{W}_{in} = \dot{m} (h_2 - h_1)$$

$$\dot{W}_{in} = \dot{m} (h_4 - h_3)$$

$$\dot{m} h_5 - \dot{m} h_6 - \dot{W}_{out} = 0$$

$$\dot{W}_{out} = \dot{m} (h_6 - h_5)$$

$$\dot{W}_{out,net} = \dot{m} (h_6 - h_5) - [\dot{m} (h_2 - h_1) + \dot{m} (h_4 - h_3)]$$

$$T_5 = T_{max}$$

$$T_1 = T_{min}$$

$$T_1 = T_3$$

$$T_4 = T_2$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^\alpha$$

$$T_2 = T_1 r_{p1}^\alpha$$

$$\frac{T_4}{T_5} = \frac{P_4}{P_5}^\alpha = r_{p2}^\alpha$$

$$T_4 = T_5 r_{p2}^\alpha$$

$$\dot{W}_{out,net} = \dot{m} c_p (T_6 - T_5) - \dot{m} c_p (T_2 + T_4 - T_1 - T_3)$$

$$= \dot{m} c_p (T_6 - T_5) - \dot{m} c_p (2T_2 - 2T_1)$$

$$= \dot{m} c_p T_{max} (r_{p1}^\alpha - 1) - 2 \dot{m} c_p T_{min} (r_{p1}^\alpha - 1)$$

$$= \dot{m} c_p (\dot{m} c_p T_{max} (r_{p1}^\alpha - 1) + 2 \dot{m} c_p T_{min} (1 - r_{p1}^\alpha))$$