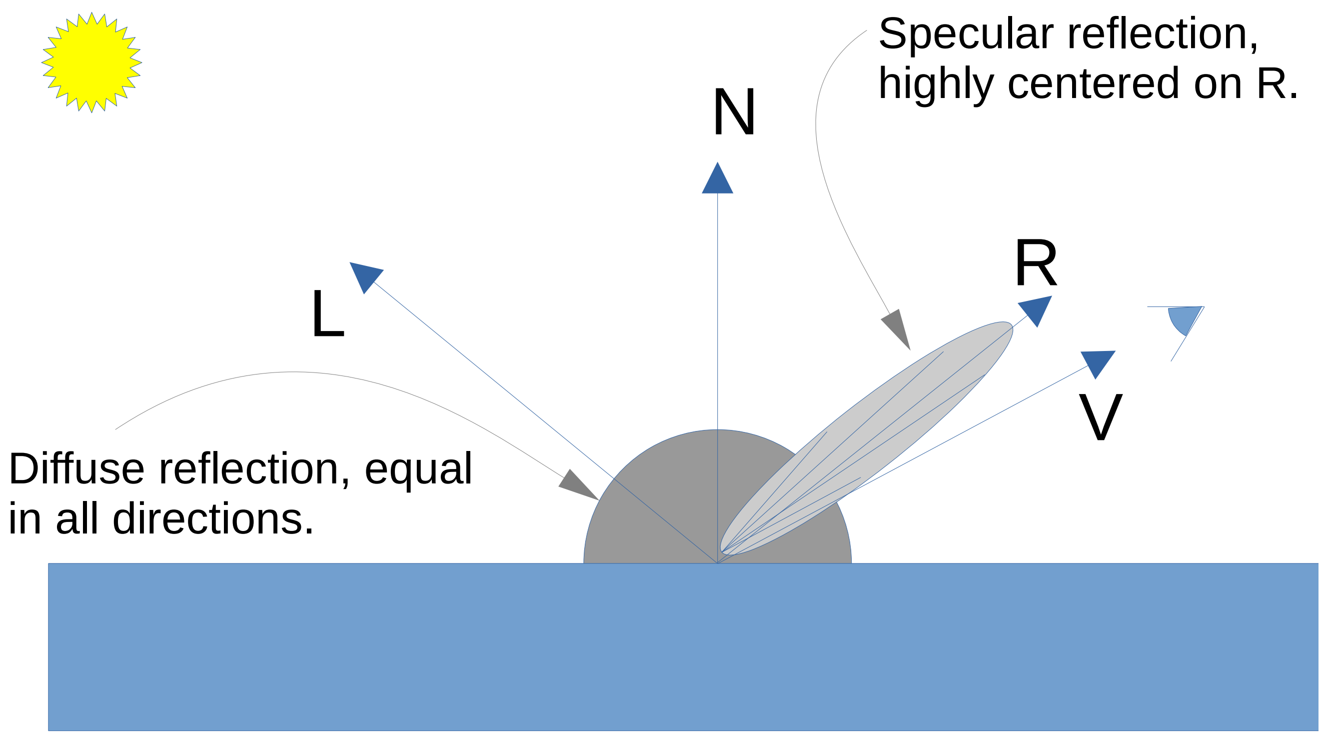
BRDF
Bidirectional Reflectance Distribution Function



Micro-facet BRDF Lighting

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The general BRDF lighting equation is

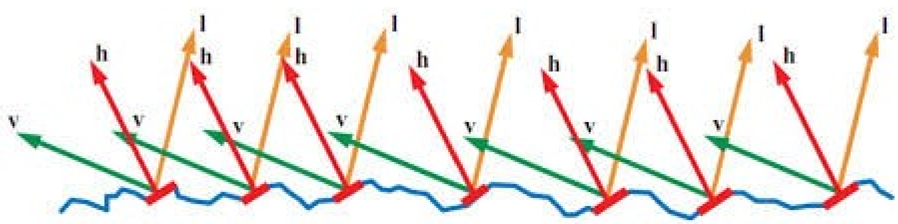
$$I_o = I_i \, (N \cdot L)_+ \; BRDF$$

where the BRDF portion is:

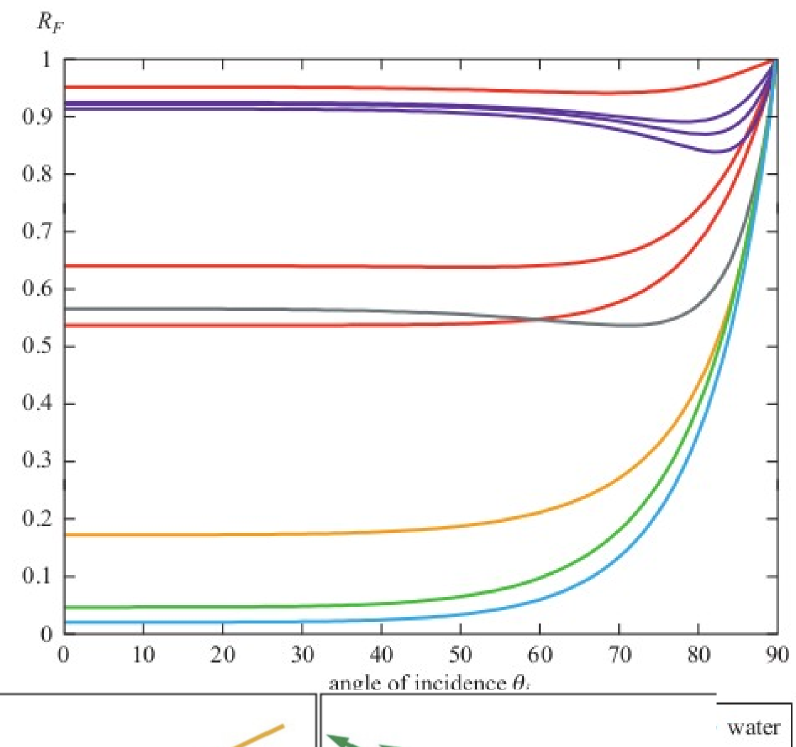
$$BRDF = \frac{K_d}{\pi} + \frac{D(H) \;\; F(L, H) \;\; G(L, V, H)}{4 \, (L \cdot N) \, (V \cdot N)}$$

and where H is half way between L and V.

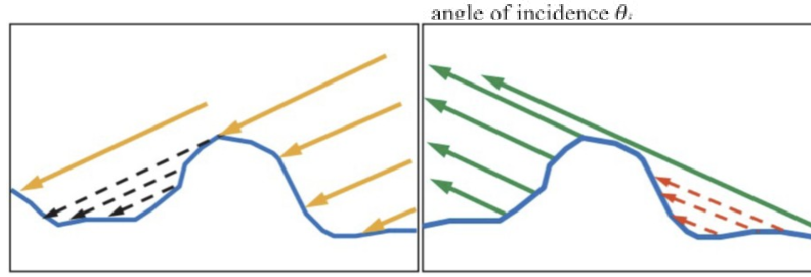
- **D term (distribution):** The fraction of a surface aligned with H, so it reflect from L to V. This is a probability distributions, and so must integrate to 1.



- **F term (Fresnel):** The fraction of light reflected from (not absorbed by) a micro-facet. Calculated theoretically. Varies greatly with angle of incidence.



- **G term:** Accounts for self-shadowing and self-occlusion between micro-facets. Function is mostly 1, but falls toward 0 at extreme angles.



Functions for D, F, and G

A nice **starter set** for D, G, and F (but see the next page for more):

$$BRDF = \frac{K_d}{\pi} + \frac{D(H)\ F(L,H)\ G(L,V,H)}{4\,(L\cdot N)\,(V\cdot N)}$$

- Phong-BRDF with roughness parameter α : 0(rough) ... ∞ (mirror)

  $$D(H) = \frac{\alpha+2}{2\pi}(N\cdot H)^{\alpha}$$

- Schlick's approximation of the Fresnel equation:

  $$F(L,H) = K_s + (1-K_s)(1-L\cdot H)^5$$

- A well known approximation to more carefully derived shadow/occlusion term:

  $$\frac{G(L,V,H)}{(L\cdot N)\,(V\cdot N)} \approx \frac{1}{(L\cdot H)^2}$$

MicroFacet BRDFs (Phong, Beckman, GGX)

All microfacet BRDFs have this general form:

$$I_o = I_i\,(N\cdot L)_+\ BRDF$$

$$\text{BRDF} = \frac{K_d}{\pi} + \frac{F(L,H)\ G(L,V,H)\ D(H)}{4\,(L\cdot N)\,(V\cdot N)}$$

where N, L, and V are unit length vectors for surface orientation and light and eye directions.

These sub expressions occur in several places:

$H=(L+V)/\|L+V\|$ is the so called **half** vector

$\tan\theta_v=\sqrt{(1.0-(v\cdot N)^2)}\,/\,(v\cdot N)$ for an arbitrary vector v. (Which may be H, L, or V).

F term

F is the Fresnel (reflection) is usually approximated by Schlick as

$$F(L,H) = K_s + (1-K_s)(1-L\cdot H)^5$$

where K_s is the specular reflection color at $L=V=N=H$.

The exact formulation (if you are interested) is

$$F(L,H) = \frac{1}{2}\ \frac{(g-c)^2}{(g+c)^2}\left(1 + \frac{(c(g+c)-1)^2}{(c(g-c)+1)^2}\right)$$

where

$$g\ =\ \sqrt{\eta_t^2/\eta_i^2 - 1 + c^2}\ ,$$

$$c = |L\cdot H|$$

and η_i and η_t are indices of refraction of the two materials.

D term

D is the micro-facet distribution, each controlled by it’s own roughness/shininess parameter
Many exist:

Phong: $D_p(H)\ =\ \dfrac{\alpha_p+2}{2\pi}(N\cdot H)^{\alpha_p}$

(α_p : $1..\infty$; increasing means smoother surface)

Beckman: $D_b(H) = \dfrac{1}{\pi\alpha_b^2(N\cdot H)^4}\ e^{\frac{-\tan^2\theta_H}{\alpha_b^2}}$

(α_b : $0..1$; increasing means rougher surface)

similar to Phong for smooth surfaces using $\alpha_p = 2\alpha_b^{-2} - 2$

GGX: $D_g(H) = \dfrac{\alpha_g^2}{\pi(N\cdot H)^4(\alpha_g^2+\tan^2\theta_H)^2}$ or equivalently $D_g(H) = \dfrac{\alpha_g^2}{\pi\left((N\cdot H)^2(\alpha_g^2-1)+1\right)^2}$

Roughness/shininess parameters in D and conversions.

Phong’s shininess parameter α_p ranges over $[0,\infty]$ for rough to shiny surfaces
Beckman and GGX roughness parameters α_b and α_g range over $[0,1]$ for smooth to rough surfaces.

Conversions between α_p and either of α_b or α_g :

$$\alpha_g\ =\ \sqrt{\frac{2}{\alpha_p+2}}$$

and

$\alpha_p = -2 + 2/\alpha_g^2$

G is the self occluding and self-shadowing geometry term

Many exist in the Smith form: $G(L, V, H) = G_1(L, H)\, G_1(V, H)$
where $G_1(v, H)$ is:

Beckman uses this very accurate rational approximation

$$G_1(v, H) = \begin{cases} \dfrac{3.535a + 2.181a^2}{1.0 + 2.276a + 2.577a^2} & \text{if } a < 1.6 \\ 1 & \text{otherwise} \end{cases}$$

with

$$a = 1/(\alpha_b \tan\theta_v)$$

Phong:

Same G_1 as Beckman, but with $a = (\sqrt{\alpha_p/2+1}) / \tan\theta_v$

Beware roundoff errors in calculating the G_1 ***function:***

- The value of $(v \cdot N)$ may round up to greater than 1.0 (it shouldn't, but it does). If so, return $G_1(\ldots) = 1.0$.
- The calculation of $\tan\theta_v$ may be zero. If so, don't divide by it, instead return $G_1(\ldots) = 1.0$.

GGX:

$$G_1(v, m) = \frac{2}{1 + \sqrt{1 + \alpha_g^2 \tan^2\theta_v}}$$

Sometimes G is combined with most of the denominator and called the **visibility** term

$$V(L, V) = \frac{G(L, V, H)}{(L \cdot N)(V \cdot N)}$$

A simple approx is: $V(L, V) = 1$
is not too bad – darkens too fast and is independent of roughness.

Better approx is: $V(L, V) \approx 1/(L \cdot H)^2$
often considered good enough for real time graphics.

Scan Conversion and Normals