Computing Temporal Order without an Analytical Solution

Tony Saad

December 31, 2014

It is possible to compute the temporal order of accuracy without an analytical solution. The order of accuracy is defined as the exponent of the leading order term of the truncation error. Using a general Taylor series expansion for a quantity f, we have for a p-th order scheme

$$f_1 = \tilde{f} + a_p \Delta t^p + O(\Delta t^{p+1}) \tag{1}$$

The same formula may be applied for two more refinement levels

$$f_2 = \tilde{f} + a_p (r\Delta t)^p + O(r^{p+1} \Delta t^{p+1}) \tag{2}$$

$$f_3 = \tilde{f} + a_p (r^2\Delta t)^p + O(r^{2p+2} \Delta t^{p+1}) \tag{3}$$

where $r = \frac{\Delta t_{\text{fine}}}{\Delta t_{\text{coarse}}} < 1$. If the step size is small enough and the higher order terms are negligible, it is possible to estimate the observed order of accuracy as

$$\frac{f_3 - f_2}{f_2 - f_1} = \frac{a_p r^p \Delta t^p (r^p - 1)}{a_p \Delta t^p (r^p - 1)} = r^p \tag{4}$$

or

$$p = \frac{\ln\left(\frac{f_3 - f_2}{f_2 - f_1}\right)}{\ln r} \tag{5}$$