Jacob Holmes

CA 2 Documentation

Problem 2:

Selected nodes in the maximum weight independent set:

Node 1 with weight 55

Node 4 with weight 77

Node 10 with weight 21

Node 8 with weight 38

Node 7 with weight 40

Total weight of selected nodes: 231.0

CPLEX Output:

graph\_ws1000:

Upper bound: 7091.0726737

Lower bound: 6355

Absolute optimality gap: 736.073

Relative optimality gap: 11.58%

graph\_er5000:

Upper bound: 96075.049531

Lower bound: 55592

Absolute optimality gap: 40483

Relative optimality gap: 72.82%

Description of values:

Based on the branch and bound method used with MIP problems in CPLEX and the maximization case, the lower bound value is the objective value from the best MIP solution found so far. The upper bound is the objective of the LP relaxation problem which decreases as “branches” that are non-optimal are identified. The absolute optimality gap is the numeric difference between these two numbers, and the relative optimality gap is the percentage difference based on the lower bound in this maximization case. This relative optimality gap is useful for determining how close these two bounds are in terms of degree of magnitude and helps normalize how far apart these two are. For example, even though the absolute optimality gap of the 1000-node graph is around 736, its relative optimality gap is 11.58%, which means that the best MIP solution objective value found (6355) is pretty close to the optimal LP relaxation problem objective value (~7091).

Heuristic Output:

graph\_ws1000:

Objective value: 6090

Solution: [388, 806, 659, 128, 28, 579, 589, 715, 16, 334, 615, 543, 426, 174, 747, 158, 845, 328, 441, 832, 830, 68, 201, 449, 964, 198, 267, 276, 330, 197, 412, 714, 824, 888, 537, 350, 573, 439, 31, 958, 548, 961, 604, 233, 570, 694, 298, 972, 993, 689, 130, 179, 171, 264, 741, 405, 996, 759, 84, 921, 162, 12, 282, 400, 489, 486, 693, 634, 656, 729, 460, 754, 527, 600, 359, 940, 526, 408, 743, 200, 891, 683, 655, 593, 392, 855, 849, 66, 206, 785, 950, 819, 72, 44, 108, 294, 840, 734, 26, 360, 751, 853, 220, 246, 425, 492, 627, 782, 706, 239, 999, 739, 457, 522, 147, 250, 208, 102, 667, 92, 613, 770, 422, 226, 115, 771, 766, 391, 308, 977, 638, 483, 948, 135, 7, 238, 289, 452, 5, 873, 646, 307, 253, 372, 512, 186, 440, 249, 701]

graph\_er5000:

Objective value: 53664

Solution: [3039, 3535, 779, 2895, 2926, 4782, 3737, 832, 4227, 716, 153, 3328, 138, 623, 4116, 1055, 481, 4755, 1941, 2755, 775, 3767, 2694, 4196, 1050, 1982, 3702, 3676, 2710, 3010, 2616, 2089, 2098, 3300, 98, 4122, 1876, 4649, 669, 3989, 1030, 3725, 1632, 3450, 1769, 612, 655, 4758, 3146, 4647, 449, 2205, 974, 2644, 1802, 3879, 1491, 965, 826, 1569, 4591, 3003, 3534, 4090, 2808, 2218, 2339, 4025, 3912, 1559, 3231, 70, 1547, 2019, 553, 3154, 834, 4609, 1645, 1135, 467, 3655, 2444, 3724, 1906, 1734, 2532, 4297, 3330, 4219, 3105, 3466, 2647, 4293, 4538, 1189, 1904, 3246, 3240, 2400, 2417, 1210, 4289, 3876, 3145, 858, 3838, 2699, 4738, 4129, 1985, 742, 628, 155, 2125, 1590, 1727, 1117, 731, 4323, 2900, 1142, 3999, 1083, 2008, 901, 4316, 4302, 93, 2185, 2048, 4372, 994, 312, 1399, 135, 1578, 2150, 3605, 3691, 2857, 4902, 9, 1478, 4098, 608, 606, 2514, 3320, 551, 2629, 2557, 4611, 4502, 3440, 730, 856, 4831, 3022, 1754, 2874, 2020, 3906, 1605, 4696, 4524, 1020, 4198, 3714, 1215, 1898, 4894, 2375, 2489, 215, 3717, 4615, 3163, 2966, 4657, 4688, 1333, 2965, 1724, 1329, 1505, 905, 293, 3159, 1295, 4621, 1939, 2108, 2287, 4449, 4650, 1940, 3660, 194, 3508, 1152, 4628, 3994, 3993, 3410, 3736, 4415, 60, 758, 4322, 504, 815, 4263, 4842, 3690, 1938, 118, 3757, 3666, 2077, 4037, 3823, 2393, 2122, 1465, 872, 2880, 2076, 851, 1849, 1883, 4683, 3732, 3255, 2742, 818, 84, 533, 3852, 3976, 2273, 1197, 1607, 1466, 2977, 3286, 2256, 3507, 1582, 3744, 2322, 1172, 4013, 4932, 4974, 4722, 2622, 2834, 3189, 2896, 113, 1899, 1784, 4671, 1033, 193, 1204, 3412, 1007, 4732, 4026, 1362, 1841, 4202, 4238, 525, 2338, 3065, 1153, 3663, 3501, 2457, 841, 1269, 3217, 4903, 3226, 1535, 3936, 1808, 3608, 2631, 4437, 1596, 52, 3967, 3553, 1363, 1256, 1071, 4706, 4191, 2943, 1614, 2853, 2737, 1115, 2929, 3099, 3208, 320, 165, 950, 4789, 1146, 2172, 277, 3688, 1131, 2079, 1251, 1246, 3528, 4560, 3038, 4895, 395, 3238, 541, 613, 271, 3403, 945, 1909, 651, 4742, 1425, 1892, 3896, 7, 3034, 796, 4577, 1521, 92, 4563, 3648, 1284, 3871, 1621, 4106, 3566, 2458, 246, 361, 2258, 4610, 2465, 2898, 2298, 3021, 4486, 58, 1000, 270, 3134, 2495, 4942, 4151, 4478, 1833, 1484, 1879, 2449, 1370, 4168, 1453, 1445, 2043, 1190, 461, 2410, 4545, 4107, 3343, 2333, 4153, 3923, 2917, 4727, 2117, 2879, 626, 4321, 4303, 1565, 1421, 1037, 2835, 1591, 4640, 3364, 3937, 2935, 741, 1253, 2565, 2505, 2374, 909, 3463, 279, 455, 150, 810, 2759, 3652, 4888, 3833, 1815, 2446, 423, 2802, 1646, 2778, 2841, 296, 4071, 598, 4251, 3644, 800, 4327, 2138, 2411, 2747, 3695, 3938, 1244, 3201, 2919, 202, 2932, 3387, 385, 1618, 3187, 3398, 4174, 1968, 2160, 3293, 3126, 4279, 1442, 3458, 1063, 230, 2310, 4149, 596, 573, 3656, 1313, 3284, 1625, 1234, 2957, 1255, 1343, 4173, 2633, 1415, 1288, 4432, 302, 2986, 3308, 2561, 3313, 2140, 4878, 358, 889, 3025, 1768, 833, 3704, 508, 4103, 3107, 2685, 4387, 74, 112, 3825, 3071, 2887, 4629, 49, 3571, 2262, 654, 2168, 2194, 2250, 1268, 233, 3505, 3620, 1527, 2539, 2154, 1355, 3668, 4676, 1537, 4380, 2058, 2418, 1630, 2114, 3156, 2116, 3271, 1462, 576, 265, 4757, 4310, 1969, 4579, 18, 602, 4659, 1177, 4620, 3164, 1322, 2176, 597, 3797, 2641, 1657, 2050, 1960, 2207, 4684, 4977, 3376, 1348, 930, 4444, 2811, 2683, 4784, 45, 1261, 1173, 1270, 3943, 679, 1580, 4851, 286, 1467, 2512, 4918, 920, 4802, 1977, 2428, 3667, 2315, 1501, 1147, 1185, 3810, 518, 2187, 3554, 4968, 1900, 1753, 537, 4619, 3176, 10, 2201, 3845, 3055, 2519, 3689, 900, 3488, 3685, 4551, 627, 217, 1502, 1046, 2648, 1691, 3948, 211, 860, 2042, 885, 2580, 146, 1064, 3878, 2872, 2484, 4345, 555, 5, 4693, 78, 781, 4873, 4685, 3979, 4733, 2997, 3998, 4993, 1283, 4020, 2994, 3591, 1778, 1791, 4204, 2343, 4367, 610, 3309, 1107, 2948, 2687, 2751, 450, 4721, 3830, 1823, 4746, 4318, 4260, 2875, 4070, 4043, 4787, 3790, 4890, 3239, 1436, 3242, 2195, 2191, 683, 2725, 4390, 4012, 575, 4077, 3141, 158, 752, 3111, 3547, 2264, 4712, 3828, 692, 4981, 3088, 3405, 4639, 512, 2007, 3812, 4660, 4692, 3980, 2245, 469, 2951, 2180, 311, 3561, 2479, 3650, 2049, 4378, 2123, 3057, 1584, 2213, 1309, 2987, 1592, 1703, 4164, 2054, 370, 4065, 1178, 1562, 1766, 4356, 4641, 983, 2581, 2844, 201, 2664, 3562, 4804, 2745, 1134, 95, 3052, 801]

Reflection:

The heuristic results do not produce as good of objective values as the lower bound solutions of the CPLEX solver. However, the difference in scale of these objective values is small with relative differences of less than 5% on each based on the heuristic objectives. With this small difference in the objective values, the savings from the heuristic method are important, as the longer problem, the 5000-node heuristic solution, was solved in 1.2s compared to 30 minutes which is 1500 times faster. This heuristic solution provides a relatively good solution in a shorter amount of time than the CPLEX solution, which would require even more time to determine if it is mathematically optimal.