Contents

1 Momentum and impulse

1. The **momentum** of a body of mass m which is moving with velocity v is mv. The units of mementum can be Ns or kgms^{-1}.
2. If a constant force F acts for a time t then we define the **impulse** of the force to be Ft. The units of impulse are Ns
3. The **impulise-momentum principle:**

Impulse = final momentum - initla momentum

Impulse = change in momentum

$$I = mv - mu$$

Where m is the mass of the body, u the initial velocity and v the final velocity.

1. **Principle of conservation of momentum:**

Total momentum before impact = total momentum after impace

[] $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ [] where a body of mass m_1 moving with velocity u_1 collides with a body of mass m_2 moving with velocity u_2, v_1 and v_2 are the velocities of m_1 and m_2 after the colission respectively.

1. You can write the impulse-momentum principle and the principle of conservation of momentum as vector equations

   - $\boldsymbol{I} = m\boldsymbol{v} - m\boldsymbol{u}$

     where m is the mass of the body, $\boldsymbol{u}$ the initial velocity and $\boldsymbol{v}$ the final velocity.

   - $m_1\boldsymbol{u_1} + m_2\boldsymbol{u_2} = m_1\boldsymbol{v_1} + m_2\boldsymbol{v_2}$

     where a body of mass m_1 moving with velocity $\boldsymbol{u_1}$ collides with a body of mass m_2 moving with velocity $\boldsymbol{v_2}$, $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ are the velocities of the bodies after the collision.

2 Work, energy and power

1. You can calculate the work done by a force when its point of application along a straight line using the formula

   - work done $=$ $\begin{array}{c}\text{component of force in}\\ \text{direction of motion}\end{array} \times \begin{array}{c}\text{distance moved}\\ \text{in direction of force}\end{array}$

2. Work done against gravity $= m$, where m is the mass of the particle, g is the acceleration due to gravity and h is the vertical distance raised.

3. Kinetic energy (K.E.) $= \frac{1}{2}mv^2$, where m is the mass of the particle and v is its speed

   Potential energy (P.E.) $= mgh$, where h is the height of the particle above an arbitary fixed level

4. Work done $=$ change in kinetic energy

5. You must choose a **zero level** of potential energy before calculating a particle's potential energy.

6. **Principle of conservation of mechanical energy**

   When no external forces (other than gravity) do work on a particle during its motion, the sum of the particle's kinetic and potential energy remains constant.

   Work-energy principle

   The change in the total energy of a particle is equal to the work done on the particle.

7. Power is the rate of doing work.

   For a vehicle, power $= Fv$ where F is the driving force produced by the engine and v is the speed of the vehicle.

3 Elastric strings and springs

1. When an elastic string or spring is stretched, the tension, T, produced is proportional to the extension, x.

   - $\boldsymbol{Tx}$
   - $\boldsymbol{T = kx}$**, where k is a constant**

     The constant k depends on the unstretched length of the spring or string, l, and the **modulus of elasticity** of the string or spring λ.
   - $\boldsymbol{T = \frac{\lambda x}{l}}$

     This relationship is callled **Hooke's law**.

2. The area under a **force-distance** graph is the **work done** in stretching an elastic string or spring. The work done in stretching or compressing an elastric string or spring with modulus of elasticity

λ from its natural length, l to a length $(l + x)$ is $\dfrac{\lambda x^2}{2l}$.

When λ is measured in newtons and x and l are measured in metres, the work done is in **joules (J)**.

3. The **elastic potential energy** (E.P.E.) stored in a stretched string or spring or spring is exactly equal to the amount of work done to stretch the string or spring.

   The E.P.E. stored in a string or spring of modulus of elasticity λ which as been stretched from its natural length l, to a length $\dfrac{\lambda x^2}{2l}$.

4. When no external forces (other than gravity) act on a particle, then the sum of its kinetic energy, gravitational potential energy and elastic potential energy remains constant.

4 Elastic collisions in one dimension

1. **Newton's law of restitution** states that

   The constant e is the **coefficient of restitution** between the particles $0 \leq e \leq 1$

2. For the direct collision of a particle with smooth plane. Newton's law of restitution can be written as

   $$\frac{\text{speed of rebound}}{\text{speed of approach}} = e$$

3. The loss of kinetic energy due to impact is

   $$(\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2) - (\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2)$$

5 Elastic collisions in two dimensions

1. In an **oblique** impact betweeen a smooth sphere and a smooth fixed surface:

   - The impulse on the sphere acts perpendicular to the surface, through the center of the sphere.
   - The component ot the velocity of the sphere parallel to the surface is unchanged.

     $v \cos \beta = u \cos \alpha$
   - You can use **Newton's law of restitution** to find the component of the velocity of the sphere perpendicular to the srface.

     $v \sin \beta = eu \sin \alpha$

2. An impact between two spheres:

- The reaction between the two spheres acts along the **lines of centers**, so the impulse affecting each sphere also affects along the line of centres.
- The components of the velocities of the spheres perpendicular to the line of centres are unchanged in the impact.
- Newton's law of restitution applies to the components of the velocities of the spheres parallel to the line of centres.
- The principle of conservation of momentum applies parallel to the line of ceners.