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The modeling of Inverse Kinematics for 5 DOF manipulator

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Abstract

This paper is the part of research aimed at creating robotic manipulator controlled by means of Brain-Computer Interfacefor improving household self-reliance of persons with disabilities and expanding the scope of their activity.The robotic manipulator gives possibility of self-fulfillment in basic household functions, s.a. drinking, eating, facial hygiene. The paper presents a mathematical model of the kinematics of the robotic manipulator. The solution of the forward and inverse kinematics problems based on the geometric method are given and obtained in MATLAB for the 5DOF manipulator.

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*Keywords:*Robot manipulator, Forward Kinematics, Inverse Kinematics, Denavit-Hartenberg method, MATLAB, Brain-Computer Interface

1. Introduction

Rehabilitation is rather difficult for people with limitation of their movements or speech. One of the most important social goals right now is rehabilitation of disabled people by creating a barrier-free environment. Robotic manipulators are one of the most advanced solutionsat the moment. There are many prosthetic robotic hands controlled by myoelectric signals from the injured limb or other preserved mobility of fist or fingers [1].

At the same time dozens research teams around the world work to create the brain-computer interface (BCI) – the device converting the electroencephalogram (EEG) into control signals that could be used to control prosthetic hand, computer, or another execution unit. For now report of a breakthrough in this area have not been published [2].The existing systems have to be more robust to accomplish Real-Time control,feedback systems need further development to achieve greater efficiency of prosthetic control. Although a few tests were conducted in real

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conditions of disabled people'slife [3]. Therefore, there are still several challenges before this technology could beexploited worldwide.

Our scientific work deals with development of the robotic manipulator controlled by non-invasive BCI called Steady State Visually Evoked Potential (SSVEP) [4]. The interface menu consists of certain items flashing at a fixed frequency. Ashuman looks atthe item, BCI decodes the EEG signals and sends information about ones with matching frequency to execution unit. This control system willbe applied to developedrobotic manipulator.

Details ofthe previous workon the manipulator are givenin [5], viaformation of technical shape,calculation of constructive and energy parameters. Number of degrees of freedom (DOF) equals five. Kinematic scheme, dimensions and dairy links (a limitation of angular displacement) are based on the application given above.Within this paper, solution tothe inverse kinematics problem was considered as a way to determine movement of the manipulator.

Two kinematics problems are dealt with in this paper. The forward kinematics problem is calculation of the coordinates and orientation of the gripper for determined links configuration. The inverse kinematics problem is vice versa process: finding such links configuration for which gripper matches thegiven position and orientation.

There are several ways to solve the inverse kinematics problem.Closed-form solutions are algebraic and geometric. Basic numerical ones are the following: the Jacobian transpose method [6], the pseudoinverse method [7], cyclic coordinate descent methods [8], the Levenberg-Marquardt damped least squares methods [9], quasi-Newton and conjugate gradient methods [10], neural net and artificial intelligence methods [11, 12], and the singular value decomposition [13].

Closed-form solutionsoperate faster than numerical ones and can be used to identify all possible variants. Nonuniversality and dependency on kinematic scheme are considered to be their main disadvantage.As configuration of the 5 DOF manipulator is quite simple, closed-form solutions suits more. Based on [14], the geometric method was decided to be the most appropriate within given conditions.

2. Mathematical Model of Robot kinematics

Todefine kinematics of the manipulator axes need to be placed according to theDenavit-Hartenberg methodthe basic rules of which are:

- z-axis is in the direction of the joint axis;
- x-axis is parallel to the common normal.

Kinematics scheme of the considered 5 DOF manipulator with the systems of axesis givenin Figure 1.

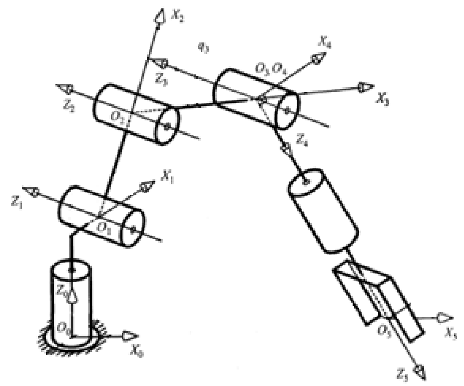

Fig. 1. – Kinematic scheme ofconsidered 5 DOF manipulator.

The Denavit-Hartenberg parametersarecorrespond to mounting configuration of links and determent based on the following rule:

- $q_i$is the angle from $x_{i-1}$to $x_i$along z_{i-1};
- $d_i$is the distance from the intersection of z_{i-1} with $x_i$to the origin of (i-1) system of axes;

- $a_i$is the shortcut between z_{i-1} and z_i;
- $\alpha_i$is the angle from $z_{i-1}$to $z_i$alongx_i.

Denavit-Hartenberg parameters are marked in Figure 2.

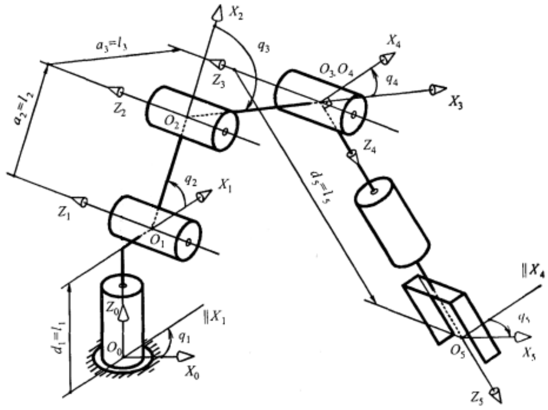

Fig. 2. – Kinematic scheme ofthe 5 DOF manipulator.

The Denavit-Hartenberg parameters for the kinematic scheme are presented in Table 1.

Table 1. Denavit-Hartenberg parameters of the 5 DOF manipulator in Fig. 1.

I	q_i	α_i	a_i	d_i
1	q_1	$\frac{-\pi}{2}$	0	d_1
2	q_2	0	a_2	0
3	q_3	0	a_3	0
4	q_4	$\frac{\pi}{2}$	0	0
5	q_5	0	0	d_5

These parameters can be used to describe the relative position and orientation of links by transition matrixes:

$$A_i = \begin{vmatrix} c_i & -c_{\alpha_i} \cdot s_i & s_{\alpha_i} \cdot s_i & a_i \cdot c_i \\ s_i & c_{\alpha_i} \cdot c_i & -s_{\alpha_i} \cdot c_i & a_i \cdot s_i \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{vmatrix}, \text{where } c_i = \cos q_i,\quad s_i = \sin q_i,\quad c_{\alpha_i} = \cos\alpha_i,\quad s_{\alpha_i} = \sin\alpha_i .$$

For the Denavit-Hartenberg parameters of the 5 DOF manipulator:

$$A_1 = \begin{vmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{vmatrix}, \tag{1}$$

$$A_2 = \begin{vmatrix} c_2 & -s_2 & 0 & a_2 \cdot c_2 \\ s_2 & c_2 & 0 & a_2 \cdot s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}, \tag{2}$$

$$A_3 = \begin{vmatrix} c_3 & -s_3 & 0 & a_3 \cdot c_3 \\ s_3 & c_3 & 0 & a_3 \cdot s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}, \tag{3}$$

$$A_4 = \begin{vmatrix} c_4 & 0 & -s_4 & 0 \\ s_2 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}, \tag{4}$$

$$A_5 = \begin{vmatrix} c_5 & -s_5 & 0 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{vmatrix}. \tag{5}$$

Calculation of the homogeneous transformation matrix T_5that connects the system of axis $O_5X_5Y_5Z_5$ with the absolute system of axes $O_0X_0Y_0Z_0$is given below:

$$T_5 = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5; \tag{6}$$

$$T_5 = \begin{vmatrix} c_1 \cdot c_{234} \cdot c_5 + s_1 \cdot s_5 & -c_1 \cdot c_{234} \cdot s_5 + s_1 \cdot c_5 & -c_1 \cdot s_{234} & c_1 \cdot (-d_5 \cdot s_{234} + a_3 \cdot c_{23} + a_2 \cdot c_2) \\ c_1 \cdot c_{234} \cdot c_5 - s_1 \cdot s_5 & -s_1 \cdot c_{234} \cdot s_5 - c_1 \cdot c_5 & -s_1 \cdot s_{234} & s_1 \cdot (-d_5 \cdot s_{234} + a_3 \cdot c_{23} + a_2 \cdot c_2) \\ -s_{234} \cdot c_5 & s_{234} \cdot s_5 & -c_{234} & d_1 - a_2 \cdot s_2 - a_3 \cdot s_{23} - d_5 \cdot c_{234}) \\ 0 & 0 & 0 & 1 \end{vmatrix}, \tag{7}$$

where $c_{ijk} = cos(q_i + q_j + q_k)$, $s_{ijk} = \sin(q_i + q_j + q_k)$.

Gripper's position is definedby $p_5$vector while orientation defined by x_5, y_5, z_5 vectors provided by theforwardkinematics.

$$p_5 = \begin{pmatrix} c_1 \cdot (-d_5 \cdot s_{234} + a_3 \cdot c_{23} + a_2 \cdot c_2) \\ s_1 \cdot (-d_5 \cdot s_{234} + a_3 \cdot c_{23} + a_2 \cdot c_2) \\ d_1 - a_2 \cdot s_2 - a_3 \cdot c_{23} - d_5 \cdot c_{234} \end{pmatrix}; \tag{8}$$

$$x_5 = \begin{pmatrix} c_1 \cdot c_{234} \cdot c_5 + s_1 \cdot s_5 \\ s_1 \cdot c_{234} \cdot c_5 + c_1 \cdot s_5 \\ -s_{234} \cdot c_5 \end{pmatrix}; \tag{9}$$

$$y_5 = \begin{pmatrix} c_1 \cdot c_{234} \cdot s_5 + s_1 \cdot c_5 \\ -s_1 \cdot c_{234} \cdot s_5 - s_1 \cdot c_5 \\ s_{234} \cdot s_5 \end{pmatrix}; \tag{10}$$

$$z_5 = \begin{pmatrix} -c_1 \cdot s_{234} \\ -s_1 \cdot s_{234} \\ c_{234} \end{pmatrix}. \tag{11}$$

Solution to the inverse kinematics problem isreducedto search of the arguments q_1, q_2, q_3, q_4, q_5 based on the gripper's position and orientation. These six equationsin five unknowns may have no solution, thoughit is possible to consider it for someinstances [15]. Equations 8-11 can be rewritten in the form:

$$x_{5x} = c_1 \cdot c_{234} \cdot c_5 + s_1 \cdot s_5; \tag{12}$$

$$x_{5y} = s_1 \cdot c_{234} \cdot c_5 + c_1 \cdot s_5; \tag{13}$$

$$x_{5z} = -s_{234} \cdot c_5; \tag{14}$$

$$y_{5x} = -c_1 \cdot c_{234} \cdot s_5 + s_1 \cdot c_5; \tag{15}$$

$$y_{5y} = -s_1 \cdot c_{234} \cdot s_5 - s_1 \cdot c_5; \tag{16}$$

$$y_{5z} = s_{234} \cdot s_5; \tag{17}$$

$$z_{5x} = -c_1 \cdot s_{234}; \tag{18}$$

$$z_{5y} = -s_1 \cdot s_{234}; \tag{19}$$

$$z_{5z} = c_{234}; \tag{20}$$

$$p_{5x} = c_1 \cdot (-d_5 \cdot s_{234} + a_3 \cdot c_{23} + a_2 \cdot c_2); \qquad (18)$$

$$p_{5y} = s_1 \cdot (-d_5 \cdot s_{234} + a_3 \cdot c_{23} + a_2 \cdot c_2); \qquad (19)$$

$$p_{5z} = d_1 - a_2 \cdot s_2 - a_3 \cdot c_{23} - d_5 \cdot c_{234}. \qquad (20)$$

Based on (12) – (20) the solution can be defined as:

$$q_1 = \tan^{-1} \frac{p_{5y}}{p_{5x}}. \tag{21}$$

$$q_2 = atan2\Big(a \cdot (a_2 + a_3 \cdot c_3) - b \cdot a_3 \cdot s_3, \;\; a \cdot a_3 \cdot s_3 + b \cdot (a_2 + a_3 \cdot c_3)\Big); \tag{22}$$

$$q_3 = arccos \frac{a^2 + b^2 - a_2^2 - a_3^2}{2 \cdot a_2 \cdot a_3}, \text{ where: } a = d_1 - d_5 \cdot c_{234} - p_{5z}; \tag{23}$$

$$b = p_{5x} \cdot c_1 + p_{5y} \cdot s_1 + d_5 \cdot s_{234}.$$

$$q_4 = q_{234} - q_2 - q_3; \tag{24}$$

$$q_5 = c_{234} \cdot q_1 - 2atan\left(x_{5y}, x_{5x}\right). \tag{25}$$

It should be noted that for solution in form (21) $p_{5x} > 0$ or $-\dfrac{\pi}{2} < q_1 < \dfrac{\pi}{2}$.

3 Results

3.1Modeling ForwardKinematics

Solution tothe forward kinematics was modeled by means of MATHLAB for the Denavit-Hanterberg parameters given in Table 2.

Table2.DeffinedDenavit-Hartenberg parameters.

i	q_i, mm	α_i, rad	a_i, mm	d_i, mm
1	q1=0	$\frac{-\pi}{2}$	0	d1=3
2	q2=-60	0	a2=2	0
3	q3=90	0	a3=2	0
4	q4=-30	$\frac{\pi}{2}$	0	0
5	q5=0	0	0	d5=1.5

The homogeneous transformation matrix T_5 connecting the system of axes $O_5X_5Y_5Z_5$ with the absolute system of axes $O_0X_0Y_0Z_0$described in form of (7) can be expressed as:

$$T_5 = \begin{vmatrix} 1 & 0 & 0 & 3.6639 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1.9824 \\ 0 & 0 & 0 & 1 \end{vmatrix}. \tag{26}$$

The results of computation are depicted in Figure 2. The gripper can be seen to reach the point with coordinates $(3.6639, \;\; 0, \;\; 1.9824)$,which corresponds to the given parameters.

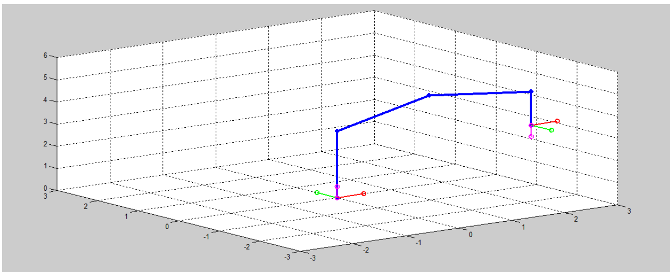

Fig. 3. – The results of modeling the forward kinematics.

3.2Modeling the Inverse Kinematics

Solution to the inverse kinematics problem was modeled by means of MATLAB.The gripper's position and orientation are definedby the homogeneous transformation matrixgiven below:

$$T_5 = \begin{vmatrix} 1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{vmatrix}. \quad (27)$$

Results of computation are presented in Table 3 and Figure 3.The gripper can be seen to reach the specified point, therefore the arguments q_1, q_2, q_3, q_4, $q_5$are found successfully.

Table3.DeffinedDenavit-Hartenberg parameters of the manipulator.

q_1	q_2	q_3	q_4	q_5
0	-49.9677	81.0107	-31.0430	0

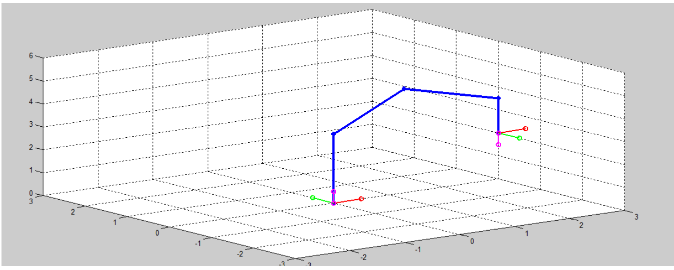

Fig. 4. – The results of modeling the inverse kinematics.

The path of the gripper is defined as a circle with the center atpoint with coordinates (2,0,2) and radius R=1. Results of computation are given at Figure 4.

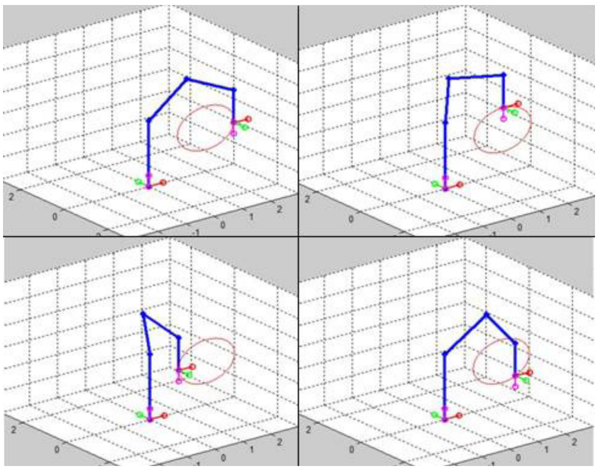

Fig. 5. – The results of modeling the inverse kinematics applied to circlepath of the gripper.

The gripper can be seen to pass the specified circle curve, therefore the arguments q1, q2, q3, q4, q5 are found successfully for every given point. Therefore, the results of the chosen solution method can be considered as appropriate ones.

4. Conclusion

Within this paper, the usage of manipulator controlled by Brain-Computer interface is verified. Review of solution methods to the inverse kinematics problem is given. The inverse kinematics problem for the 5 DOF robotic manipulator is solved withgeometric method. The solution ismodelledby means of MATLAB. Direction for further research is thedevelopment of an algorithm for path planning with environmental constrains taken into account.

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