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# Design of rank-one modification feedback controllers for second-order systems with time delay using frequency response methods

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ABSTRACT

In this note, a novel approach to design rank-one – single input – feedback controllers for second-order systems with time delay is presented. The versatile, well-known receptance modeling is combined with classical frequency response methods of control design, and the feedback gains are computed in order to achieve a given stability margin based on sensitivity specifications. The systems under study are assumed to be stable in open-loop. A heuristic optimization technique, namely, genetic algorithm is employed to impose a minimal distance from the Nyquist plot of the controlled system to the critical point, guaranteeing thereby a predefined margin of robust stability. In addition to the well known advantages of the receptance method in obtaining experimental models, the proposed design approach brings the possibility of considering time delay without approximations, by using frequency response calculations, avoiding thereby the drawbacks inherent to the *a posteriori* stability verification when pole assignment techniques are used. Numerical examples help to enlighten the merits of the proposed approach.



## 1. Introduction

In several phenomena, such as mechanical vibration or oscillations in electrical networks, the dynamic model raised from first principles is of a second-order type. Such type of ordinary differential equation modeling, which is matrix coefficient in nature, can present a series of advantages against the same model represented as first-order state-space equations, for instance, symmetry, bandness and definiteness of the matrices used in the representation. A detailed discussion regarding these advantages is presented in [1].

The solution of a second-order differential equation leads to the well-known quadratic eigenvalue problem (QEP) that, according to [1] consists of determining the eigenvalues and associated eigenvectors – the eigenstructure. If the designer is interested in applying active vibration control – AVC – to vibration suppression, state or derivative feedback techniques can be an option to reallocate the spectrum to a safe position in the complex plane. One of the significant drawbacks in carrying out the design procedure using a system of second-order differential equations is undoubtedly the (possibly) high number of degrees of freedom necessary for a sufficiently accurate finite elements model – FEM, that can go up to hundreds, or even thousands, millions. Some possibilities to approach the state-feedback design use adaptations of existing metrics,

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such as singular value decomposition and modal approximants [2]. On the other hand, the receptance approach [3], a modeling technique that can be conducted using only experimental data, has been explored since its introduction in several works for the design of AVC. The receptances can be limited to the primary degrees of freedom defined by the designer, and hence be of a very reduced order when compared with FEM.

When time delays enter the picture, the characteristic polynomial of the closed-loop system is no longer a finite dimensional polynomial, and some unexpected issues can arise regarding the closed-loop eigenvalues. Time delay is pervasive in systems with high capacitance, networked control systems – NCS, communication in sideral space systems, and others [4]. Thus, controlling systems with internal delays is a practical challenge, in spite of some possible benefits of introducing time delay as a parameter by the designer in certain cases [5–7]. In general, pole placement using receptances in systems with time delay does not guarantee stability of the closed-loop system [8,9]. Then, *a posteriori* verification of the correct assignment is needed since even unstable eigenvalues can be assigned as primary ones. Moreover, some techniques that use aproximations, such as Taylor series truncation or Padé approximants must be employed to check for stability *a posteriori* [8,10]. In this note, a novel approach, inspired by the ideas presented in [10] is proposed to design the state feedback gains for systems with single input, that is, using rank-1 modifications of the damping and stiffness matrices through frequency response techniques for linear control, such as Nyquist stability criterion, stability margins, and equivalences with sensitivity functions [11]. A minimal distance from the frequency response (Nyquist) plot to the critical point (with respect to stability) is established, and the state feedback gains are computed by minimizing a cost function based on this bound, using the well-known genetic algorithm heuristic. The use of frequency response techniques allows for considering the time delay in an exact fashion, with no need for approximations. By doing so, our approach achieves *a priori* stability certification and outperforms the pole assignment techniques, overall in systems with a large time delay. Numerical examples are offered to illustrate the merits of the proposed approach.

## 2. Preliminaries and problem statement

Consider the system of second-order ordinary differential equations given by:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{D}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the displacement vector, and consequently $\dot{\mathbf{x}}(t), \ddot{\mathbf{x}}(t) \in \mathbb{R}^n$ are respectively the velocity and acceleration vectors; $\mathbf{M} \in \mathbb{R}^{n\times n}$, symmetric, positive definiteness is the mass matrix; $\mathbf{C} \in \mathbb{R}^{n\times n}$, symmetric is the damping matrix; $\mathbf{K} \in \mathbb{R}^{n\times n}$, symmetric, positive semi-definite is the stiffness matrix; and $\mathbf{f}(t) \in \mathbb{R}^n$ is a vector of external control forces. Eq. (1) can be expressed in the Laplace domain, considering null initial conditions $\mathbf{x}(0) = 0$ and $\dot{\mathbf{x}}(0) = 0$, resulting in:

$$\mathbf{X}(s) = \mathbf{H}(s)\mathbf{F}(s) \tag{2}$$

in which $\mathbf{X}(s) = \mathscr{L}\{x(t)\}; \mathbf{F}(s) = \mathscr{L}\{\mathbf{f}(t)\}$; and $\mathbf{H}(s) = \left(\mathbf{M}s^2 + \mathbf{C}s + \mathbf{K}\right)^{-1}$ is known as system receptance, or structural receptance. The concept of receptance was introduced with the aim of applying in control of vibrating systems and structural control in the seminal works [8,12]. Structural receptance can be easily assembled from experimental data, since the forces for the actuators can be programmed in the frequency domain and the displacement responses acquired to proceed with system identification. In fact, there is no need to known the system matrices $\mathbf{M}, \mathbf{C}, \mathbf{K}$ to compute $\mathbf{H}(s)$. This key issue will be explored later in this work.

The force vector will be defined by the actuator distribution in the system, giving then the following:

$$\mathbf{f}(t) = \mathbf{B}\mathbf{u}(t-\tau), \tag{3}$$

with $\mathbf{B} \in \mathbb{R}^{n\times m}$ being the well-known influence or actuator matrix; $\mathbf{u}(t) \in \mathbb{R}^m$ is a control vector; and $\tau > 0$ is a fixed internal delay. In this work, the single input case is considered, that is $m = 1$. In the ideal case, the time delay $\tau$ is sufficiently small to be neglected. But in several practical situations, it can be large enough and its (often) undesirable effects must be taken into account. In contrast with systems without delay, the task of controlling system (1) with the control input (3) via state feedback is a little bit more involved. Consider the well-known linear state feedback control law:

$$\mathbf{u}(t) = \mathbf{f}\dot{x}(t) + \mathbf{g}x(t) + \mathbf{v}(t) \tag{4}$$

where $\mathbf{f}, \mathbf{g} \in \mathbb{R}^{1\times n}$ are the feedback gain matrices, and $\mathbf{v}(t)$ is a possible reference. By replacing (4) in (3), taking the Laplace transform and then replacing it in (2) one has:

$$\mathbf{X}(s) = e^{-\tau s}\mathbf{H}(s)\mathbf{B}(\mathbf{f}s + \mathbf{g})\mathbf{X}(s) + e^{-\tau s}\mathbf{H}(s)\mathbf{V}(s).\therefore \tag{5}$$

$$\mathbf{X}(s) = [\mathbf{I} - e^{-\tau s}\mathbf{H}(s)\mathbf{B}(\mathbf{f}s + \mathbf{g})]^{-1}e^{-\tau s}\mathbf{H}(s)\mathbf{V}(s) \tag{6}$$

Then, by defining the closed-loop receptance as:

$$\mathbf{H}_{cl}(s) = [\mathbf{I} - e^{-\tau s}\mathbf{H}(s)\mathbf{B}(\mathbf{f}s + \mathbf{g})]^{-1}e^{-\tau s}\mathbf{H}(s), \tag{7}$$

attention can be focused on the inverse of (6). That inverse can be manipulated using the Shermann-Morrison identity [13] to give:

$$\mathbf{H}_{cl}(s) = e^{-\tau s}\mathbf{H}(s) + \frac{e^{-\tau s}\mathbf{H}(s)\mathbf{B}(\mathbf{f}s + \mathbf{g})\mathbf{H}(s)}{1 - e^{-\tau s}(\mathbf{f}s + \mathbf{g})\mathbf{H}(s)\mathbf{B}}. \tag{8}$$

The roots of the following equation:

$$1 - e^{-\tau s}(\mathbf{f}s + \mathbf{g})\mathbf{H}(s)\mathbf{B} = 0 \tag{9}$$

are known as closed-loop poles. Under the assumption that the system is controllable, for the case in which $\tau = 0$, (8) has exactly $2n$ poles, and the well-known pole placement problem, that is, find the feedback matrices $\mathbf{f}$ and $\mathbf{g}$ such that the closed-loop poles are assigned as $s_1, s_2, \cdots, s_{2n}$ can be easily solved by assembling the following system of linear equations:

$$\begin{bmatrix} s_1\mathbf{r}_1^T & \mathbf{r}_1^T \\ s_2\mathbf{r}_2^T & \mathbf{r}_2^T \\ \vdots & \vdots \\ s_{2n}\mathbf{r}_{2n}^T & \mathbf{r}_{2n}^T \end{bmatrix} \begin{bmatrix} \mathbf{f}^T \\ \mathbf{g}^T \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{10}$$

where $\mathbf{r}_k = \mathbf{H}(s_k)\mathbf{B}, k = 1, 2, \cdots, 2n$. However, if $\tau \neq 0$, (9) is no longer a polynomial problem, and is known as a quasi-polynomial system of equations [14]. The major issue with such a system is that it has an infinite number of solutions, that is, the number of poles is infinity. A tentative solution for the pole placement can nevertheless be conducted, with (10) being modified as [9]:

$$\begin{bmatrix} s_1\mathbf{r}_1^T & \mathbf{r}_1^T \\ s_2\mathbf{r}_2^T & \mathbf{r}_2^T \\ \vdots & \vdots \\ s_{2n}\mathbf{r}_{2n}^T & \mathbf{r}_{2n}^T \end{bmatrix} \begin{bmatrix} \mathbf{f}^T \\ \mathbf{g}^T \end{bmatrix} = \begin{bmatrix} e^{s_1\tau} \\ e^{s_2\tau} \\ \vdots \\ e^{s_k\tau} \end{bmatrix} \tag{11}$$

Although the chosen poles are a solution of (9), they are a tiny subset of the infinite set of solutions, and the actual dominant poles can be other than those assigned, close to the imaginary axis or even unstable. Such poles are known as primary poles, and the closed-loop dynamics is mostly governed by them. To overcome this difficulty, several works dealing with full or partial pole placement using system receptance propose a procedure called *a posteriori* analysis [9]. In this scenario, the central problem can be stated as follows: given the system (1), compute the state feedback matrices for the control law in (4), such that the closed-loop system is stable and the vibration suppression is achieved as fast as possible. The solution using pole placement, though simple, does not guarantee a safe, accurate localization of the closed-loop poles. Also, no criterion for stability robustness can be directly used with pole placement.

In the next section, a method is proposed to solve the stabilization problem, using classical concepts of control theory such as stability margins and peak of the sensitivity function. These concepts are already suggested in the discussions found in [10], and are reissued here to allow for the derivation of a systematic procedure to compute stabilizing feedback matrices.

## 3. The proposed approach

### 3.1. Problem formulation

The frequency response methods are known by their ability to deal directly with time delays in dynamic systems, with no need to resort to approximations [11]. This feature inspired Araujo [10] to discuss the use of frequency response methods such as Nyquist plots, stability margins or even the Padé approximation of the delay to deliver a fruitful discussion on *a posteriori* analysis methods proposed in [9]. The Nyquist plot is a classical frequency response tool adopted in the present proposal. It consists in a plot of the imaginary versus real parts of the loop transfer function of a given control system. If, by assumption, the system is open-loop stable and of minimum phase, closed-loop stability can be determined by analyzing the number of encirclement of the point $(-1, 0)$ in the complex plane by the Nyquist plot. Moreover, a number of performance indexes can be computed using the notions of sensitivity function and complementary sensitivity function [11]. In Fig. 1, a Nyquist plot is shown to be constrained by the circle of radius $M_s^{-1}$, in which $M_s$ is the peak of the sensitivity function defined as:

$$M_s = \left\| \frac{1}{1 + L(s)} \right\|_\infty, \tag{12}$$

in which $L(s)$ is the loop transfer function. The $M_s$ function is closely related to gain (GM) and phase (PM) margins of single-input, single-output control systems:

$$GM \geqslant \frac{M_s}{M_s - 1}, \ PM \geqslant 2sin^{-1}\left(\frac{1}{2M_s}\right). \tag{13}$$

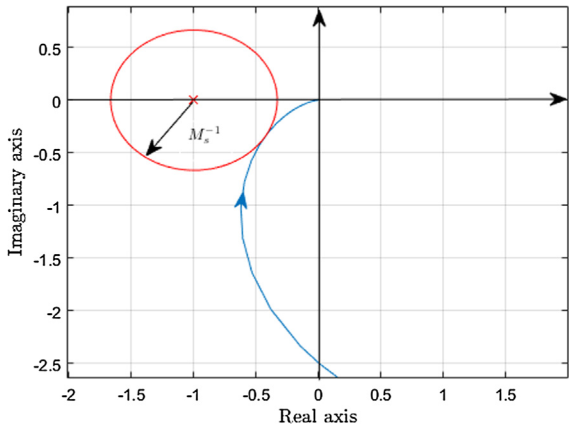


**Fig. 1.** A Nyquist diagram with an $M_s$ circumference.

From (13), one can see that these two relevant robustness criteria can be assigned with a suitable choice of $M_s$. For instance, if one chooses $M_s < 2$, the margins are bounded as $GM > 2$ and $PM > 30^o$, which are common rules of thumb [11]. The larger $M_s$, the closer the Nyquist plot is allowed to approach the critical point $(-1, 0)$, reducing system robustness and generally resulting in more oscillatory responses. Small values of $M_s$ lead to more robust solutions, but result in generally slower time responses. Typical values of $M_s$ lie in the range $1.2 \leqslant M_s \leqslant 1.8$.

A close look at the denominator of the second term in (9) reveals that the loop transfer function can be adopted as:

$$L(s) = -e^{-\tau s}(\mathbf{f}s + \mathbf{g})\mathbf{H}(s)\mathbf{B} \tag{14}$$

Considering the Nyquist plot of (14), the solution for the central problem of closed-loop stability can be formulated as the following optimization problem:

$$\min_{\mathbf{f},\,\mathbf{g}} \quad h(\mathbf{f},\,\mathbf{g}) = \left(\min_{\omega_i}|L(j\omega)+1| - M_s^{-1}\right)^2 \tag{15}$$

$$s.t.: \ \ \mathrm{Re}\{L(j\omega_i)\} \geqslant -1 + M_s^{-1} \ \forall \mathrm{Im}\{L(j\omega_i)\} = 0 \tag{16}$$

$$\min_{\omega_i} \mathrm{Re}\{L(j\omega_i)\} = -1 + M_s^{-1} \ \forall \mathrm{Im}\{L(j\omega_i)\} = 0 \tag{17}$$

with $L(j\omega_i) = -(\mathbf{g} + j\omega_i\mathbf{f})^T\mathbf{H}(j\omega_i)\mathbf{B}e^{-j\omega_i\tau}$.

The subscript $i$ denotes the i-th element of a vector of frequencies $\omega$, in an interval from $\omega_1 = 0$ to a frequency large enough to assure $||L(j\omega)|| \simeq 0$. In Eq. (15) the control objective is represented, which seeks to make the Nyquist plot for $L(j\omega)$ be tangent to the circumference of radius $M_s^{-1}$ with center in the point $(-1, 0)$. Constraint (16) is introduced to enforce that all points $L(j\omega_i)$ of the Nyquist plot, whose imaginary part is null, must have its real part to the right of the point $\left(-1 + M_s^{-1}, 0\right)$.

Constraint (17) is imposed in order to make the tangent point lie as close as possible to the the point $\left(-1 + M_s^{-1}, 0\right)$. Since the relationship between the gain margin $GM$ of the system and $M_s$ is given as in Eq. (13) a fast transient response with guaranteed gain margin is expected if the Nyquist plot is tangent to the circumference at the point $\left(-1 + M_s^{-1}, 0\right)$, where (13) becomes an equality. This constraint is expected to enforce the highest feedback gains with a comfortable guaranteed gain margin.

Duo to the complexity of this optimization problem, it can be hard to derive gradient-based numerical methods for its solution. The use of a meta-heuristic approach is a natural choice to cope with this kind of problem. Genetic Algorithm (GA) can deal with non-smooth cost functions and constraints such as those in problem (15). It is a stochastic algorithm in nature, based on the search in a population, using the concepts of mutation and crossing between its members, in a metaphor with genetic evolution [15]. In view of Nyquist stability criterion, the solutions given by GA for the problem (15)–(17) if existent, guarantee closed-loop stability.

### *3.2. Numerical procedure*

In order to prevent numerical and convergence issues, the following steps are proposed for implementation of the method.

### 3.2.1. Initial population

In spite of its stochastic nature, GA can experiment slow convergence if an initial population for the feedback gains is randomly chosen. A considerable number of such gains is likely to result in closed-loop unstable systems. Then, the crossing and mutation operations can deliver best new individuals at a poor rate. In order to obtain better results, with shorter run times, an initial population with only stable individuals can be generated following these steps:

- Compute the feedback matrices $\tilde{\mathbf{f}}, \tilde{\mathbf{g}}$ for a pole placement design without delay ($\tau = 0$), using, e.g, the method in [9];
- Compute $\tilde{L}(s) = -\left(\tilde{\mathbf{f}}s + \tilde{\mathbf{g}}\right)\mathbf{H}(s)\mathbf{B}$
- By considering the fixed known time delay $\tau$, detune the feedback gains, by seeking for the maximum $\alpha_m \in [0, 1]$ using the simplified criterion for the maximum admissible delay proposed in [16]:

$$||j\omega\tilde{\mathbf{L}}(j\omega)\left[\alpha_m^{-1}\mathbf{I} - \tilde{\mathbf{L}}(j\omega)\right]^{-1}||_\infty \tau = 1. \tag{18}$$

- Generate a suitable population $\alpha\tilde{\mathbf{f}}, \alpha\tilde{\mathbf{g}}$, using random values of $\alpha \in [0, \alpha_m)$.

According to [16], since we assume there is no open-loop eigenvalue in the right half-plane, there exists a maximal value of $\alpha$, here called $\alpha_m$, given by Eq. (18), such that $\alpha_m\left[\tilde{\mathbf{f}}\ \tilde{\mathbf{g}}\right]$ stabilizes the delayed system. A initial population of stabilizing gains $\alpha_k\left[\tilde{\mathbf{f}}\ \tilde{\mathbf{g}}\right]$ is then created, by generating random values of $\alpha_k \in [0, \alpha_m)$.

### 3.2.2. Running the GA solver

The cost function (15) together with the constraints (16) and (17) and also the initial population generated are then implemented using the genetic algorithm function of MATLAB©. The designer can choose the crossover and mutation parameters at his/her convenience. In the examples of the next section, the default values of the MATLAB© were used for the sake of simplicity.

## 4. Numerical experiments

In this section, three numerical examples are worked out, and the results are compared with the full [9] or partial [3] pole placement techniques.

### 4.1. Example 1

This example is a benchmark for several works, such as [9,10,17] and [16]. It consists of a one DoF mass-damper-spring, depicted at Fig. 2. The system parameters are $M = 1,\ C = 0.01,\ K = 5$, and $B = 1$. The fixed delay is $\tau = 0.1$. For this experiment, three situations are considered: the constraint (16) is relaxed (Case I) and adopted (Case II). In both cases, $M_s = 1.66$ and they have been compared together with the direct pole placement method (DPP) given by Eq. (11), with target closed-loop poles given by $-1, -47$ as in [9]. Three different values of circumference radii are considered: $M_s = 1.33, 1.66$, and 2.00.

In Fig. 3, the results for Case I, Case II, and DPP are displayed. The Nyquist plots for Case II have a tangent point closer to the point $\left(-1 + M_s^{-1}, 0\right)$ as expected. The DPP solution is unstable since the Nyquist plot has one encirclement of the point $(-1, 0)$. As expected, in the Fig. 4 the time response for the Case II is faster when compared with Case I. The unstable response of DPP is also seen at this figure, thereby confirming the predictions from the Nyquist stability criterion. The pole cloud is displayed in Fig. 5, reinforcing the obtained results and conclusions.

The computed feedback gains referring to this part of the experiment are displayed in Table 1.

For the part of the experiment in which the influence of the circumference radius is studied, the constraint (17) is adopted. In Fig. 6, the Nyquist plots are displayed. In the time domain, Fig. 7 illustrate that better responses are obtained

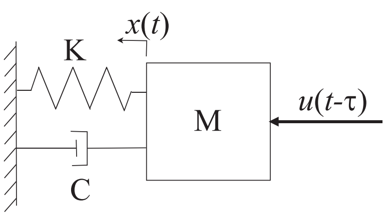


**Fig. 2.** SDOF System of Example I.

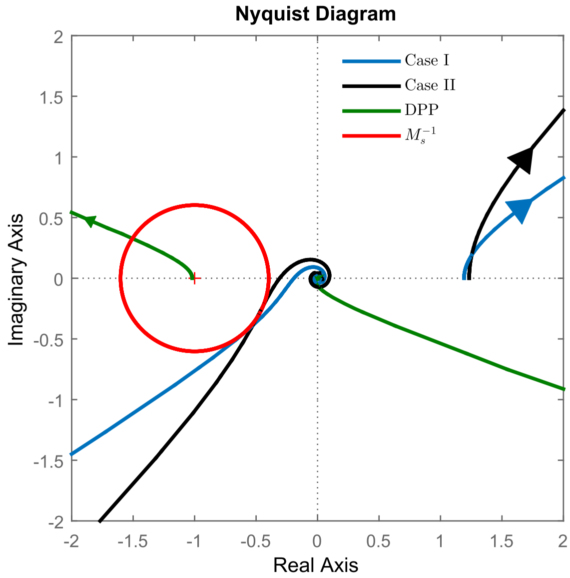


**Fig. 3.** Nyquist diagrams for the Example I.

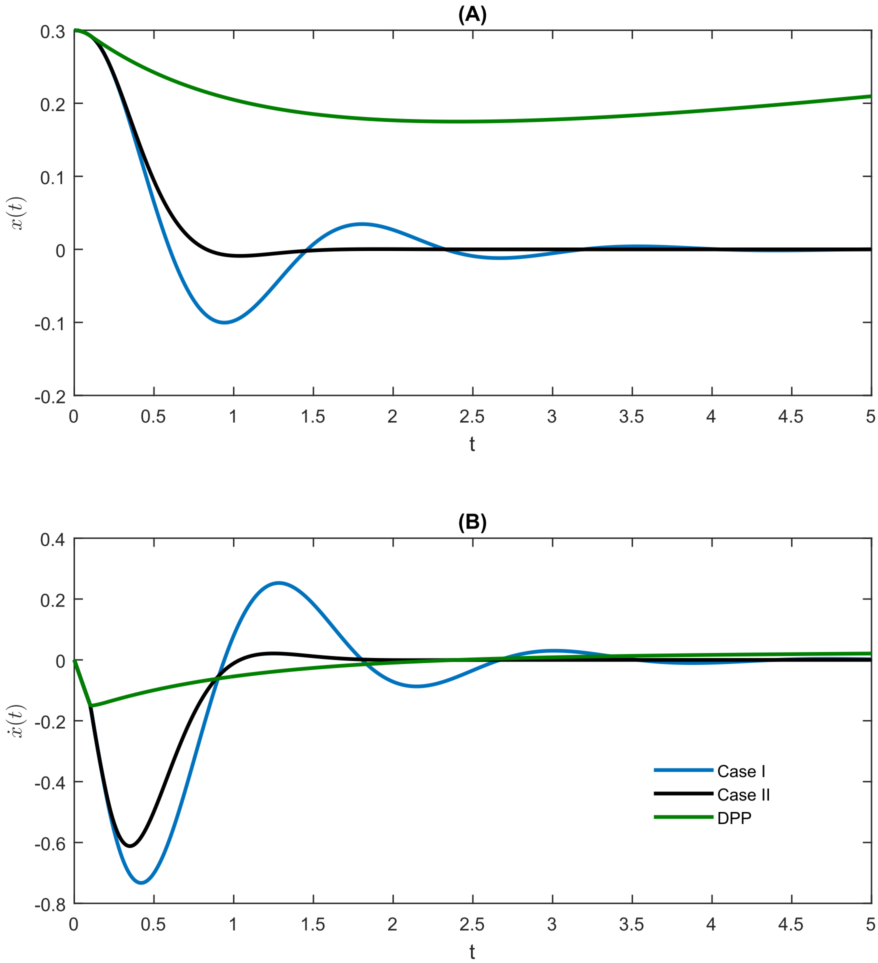


**Fig. 4.** Time domain responses of the Example I: (A) displacement (B) velocity.

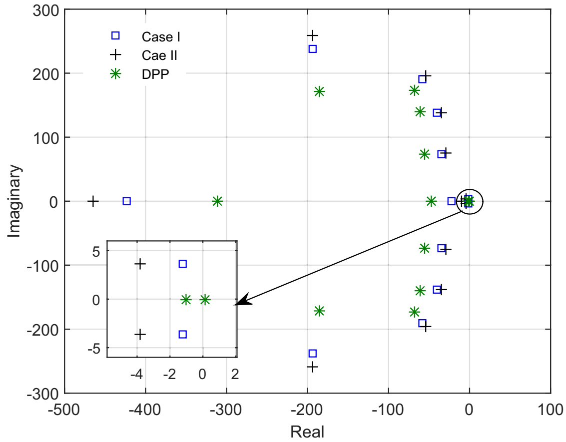


**Fig. 5.** Pole cloud of the Example I, with zooming on the primary poles.

**Table 1**
Feedback gains for the Example 1.

| Gain | Method | | |
|---|---|---|---|
| | Case I | Case II | DDP |
| **f** | −2.6097 | −4.4300 | −0.3198 |
| **g** | −5.9599 | −6.1675 | 5.1001 |

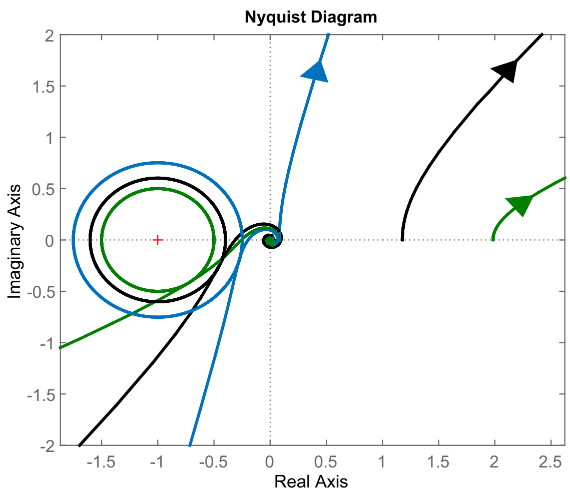


**Fig. 6.** Nyquist plots for different values of $M_s$.

with greater values of radius $M_s^{-1}$. The pole cloud in closed-loop is displayed in Fig. 8. The gains as function of the radii are displayed in Table 2.

### 4.2. Example 2

This example is taken from [18], with a modification in the influence matrix to give a single input design. The system matrices are:

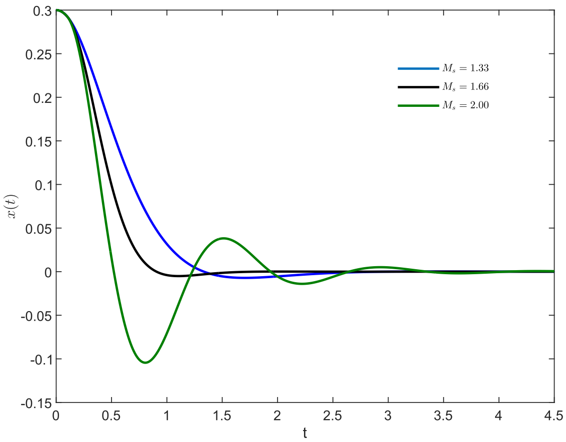


**Fig. 7.** Transient responses of displacement $x(t)$ for different values of $M_s$.

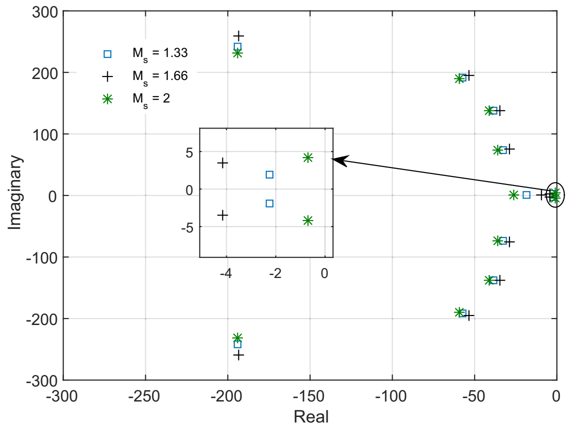


**Fig. 8.** Pole cloud of the Example I for different values of $M_s$.

**Table 2**
Gains as function of Circumference radius $M_s^{-1}$

| Gain | Circumference | | |
|---|---|---|---|
| | $M_s = 1.33$ | $M_s = 1.66$ | $M_s = 2.00$ |
| **f** | −2.9997 | −4.4924 | −3.2829 |
| **g** | −0.4081 | −5.8803 | −9.9175 |

$$\mathbf{M} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix},$$

$$\mathbf{K} = \begin{bmatrix} 1500 & -500 & 0 \\ -500 & 600 & -100 \\ 0 & -100 & 100 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

The fixed delay is $\tau = 0.7$. DPP using (7) is applied with target closed-loop poles $-0.2 \pm j2.5$, $-0.4 \pm j6$, and $-0.6 \pm j10$. The computed gains result in an unstable close-loop response. That means that the target poles are not the primary (dominant) ones, but secondary ones. On the other hand, the proposed approach of minimizing (15) with constraints (16) and (17) by taking $M_s = 2$ give a stable closed-loop response, confirmed by the Nyquist plot displayed in Fig. 9 and also by the time domain evolution of the displacement vector, for the given initial condition $\mathbf{x}(0) = [1\ 0\ 0]^T$ displayed in Fig. 10. The primary closed-loop poles (dominant) are displayed in Fig. 11 The feedback matrices for this case were:

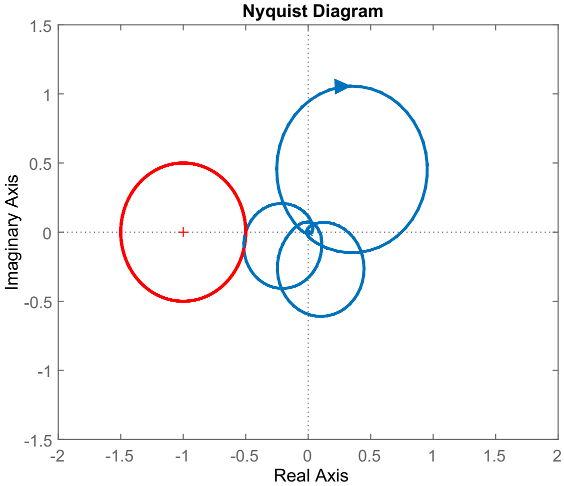


**Fig. 9.** Nyquist plot of the Example 2.

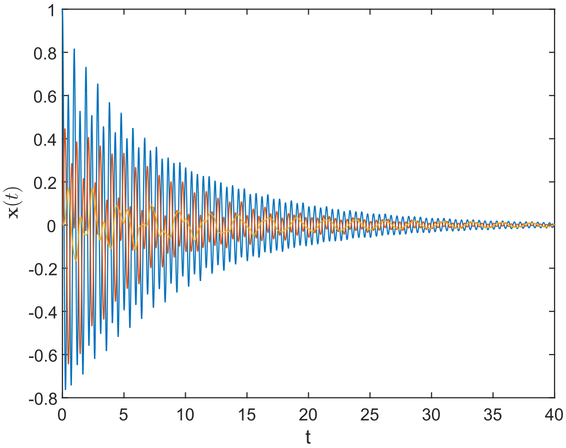


**Fig. 10.** Transient response for Example 2.

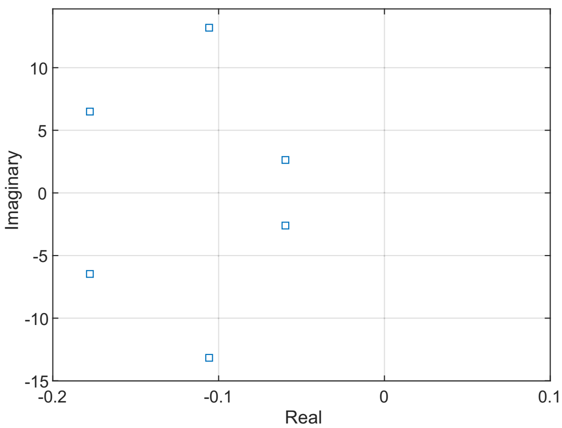


**Fig. 11.** Primary poles for Example 2.

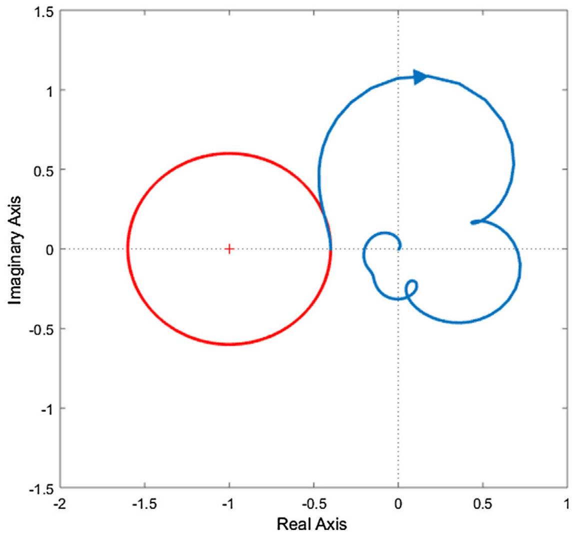


**Fig. 12.** Nyquist plot for the design of Example 3.

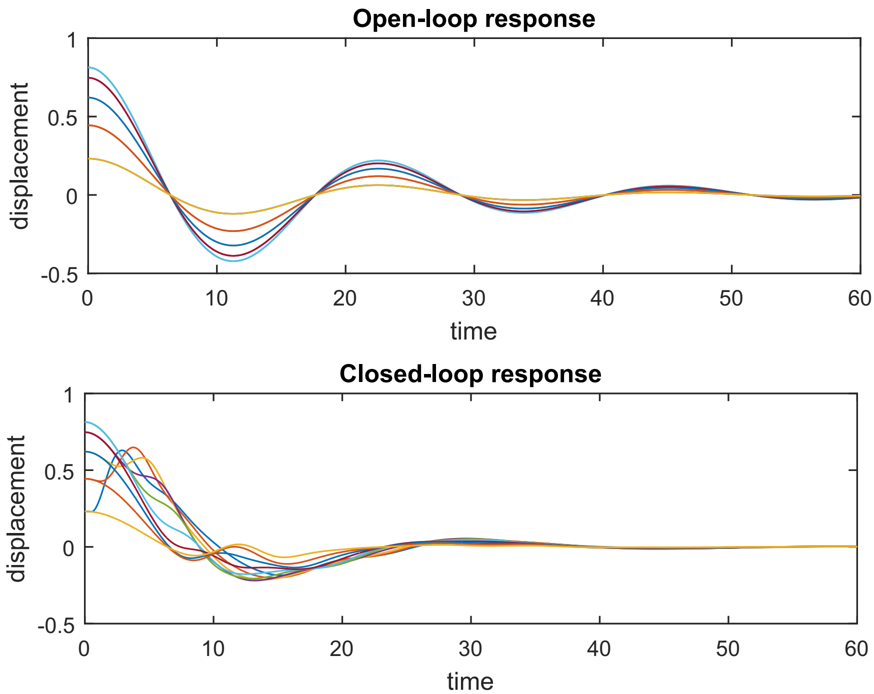


**Fig. 13.** Time domain responses for Example 3.

$$\mathbf{f} = [0.0147 \quad 6.9692 \quad 6.4320], \; \mathbf{g} = [3.0064 \quad -3.9902 \quad 8.8871].$$

### 4.3. Example 3

For this case, a moderate-higher order system is considered, with 10 DoF, to challenge the method in a more complicated case from the computational point of view. The system matrices are given below, and they resemble various structural models in mechanic systems or electric cables/transmission lines in electric systems [19,20]:

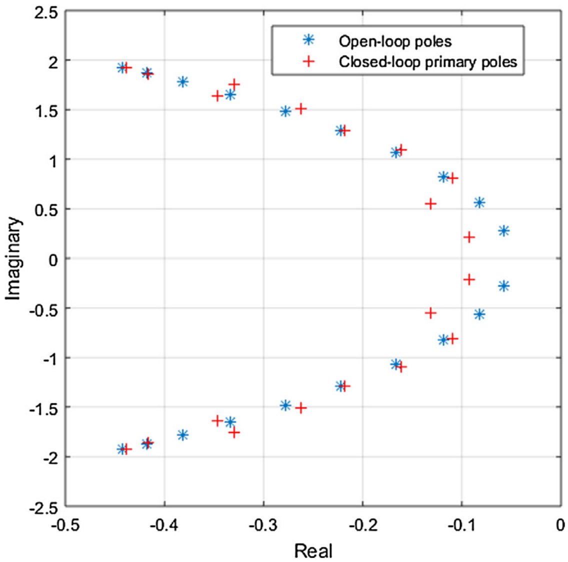


**Fig. 14.** Open-loop and closed-loop poles of Example 3.

$$\mathbf{M} = \mathbf{I}_{10}, \mathbf{C} = \begin{bmatrix} 0.5 & -0.2 & 0 & \cdots & 0 & 0 \\ -0.2 & 0.5 & -0.2 & \cdots & 0 & 0 \\ 0 & -0.2 & 0.5 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -0.2 & 0.5 \end{bmatrix},$$

$$\mathbf{K} = \begin{bmatrix} 2 & -1 & 0 & \cdots & 0 & 0 \\ -1 & 2 & -1 & \cdots & 0 & 0 \\ 0 & -1 & 2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

A time delay $\tau = 0.5$ is considered. The gains for the proposed approach were then computed. The Nyquist plot of the proposed design is displayed in Fig. 12. A simulation in time domain was carried out, with an initial displacement. The closed-loop performance is better than the open-loop response, as can be seen in Fig. 13. The closed-loop dominant poles with the proposed approach are displayed in Fig. 14, where the relative stability improvement is evident in spite of the severe delay that plagues the system.

## 5. Conclusions

In this note, a novel approach to designing state feedback controllers for second-order systems with time delay using frequency response was proposed. Using the concept of receptance, the Nyquist stability criterion is used together with a robustness measure in an optimization procedure to ensure both stability and performance. The method guarantees stability *a priori*, in contrast with the direct pole placement method, in which *a posteriori* analysis must be conducted. The method can cope with time delay with no approximations and uses the versatility of system receptance, that can be entirely obtained with an experimental procedure. Numerical examples help to show that the proposal is effective in the cases where direct pole placement fails to deliver the desired performance. Future work should study the expansion of the method to the multi-input case.

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