FORMULAÇÃO MATEMÁTICA

$$\boxed{u(s) = K_p + \frac{K_I}{s} + K_d s \cdot c \cdot X(s) \cdot e^{-\theta t}}$$

$$M\ddot{x} + D\dot{x} + Kx = Bu$$

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \quad D = \begin{bmatrix} 2d & -d \\ -d & d \end{bmatrix} \quad K = \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

TRANSFORMANDO 2° ordem PARA 1° ordem

$$\dot{X} = AX + Bu$$

$$x = \tilde{x}_1 \quad \dot{x} = \tilde{x}_2$$

$$\dot{\tilde{x}}_1 = \tilde{x}_2$$

$$\dot{\tilde{x}}_2 = -M^{-1}K\tilde{x}_1 - M^{-1}D\tilde{x}_2 + M^{-1}Bu$$

$$\dot{\tilde{X}} = \begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}B \end{bmatrix} u$$

$$\dot{\tilde{x}} = A\tilde{x} + Bu$$

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix} \quad \boxed{B = \begin{bmatrix} 0 \\ M^{-1}B \end{bmatrix}}$$

$$M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = Bu(t-\tau)$$

$$u(t-\tau) = K_p \cdot e(t-\tau) + K_i \cdot \int_{\tau=0}^{t} e(t-\tau)\,d\tau + K_d \frac{de(t)}{d\tau}$$

$$u(s-\tau) = K_p \cdot E(s-\tau) + \frac{K_I \cdot E(s-t)}{s} + K_d \cdot E(s-t) \cdot s$$

$$u(s-\tau) = \left[K_p + \frac{K_I}{s} + K_d \cdot s \cdot c \cdot X(s)\right] \cdot E(s-\tau)$$

I

$$\boxed{M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = Bu(t)}$$

II

ONDE $M, D, K \in R^{n \times n}$; $B, K_p, K_I, K_d \in R^n$;

$x \in \mathbb{R}^n$, $M = M^T$, $D = D^T$, $K = K^T$; $v^T M v > 0$,

$v^T D v \geq 0$ e $v^T K v \geq 0$ PARA qualquer $v \neq 0$, $v \in \mathbb{R}^n$

ASSUMINDO $x(t) = z e^{st}$, e z um VETOR CONSTANTE

E S A VARIÁVEL DE LAPLACE

SUBSTITUINDO I EM II:

$$\left(Ms^2 + D.s + K - B\left(K_p + \frac{K_I}{s} + K_d \cdot S \cdot C \cdot X(s)\right)\right) z e^{st}$$

$$\left[M.s^2 + Ds + K - e^{-s\tau}B.K_P - e^{-s\tau}B\frac{K_I}{s} - e^{-s\tau}B.Kd..s.c.X(s)\right]_{z=0}$$

$$\left[M.s^2 + s\left(D - B.Kd.e^{-s\tau}c.X(s)\right) + K - e^{-s\tau}B\left[\frac{K_I}{s} + K_P\right]\right]_{z=0}$$

$$\left[Ms^2 + s\left(D - BKd.c.X(s)\right) + K - e^{-s\tau}B\left[K_I s^{-1} + K_P\right]\right]_{z=0}$$

III

APLICANDO A FORMULA DE SHERMAN - MORRISON

III

$$\boxed{(A + uV^T)^{-1} = A^{-1} - \frac{A^{-1}uV^T A^{-1}}{1 + V^T A^{-1} u}}$$

$$A = (Ms^2 + Ds + K) \qquad u = B$$

$$V = -\left(K_P + \frac{K_I}{s} + K.ds.c.X(s)\right).e^{-s\tau}$$

$$\hat{H}(s) = H(s) + \frac{H(s)\,B.\left(\left[K_P + \frac{K_I}{s} + Kd.s.c.X(s)\right].e^{-s\tau}\right)H(s)}{1 - \left(\left[K_P + \frac{K_I}{s} + Kd.s.c.X(s)\right]e^{-s\tau}\right)H(s).B}$$

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$$\hat{H}(s) = \left(Ms^2 + D.s + K - e^{-s\tau}B\left(K_P + \frac{K_I}{s} + K_d.s.c.X(s)\right)\right)^{-1}$$

↳ MATRIZ DE RECEPTANCIA DE MALHA FECHADA

$$H(s) = (Ms^2 + D.s + K)^{-1}$$

↳ MATRIZ DE RECEPTÂNCIA DE MALHA ABERTA

EQUAÇÃO CARACTERISTICA

$$1 - \left(K_P + \frac{K_I}{s} + K_d.s.c.X(s).\right)H(s).B.e^{-s\tau} = 0$$

$$\boxed{\left(K_P + \frac{K_I}{s} + K_d.s.c.X(s)\right).H(s).B = e^{s\tau}}$$

↳ EQUAÇÃO CARACTERISTICA

$$\boxed{\begin{bmatrix} 1\Omega_1 & \frac{1}{s}\Omega_1 & \Omega_1 s.c.X(s) \\ \Omega_2 & \frac{1}{s}\Omega_2 & \Omega_2 s.c.X(s) \\ \vdots & & \\ \Omega_{2n} & \frac{1}{s}\Omega_{2n} & \Omega_{2n}.c.X(s) \end{bmatrix} \begin{bmatrix} K_p \\ K_I \\ K_d \end{bmatrix} = \begin{bmatrix} e^{s_1\tau} \\ e^{s_2\tau} \\ \vdots \\ e^{s_{2n}\tau} \end{bmatrix}}$$

$$\boxed{\Omega_K = H(s_K).B}$$