Assignment 1

Reading Material:

1. Chapter 1: Mathematical Review;
2. Chapter 2: Combinatorics.

Problems:

1. **Reliability of a k-out-of-n system.** A system consists of n identical components that are operational with probability p, independently of other components. The system is operational if at least k out of the n components are operational. What is the probability that the system is operational?
2. We deal from a well-shuffled 52-card deck. Calculate the probability that the 13th card is the first king to be dealt.
3. Twenty distinct cars park in the same parking lot every day. Ten of these cars are US-made, while the other ten are foreign-made. The parking lot has exactly twenty spaces, all in a row, so the cars park side by side. However, the drivers have varying schedules, so the position any car might take on a certain day is random.
   (a) In how many different ways can the cars line up?
   (b) What is the probability that on a given day, the cars will park in such a way that they alternate (no two US-made are adjacent and no two foreign-made are adjacent)?
4. An academic department offers 8 lower level courses: $\{L_1, L_2, \ldots, L_8\}$ and 10 higher level courses: $\{H_1, H_2, \ldots, H_{10}\}$. A valid curriculum consists of 4 lower level courses, and 3 higher level courses.
   (a) How many different curricula are possible?
   (b) Suppose that $\{H_1, \ldots, H_5\}$ have L_1 as a prerequisite, and $\{H_6, \ldots, H_{10}\}$ have L_2 and L_3 as prerequisites, i.e., any curricula which involves, say, one of $\{H_1, \ldots, H_5\}$ must also include L_1. How many different curricula are there?
5. We draw the top 7 cards from a well-shuffle standard 52-card deck. Find the probability that:
   (a) The 7 cards include exactly 3 aces.
   (b) The 7 cards include exactly 2 kings.
   (c) The probability that the 7 cards include exactly 3 aces or exactly 2 kings.
6. A well-shuffled 52-card deck is dealt to 4 players. Find the probability that each of the players gets an ace.