

Assignment 7

Reading Assignment:

1. Chapter 8: Continuous Random Variables.

Problems:

1. Two fair dice are rolled. Find the joint probability mass function of X and Y when
 - (a) X is the largest value obtained on any die and Y is the sum of the values;
 - (b) X is the value on the first die and Y is the larger of the two values;
 - (c) X is the smallest and Y is the largest value obtained on the dice.
2. N people arrive separately to a professional dinner. Upon arrival, each person looks to see if he or she has any friends among those present. That person then either sits at the table of a friend or at an unoccupied table if none of those present is a friend. Assuming that each of the $\binom{N}{2}$ pairs of people are, independently, friends with probability p , find the expected number of occupied tables. *Hint:* Let X_i equal 1 or 0 dependent on whether the i th arrival sits at a previously unoccupied table.
3. Cards from an ordinary deck of 52 playing cards are turned face up one at a time. If the first card is an ace, or the second a deuce, or the third a three, or, \dots , or the thirteenth a king, or the fourteenth an ace, and so on, we say that a match occurs. Note that we do not require that $(13n + 1)$ th card be any particular ace for a match to occur but only that it be an ace. Compute the expected number of matches that occur.
4. The number of people who enter an elevator on the ground floor is a Poisson random variable with mean 10. If there are N floors above the ground floor and if each person is equally likely to get off at any one of these N floors, independently of where the others get off, compute the expected number of stops that the elevator will make before discharging all of its passengers.
5. Let X have the PDF
$$f_X(x) = \frac{\lambda}{2} e^{-\lambda|x|},$$
where λ is a positive scalar. Verify that f_X satisfies the normalization condition, and evaluate the mean and variance of X .
6. Consider a triangle and a point chosen within the triangle according to the uniform probability law. Let X be the distance from the point to the base of the triangle. Given the height of the triangle, find the CDF and the PDF of X .