Journal of Ocean Engineering and Science xxx (xxxx) xxx

Contents lists available at ScienceDirect

Journal of Ocean Engineering and Science

journal homepage: www.elsevier.com/locate/joes

New exact solutions for the reaction-diffusion equation in mathematical physics

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A R T I C L E I N F O

Article history:
Received 25 December 2019
Revised 4 May 2022
Accepted 6 May 2022
Available online xxx

Keywords:
The reaction-diffusion equation
Exp$(-\varphi(\xi))$-expansion technique
Sine-cosine technique
Riccati–Bernoulli sub-ODE technique
Exact solutions

A B S T R A C T

We utilize three mathematical techniques, specifically $\exp(-\varphi(\xi))$-expansion (EXP), sine-cosine (SCT) and Riccati–Bernoulli sub-ODE techniques (RRT) to obtain soliton solutions of the reaction-diffusion equation. These methods are applied to construct new exact-periodic and soliton solutions for the reaction-diffusion equation. Furthermore, these mentioned methods give a robust, efficient mathematical tool for solving various models of nonlinear partial differential equations in nonlinear science. Indeed the reported results are very vital in explaining the physical phenomena of the proposed model.

1. Introduction

The nonlinear partial differential equations (NPDEs) are often presented to describe the complex phenomena in many fields such as optics, signal processing, condensed matter physics, biology and fluid dynamics [1–11]. According to an important role of these NPDEs, many strong techniques have been suggested to find exact solutions to these equations, such as the [12–35].

Our objective here is to stress the power of the functional variable methods for getting various novel solutions of the reaction-diffusion equation. Namely, we present the EXP, SCT and RRT techniques. Diffusion and response mechanisms play the essential roles in many systems' dynamics in semiconductor physics, plasma and chemistry. We give new solutions and show that the proposed methods are robust, efficient and sufficient to solve such kinds of NPDEs. Indeed, RRT technique introduces an infinite solutions. In fact, the basic motivation behind the reported solutions comes from the examination of some physical phenomena.

The rest of this paper is organized as follows. Section 2 gives the description of the three powerful methods. These methods are applied for finding the solutions for the reaction-diffusion equation in Section 3. Some important conclusions are given in Section 4.

2. Description of the methods

Given NPDEs as comes next

$$H(\chi, \chi_t, \chi_x, \chi_{tt}, \chi_{xx}, \ldots) = 0, \tag{1}$$

where H is a polynomial in $\chi(x,t)$ and its partial derivatives. By means of the wave transformation,

$$\chi(x,t) = \chi(\xi), \qquad \xi = x - ct, \tag{2}$$

Eq. (1) will be reduced to

$$D(\chi, \chi', \chi'', \chi''', \ldots) = 0, \tag{3}$$

2.1. The EXP technique

Assumed Eq. (3) has a solution as He and Wu [15], Aminikhad et al. [16]:

$$\chi(\xi) = \sum_{m=0}^{n} a_m (\exp(-\varphi(\xi)))^m, \quad a_n \neq 0, \tag{4}$$

where $\varphi(\xi)$ satisfies

$$\varphi'(\xi) = \exp(-\varphi(\xi)) + \mu \exp(\varphi(\xi)) + \lambda. \tag{5}$$

Eq. (5) has the following solutions, for an arbitrary constant k :

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https://doi.org/10.1016/j.joes.2022.05.006

1. At $\mu \neq 0, -4\mu + \lambda^2 > 0$,

$$\varphi(\xi) = ln\left(\frac{-\sqrt{-4\mu+\lambda^2}\,\tanh\left(\frac{\sqrt{-4\mu+\lambda^2}}{2}(\xi+K)\right)-\lambda}{2\mu}\right). \tag{6}$$

2. At $\lambda^2 - 4\mu < 0, \mu \neq 0$,

$$\varphi(\xi) = ln\left(\frac{\sqrt{4\mu-\lambda^2}\,\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+K)\right)-\lambda}{2\mu}\right), \tag{7}$$

3. At $\mu = 0, \lambda \neq 0, \lambda^2 - 4\mu > 0$,

$$\varphi(\xi) = -ln\left(\frac{\lambda}{\exp(\lambda(\xi+K))-1}\right), \tag{8}$$

4. At $\lambda \neq 0, \lambda^2 - 4\mu = 0, \mu \neq 0$,

$$\varphi(\xi) = ln\left(-\frac{2(\lambda(\xi+K)+2)}{\lambda^2(\xi+K)}\right), \tag{9}$$

5. At $\lambda = 0, \lambda^2 - 4\mu = 0, \mu = 0$,

$$\varphi(\xi) = ln(\xi+K). \tag{10}$$

Finally, superseding Eq. (4) with Eq. (5) into Eq. (3) and aggregating all terms of the same power $\exp(-m\varphi(\xi))$, $m = 0, 1, 2, 3, \ldots$. Then, algebraic equations can be obtained by equating every term to zero. These equations may be solved to find the values of a_i. This implies the solutions of (4) is reached.

2.2. The SCT technique

The solutions of Eq. (3) can be expressed in the form Wazwaz [36], Tascan and Bekir [37]

$$\chi(x,t) = \begin{cases} \lambda\sin^r(\mu\xi), & |\xi| \leq \frac{\pi}{\mu}, \\ 0, & \text{otherwise}, \end{cases} \tag{11}$$

or

$$\chi(x,t) = \begin{cases} \lambda\cos^r(\mu\xi), & |\xi| \leq \frac{\pi}{2\mu}, \\ 0, & \text{otherwise}, \end{cases} \tag{12}$$

with λ, μ and $r \neq 0$, depicting parameters determined in sequel. From (11) we have

$$\begin{aligned} \chi(\xi) &= \lambda\sin^r(\mu\xi), \\ \chi^n(\xi) &= \lambda^n\sin^{nr}(\mu\xi), \\ (\chi^n)_\xi &= n\mu r\lambda^n\cos(\mu\xi)\sin^{nr-1}(\mu\xi), \\ (\chi^n)_{\xi\xi} &= -n^2\mu^2 r\lambda^n\sin^{nr}(\mu\xi) + n\mu^2\lambda^n r(nr-1)\sin^{nr-2}(\mu\xi), \end{aligned} \tag{13}$$

and from (12) we have

$$\begin{aligned} \chi(\xi) &= \lambda\cos^r(\mu\xi), \\ \chi^n(\xi) &= \lambda^n\cos^{nr}(\mu\xi), \\ (\chi^n)' &= -n\mu r\lambda^n\sin(\mu\xi)\cos^{nr-1}(\mu\xi), \\ (\chi^n)'' &= -n^2\mu^2 r\lambda^n\cos^{nr}(\mu\xi) + n\mu^2\lambda^n r(nr-1)\cos^{nr-2}(\mu\xi). \end{aligned} \tag{14}$$

Finally, imposing Eqs. (13) and (14) into Eq. (3), then balance the terms of the cosine functions (14) or the sine functions (13). Hence, we sum all terms with the same power in $\cos^k(\mu\xi)$ or $\sin^k(\mu\xi)$ and equating their coefficients to zero in order to obtain an algebraic equations in the unknowns μ, λ and r. Solving this system yields these unknown constants.

2.3. RRT technique

According to RRT method [17–19,21,22], Eq. (3) has a solution given by:

$$\chi' = a\chi^{2-n} + b\chi + c\chi^n. \tag{15}$$

Using Eq. (15), one has

$$\begin{aligned} \chi'' &= ab(3-n)\chi^{2-n} + a^2(2-n)\chi^{3-2n} \\ &\quad + nc^2\chi^{2n-1} + bc(n+1)\chi^n + (2ac+b^2)\chi, \end{aligned} \tag{16}$$

$$\begin{aligned} \chi''' &= (ab(3-n)(2-n)\chi^{1-n} + a^2(2-n)(3-2n)\chi^{2-2n} \\ &\quad + n(2n-1)c^2\chi^{2n-2} + bcn(n+1)\chi^{n-1} + (2ac+b^2))\chi'. \end{aligned} \tag{17}$$

The solitary solutions $\chi_i(\xi)$ of Eq. (15) are:

1. At $n = 1$

$$\chi(\xi) = \mu e^{(a+b+c)\xi}. \tag{18}$$

2. At $n \neq 1$, $c = 0$, and $b = 0$

$$\chi(\xi) = (a(n-1)(\xi+\mu))^{\frac{1}{n-1}}. \tag{19}$$

3. At $n \neq 1$, $c = 0$, and $b \neq 0$

$$\chi(\xi) = \left(\frac{-a}{b} + \mu e^{b(n-1)\xi}\right)^{\frac{1}{n-1}}. \tag{20}$$

4. At $b^2 - 4ac < 0$, $n \neq 1$, and $a \neq 0$

$$\chi(\xi) = \left(\frac{-b}{2a} + \frac{\sqrt{4ac-b^2}}{2a}\tan\left(\frac{(1-n)\sqrt{4ac-b^2}}{2}(\xi+\mu)\right)\right)^{\frac{1}{1-n}} \tag{21}$$

and

$$\chi(\xi) = \left(\frac{-b}{2a} - \frac{\sqrt{4ac-b^2}}{2a}\cot\left(\frac{(1-n)\sqrt{4ac-b^2}}{2}(\xi+\mu)\right)\right)^{\frac{1}{1-n}}. \tag{22}$$

5. At $b^2 - 4ac > 0$, $n \neq 1$, and $a \neq 0$

$$\chi(\xi) = \left(\frac{-b}{2a} - \frac{\sqrt{b^2-4ac}}{2a}\coth\left(\frac{(1-n)\sqrt{b^2-4ac}}{2}(\xi+\mu)\right)\right)^{\frac{1}{1-n}} \tag{23}$$

and

$$\chi(\xi) = \left(\frac{-b}{2a} - \frac{\sqrt{b^2-4ac}}{2a}\tanh\left(\frac{(1-n)\sqrt{b^2-4ac}}{2}(\xi+\mu)\right)\right)^{\frac{1}{1-n}}. \tag{24}$$

6. At $b^2 - 4ac = 0$, $n \neq 1$, and $a \neq 0$

$$\chi(\xi) = \left(\frac{1}{a(n-1)(\xi+\mu)} - \frac{b}{2a}\right)^{\frac{1}{1-n}}. \tag{25}$$

2.3.1. Bäcklund transformation

Given the solutions $\chi_{m-1}(\xi)$ and $\chi_m(\xi)(\chi_m(\xi)=\chi_m(\chi_{m-1}(\xi)))$ of Eq. (15), then we have

$$\frac{d\chi_m(\xi)}{d\xi}=\frac{d\chi_m(\xi)}{d\chi_{m-1}(\xi)}\frac{d\chi_{m-1}(\xi)}{d\xi}=\frac{d\chi_m(\xi)}{d\chi_{m-1}(\xi)}(a\chi_{m-1}^{2-n}+b\chi_{m-1}+c\chi_{m-1}^{n}),$$

namely

$$\frac{d\chi_m(\xi)}{a\chi_m^{2-n}+b\chi_m+c\chi_m^{n}}=\frac{d\chi_{m-1}(\xi)}{a\chi_{m-1}^{2-n}+b\chi_{m-1}+c\chi_{m-1}^{n}}. \tag{26}$$

Solving Eq. (26) gives

$$\chi_m(\xi)=\left(\frac{-cK_1+aK_2(u_{m-1}(\xi))^{1-n}}{bC_1+aC_2+aC_1(\chi_{m-1}(\xi))^{1-n}}\right)^{\frac{1}{1-n}}. \tag{27}$$

Here C_1 and C_2 are arbitrary constants. Utilizing Eq. (27), the infinite solutions for Eq. (15), and Eq. (1) are reach.

3. The reaction-diffusion equation

Here we examine the reaction-diffusion model [38–40], given as follows:

$$u_{tt}+\alpha\,u_{xx}+\beta\,u+\gamma\,u^3=0, \tag{28}$$

where α,β and γ are nonzero constants.

Using the wave transformation

$$u(x,t)=u(\xi),\qquad \xi=x+\omega t, \tag{29}$$

where ω is arbitrary constants, the Eq. (28) is transformed into equation:

$$u''+\delta\,u^3+\eta\,u=0, \tag{30}$$

where $\delta=\frac{\gamma}{\alpha+\omega^2}$, $\eta=\frac{\beta}{\alpha+\omega^2}$.

3.1. Solving Eq. (28) using the EXP technique

By EXP technique, Eq. (30) admits

$$u=A_0+A_1\exp(-\varphi), \tag{31}$$

whereA_0 and A_1 are constants and $A_1\neq 0$. One may easily reach

$$u''=A_1\big(2\,\exp(-3\varphi)+3\lambda\,\exp(-2\varphi)+(2\mu+\lambda^2)\,\exp(-\varphi)+\lambda\mu\big), \tag{32}$$

$$u^3=A_1^3\,\exp(-3\varphi)+3A_0A_1^2\,\exp(-2\varphi)+3A_0^2A_1\,\exp(-\varphi)+A_0^3. \tag{33}$$

Superseding u, u'', u^3 into Eq. (30), we obtain

$$A_1\lambda\mu+\delta A_0^3+\eta A_0=0, \tag{34}$$

$$A_1(\lambda^2+2\mu)+3\delta A_0^2A_1+\eta A_1=0, \tag{35}$$

$$3A_1\lambda+3\delta A_0A_1^2=0, \tag{36}$$

$$2A_1+\delta A_1^3=0. \tag{37}$$

Solving Eqs. (34)–(37), we get

$$\eta=\frac{1}{2}(\lambda^2-4\mu),\ A_0=\pm\frac{\lambda}{\sqrt{-2\delta}},\ A_1=\pm\frac{\sqrt{2}}{\sqrt{-\delta}}.$$

We consider only one case, whenever the other cases follow similarly. Superseding the values of A_0,A_1 into Eq. (31) gives

$$u(\xi)=\pm\frac{1}{\sqrt{-2\delta}}(\lambda+2\exp(-\varphi(\xi))). \tag{38}$$

Superseding Eqs. (6) and (7) into Eq. (38), we obtain:

Case 1. At $\mu\neq 0,\lambda^2-4\mu>0$,

$$u_{1,2}(x,t)=\pm\frac{1}{\sqrt{-2\delta}}\left(\lambda-\frac{4\mu}{\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(x+wt+K)\right)+\lambda}\right), \tag{39}$$

where δ,w,λ,μ and K are arbitrary constants.

Case 2. At $\lambda^2-4\mu<0,\mu\neq 0$,

$$u_{3,4}(x,t)=\pm\frac{1}{\sqrt{-2\delta}}\left(\lambda+\frac{4\mu}{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(x+wt+K)\right)-\lambda}\right). \tag{40}$$

Case 3. At $\lambda^2-4\mu>0,\mu=0,\lambda\neq 0$

$$u_{5,6}(x,t)=\pm\frac{1}{\sqrt{-2\delta}}\left(\lambda+\frac{2\lambda}{\exp\left(\lambda(x+wt+K)\right)-1}\right). \tag{41}$$

Case 4. At $\lambda^2-4\mu=0,\mu\neq 0,\lambda\neq 0$,

$$u_{7,8}(x,t)=\pm\frac{1}{\sqrt{-2\delta}}\left(\lambda-\frac{\lambda^2(x+w\,t+K)}{\lambda(x+w\,t+K)+2}\right). \tag{42}$$

Case 5. At $\lambda^2-4\mu=0,\mu=0,\lambda=0$,

$$u_{9,10}(x,t)=\pm\frac{1}{\sqrt{-2\delta}}\left(\frac{1}{x+w\,t+K}\right), \tag{43}$$

where δ,w and K are constants.

3.2. Solving Eq. (28) using the SCT technique

According to SCT technique, subtitling Eq. (13) into Eq. (30), gives

$$\left(-\mu^2r^2\lambda\sin^r(\mu\xi)+\mu^2\lambda r(r-1)\sin^{r-2}(\mu\xi)\right)+\delta\lambda^3\sin^{3r}(\mu\xi)+\eta\lambda\sin^r(\mu\xi)=0. \tag{44}$$

Thus by comparing the coefficients of the sine functions, we get

$$r-1\neq 0,\qquad r-2=3r,$$

$$\mu^2\lambda r(r-1)+\delta\lambda^3=0, \tag{45}$$

$$-\mu^2r^2\lambda+\eta\lambda=0.$$

We then reach

$$r=-1,\qquad \lambda=\pm i\sqrt{\frac{2\eta}{\delta}},\qquad \mu=\pm\sqrt{\eta}, \tag{46}$$

for $\frac{\eta}{\delta}>0$ and $\delta\neq 0$. If we use the cosine terms (14), we get the same outcome. Periodic alternatives are therefore

$$\tilde{u}_{1,2}(x,t)=\pm i\sqrt{\frac{2\eta}{\delta}}\sec(\sqrt{\eta}(x+wt)),\qquad |\sqrt{\eta}(x+wt)|<\frac{\pi}{2} \tag{47}$$

and

$$\tilde{u}_{3,4}(x,t) = \pm i\sqrt{\frac{2\eta}{\delta}}\csc(\sqrt{\eta}(x+wt)), \quad 0 < \sqrt{\eta}(x+wt) < \pi. \tag{48}$$

However, for $\frac{\eta}{\delta} < 0$ and $\delta \neq 0$, we obtain the soliton and complex solutions

$$\tilde{u}_{5,6}(x,t) = \pm i\sqrt{\frac{2\eta}{\delta}}\,\mathrm{sech}\big(\sqrt{-\eta}(x+wt)\big) \tag{49}$$

and

$$\tilde{u}_{7,8}(x,t) = \pm i\sqrt{\frac{2\eta}{\delta}}\,\mathrm{csch}(\sqrt{\eta}(x+wt)). \tag{50}$$

3.3. Solving Eq. (28) using the RRT technique

According to RRT technique, inserting Eq. (16) into Eq. (30), we get

$$\big(ab(3-n)u^{2-n} + a^2(2-n)u^{3-2n} + nc^2u^{2n-1} + bc(n+1)u^n + (2ac+b^2)u\big) + \delta u^3 + \eta u = 0. \tag{51}$$

Putting $n = 0$, Eq. (51) becomes

$$(3abu^2 + 2a^2u^3 + bc + (2ac+b^2)u) + \delta u^3 + \eta u = 0. \tag{52}$$

Setting each coefficient of $u^i (i = 0, 1, 2, 3)$ to zero, we get

$$bc = 0, \tag{53}$$

$$(2ac + b^2) + \eta = 0, \tag{54}$$

$$3ab = 0, \tag{55}$$

$$2a^2 + \delta = 0. \tag{56}$$

Solving Eqs. (53)–(56) gives

$$b = 0, \tag{57}$$

$$c = \mp\frac{\eta}{\sqrt{-2\delta}}, \tag{58}$$

$$a = \pm\sqrt{\frac{-\delta}{2}}. \tag{59}$$

Hence, the solutions for Eq. (30) given as follows

1. For $c = 0$, $b = 0$ the solution of Eq. (30) is

$$\hat{u}_1(x,t) = (-a(x+wt+\mu))^{-1}. \tag{60}$$

where w, μ are arbitrary constants.

2. When $\eta < 0$, superseding Eqs. (57)–(59) and (29) into Eqs. (21) and (22), then the exact solutions of Eq. (28) are

$$\hat{u}_{2,3}(x,t) = \pm\sqrt{\frac{\eta}{\delta}}\tan\big(\sqrt{-\eta}(x+wt+\mu)\big) \tag{61}$$

and

$$\hat{u}_{4,5}(x,t) = \pm\sqrt{\frac{\eta}{\delta}}\cot\big(\sqrt{-\eta}(x+wt+\mu)\big). \tag{62}$$

where w, μ are arbitrary constants.

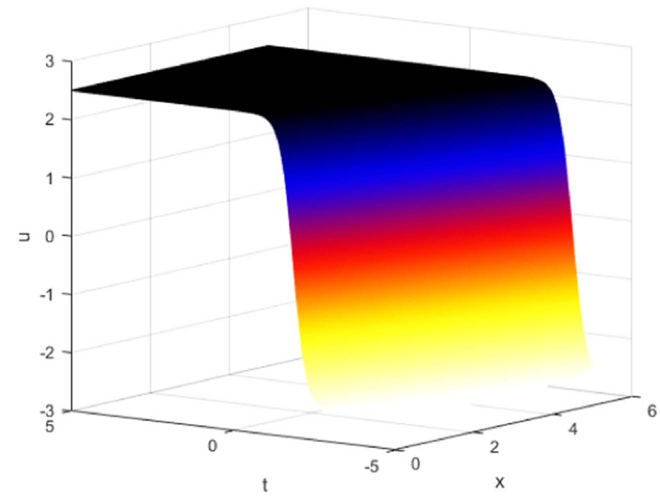

Fig. 1. Graph of u_1 in (39) with $\alpha = 2, \gamma = -3, \lambda = 3.5, \mu = 1, w = 2, \delta = -0.5, k = 3$ and $0 \leq t \leq 5$, $-5 \leq x \leq 5$.

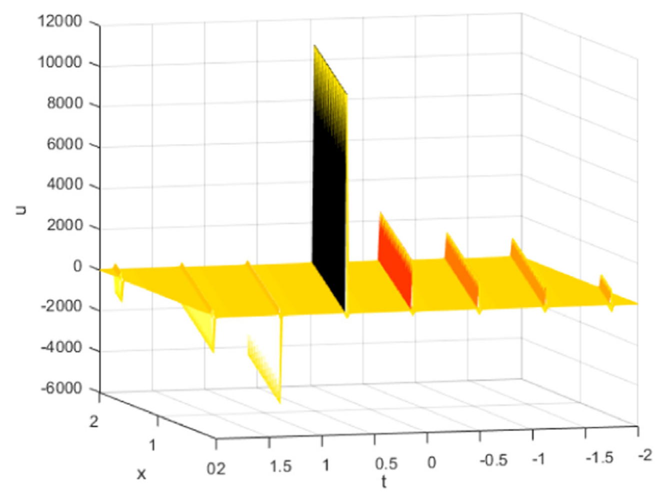

Fig. 2. Graph of u_3 in (40) with $\alpha = 1.5, \alpha = -2.5, \lambda = 2, \mu = 5, w = 2.5, \delta = -0.3226, k = 1$ and $0 \leq t \leq 2$, $-2 \leq x \leq 2$.

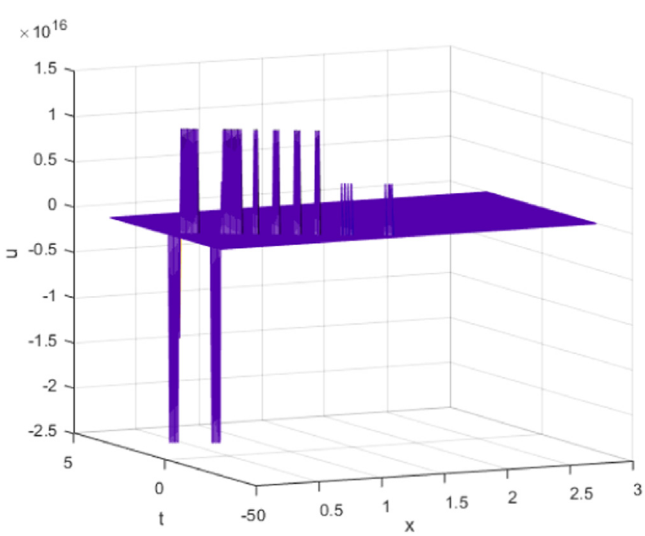

Fig. 3. Graph of u_3 in (40) with $\alpha = 1, \gamma = -2, \lambda = 2.3, \mu = 0, w = 0.5, \delta = -1.6, k = 1$ and $0 \leq t \leq 3$, $-3 \leq x \leq 3$.

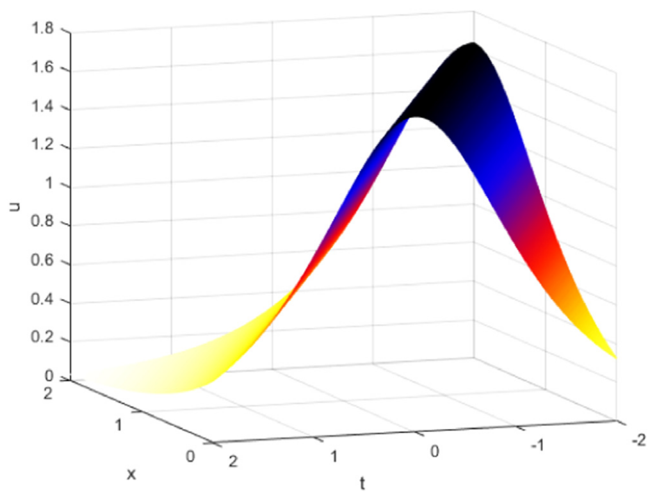

Fig. 4. Graph of $u = \tilde{u}_1$ in (47) with $\alpha = 2, \beta = -1, \gamma = -3, w = 1, \delta = -1, \eta = -1.3333$ and $0 \le t \le 2$, $-2 \le x \le 2$.

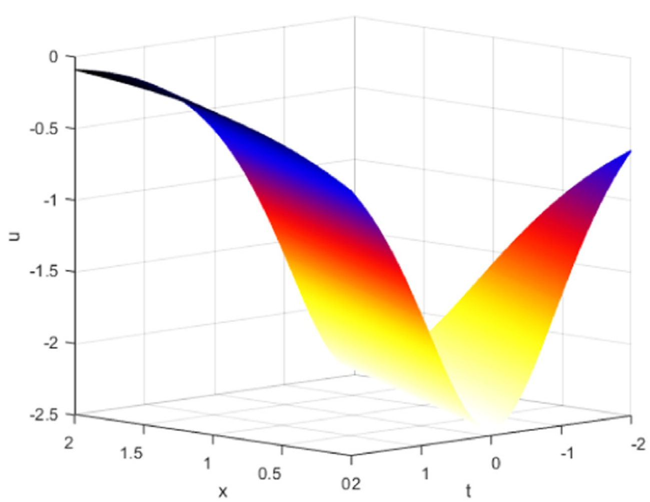

Fig. 5. Graph of $u = \tilde{u}_5$ in (49) with $\alpha = 2, \beta = -3, \gamma = 1, w = 1, \delta = 0.3333, \eta = -1$ and $0 \le t \le 2$, $-2 \le x \le 2$.

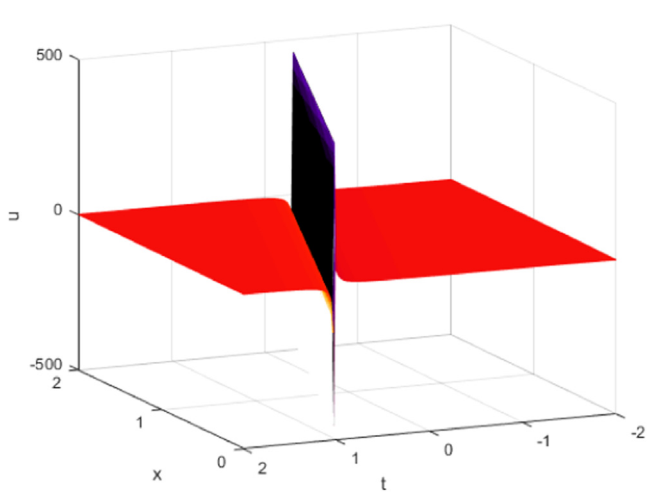

Fig. 6. Graph of $u = \hat{u}_2$ in (61) with $\alpha = 3, \beta = -2, \gamma = -1, w = 1.5, \delta = -0.1905, \eta = -0.3810, \mu = 1$ and $0 \le t \le 2$, $-2 \le x \le 2$.

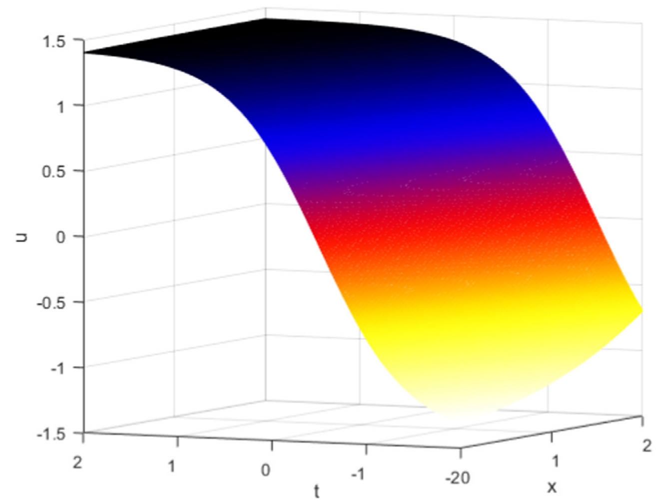

Fig. 7. Graph of $u = \hat{u}_6$ in (63) with $\alpha = 3, \beta = 2, \gamma = -1, w = 2, \delta = -0.1429, \eta = 0.2857, \mu = 1$ and $0 \le t \le 2$, $-2 \le x \le 2$.

3. When $\eta > 0$, superseding Eqs. (57)–(59) and (29) into Eqs. (23) and (24), then the exact solutions of Eq. (28) are

$$\hat{u}_{6,7}(x,t) = \pm\sqrt{\frac{-\eta}{\delta}}\tanh(\sqrt{\eta}(x + wt + \mu)) \tag{63}$$

and

$$\hat{u}_{8,9}(x,t) = \pm\sqrt{\frac{-\eta}{\delta}}\coth(\sqrt{\eta}(x + wt + \mu)). \tag{64}$$

Here w, μ are arbitrary constants.

Remark 3.1. Using Eq. (27) for $u_i(x,y,t)$, i =1,...,9, once, then Eq. (30) as well as for Eq. (28) has an infinite solutions. In sequence, by applying this process again, we get new families of solutions.

$$u_1^\star(x,y,t) = \frac{B_3}{-aB_3(x + wt + \mu) \pm 1}, \tag{65}$$

$$u_{2,3}^\star(x,y,t) = \frac{\pm\frac{\eta}{\delta} \pm B_3\sqrt{\frac{\eta}{\delta}}\,\tan(\sqrt{-\eta}(x + wt + \mu))}{B_3 \pm \sqrt{\frac{\eta}{\delta}}\,\tan(\sqrt{-\eta}(x + wt + \mu))}, \tag{66}$$

$$u_{4,5}^\star(x,y,t) = \frac{\pm\frac{\eta}{\delta} \pm B_3\sqrt{\frac{\eta}{\delta}}\,\cot(\sqrt{-\eta}(x + wt + \mu))}{B_3 \pm \sqrt{\frac{\eta}{\delta}}\,\cot(\sqrt{-\eta}(x + wt + \mu))}, \tag{67}$$

$$u_{6,7}^\star(x,y,t) = \frac{\pm\frac{\eta}{\delta} \pm B_3\sqrt{\frac{-\eta}{\delta}}\tanh(\sqrt{\eta}(x + wt + \mu))}{B_3 \pm \sqrt{\frac{-\eta}{\delta}}\tanh(\sqrt{\eta}(x + wt + \mu))}, \tag{68}$$

$$u_{8,9}^\star(x,y,t) = \frac{\pm\frac{\eta}{\delta} \pm B_3\sqrt{\frac{-\eta}{\delta}}\coth(\sqrt{\eta}(x + wt + \mu))}{B_3 \pm \sqrt{\frac{-\eta}{\delta}}\coth(\sqrt{\eta}(x + wt + \mu))}, \tag{69}$$

where B_3, w, μ are arbitrary constants.

Remark 3.2.

1. Comparing the results in this paper concerning Eq. (28) with the results in Khater et al. [39], Mei et al. [40], show that the results given in this article are new and inclusive. Furthermore, by taking appropriate values for the parameters, we achieve similar solutions.
2. Further a substantial feature, that Riccati–Bernoulli sub-ODE technique gives infinite solutions.
3. Therefore, this method is powerful and sufficient for solving such types NPDEs.

Here 3D figures for some solutions have been presented, specifically Figs. 1–7.

It is remarkable that our new outcomes may be utilized in several practical domains such as describing wave phenomena, optics, plasma, fluid mechanics, ocean engineering, and so on. The soliton type solutions and wave solutions play important role in the study of surface water waves which has various applications such as coastal engineering and ship building to wave forecasting. Traveling wave solutions which include in some of our obtained results are applicable in the most of ocean engineering field such as ship waves on water, ocean waves from storms, traveling waves and their breaking on beaches and so on [41–45].

4. Conclusions

To search for exact solutions of the reaction-diffusion equation, three strong techniques have been effectively implemented. As a consequence, different fresh solutions for traveling waves are acquired. For physicists, these solutions are so essential to explain some physical phenomena. Indeed these methods are not only efficient and robust but also have the merit of being widely applicable. Therefore, these m can be introduced to additional nonlinear partial differential equations and this will be achieved elsewhere.

Declaration of Competing Interest

There are no political, personal, religious, ideological, academic, and intellectual competing interests. The authors declare that they have no competing interests.

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