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Homework 5

1. $P \Rightarrow \neg Q$, $Q \Rightarrow \neg P$

P	Q	$(\neg P \vee \neg Q)$ $P \Rightarrow \neg Q$	$(\neg Q \vee \neg P)$ $Q \Rightarrow \neg P$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

Since the truth table shows the same values for both sentences for all values of P and Q, the sentences are equivalent.

$P \Leftrightarrow \neg Q$, $((P \wedge \neg Q) \vee (\neg P \wedge Q))$

P	Q	$(\neg P \vee \neg Q)$ $P \Rightarrow \neg Q$	$(Q \vee P)$ $\neg Q \Rightarrow P$	$P \Leftrightarrow \neg Q$	$P \wedge \neg Q$	$\neg P \wedge Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
T	T	F	T	F	F	F	F
T	F	T	T	T	T	F	T
F	T	T	T	T	F	T	T
F	F	T	F	F	F	F	F

The truth table shows that the sentences are equivalent.

2. • $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$

Smoke	Fire	$(\neg Smoke \vee Fire)$ $Smoke \Rightarrow Fire$	$(Smoke \vee \neg Fire)$ $\neg Smoke \Rightarrow \neg Fire$	$(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

This sentence is neither valid nor unsatisfiable since it holds in some worlds but not others.

- $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \vee Heat) \Rightarrow Fire)$

$(\neg Smoke \vee Fire)$ $(\neg(Smoke \vee Heat) \vee Fire)$

Smoke	Fire	Heat	Smoke ⇒ Fire	(Smoke ∨ Heat ⇒ Fire)	(Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	F	F
F	F	F	T	T	T

This sentence is neither valid nor unsatisfiable since it holds in some worlds but not others.

- $((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$

$(\neg(S \wedge H) \vee F)$ $(\neg S \vee F)$ $(\neg H \vee F)$

S	F	H	(S∧H)⇒F	S⇒F	H⇒F	(S⇒F)∨(H⇒F)	((S∧H)⇒F)⇔((S⇒F)∨(H⇒F))
T	T	T	T	T	T	T	T
T	T	F	F	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Since this sentence holds for all worlds, it is valid.

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Homework 5

3. a) Abbreviations: My - mythical, Mo - mortal, Ma - mammal, Mag - magical
H - horned.

Knowledge base: 1. $My \Rightarrow \neg Mo$
2. $\neg My \Rightarrow Mo \wedge Ma$
3. $(\neg Mo \vee Ma) \Rightarrow H$
4. $H \Rightarrow Mag$

b) 1. $\neg My \vee \neg Mo$
2. $My \vee (Mo \wedge Ma) \rightarrow (My \vee Mo) \wedge (My \vee Ma)$
3. $\neg(\neg Mo \vee Ma) \vee H \rightarrow (Mo \wedge \neg Ma) \vee H \rightarrow (Mo \vee H) \wedge (\neg Ma \vee H)$
4. $\neg H \vee Mag$

CNF: $(\neg My \vee \neg Mo) \wedge (My \vee Mo) \wedge (My \vee Ma) \wedge (Mo \vee H) \wedge (\neg Ma \vee H) \wedge (\neg H \vee Mag)$

c) We start by listing all the clauses in the CNF.
1. $\neg My \vee \neg Mo$
2. $My \vee Mo$
3. $My \vee Ma$
4. $Mo \vee H$
5. $\neg Ma \vee H$
6. $\neg H \vee Mag$

Then we can use resolution while negating horned, magical, and mythical.
7. $\neg Mo \vee Ma$ (1,3)
8. $\neg Mo \vee H$ (5,7)
9. H (4,8)
10. Mag (6,9)
11. $\neg H$ (assuming negation of horned)
12. $\neg Mag$ (assuming negation of magical)
13. empty clause (9,11)
14. empty clause (10,12)
15. $\neg My$ (assuming negation of mythical)
16. Mo (2,15)
17. Ma (3,15)

The two contradictions show that the unicorn is magical and horned. Since no contradiction can be found for ¬My, whether the unicorn is mythical cannot be proved.