Calculate:

$$L = \lim_{x\to 0}\left(\frac{\sqrt{2}-\sqrt{1+\cos(7x)}}{1-\cos(5x)}\right)$$

Solution

$$L = \lim_{x\to 0}\left(\left(\frac{\sqrt{2}-\sqrt{1+\cos(7x)}}{1-\cos(5x)}\right)\cdot\left(\frac{\sqrt{2}+\sqrt{1+\cos(7x)}}{\sqrt{2}+\sqrt{1+\cos(7x)}}\right)\right)$$

$$L = \lim_{x\to 0}\left(\frac{1-\cos(7x)}{(1-\cos(5x))\cdot\left(\sqrt{2}+\sqrt{1+\cos(7x)}\right)}\right)$$

$$L = \lim_{x\to 0}\left(\frac{1}{\sqrt{2}+\sqrt{1+\cos(7x)}}\right)\cdot\lim_{x\to 0}\left(\frac{1-\cos(7x)}{1-\cos(5x)}\right)$$

$$L = \frac{1}{2\sqrt{2}}\cdot\lim_{x\to 0}\left(\frac{1-\cos(7x)}{1-\cos(5x)}\right) = \frac{1}{2\sqrt{2}}\cdot\frac{\lim\limits_{x\to 0}(1-\cos(7x))}{\lim\limits_{x\to 0}(1-\cos(5x))}$$

$$L = \frac{1}{2\sqrt{2}}\cdot\frac{49\lim\limits_{x\to 0}\left(\frac{1-\cos(7x)}{(7x)^2}\right)}{25\lim\limits_{x\to 0}\left(\frac{1-\cos(7x)}{(5x)^2}\right)} = \frac{49}{50\sqrt{2}}$$

Where

$$\lim_{x\to 0}\left(\frac{1-\cos(7x)}{(7x)^2}\right) = 1$$

$$\lim_{x\to 0}\left(\frac{1-\cos(7x)}{(7x)^2}\right) = 1$$

Therefore

$$\therefore\quad L = \lim_{x\to 0}\left(\frac{\sqrt{2}-\sqrt{1+\cos(7x)}}{1-\cos(5x)}\right) = \frac{49}{50\sqrt{2}}$$