C.3. Continuous setting.

Consider the following generative model:

$$b_k \sim \text{Bern}(p), \tag{62}$$

$$\lambda_k \mid b_k \sim \begin{cases} \delta_0 & \text{if } b_k = 0 \\ g(\cdot) & \text{otherwise} \end{cases} \tag{63}$$

$$y_k \mid b_k, \lambda_k, c_k \sim \begin{cases} \text{Unif}(a_0, b_0) & \text{if } b_k = 0 \\ F_{c_k \lambda_k}(\cdot) & \text{otherwise.} \end{cases} \tag{64}$$

After applying the convolution-closed augmentation scheme, the complete conditional for $b_k \mid (a < y_k < b)$ becomes

$$P(b_k = 1 \mid \bar{y}_k, \bar{c}) = \frac{P(b_k = 1)P(\bar{y}_k \mid \bar{c}, b_k = 1)}{P(b_k = 1)P(\bar{y}_k \mid \bar{c}, b_k = 1) + P(b_k = 0)\text{Unif}(\bar{y}_k; a_0, b_0)} \tag{65}$$

$$= \frac{p \int F_{\bar{c}\lambda_k}(\bar{y}_k) g(\lambda_k) d\lambda_k}{p \int F_{\bar{c}\lambda_k}(\bar{y}_k) g(\lambda_k) d\lambda_k + (1-p)\frac{1}{b-a}} \tag{66}$$

$$= \frac{p f(\bar{c}, \bar{y}_k)}{p f(\bar{c}, \bar{y}_k) + (1-p)\frac{1}{b-a}}. \tag{67}$$

Here, $f(\bar{c}, \bar{y}_k) = \int F_{\bar{c}\lambda_k}(\bar{y}_k) g(\lambda_k) d\lambda_k$ is the pdf of the marginal of $\bar{y}_k$. If $f(\bar{c}, \bar{y}_k) \approx f(\bar{c}, \bar{y}_{k'})$ for $\bar{y}_k, \bar{y}_{k'} \in (a, b)$, $P(b_k = 1 \mid \bar{y}_k, \bar{c}) \approx P(b_{k'} = 1 \mid \bar{y}_{k'}, \bar{c})$. Assuming equality, we can sample from the complete conditionals jointly, as done in the discrete case, where we first sampling a binomial, and then select the k uniformly at random without replacement.

As an example, suppose the marginal distribution $N(0, \sigma^2)$, such that $f(\bar{c}, \bar{y}_k) = \frac{1}{\sqrt{2\pi}\sigma} \exp(\frac{-\bar{y}_k^2}{2\sigma^2})$, $a_0 = -a, b_0 = a$. Then $P(b_k = 1 \mid \bar{y}_k, \bar{c})$ achieves a maximum at $\bar{y}_k = 0$ and a minimum at the endpoints, $\bar{y}_k = \pm a$. However, the difference (max - min) is quite small. For example, $a = 0.1, \sigma = 1, p = 0.5$, the difference is $0.0739 - 0.0736 = 0.0003$, and we should be able to derive a formula from the closed form expressions.