

ON THE NATURE OF DYNAMICAL INSTABILITIES IN INTERACTING BINARY SYSTEMS

By Jacob Hooey

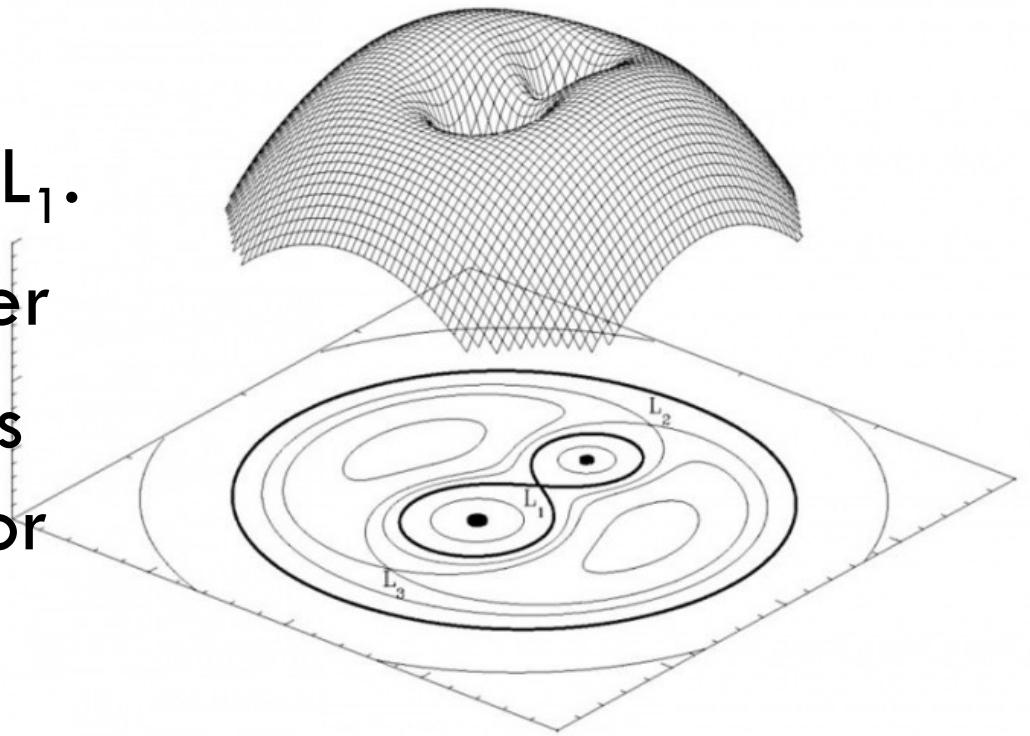
Cataclysmic Variables

Why Study Stability Conditions?

- Because of Type Ia Supernovae (Ia SNe)
- Cataclysmic Variables for which the mass of the white dwarf accretor had been pushed beyond the Chandrasekhar Limit of 1.4 solar masses
- Classical Novae on the surface of the white dwarf accretor
- Supersoft X-ray Sources (SSXSs) - the source of the X-rays was due to a steady nuclear burning on the surface of a white dwarf star

Roche Lobe Geometry

- The Roche Lobe is defined by the critical equipotential surface that intersects at the inner Lagrange point L_1 .
- $R_2 < R_L$ – no mass transfer
- $R_2 \text{ approx. } > R_L$ – mass is transferred from the donor to the WD.



Eddington Limit

- For a particle (m) in a radiative and gravitational field, the Eddington Luminosity is defined as the point at which the gravitational force inwards equals the radiative force outwards.
- This corresponds to the point when the accretion flow inwards would be completely choked off by the radiative pressure outwards.
- In our case we will be using a WD of 1.4 solar masses as our limiting case
- Which has a approximate radius of $R=10^3\text{km}$.
- Therefore the resultant Eddington limit is approximately

$$\dot{M} = 1.5 \times 10^{-6} M_{\odot} \text{yr}^{-1}$$

Analytic Stability Solution

- This is the analytic formula that demonstrates the relation between angular momentum loss and mass loss (I derived this in an earlier lecture, and I don't have time to address the derivation today).

$$\frac{\dot{m}_2}{m_2} = \frac{\left(\frac{\dot{J}}{J}\right)_{AML}}{\left[\frac{5}{6} - q\beta - \left(f + \frac{1}{3}\right)\left(\frac{q}{q+1}\right)(1-\beta) + \frac{\xi}{2}\right]}$$

Where,

$$q = \frac{m_2}{m_1} \quad \text{and} \quad \beta = -\frac{\dot{m}_1}{\dot{m}_2}$$

We want to analyze the situation where $\frac{\dot{m}_2}{m_2}$ is undefined.
i.e. where the denominator = 0

$$\left[\frac{5}{6} - q\beta - \left(f + \frac{1}{3}\right)\left(\frac{q}{q+1}\right)(1-\beta) + \frac{\xi}{2}\right] = 0$$

We will consider the case with zero mass conservation

$$\beta = 0$$

Analytic Stability Solution

- To find f I use a previously derived formula for

$$\left(\frac{\dot{J}}{J}\right)_{MT} = \left(\frac{M_2}{M_1}\right) \left(\frac{\dot{M}_T}{M_T}\right) = q \left(\frac{\dot{M}_T}{M_T}\right)$$

- Then by taking my definition of $\left(\frac{\dot{J}}{J}\right)_{MT} = f \left(\frac{\dot{M}_T}{M_T}\right)$
- Therefore $f = q_{cr}$
- We will consider the case of a fully convective star, therefore we have an stellar index $\xi = -1/3$
- Therefore

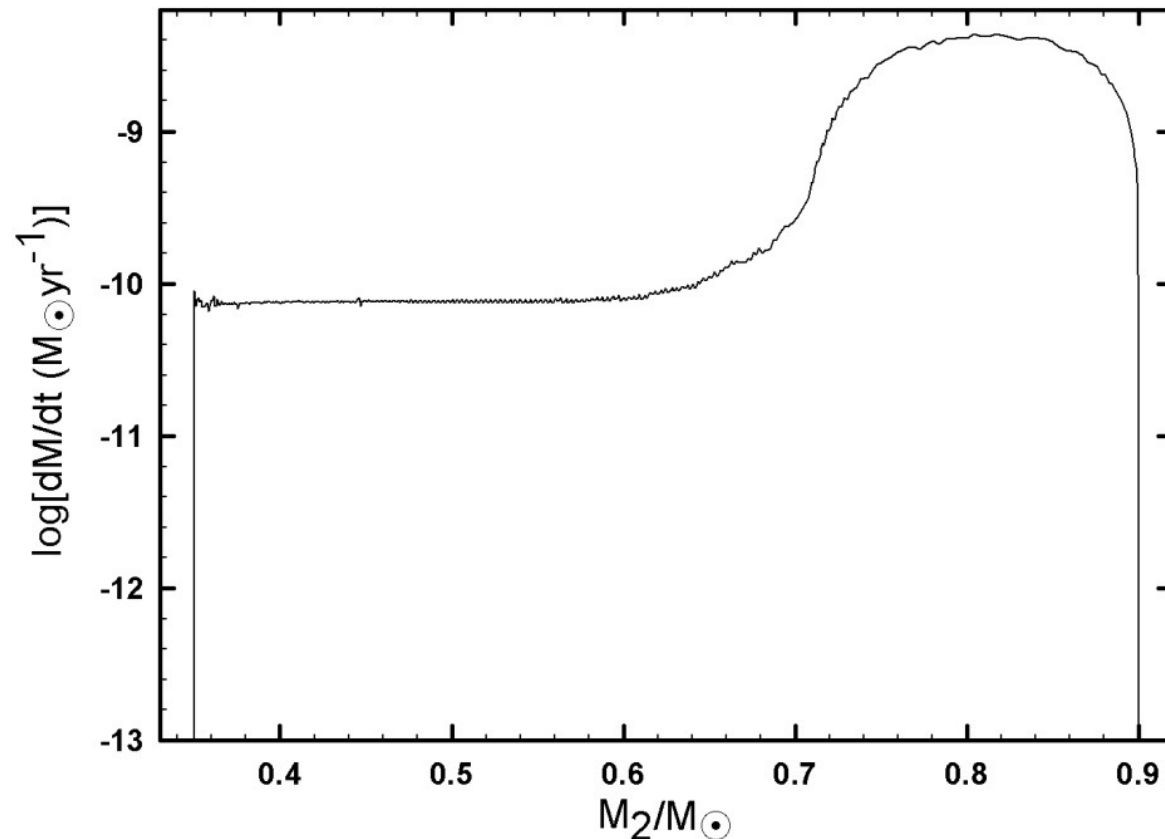
$$q_{cr} = 1$$

Stable Mass Transfer

- Stable mass transfer is the case where the donor transfers mass steadily until its evolution is complete.

$$M_1/M_\odot = 0.50, \quad M_{2,0}/M_\odot = 0.90$$

$\log(dM/dt)$ vs M_2/M_\odot

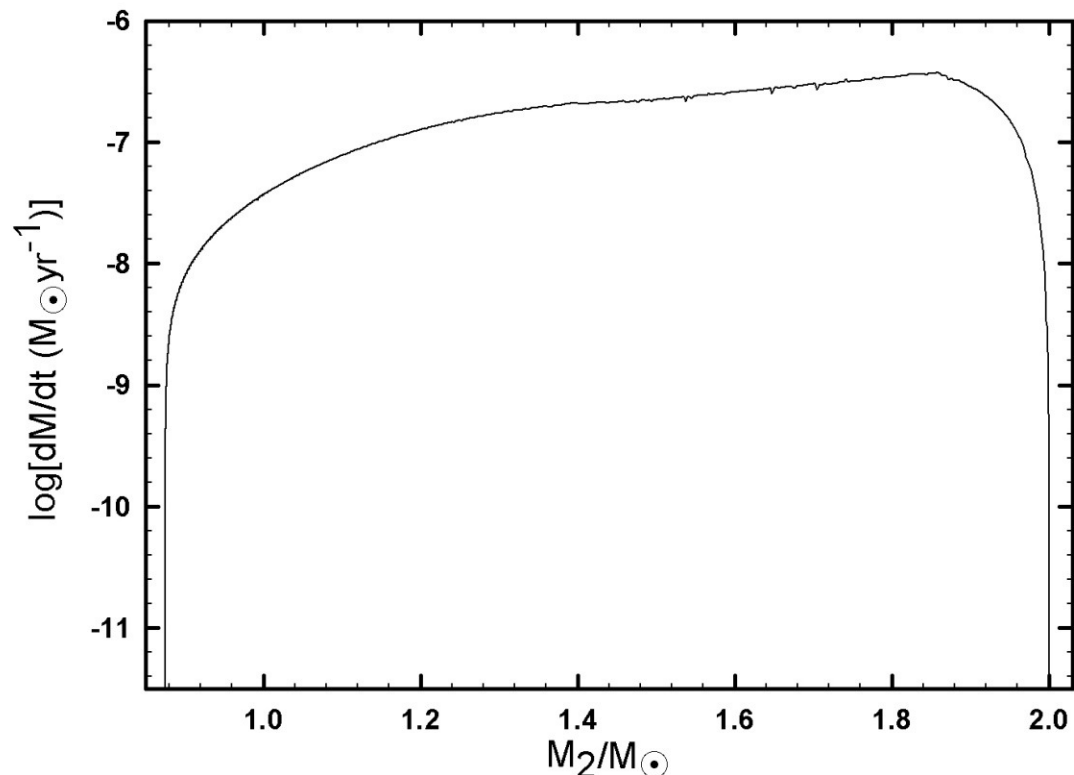


Thermal Timescale Mass Transfer (TTMT)

- Maintains a mass loss rate in the approximate range $10^{-8} \leq \dot{M}_2 < 10^{-6}$ for a length of time that allows at least a mass loss of $\approx 0.1M_{\odot}$

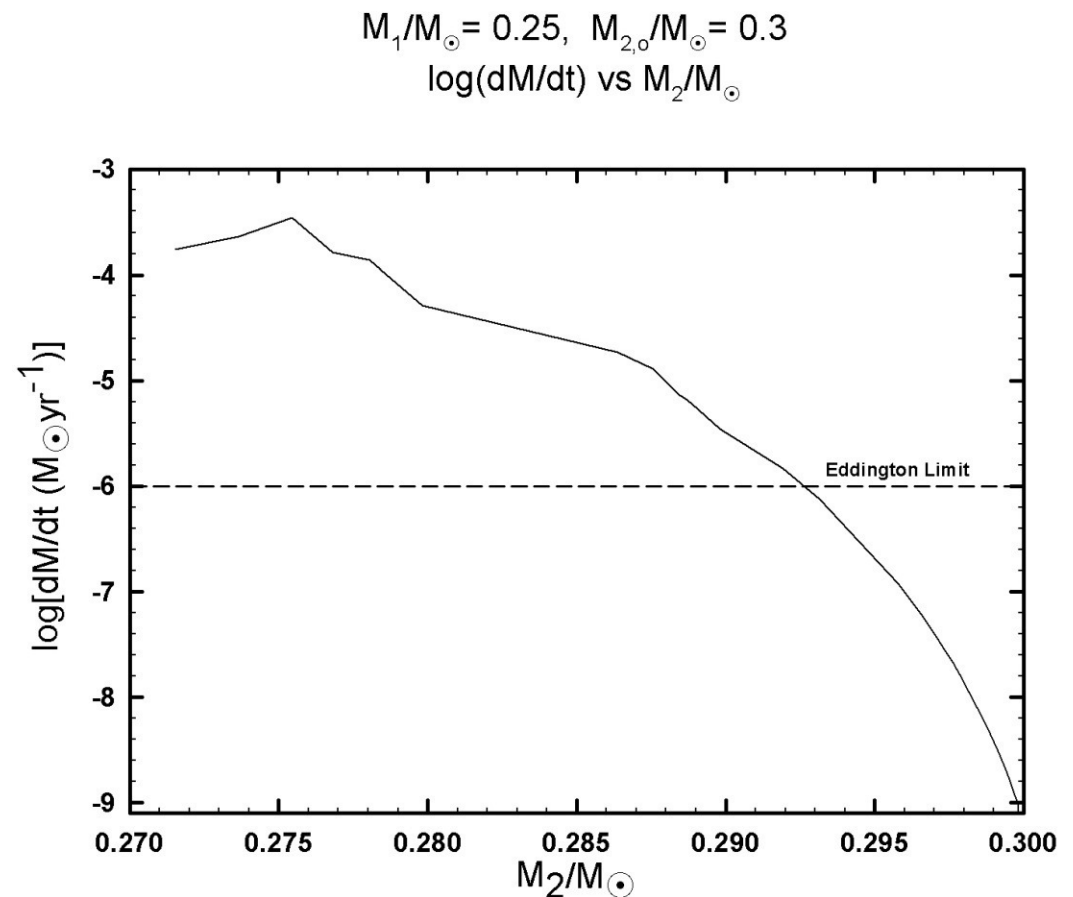
- The time during which a system exhibits a very high rate of mass transfer is referred to as a Kelvin-Helmholtz or thermal timescale.

$M_1/M_{\odot} = 0.85$, $M_{2,0}/M_{\odot} = 2.00$
 $\log(dM/dt)$ vs M_2/M_{\odot}



Dynamical Instability (DI)

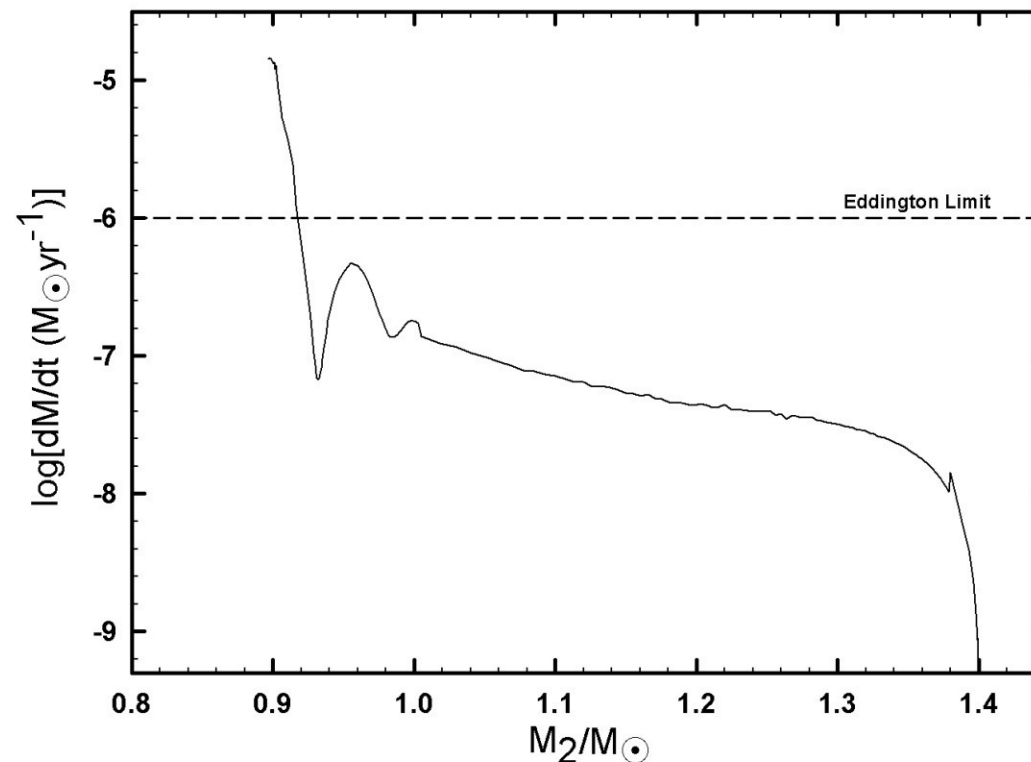
- Dynamical instability (DI) occurs early on in the CVs evolution
- The mass transfer rate increases rapidly and reaches the Eddington Limit almost immediately
- As the plot shows, the system reaches the Eddington limit before the donor was able to lose more than of its mass



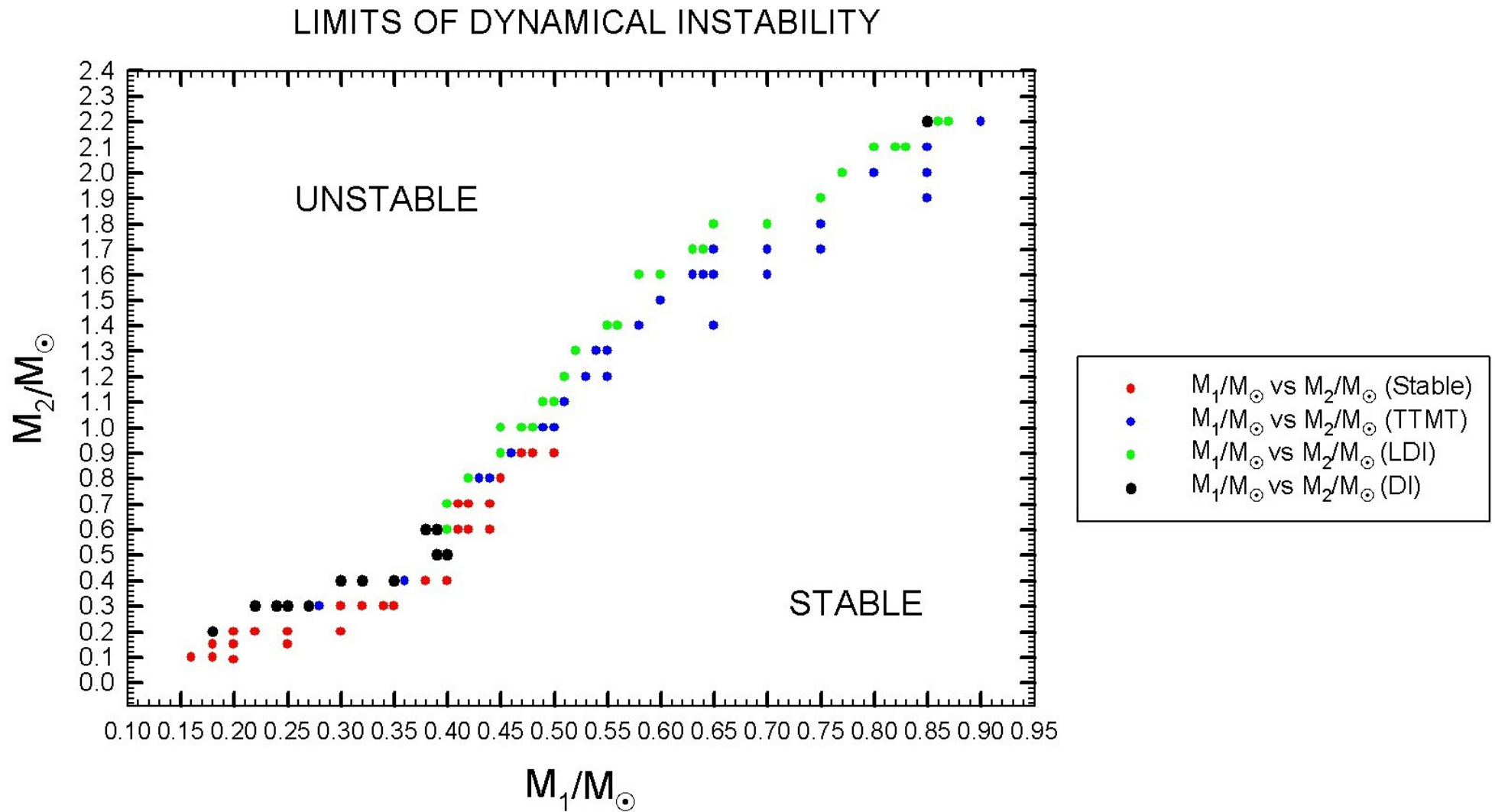
Latent Dynamical Instability (LDI)

- The system reaches the Eddington limit after a long steady high mass loss period

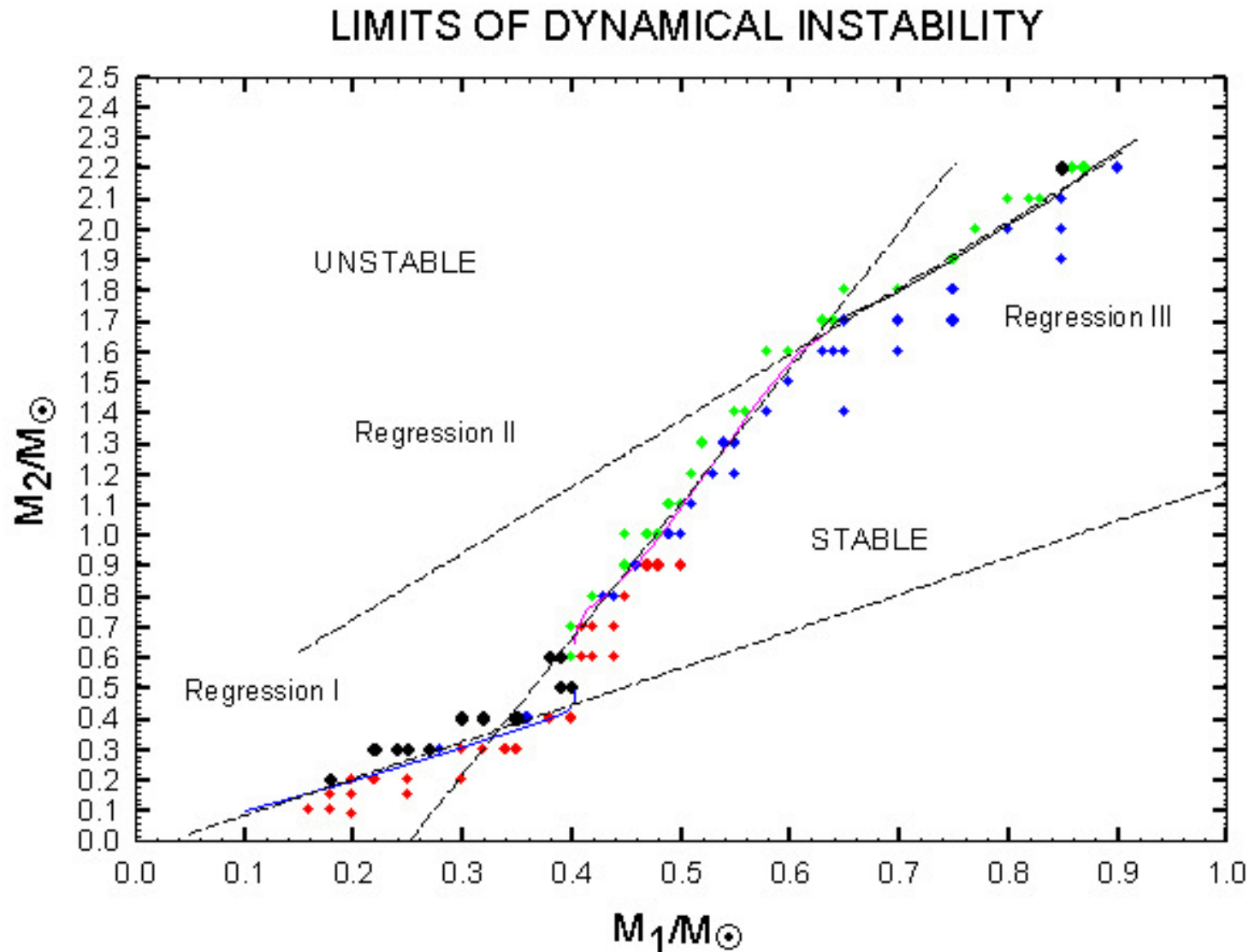
$M_1/M_\odot = 0.55$, $M_{2,0}/M_\odot = 1.40$
 $\log(dM/dt)$ vs M_2/M_\odot



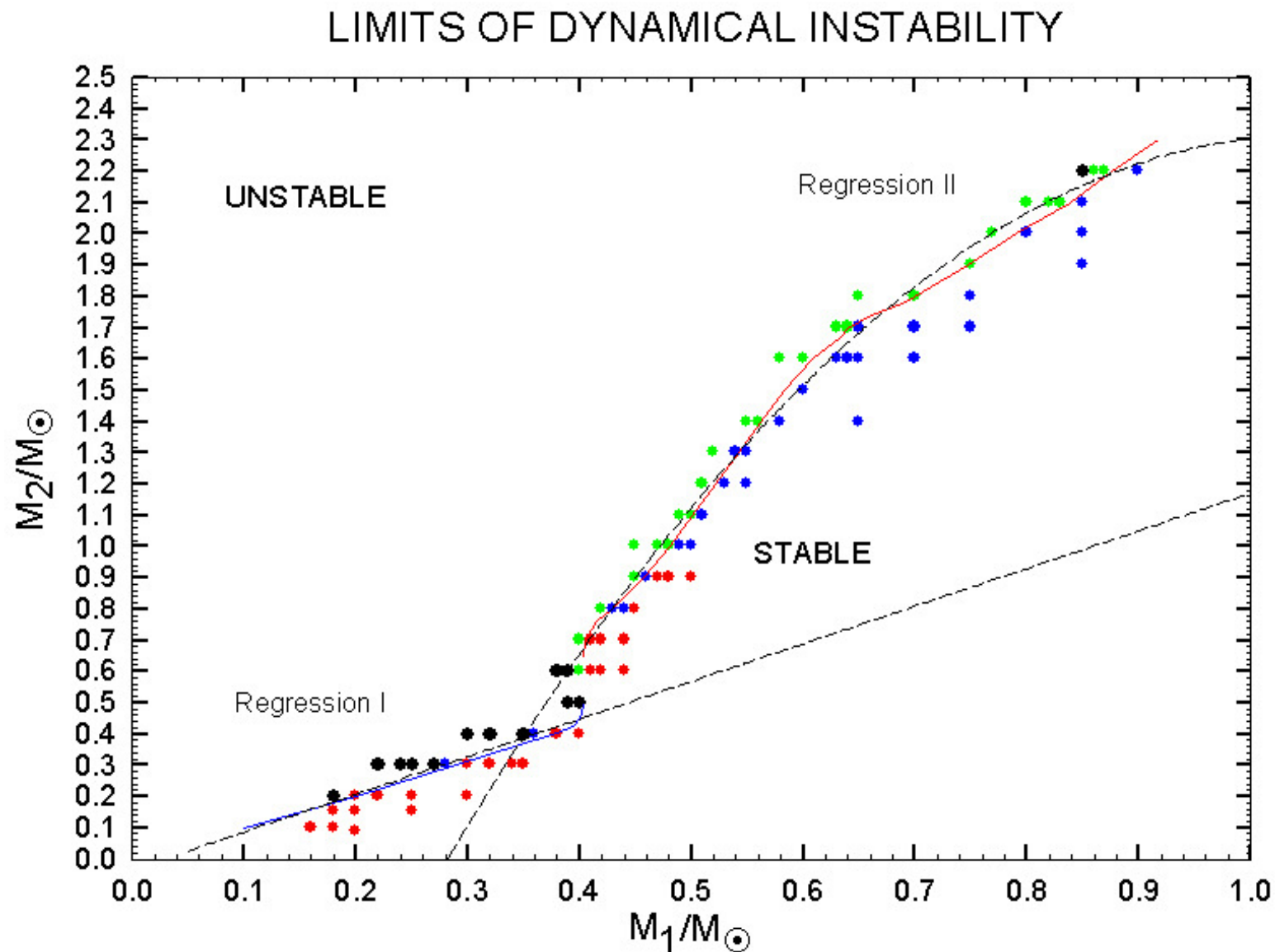
Computed CV Tracks



Three Part Linear Regression



Linear and Quadratic Regression Combination



Results for $q=M_2/M_1$

Three Part Linear Regression Results

$$\square q_{cr} \begin{cases} 1.20 & \text{for } 0.16M_{\odot} \leq M_1 < 0.40M_{\odot} \\ 4.43 & \text{for } 0.40M_{\odot} < M_1 \leq 0.62M_{\odot} \\ 2.17 & \text{for } 0.62M_{\odot} \leq M_1 \leq 0.90M_{\odot} \end{cases}$$

Linear and Quadratic Fit

$$\square q_{cr} = 1.20 \quad \text{for } 0.16M_{\odot} \leq M_1 < 0.40M_{\odot}$$

$$\square q_{cr} = 8.22 - \frac{2.01M_{\odot}}{M_1} - 3.91\frac{M_1}{M_{\odot}} \quad \text{for } 0.40M_{\odot} < M_1 \leq 0.90M_{\odot}$$