Mass Transfer in Binary Star Systems

Dynamical Instability

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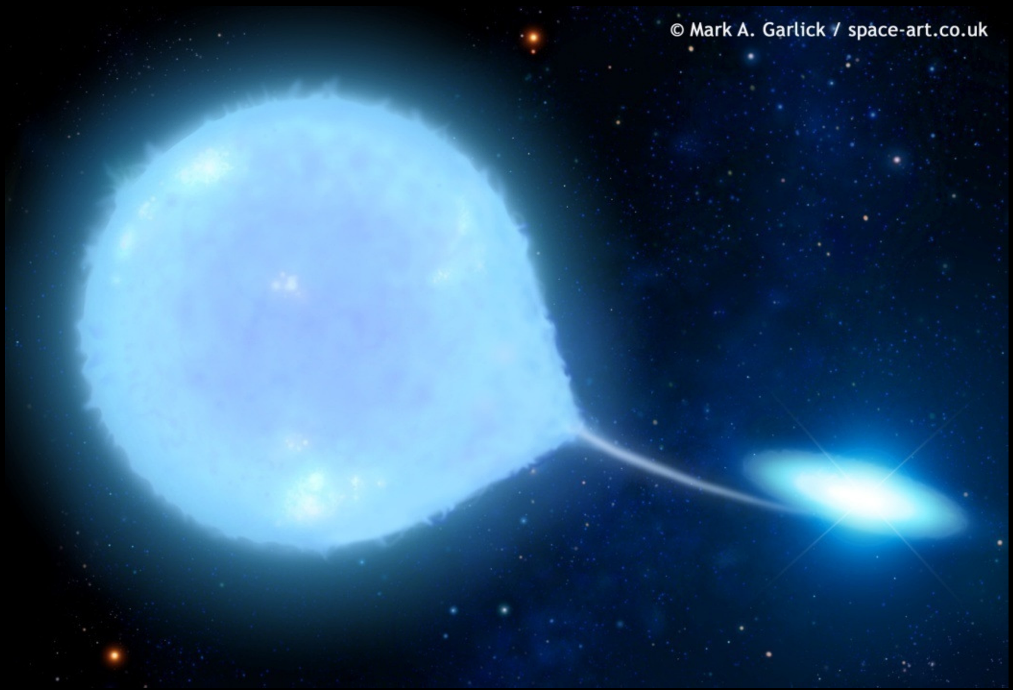
What Binary System is being studied?

- Cataclysmic Variables (CVs)
- Meaning that the binary system contains a compact object, in our case, a white dwarf.



- Also, when dealing with CVs, mass transfer must be present

Thesis Objective

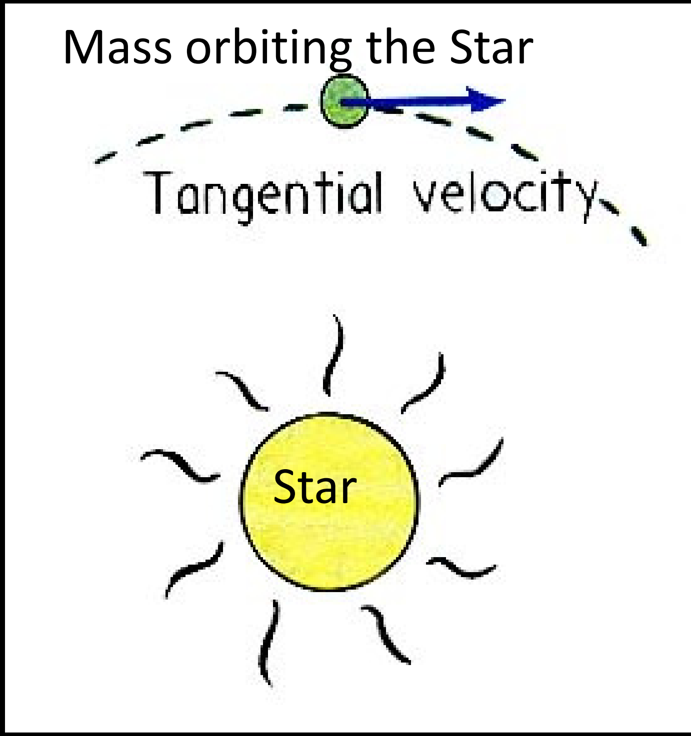
- To understand the boundary conditions under which mass transfer within a binary star system is stable.
- These boundary conditions are defined by three variables:
    1. M_1 = Mass of the Primary Star
    2. M_2 = Mass of the Secondary Star
    3. Evolutionary State (Age)

Assumptions

1. $R_2=R_L$ where R_2 = radius of the larger Star
   and R_L = Roche Lobe Radius
2. The binary system's orbits are approximately circular
3. The white dwarf is considered to be centrally condensed.
4. The response of a star is hydrostatic

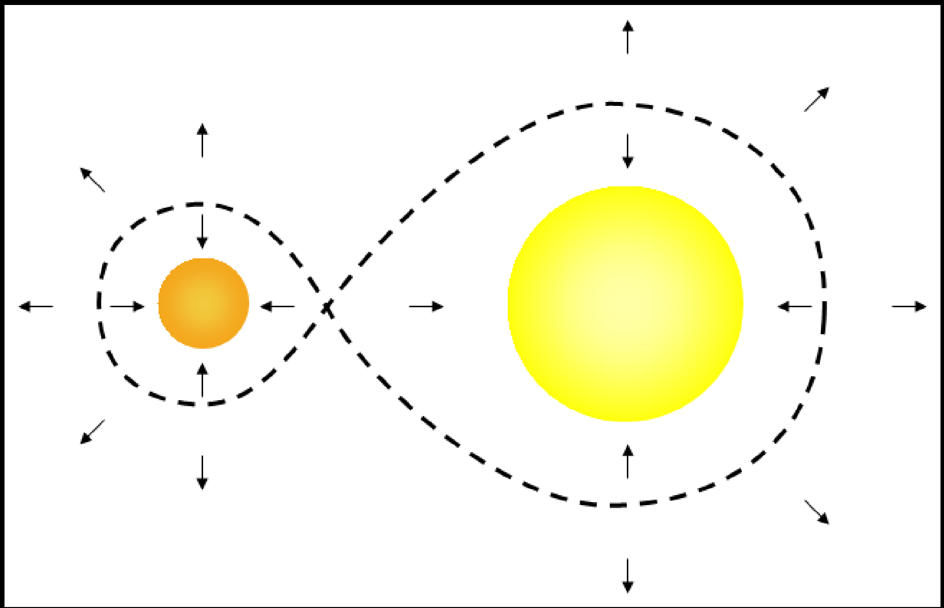
Binary Stars Roche Lobe

- When binary stars rotate, mass has a tendency to be pulled away from the axis of rotation.
- This is due to the inertia of the particles, which dictates that the particles should continue to move in a straight line.
- In a binary system, the effect becomes stronger the greater the distance from the center of motion.



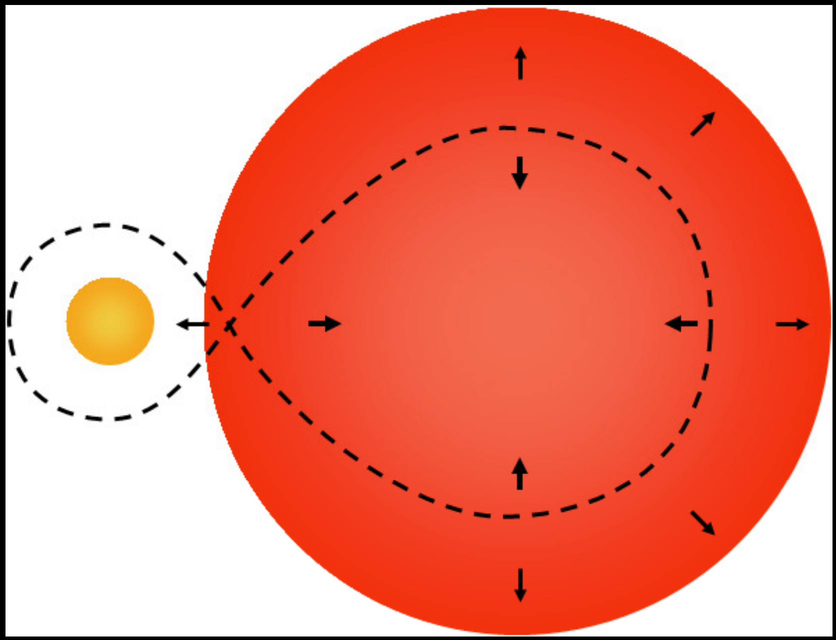
Roche Lobe

- If we calculate the distance where the stars' gravity balances this imaginary "centrifugal" force, we find a boundary like a figure-eight surrounding the stars called its Roche Lobe.
- Edouard Roche was the first to discover this effect in the late 1800's
- Inside the Roche lobes, matter feels a force inward toward the center of each star. Outside the lobes, matter flies away and is lost from the system.



Evolving Binary Stars

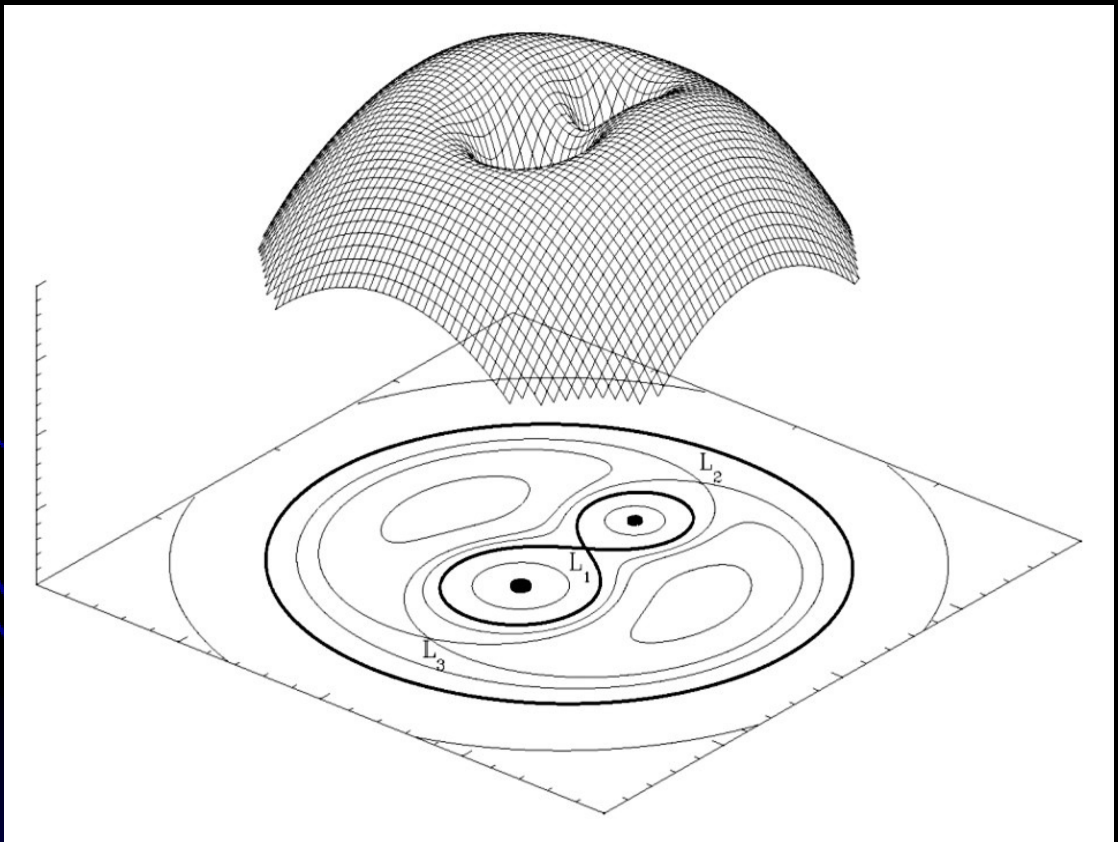
- When the main sequence star evolves into a red giant and fills its Roche lobe



- The part of the atmosphere outside the Roche lobe blows away.
- The part of the atmosphere that is now inside the other stars Roche lobe falls onto the other star.

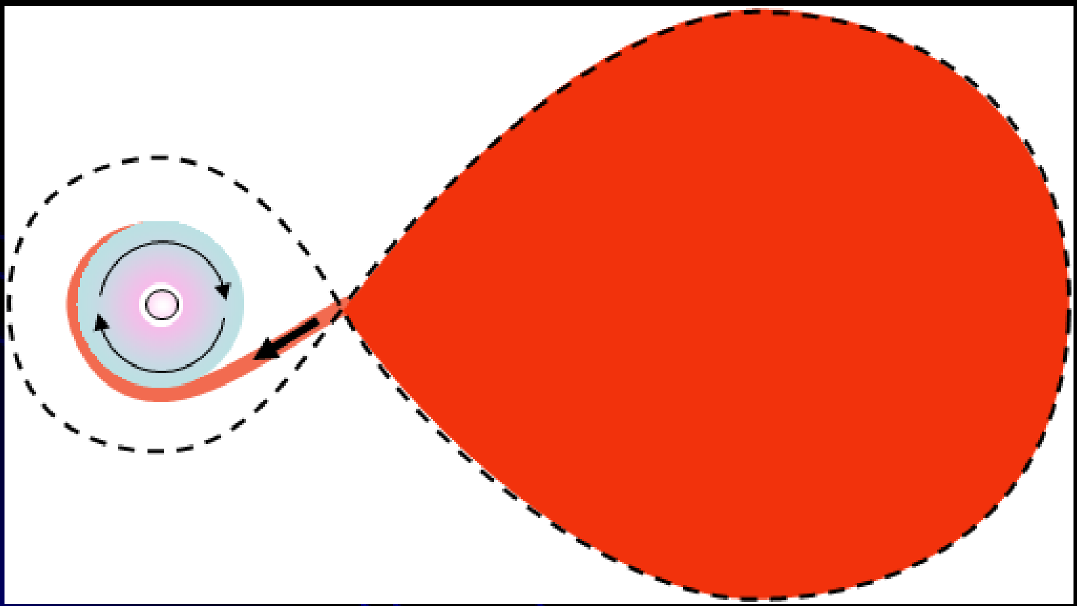
Roche lobes and Lagrange points

- L1, L2 and L3 are the Lagrangian points where forces cancel out.
- Mass can flow through the saddle point L1 from one star to its companion, if the star fills its Roche lobe.



$R_2 = R_L$

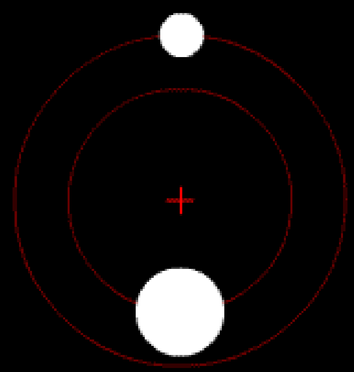
- The size of the expanding giant star is limited by the presence of the companion star.
- The larger star now transfers mass to the smaller star.



- The mass accreting star is a centrally condensed white dwarf
- The white dwarf receives a stream of hydrogen from the red giant and forms an accretion disk

Circular Orbits

- Implies that the orbital separation (A) is approximately constant over 1 complete orbit.
- Therefore the approximate analytical formula for the Roche Lobe depends only on the ratio of both stars masses and the Orbital Separation.



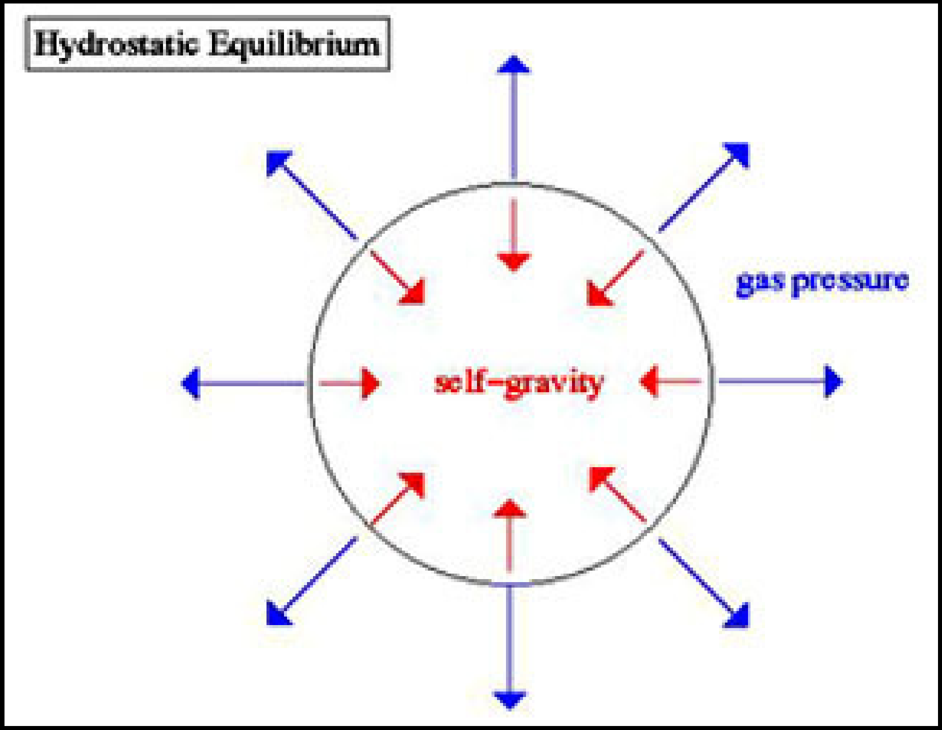
$$R_L \approx 0.46A\left(\frac{m_2}{m_1+m_2}\right)^{1/3} = 0.46A\left(\frac{m_2}{m_T}\right)^{1/3}$$

Centrally Condensed White Dwarf

- Implies that the effects of tidal forces can be ignored.
- When a body rotates while subject to tidal forces, internal friction results in the gradual dissipation of its rotational kinetic energy as heat.
- We can also consider it to be a gravitational point source.

Hydrostatics

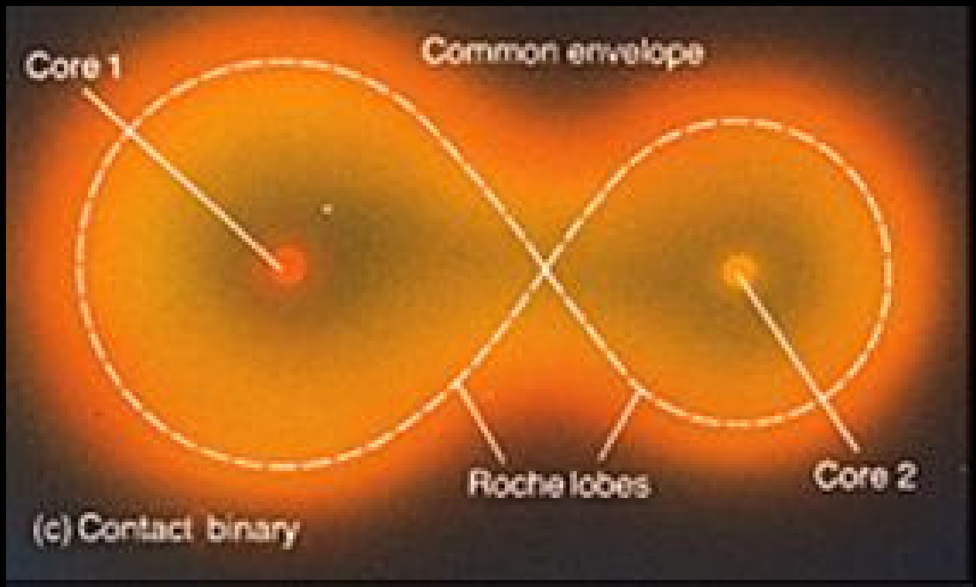
- Is the science of fluids at rest
- All main sequence stars are in Hydrostatic Equilibrium.
- Hydrostatic equilibrium is the reason stars don't implode, or explode.
- This means that the thermal pressure in the core is balanced by the pressure from the overlying mass.



Eddington Limit

- The theoretical limit at which the radiation pressure of a light-emitting body would exceed the body's gravitational attraction.
- A star emitting radiation at greater than the Eddington limit would break up.

$$L = 4\pi \frac{GMm_P c}{\sigma_T}$$



G is the gravitational constant,
M is the mass of the central object,
m_p is the mass of a proton,
c = speed of light,
σ_T is the effective area

Angular Momentum

- The equation for the instantaneous orbital angular momentum in a synchronized binary system with circular orbit about its center of mass is,

$$J^2 = G\frac{(m_1 \cdot m_2)^2}{(m_1 + m_2)}a = G\frac{(m_1^2 \cdot m_2^2)}{m_T}a$$

J = angular momentum

G = gravitational constant

m_1 = white dwarf (primary)

m_2 = secondary star

a = orbital separation

Working with Rates of Change

- From the equation of angular momentum, we get,

$$2\frac{\dot{J}}{J} = 2\frac{\dot{m}_1}{m_1} + 2\frac{\dot{m}_2}{m_2} + \frac{\dot{a}}{a} - \frac{\dot{m}_T}{m_T}$$

- From the equation of the Roche lobe

$$R_L = R_2 = 0.46a\left(\frac{m_2}{m_T}\right)^{1/3}$$

- We get,

$$\frac{\dot{a}}{a} = \frac{\dot{R}_2}{R_2} - \frac{1}{3}\frac{\dot{m}_2}{m_2} + \frac{1}{3}\frac{\dot{m}_1}{m_1}$$

Reorganizing the Equations

- We get, $$2\frac{\dot{J}}{J} = 2\frac{\dot{m}_1}{m_1} + \frac{5}{3}\frac{\dot{m}_2}{m_2} - \frac{2}{3}\frac{\dot{m}_T}{m_T} + \frac{\dot{R}_2}{R_2}$$
- Using,

$$\beta = -\frac{\dot{m}_1}{\dot{m}_2} \quad ; \quad \frac{\dot{m}_1}{m_1} = \frac{\dot{m}_2}{m_1}\left(\frac{\dot{m}_1}{\dot{m}_2}\right) = -\frac{\dot{m}_2}{m_1}\beta$$

$$\frac{\dot{m}_T}{m_T} = \frac{\dot{m}_2(1-\beta)}{m_T} \qquad \frac{\dot{R}_2}{R_2} = \xi\frac{\dot{m}_2}{m_2} \quad \xi = \text{adiabatic index}$$

Adiabatic index is the response of the radius to the change in mass

Momentum

$$2\frac{\dot{J}}{J} = 2\frac{\dot{m}_2}{m_2}\left[\frac{5}{6} - \frac{m_2}{m_1}\beta - \frac{1}{3}\frac{m_2}{m_T}(1-\beta) + \frac{\xi}{2}\right]$$

- Now if we consider the total momentum

$$\frac{\dot{J}}{J} = \left(\frac{\dot{J}}{J}\right)_{\text{angular momentum}} + f(m_1, m_2, \beta)\frac{\dot{m}_T}{m_T}$$

- We get,

$$\frac{\dot{m}_2}{m_2} = \frac{\left(\frac{\dot{J}}{J}\right)_{AML}}{\left[\frac{5}{6} - q\beta - \left(f + \frac{1}{3}\right)\left(\frac{q}{q+1}\right)(1-\beta) + \frac{\xi}{2}\right]}$$

- Where

$$q = \frac{m_2}{m_1}$$

Stability Condition q

- We want to analyze the situation where $\dfrac{\dot{m}_2}{m_2}$ is undefined.
- i.e. where the denominator = 0

$$\left[\frac{5}{6} - q\beta - \left(f + \frac{1}{3}\right)\left(\frac{q}{q+1}\right)(1-\beta) + \frac{\xi}{2}\right] = 0$$

- We will consider the special case where $\beta = 1$ (that is all mass lost from secondary is accreted by the primary)
- Therefore, we get

$$q = \frac{2}{3}$$

What I'm working on now

- Numerical Simulations modeling the mass transfer between varying masses of stars.
- These simulations will hopefully help us define the boundary conditions for stability.
- Also to investigate the possibility of stable mass transfer between super massive stars (5 solar masses) that are coupled with white dwarfs.