Signals & Systems

Homework 2

ECE 315 - Fall 2020

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Problem 1

Graph $x[n]$*,*

Equation 1, plot parts a-d by hand

Equation 1

$$x[n] = \begin{cases} \text{x[n]}, & -5 \le n \le 6 \\ 0, & -5 \ge n \ge 6 \end{cases}$$

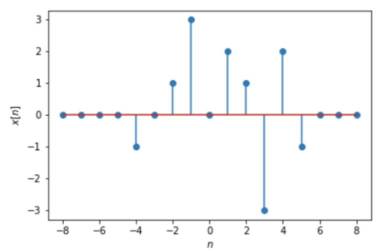


Figure 1) Graph for problem 1, Equation 1.

Part (a)

$$2x[n+3]$$

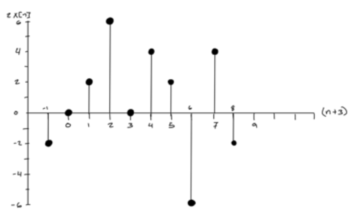


Figure 2

Part (b)

$$\frac{1}{2}x[3n]$$

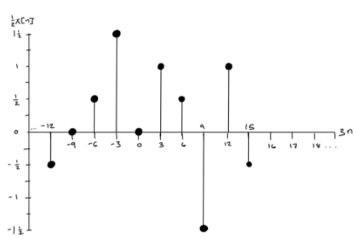


Figure 3

Part (c)

$$x\left[\frac{n}{2}\right]$$

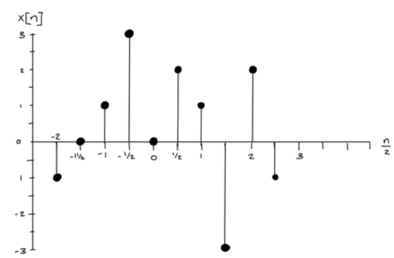
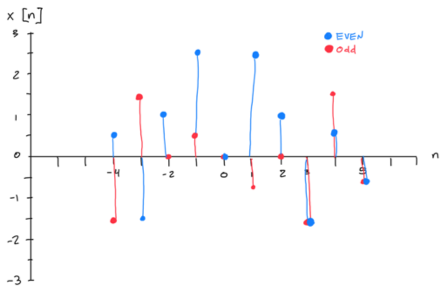


Figure 4

Part (d) plot the even and odd parts of $x[n]$


x [n]
EVEN
Odd
n
3
2
1
0
-1
-2
-3
-4
-2
0
2

Problem 2 MATLAB

Part (a)

(i) **Compute** and **plot;** Equation 2 Equation 3, and their **backward differences**:

Equation 2

$$x_1[n] = \begin{cases} 20n - n^2, & 0 \le n \le 20 \\ 0, & else \end{cases}$$

Equation 3

$$x_2[n] = \begin{cases} 1 + 20n - n^2, & 0 \le n \le 20 \\ 1, & else \end{cases}$$

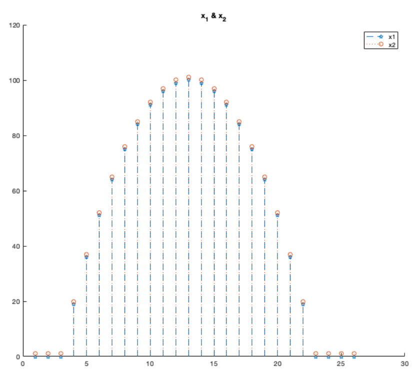


Figure 5 responses of x1 and x2

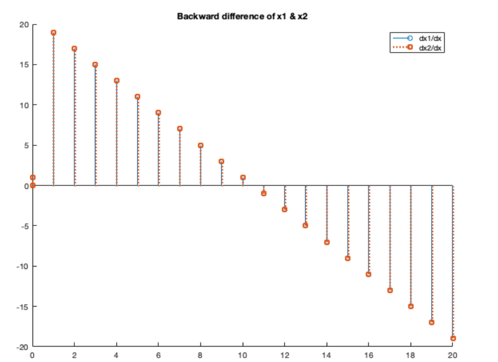


Figure 6 stem plot of backward differences for x1 and x2

(ii) How do the backward differences in Equation 2 and Equation 3 relate?

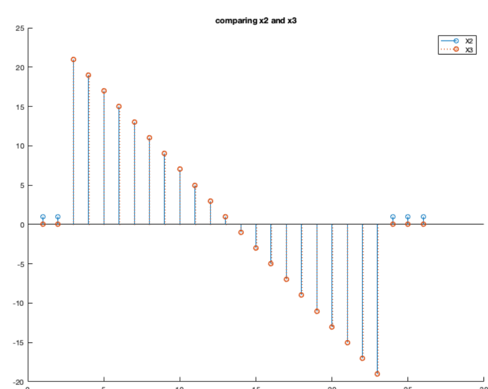


Figure 7

The backward and forward differences in Equation 2 and Equation 3 are identical.

Part (b)

(i) Compute and plot Equation 4 and its backward difference.

Equation 4

$$x_3[n] = \begin{cases} 1 + 20n - n^2, & 0 \leq n \leq 20 \\ 0, & else \end{cases}$$

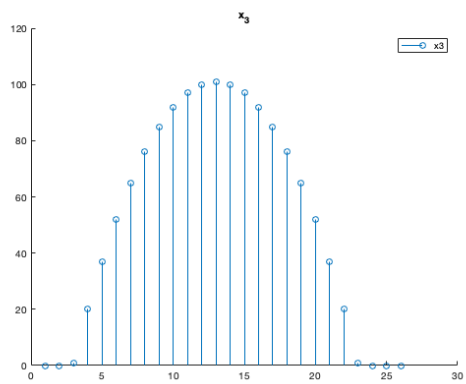


Figure 8 a plot of Equation 4

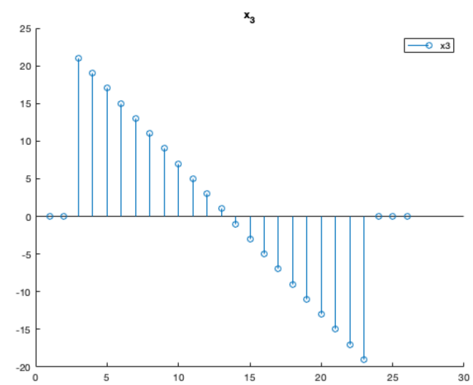


Figure 9 plotted backward difference of Equation 4

(ii) How does the backward difference of signal Equation 4 compare to that of signal Equation 2?

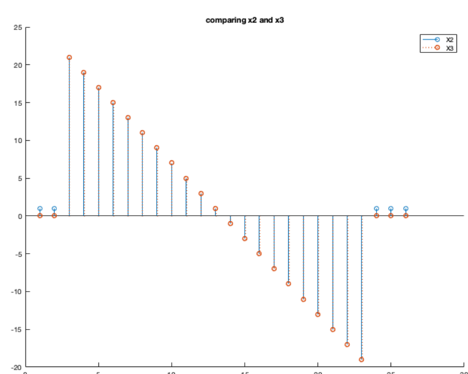


Figure 10 comparison of the Equation 2and Equation 44

The backward difference of equation 2 and equation 4 are all the same.

Part (c)

(i) With Equation 5 compute and plot the accumulation of the backward difference of Equation 3.

Equation 5

$$x[n] = \sum_{m=-\infty}^{n} y[m]$$

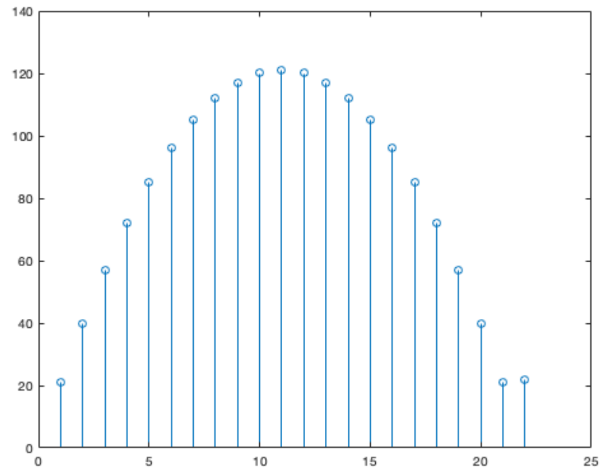


Figure 11 graphs the accumulation of the backward difference of, $x_2[n]$, $y[m] = X_2[n]$.

(ii) Which of the three: Equation 2, Equation 3, Equation 4 does Equation 5 match?

The accumulation of the backward difference of x_2 closely resembles the original function Equation 3 or x_2.

Problem 3

Determine whether the following signals are periodic and plot them using MATLAB.

Part (a)

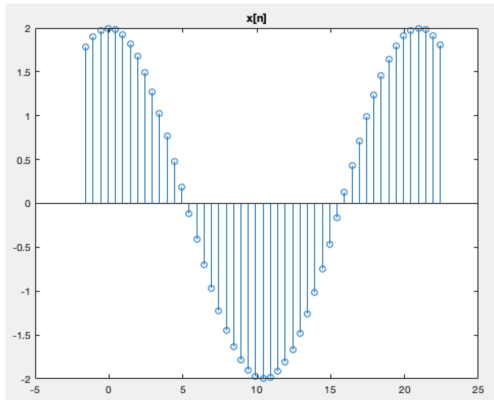
$$x[n] = \underbrace{2\sin\left(\frac{3}{10}n + \frac{\pi}{2}\right)}_{Real}$$

$$= 2\sin\left(\frac{3}{10}\left(\pi(n+T)\right)_{periodicity} + \frac{\pi}{2}\right)$$

$$\boldsymbol{T_0} = \frac{20\pi}{3}s, \qquad \boldsymbol{f_0} = \frac{3}{20\pi}Hz, \qquad \boldsymbol{\omega_0} = 2\pi \bullet \frac{3}{20\pi}\frac{rad}{s}$$

$$T_0 \text{ is not an integer } \therefore x[n] \text{ cant be periodic}$$

This signal is purely **real and a-periodic**.

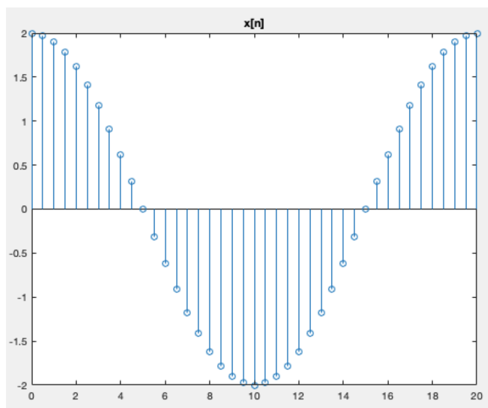


Part (b)

$$x[n] = \underbrace{2\sin\left(\frac{\pi n}{10} + \frac{\pi}{2}\right)}_{Real},$$

$$\boldsymbol{T_0} = 20s = N, \qquad \boldsymbol{f_0} = \frac{1}{20}Hz, \qquad \boldsymbol{\omega_0} = \frac{\pi}{10}\frac{rad}{s}$$

$$\boldsymbol{P_a} = \frac{1}{2N}\sum_{n=20}|x[n]|^2 = \frac{1}{2 \bullet 20}\sum_{n=20}|x[n]|^2 = 2.1W$$



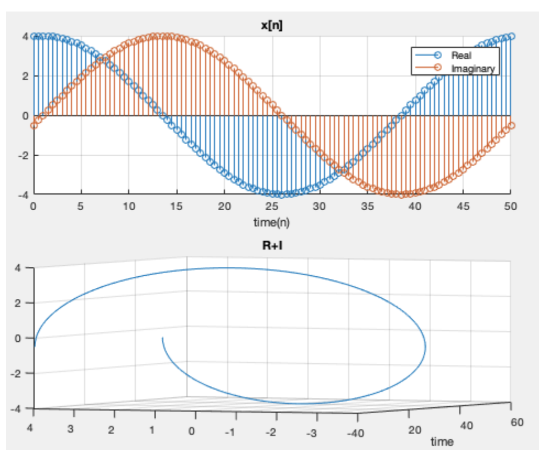
Part (c)

$$[n] = 4e^{i\left(\frac{\pi n}{25} - \frac{\pi}{5}\right)}$$

$$= |4|\left(\underbrace{\cos\left(\frac{\pi n}{25} - \frac{\pi}{5}\right)}_{real} - \underbrace{jsin\left(\frac{\pi n}{25} - \frac{\pi}{5}\right)}_{imaginary}\right)$$

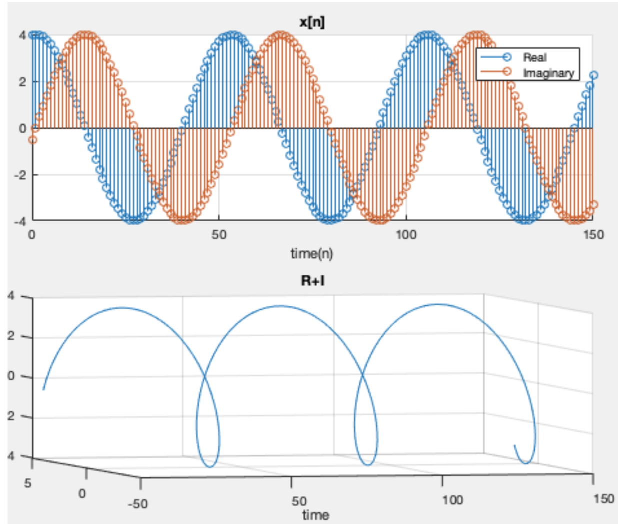
$$\boldsymbol{T_0} = 50s, \qquad \boldsymbol{f_0} = \frac{1}{50}Hz, \qquad \boldsymbol{\omega_0} = \frac{\pi}{25}\frac{rad}{s}$$

$$\boldsymbol{P_a} = 16.16W$$



Part (d)

$$x[n] = 4e^{i\left(\frac{3n}{25}-\frac{\pi}{5}\right)}, \qquad \boldsymbol{a-periodic}$$



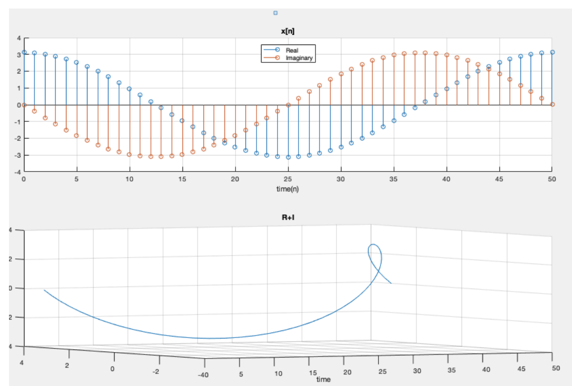
Part (e)

$$x[n] = 4e^{\left(-\frac{1}{4}+\frac{j\pi n}{25}\right)}$$

$$= \left|4e^{-\frac{1}{4}}\right|\left(\underbrace{\cos\left(\frac{\pi n}{25}\right)}_{R} + \underbrace{jsin\left(\frac{\pi n}{25}\right)}_{Im}\right)$$

$$T_0 = 50s, \qquad f_0 = \frac{1}{50}Hz, \qquad \omega_0 = \frac{\pi}{25}\frac{rad}{s},$$

$$P_a = 4.949W$$



Problem 4

Plot

Equation 7

$$x_2[n] = 2\cos\left(-\frac{13\pi n}{7} - \frac{\pi}{4}\right)$$

Equation 7 and **Error! Reference source not found.**:

Equation 6

$$x_1[n] = 2\cos\left(\frac{\pi n}{7} - \frac{\pi}{4}\right)$$

Part 1:

MATLAB plots

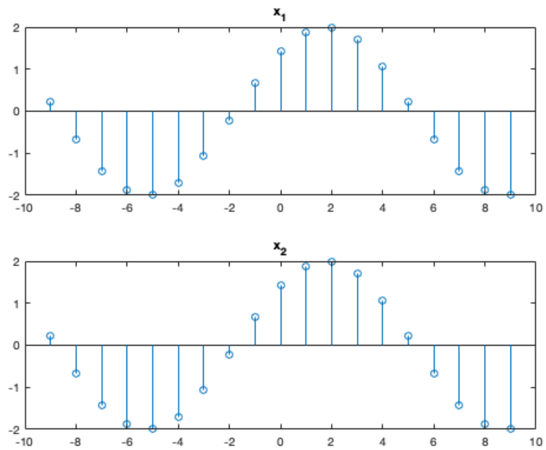


Figure 12) graph of Equation 6 and Equation 7 with the same sample rate

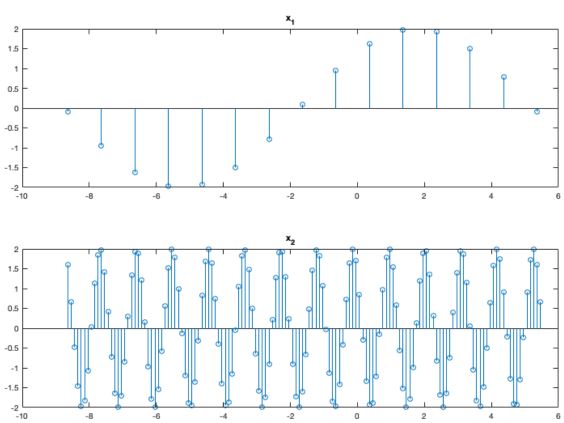


*Figure 13) Graph of **Error! Reference source not found.** and Equation 7 with an increased sample rate for Equation 7.*

Part 2:

Describe how the two, ***Error! Reference source not found.*** *and*

Equation 7, have one key difference, their frequencies:

$$T_{0_{x_1}} = 14s, f_{x_1} = \frac{1}{14}Hz, \qquad \&, \qquad T_{0_{x_2}} = \frac{14}{13}s, f_{x_2} = -\frac{13}{14}\ Hz$$

When sampled at the same rate the two functions are identical. Only when increasing the sample rates do the two functions differentiate themselves.

Problem 5

For each equation, determine whether it has the listed properties:

a) Linearity properties:
   i) Additive: $y(ab) = y(a) + y(b), f(1) = 0$
   ii) Homogeneity(scaling of degree 1): $y(at) = a \bullet y(t), \quad for\ all\ a$
b) Time invariance:
   i) $y(t) = x(t),\ y(t - t_0) = x(t - t_0)$
c) BIBO stable has a finite signal integral or sum.
d) Memoryless: a present system output depends only on the present input.
   i) $x(t)$ is memoryless
   ii) $x(t - t_0)$ has memory
e) Causal system response only depends on values of the input at current time and occasionally past time.
f) Invertible if unique excitations produce unique zero state responses

Part (a)

$$\boldsymbol{y[n] = |x[n]|}$$

Properties	Yes/No
a) Linear?: i) Additive: no $x_0 + x_1 = y(t_0 t_1), \qquad 1 + 2 \neq 1 \bullet 2$ ii) Homogeneous: yes $y[an] = a \bullet \lvert x[n] \rvert$	No
b) Time invariant?: i) Invariant under time reversal (1) $y[n] = \lvert x[n - n_0] \rvert$	Yes
c) BIBO stable?: i) $\sum_{n=-\infty}^{\infty} \lvert x[n] \rvert \nless \infty$	No
d) Memoryless?: present system output depends only on the present input i) $y[n]$ does not depend on the past or future.	Yes
e) Causal?: i) $y[n]$ does not depend on the future.	Yes
f) Invertible?: for $y[n], n = -5{,}5 = 5\ which\ is\ not\ unique$ i) Iff invertible, find its inverse	No

Part(b)

$$y(t) = 2x(t) + 2$$

Properties	Yes/No
a) Linear?: i) Additive: $y(ab) = 2x(a) + 2 + 2x(b) + 2, y(0 \bullet 2) \neq 2 + 6$, No	No
b) Time invariant?: i) $y(t) = 2x(t) + 2 = 2x(t - t_0) + 2$	Yes
c) BIBO stabile?: time t is unbounded therefore possibly infinite. i) $y(t) = \int_{-\infty}^{\infty} 2x(t) + 2 \nless \infty$	No
d) Memoryless?: Yes i) present system output depends only on the present input	Yes
e) Causal?: i) The system response only depends on values of input at current or past time.	Yes
f) Invertibility: for $y(-5) \neq y(5)$ i) Iff invertible, find its inverse $y(t)^{-} = \dfrac{x(t) - 2}{2}$	yes

Part(c)

$$y[n] = x[n+1] - x[n-1]$$

Properties	Yes/No
a) Linear?: Yes i) Additive?: $y_1 + y_2 = (x[n+1] - x[n-1])_1 + (x[n+1] - x[n-1])_2$ ii) Homogeneous?:yes	yes
b) Time invariant?: i) $y[n-n_0] \neq x[n-n_0+1] - x[n-n_0-1]$	No
c) BIBO stability?: i) $y[n] = \sum_{n=-\infty}^{\infty} \lvert x[n] \rvert < \infty$	Yes
d) Memoryless?: present system output depends only on the present input, no. i) $y[0]$ depends on past time.	No
e) Causal?: The system response only depends on values of input at current or past time. i) $y[0]$ depends on future time.	no
f) Invertible?: $(x[-4] - x[-6])_{n=-5} = (x[6] - x[4])_{n=5}$ i) Iff invertible, find its inverse	no

Part(d)

$$\boldsymbol{y(t) = \int_0^t x(\tau)\, d\tau = \frac{t^2}{2}}$$

Properties	Yes/No
a) Linear: yes by definition integrals are additive and homogeneous.	yes
b) Time invariant?: i) $y(t - t_0) = \int_0^{t-t_0} x(\tau)\, d\tau$	Yes
c) BIBO stable?: The system is bounded by undefined time t which is not defined as ∞. i) $y(t) = \int_0^t x(\tau)\, d\tau < \infty$	Yes
d) Memoryless: Has memory i) present system output depends only on the present input	No
e) Causal: i) The system response only depends on values of input at current or past time.	yes
f) Invertibility: $t < 0, \therefore\ y(t)\ is\ invertable$ i) Iff invertible, find its inverse ii) $\frac{dy(t)}{d\tau} = \frac{d}{d\tau}\left(\int_0^t x(\tau)\, d\tau\right)$	yes

Part(e)

$$\boldsymbol{y[n] = \frac{n^2+2}{n^2+3}x[n]}$$

Properties	Yes/No
a) Linear?: i) Additive: $y_1[n] + y_2[n] = \left(\frac{n^2+2}{n^2+3}x[n]\right)_1 + \left(\frac{n^2+2}{n^2+3}x[n]\right)_2$ ii) Homogeneous: a•$y[n] = a \bullet \left(\frac{n^2+2}{n^2+3}x[n]\right) = \frac{n^2+2}{n^2+3}(a \bullet x[n])$	Yes
b) Time invariant?: i) $y[n-n_0] = \frac{n^2+2}{n^2+3}x[n] \neq \frac{(n-n_0)^2+2}{(n-n_0)^2+3}x[n-n_0]$	No
c) BIBO stabile?: y is an unbounded system therefore it is unstable. i) $\sum_{n=-\infty}^{\infty}\lvert y[n]\rvert \nless \infty$	No
d) Memoryless?: i) Only depends on current time.	yes
e) Causal?: i) The system response only depends on values of input at current or past time.	yes
f) Invertible?: Yes, all n values are unique. i) Iff invertible, find its inverse ii) $\frac{n^2+3}{(n^2+2)}x[n]$	yes

Part(f)

$$y(t) = t\,x(t)\frac{d}{dt}$$

	Properties	Yes/No
a)	Linear?: i) Additive: $y_1 + y_2 = tx_1(t)\frac{d}{dt} + tx_2(t)\frac{d}{dt} = t\left(\frac{d}{dt}(x_1 + x_2)\right)$ ii) Homogeneous: $a \bullet y(t) = a\left(t\,x(t)\frac{d}{dt}\right)$	yes
b)	Time invariant?: i) $y(t) = t\,x(t)\frac{d}{dt} \neq (t - t_0)x(t)\frac{d}{dt}$	No
c)	BIBO stabile?: i) $\sum_{n=-\infty}^{\infty}\lvert x[n]\rvert \nless \infty$	No
d)	Memoryless?: i) present system output depends only on the present input	yes
e)	Causal?: i) The system response only depends on values of input at current or past time.	yes
f)	Invertible?: Integral would require a unknown constant i) Iff invertible, find its inverse	No