Signals & Systems Homework #1

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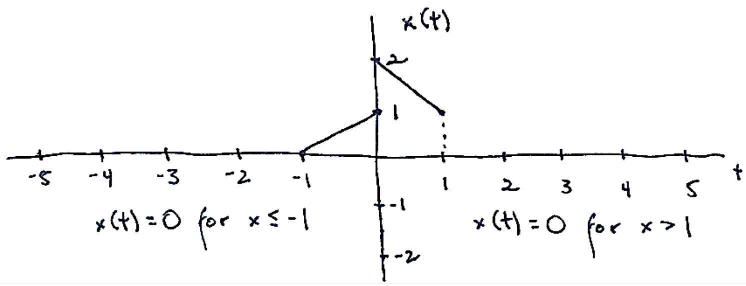
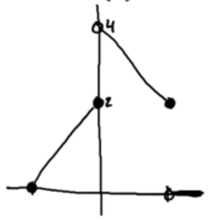
ECE 315 – Fall 2020

Due Monday, October 12, 2020

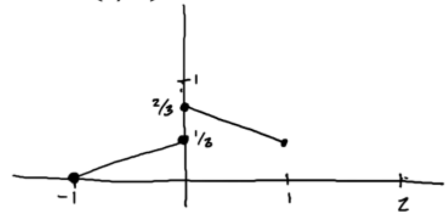
1. Consider the signal $x(t)$ shown below. All plots must be done by hand. (No credit for using an electronic device to do the plotting for you.)



a. Plot $2x(t) \rightarrow$



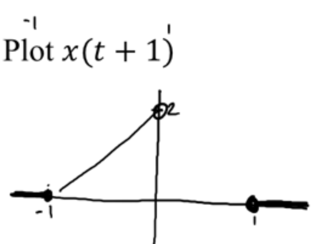
c. Plot $x(t/3)$



d. Write $x(t)$ as a function.

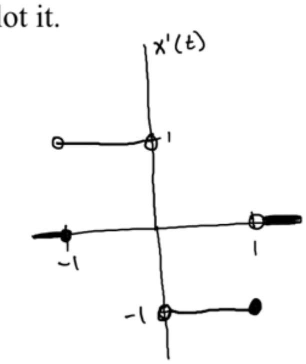
$$x(t) = \begin{cases} t+1, & -1 < t < 0 \\ 2-t, & 0 < t \leq 1 \\ 0, & -1 \geq t > 1 \end{cases}$$

b. Plot $x(t+1)$



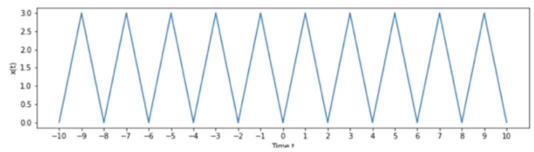
e. Determine the generalized derivative of x(t) and plot it.

$$\frac{d}{dt}x(t) = \begin{cases} 1, & -1 < t < 0 \\ -1, & 0 < t \leq 1 \\ 0, & -1 \geq t > 1 \end{cases}$$



2.

a. Write a MATLAB script that recreates the plot of the signal $x(t)$ between $t = -10$ and $t = 10$. Make the fundamental period T_0 and the height of the peaks H variables in your script.



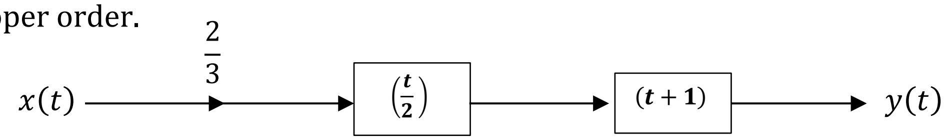
```
H = 1.5;
T = 2*pi*1/2;
t = -10:10;
x1 = H*sawtooth(T*t,1/2)+H;
```

Figure 1: A periodic signal $x(t)$

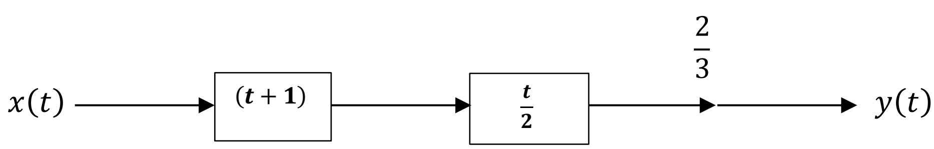
b. Plot between t = −10 and t = 10 by hand or using your MATLAB script.

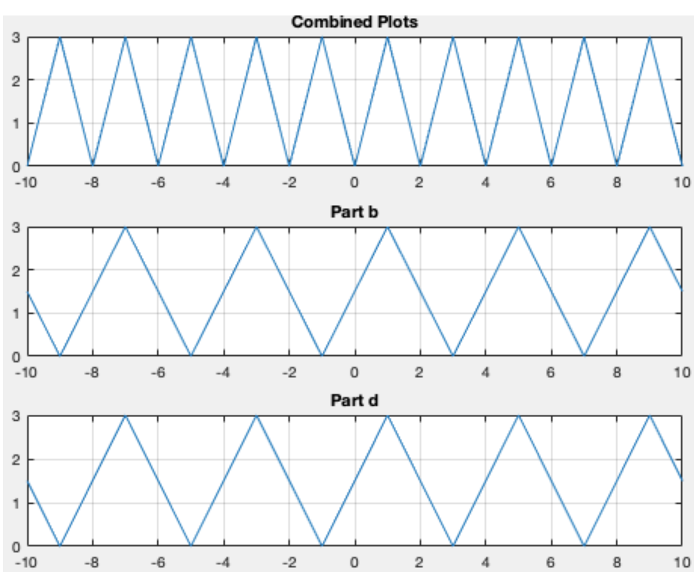
$y(t) = \frac{2}{3}x\left(\frac{t+1}{2}\right)$ **See graph in part d.**

c. Create a block diagram showing the simple transformations involved in Part (b) in a proper order.



d. Draw the block diagram that results when you interchange time shifting and time scaling in Part (c) and plot the signal y˜(t) that results from this altered transformation between t = −10 and t = 10 by hand or using your MATLAB script.






```
H = 1.5;
T = 2*pi*1/2;
t = -10:10;
x1 = H*sawtooth(T*t,1/2)+H;
subplot(3,1,1)
plot(t,x1)
title('Combined Plots');
grid on

x2 = H*sawtooth(T*((t+1)/2),1/2)+H;
subplot(3,1,2)
plot(t,x2)
title('Part b');
grid on

x4 = H*sawtooth(T*((t/2)+1/2),1/2)+H;
subplot(3,1,3)
plot(t,x4)
title('Part d');
grid on
```

3. Calculate

a.

$$\frac{d}{dt}\frac{t^2+4}{3t^2-2t} = \frac{d}{dt}\left(\frac{t^2+4}{t(3t-2)}\right)$$

$$= -\frac{2(t^2+12t-4)}{(2-3t)^2(t^2)}$$

```
>> syms t;
f = (t.^(2)+4)/(3.*t.^(2)-2)
diff(f)

f = (t^2 + 4)/(3*t^2 - 2)
ans = (2*t)/(3*t^2 - 2) - (6*t*(t^2 + 4))/(3*t^2 - 2)^2
```

b.

$$\int t\cos(2\pi t)dt = \frac{\mathrm{t} \bullet \sin(2\pi t)}{2\pi} - \int \frac{\sin(2\pi t)}{2\pi}dt$$

$$= \frac{\mathrm{t} \bullet \sin(2\pi t)}{2\pi} + \frac{\cos(2\pi t)}{4\pi^2} + c$$

```
>> syms t
expr = t.*cos(2.*pi.*t);
F = int(expr)
F = (cos(2*pi*t) + 2*t*pi*sin(2*pi*t))/(4*pi^2)
```

c.

$$\int_0^\infty t^2 e^{-\frac{t}{2}}dt = -2e^{-\frac{t}{2}}t^2 + 4\left(-2e^{-\frac{t}{2}}t - 4e^{-\frac{t}{2}}\right)\Big|_0^\infty$$

$$= 0 - (-16)$$

$$= 16$$

```
>> fun = @(t) exp(-t/2).*t.^2;
q = integral(fun,0,inf)
q =16.0000
```

4. Calculate:

a.

$$\int_{-\infty}^{\infty} \delta(t-3)\cos(\pi t)\,dt = \cos(\pi t)\int_{-\infty}^{\infty} \delta(t-3)dt$$

$$= \cos(\pi t) \bullet \delta(t-3)$$

$$= \begin{cases} -1, & t=3 \\ 0, & else \end{cases}$$

b.

$$\int_{-5}^{5} \delta\left(\frac{t+3}{2}\right)(t^2-2t+1)dt$$

$$= (-3^2-2(-3)+1) \bullet 1$$

$$= \begin{cases} 16, & t=-3 \\ 0, & else \end{cases}$$

c.

$$\int_{0}^{\infty} \delta(t+2)e^{6t^2}dt = e^{6\bullet -2^2}$$

$$= e^{24}$$

$$= \begin{cases} -12.65E10, & t=-2 \\ 0, & else \end{cases}$$

$$\therefore over \lim_{t\to\infty} \delta(t+2) = 0$$

d. Prove the scaling property. Use the definition of the unit impulse in terms of how it acts and a change of variable

$$\delta\big(a(t-t_0)\big) = \frac{1}{|a|}\delta(t-t_0)$$

Consider the dirac delta 'function' to be defined as:

$$\delta(t) = \begin{cases} \infty, & t=0 \\ 0, & else \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

$$furthermore$$

$$|a| \bullet \delta(a \bullet t) = \begin{cases} |a| \bullet \infty, & a \bullet t = 0 \\ |a| \bullet 0, & a \bullet else \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

$$a \in \mathbb{R}$$

From $|a|\delta(at)$

$$\int_{-\infty}^{\infty} |a|\delta(at))dt = \frac{|a|}{a}\int_{-\infty}^{\infty} \delta(t)dt$$

$$= 1$$

$$\therefore$$

By manipulating of properties of ∞:

$$|a|\delta(at) = \delta(t)$$

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

Simply put, a•0=1, and ∞/|a|=∞, while both arguments are irrational this is the definition of the dirac 'function'.

By manipulating the properties of integration by a variable t, we see that a function f(t)=a by the integrated dirac function, f(t) would then output the value of that function at t only.

5. Calculate:

$$\int e^{j\omega t} dt$$

a. by expressing the complex sinusoid in terms of its real and imaginary parts.

$$\int e^{j\omega t} dt = \int \underbrace{\cos(\omega t)}_{real} + \underbrace{j\sin(\omega t)}_{imaginary} dt$$

$$= \frac{1}{\omega}\big(\sin(\omega t) - jcos(\omega t)\big) + c$$

b. Express the result of part a. in terms of complex sinusoids.

$$\frac{1}{\omega}\big(\sin(\omega t) - jcos(\omega t)\big) + c = \frac{1}{\omega}\left(\left(\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right) - j\left(\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right)\right) + c$$

$$= -\frac{1}{\omega} je^{j\omega t} + c$$

c. How does this compare to $\int e^{a\omega t} dt$ where a is a real constant?

$$\int e^{a\omega t} dt = \frac{e^{a\omega t}}{a\omega} + c$$

Part b. is rationalized therefore has a negative value where as the constant needs no rationalizing. Theoretically they are arguing the same thing however $i = \sqrt{-1}$ is irrational in the denominator.

6. Determine whether the following signals are periodic and, if so, determine the fundamental period, fundamental frequency, and fundamental angular frequency. Feel free to use software to determine the LCM, if needed.

   a. $x(t) = \frac{3}{2}\cos\left(2t + \frac{\pi}{3}\right) + \frac{1}{2}\sin\left(\frac{6\pi}{7}t - \frac{1}{3}\right) = \frac{3}{2}\cos\left(2\pi \bullet \frac{t}{\pi} + \frac{\pi}{3}\right) + \frac{1}{2}\sin\left(2\pi \frac{\pi}{\frac{7\pi}{3}} t - \frac{1}{3}\right)$

      i. $T_0 = 3.1275 = \frac{T_{01}}{T_{02}} = \frac{\pi}{\frac{7}{3}}$

      $\frac{3\pi}{7} LCM = 7\pi$

      $\therefore x(t)$ is not periodic

   b. $x(t) = 2\sin\left(\frac{4\pi}{5}t - \frac{1}{2}\right) - \sin\left(\frac{\pi}{6} - \frac{2\pi}{3}t\right) \implies T_{01} = \frac{5}{2},\ T_{02} = 3 \therefore T_0 = 15, f_0 = \frac{1}{15}, \omega_0 = \frac{2\pi}{15}$

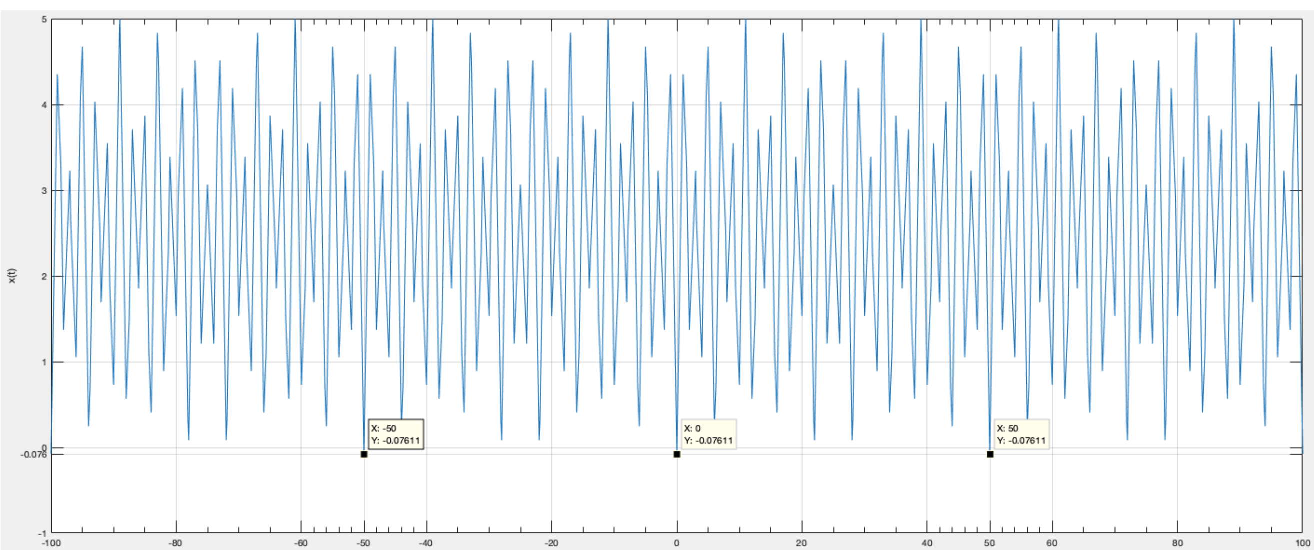
   c. $x(t) = x_1(t) + x_2(t)$, where

$$x_1(t) = \arcsin\left(\sin(\pi t + \frac{3}{2}\pi)\right) + \frac{3}{2} \rightarrow T_{01} = 2$$

$$x_2(t) = \frac{16}{25}\arcsin\left(\sin\left(\frac{16}{25}\pi t - \frac{\pi}{2}\right)\right) + 1 \rightarrow T_{02} = \frac{25}{8}$$

$$T_0 = 50, f_0 = \frac{1}{50}, \omega_0 = \frac{\pi}{25}$$

If my very rough approximations are remotely accurate there would be a fundamental period of 50.



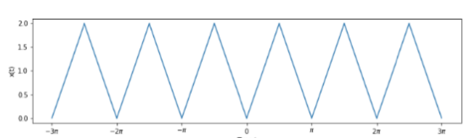
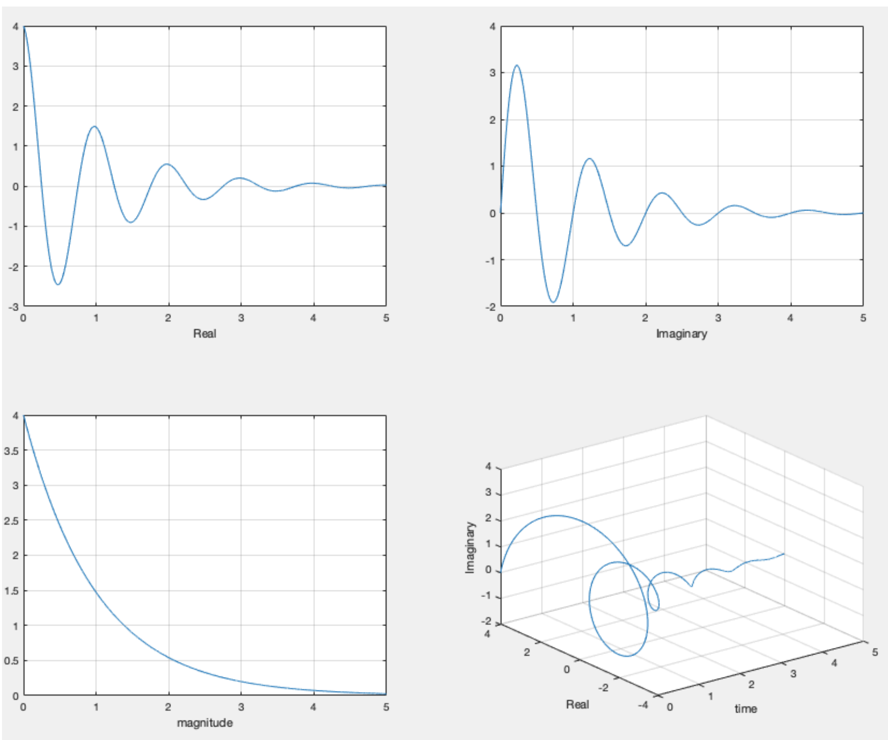


Figure 2: Another periodic signal

7. Consider the signal
   a. Determine the real and imaginary parts, magnitude, and phase of z(t).

$$z(t) = 4e^{(-1+2\pi j)t} = \underbrace{4e^{-t}}_{|z|}\,\underbrace{e^{2\pi jt}}_{\phi} = \underbrace{4e^{-t}}_{|z|}\,\underbrace{\left(\underbrace{\cos(2\pi t)}_{real} + \underbrace{j\sin(2\pi t)}_{imaginary}\right)}_{\phi}$$

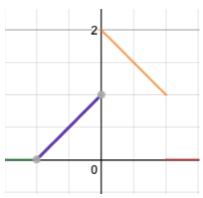
   b. Use MATLAB to plot the real and imaginary parts and the magnitude of z(t) as functions of time.
   c. Use MATLAB to plot z(t) as a function of time in the 3-dimensional space with axes corresponding to the real and imaginary parts of z and time t.



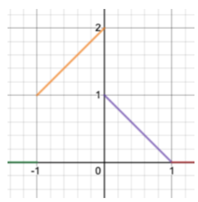

```
%statements
t = 0:.001:5;
M = 4.*exp(-t);
R = M.*cos(2.*pi.*t);
I = M.*sin(2.*pi.*t);
%Plotting
subplot(2,2,1);
plot(t,R);
grid on;
xlabel('Real');

subplot(2,2,2);
plot(t,I);
xlabel('Imaginary');
grid on;

subplot(2,2,3);
plot (t,M);
xlabel('magnitude');
grid on;

subplot(2,2,4)
plot3(t,R,I);
grid on;
xlabel('time');
ylabel('Real');
zlabel('Imaginary');
```

8. Find the even and odd parts of:
   a. the signal x(t) from Problem 1.

$$x(t) = \begin{cases} t+1, & -1 < t < 0 \\ 2-t, & 0 < t \le 1 \\ 0, & -1 \ge t > 1 \end{cases}$$



$$x(-t) = \begin{cases} 1-t, & 0 < t < 1 \\ 2+t, & -1 \le t < 0 \\ 0, & -1 > t \ge 1 \end{cases}$$



Even

$$E(t) = \frac{x(t) + x(-t)}{2}$$
$$= \begin{cases} \dfrac{(t+1)_{x(t)} + (t+2)_{x(-t)}}{2} = \boldsymbol{t + \frac{3}{2}}, & -1 < t < 0 \\ \dfrac{(2-t)_{x(t)} + (1-t)_{x(-t)}}{2} = \boldsymbol{\frac{3}{2} - t}, & 0 < t \le 1 \\ 0, & -1 \ge t > 1 \end{cases}$$

Odd

$$O(t) = \frac{x(t) - x(-t)}{2}$$
$$= \begin{cases} \dfrac{(t+1)_{x(t)} - (t+2)_{x(-t)}}{2} = -\boldsymbol{\frac{1}{2}}, & -1 < t < 0 \\ \dfrac{(2-t)_{x(t)} - (1-t)_{x(-t)}}{2} = \boldsymbol{\frac{1}{2}}, & 0 < t \le 1 \\ 0, & -1 \ge t > 1 \end{cases}$$

   b. The signal

$$x(t) = u(t+5) - u(-t-3) \to |x| = 2$$

$$x(-t) = u(5-t) - u(t-3) \to |x| = 2$$

Even

$$E(t) = \frac{x(t) + x(-t)}{2}$$
$$= \frac{\big(u(t+5) - u(-t-3)\big) + \big(u(5-t) - u(t-3)\big)}{2}$$
$$= \left|\frac{u(t+5) - u(t+3) + u(5-t) - u(t-3)}{2}\right| = 1$$

Odd

$$O(t) = \frac{x(t) - x(-t)}{2}$$
$$= \frac{\big(u(t+5) - u(t+3)\big) - \big(u(5-t) - u(t-3)\big)}{2}$$
$$= \left|\frac{u(t+5) + u(t+3) - u(5-t) - u(t-3)}{2}\right| = 0$$

   c. the unit ramp function $ramp(t)$.

$$ramp(t) = \begin{cases} t, & 0 < t \\ 0, & t < 0 \end{cases}, \quad ramp(-t) = \begin{cases} t, & t < 0 \\ 0, & 0 < t \end{cases}$$

Even:

$$E(t) = \frac{1}{2} \bullet \big(ramp(t) + ramp(-t)\big)$$
$$= \frac{1}{2} \bullet t - t$$
$$= 0$$

Odd:

$$O(t) = \frac{1}{2} \bullet \big(ramp(t) - ramp(-t)\big)$$
$$= \frac{1}{2} \bullet t - (-t)$$
$$= t$$

   d. the signal $x(t) = \frac{\cos(\pi t)}{\pi t} + t^2 sin(2\pi t)$
   I used a calculator to compute this.

$$E(t) = \frac{x(t) + x(-t)}{2} = 0$$
$$O(t) = \frac{x(t) - x(-t)}{2} = \frac{\cos(\pi t)}{\pi t} + t^2 \sin(2\pi t)$$

9. Use the symmetry properties of the integrands to evaluate the following integrals in the simplest ways.

a. A is even

$$\int_{-2}^{2} ((t)^4 + 5(t)^3 - 5(t)^2 + 2(t) - 12)dt = 2\int_0^2 \frac{1}{2}\big(x(t) + x(-t)\big)dt$$

$$= \int_0^2 (2t^4 - 10t^2 - 24)\,dt$$

$$= -\frac{928}{15}$$

b.

$$\int_{-\frac{5}{6}}^{\frac{5}{6}} \left(2\cos(2\pi t) - \frac{1}{2}\sin(4\pi t)\right) dt = 2\int_0^{\frac{5}{6}} \frac{1}{2}\big(x(t) + x(-t)\big)dt$$

$$= \int_0^{\frac{5}{6}} 4\cos(2\pi t)\,dt$$

$$= -\frac{\sqrt{3}}{\pi}$$

c. Product of an even and odd function is an odd sinusoid, the integral of an odd function is zero therefore $X(t) - X(-t) = 0$

$$x(t) = \int_{-\pi}^{\pi} \underbrace{(12t)_O(\cos(6t))_E}_{O\bullet E=O}\,dt$$

$$= X(t) - X(-t)$$

$$= 0$$

10. .

a. Find the energy of

$$x(t) = 5e^{-4t}u(t)$$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

$$= \int_{-\infty}^{\infty} \left| 5e^{-4t} \underbrace{u(t)}_{\lim_{t\to\infty} u(t)=1} \right|^2 dt$$

$$= \int_{0}^{\infty} 25e^{-8t} + \underbrace{\int_{-\infty}^{0} x(t)\, dt}_{\lim_{t\to-\infty} x(t)=0}$$

$$= 25\int_{0}^{\infty} e^{-8t}\, dt$$

$$= \frac{25}{8} e^{-8t} \Big|_{0}^{\infty}$$

$$= \frac{25}{8}$$

$$x(t) = \underbrace{2e^{-j\left(\frac{3\pi}{2}\right)t}}_{|z|}$$

$$= 2e^{2\pi i t}$$

$$= 2 * 1$$

$$P_x = \frac{1}{\frac{4}{3}} \int_{t_0}^{t_0+\frac{4}{3}} |2|^2 dt$$

$$= \frac{3}{4}\int_{t}^{t+\frac{3}{4}} 4dt = \frac{3}{4}\left(\frac{16}{3}\right)$$

$$= 4$$

b. Find the average power of

c. Is $x(t) = 2e^{-j\left(\frac{3\pi}{2}\right)t} rect(t)$ an energy or a power signal? Why?

The function, $x(t)$, is considered finite since the function decays to 0 as the limit of t approaches infinity.

$$x(t) = 2e^{-j\left(\frac{3\pi}{2}\right)t} rect(t)$$

The definition of an energy signal is any signal with finite energy is called an energy signal

Therefore this signal is considered an **energy signal** due to its finite nature.