ECE 316: Signals and Systems II **Spring 2021**

Homework 3
Due: Friday April 16, 11:59PM PT

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In order to receive credit for these problems, you must show all your work (write out all the steps you worked through). Partial credit will only be awarded if your reasoning (steps you worked through) is correct, and problems with correct answers but no work shown will receive 0 points.

NOTE:

If you work with other students for this assignment, please write the names (first and last) of every student at the top of your assignment (below your name and the assignment title). As well as a couple sentences on how you worked together. Example follows:

Classmates worked with:

Details:

I worked with Luke Tutino, Nick Lekas and David L. We again, worked simultaneously when a issue arose we would help reference the correct materials.

We were all unsure about how much detail was necessary for each solution.

I showed everyone how I solved the differencing property problem. We were not sure if I was correct but it was the best we could all come up with so some of us just went with it.

Problem 1 (4 pts)

The Z-transform can be thought of as a special case of the Laplace transform. What is this case?
The Z-transform converts a discrete-time signals, consisting of real or imaginary components, into a complex frequency representation. In short the Z-transform is the discrete counterpart to the Laplace transform.

where the form of the bilateral equation is:

$$\sum_{n=-\infty}^{\infty} x[n]z^{-n} \tag{1}$$

Where $z = re^{j\Omega}$

Describe how the transforms are related, as well as how they differ.
Both consist of the complex plane, real and imaginary parts to probe a input with sinusoids. The biggest difference is in the manor in which the mapping is done S is continuous were z is discrete where regions of convergence sharing similar properties but with s-plane being rectangular and including the entire - plane while the z-plane is polar in.

When would you use which? You may describe in words, use annotated diagrams/plots, etc.
Laplace would be used for analog systems and the z-transform would be used for digital systems i.e. solving difference equations.

Problem 2 (4 pts)

Find the z-transform of the following discrete time signals. Remember to specify region of convergence (ROC) in terms of $|z|$, if it exists. Show all work when determining the transform and ROC.

1. $x[n] = (5/6)^n u[n-2]$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \tag{2}$$

Applying properties of time shift to find the z-transform

$$\begin{aligned}
\sum_{n=\infty}^{\infty} (5/6)^n u[n-2] z^{-n} &= \sum_{n=2}^{\infty} (5/6)^n z^{-n} \\
&= \sum_{k=n-2=0}^{\infty} \left(\frac{5}{6z}\right)^{k=n-2=0} \\
&= \sum_{k=0}^{\infty} \left(\frac{5}{6z}\right)^{k+2} \\
&= \left(\frac{5}{6z}\right)^2 \sum_{k=0}^{\infty} \left(\frac{5}{6z}\right)^n \\
&= \left(\frac{5}{6z}\right)^2 \left(\frac{z}{z-\frac{5}{6}}\right) \\
&= \frac{25}{(6z)(6z-5)}
\end{aligned} \tag{3}$$

ROC *See Matlab*:

$$
\begin{aligned}
\frac{25}{(z)(6z-5)} &= \frac{A}{z} + \frac{B}{6z-5} \\
&= \frac{A6(6z)(6z-5)}{6z} + \frac{B(6z)(6z-5)}{6z-5} \\
&= A(6z-5) + B(6z), \rightarrow A = -5/6 \; B = 5 \\
&= \frac{5}{6z-5} - \frac{5}{6z}, |z| > \frac{5}{6}
\end{aligned} \tag{4}
$$

Poles and zeros:
Since this is an causal/right-sided geometric series the ROC eminates outward from the outermost pole.

Zeroes= Na

Poles= 0, 5/6

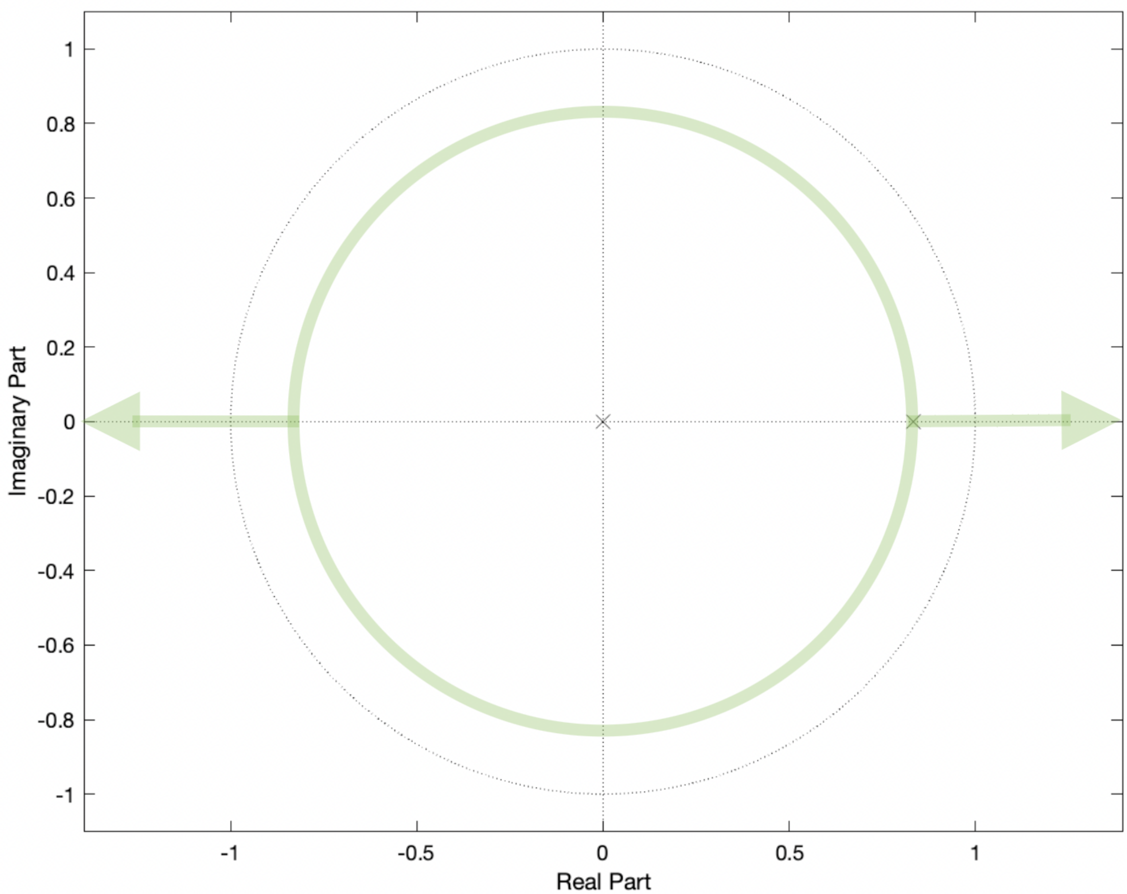


Figure 1: Matlab generated pzmap with ROC outward frm the outermost pole

2. $x[n] = ((-n)^3 + 3^n)(u[n-3] - u[n-1])$

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} ((-n)^3 + 3^n)(u[n-3] - u[n-1])z^{-n} \\ &= \sum_{n=1}^{2} ((-n)^3 + 3^n)(-1)z^{-n} \\ &= -(((-1^3 + 3^1)z^{-1})_{n=1} + ((-2^3 + 3^2)z^{-2})_{n=2}) \\ &= -\left(\frac{2}{z} + \frac{1}{z^2}\right) \\ &= \boxed{-\frac{2z+1}{z^2}} \end{aligned} \tag{5}$$

One Zero at $z = 1/2$ with no poles

This means we have a causal finite system which includes all point on the Z-plane, with exception to z=0.

Problem 3 (4 pts)

Exercise 9.7 Using the differencing property and the z transform of the unit sequence, find the z transform of the unit impulse and verify your result by checking the z-transform table. Recall, the unit impulse is $\delta[n] = u[n] - u[n-1]$

- **differencing property states:**

$$y[n] - y[n-1] = x[n] * (h[n] - h[n-1]) \tag{6}$$

- **unit-sequence definition** The discrete-time function that corresponds to the continuous-time unit step is the unit-sequence function

$$u[n] = \begin{cases} 1, & n \ \geq \ 0 \\ 0, & n \ < 0 \end{cases} \tag{7}$$

Starting from the given definition of $\delta[n]$

$$x[n] = \delta[n] = u[n] - u[n-1] = 1 \tag{8}$$

Applying the properties of the z-transform follows

$$\begin{aligned}
X(z) &= \sum_{n=0}^{\infty} x[n] z^{-n} \\
&= \sum_{n=0}^{\infty} (u[n] - u[n-1]) z^{-n} \\
&= \sum_{n=0}^{\infty} u[n] z^{-n} - \sum_{n=0}^{\infty} u[n-1]) z^{-n} \\
&= \sum_{k=0}^{\infty} z^{-k} - \sum_{k=0}^{\infty} z^{-(k=n-1=0} \\
&= \sum_{k=0}^{\infty} z^{-k} - \sum_{k=0}^{\infty} z^{-(k+1)} \\
&= \sum_{k=0}^{\infty} \left(\frac{1}{z}\right)^{k} - \frac{1}{z} \sum_{k=0}^{\infty} \left(\frac{1}{z}\right)^{k} \\
&= \frac{z}{z-1} - \frac{1}{z} \frac{z}{z-1} \\
&= \frac{z}{z-1} - \frac{1}{z-1} \\
&= \frac{z-1}{z-1} \\
&= 1
\end{aligned} \tag{9}$$

Here it is clear that the definition of the delta at n is $\delta[0] = 1$

Problem 4 (6 pts)

Exercise 9.22 b and c in Roberts. For each block diagram in Figure E.22, write the difference equation and find and graph the response y[n] of the system for discrete times $n \geq 0$ assuming no initial energy storage in the system and impulse excitation $x[n] = \delta[n]$.

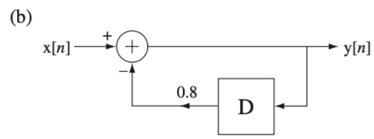


Figure 2: Roberts Exercise 9.22.b

Simplified linear interpretation:

$$x[n] \to \oplus \to D \to -0.8 \to \oplus \to y[n] \tag{10}$$

Difference equation

$$y[n] = x[n] - (0.8)y[n-1] \tag{11}$$

Initial conditions:

$$\begin{aligned} y[0] &= \delta[0] - (0.8)y[0-1] = 1 - 0 \\ &= 1 \\ y[1] &= \delta[1] - (0.8)y[1-1] = 0 - 0.8 \\ &= -0.8 \end{aligned} \tag{12}$$

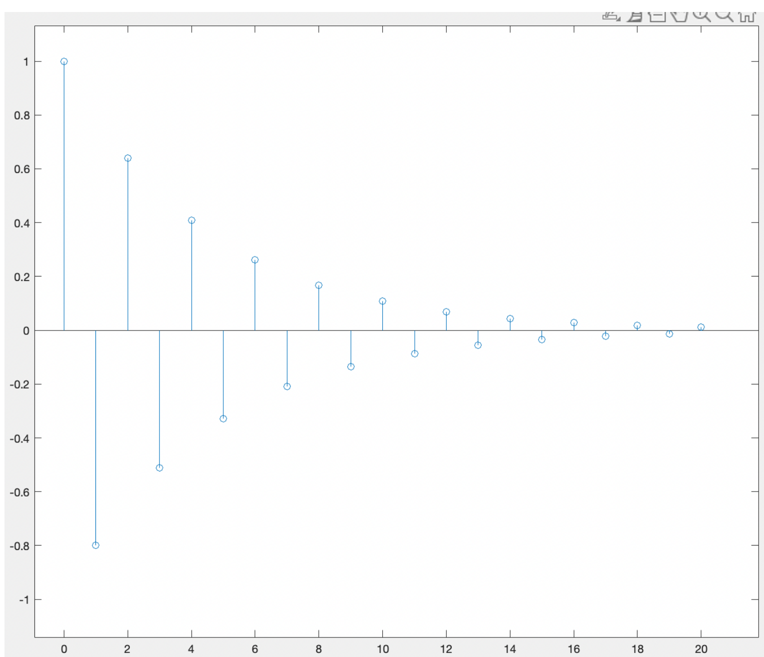


Figure 3: Matlab plot of the

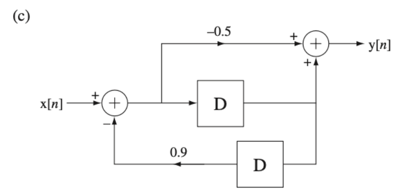


Figure 4: Roberts Exercise 9.22.c

y[n] $\rightarrow Y1$

$$\begin{aligned} Y &= Y1(z^{-1} - 0.5) \\ 1/X &= 1/Y1(1 + 0.9z^{-2}) \\ &= \frac{1/z + .45z^{-2}}{1 + 0.9z^2} - .5 \\ inverse - z &= exp(-0.0526803. * n). * (1.05409. * sin((pi. * n)/2) - 0.5. * cos((pi. * n)/2), usedwolfram \end{aligned} \tag{13}$$

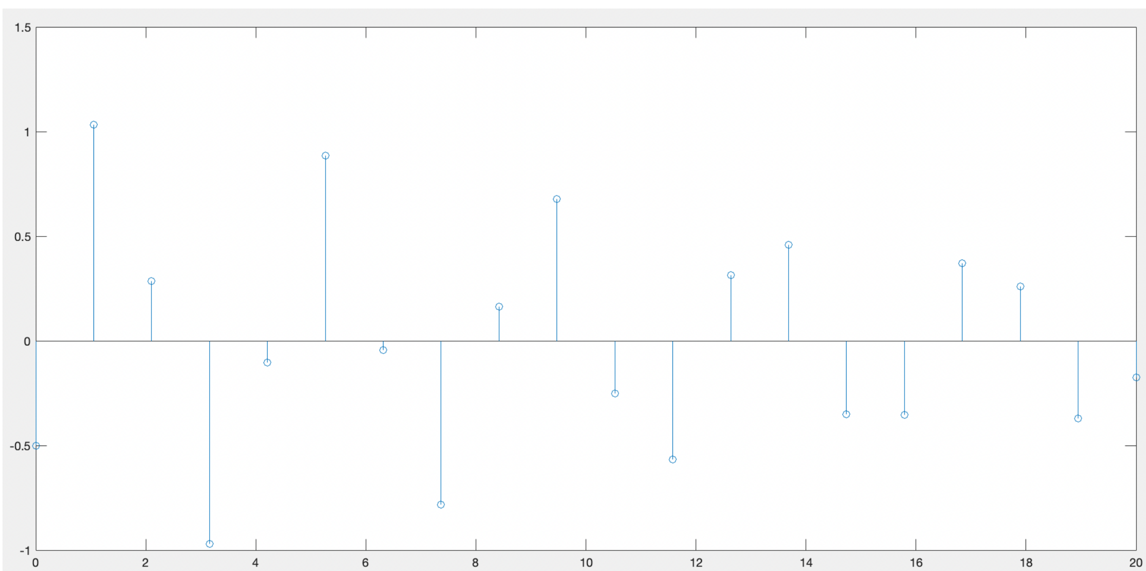
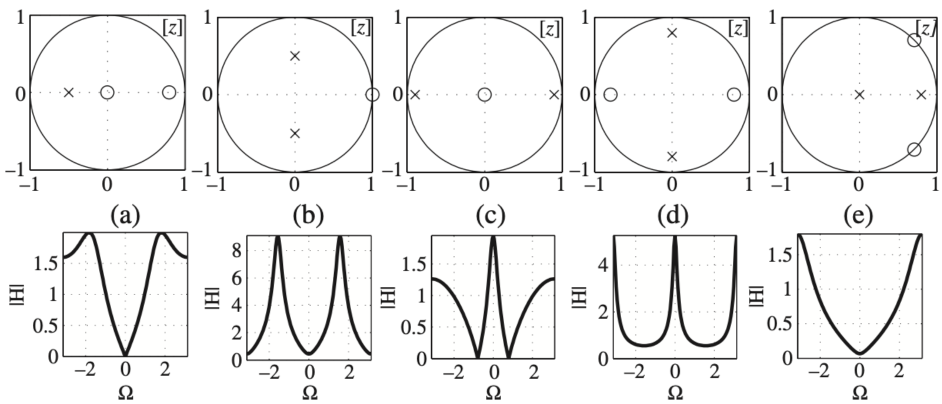


Figure 5: Matlab plot of the given block diagram

Problem 5 (5 pts)

Exercise 9.42 Match the pole-zero plots in Figure E.42 to the corresponding magnitude frequency responses. I numbered the top pole zero plots from 1 to 5, left to right.



1. e

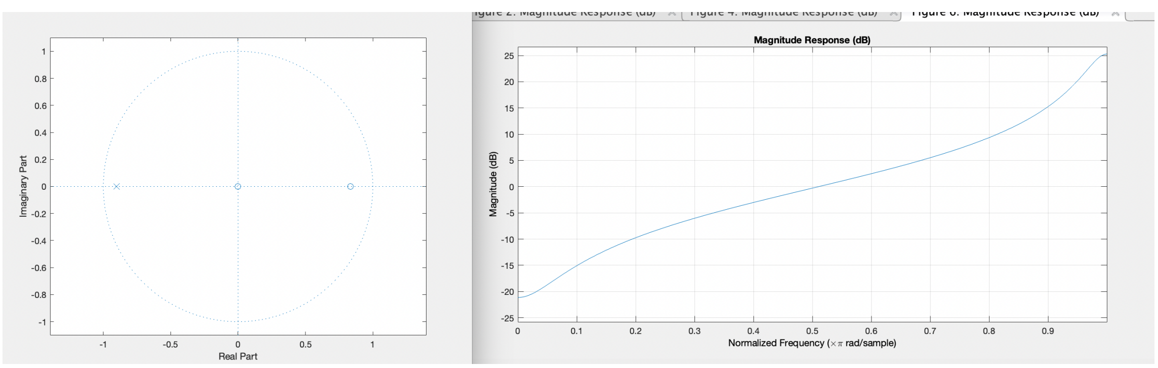


Figure 6: 1.e

2. a

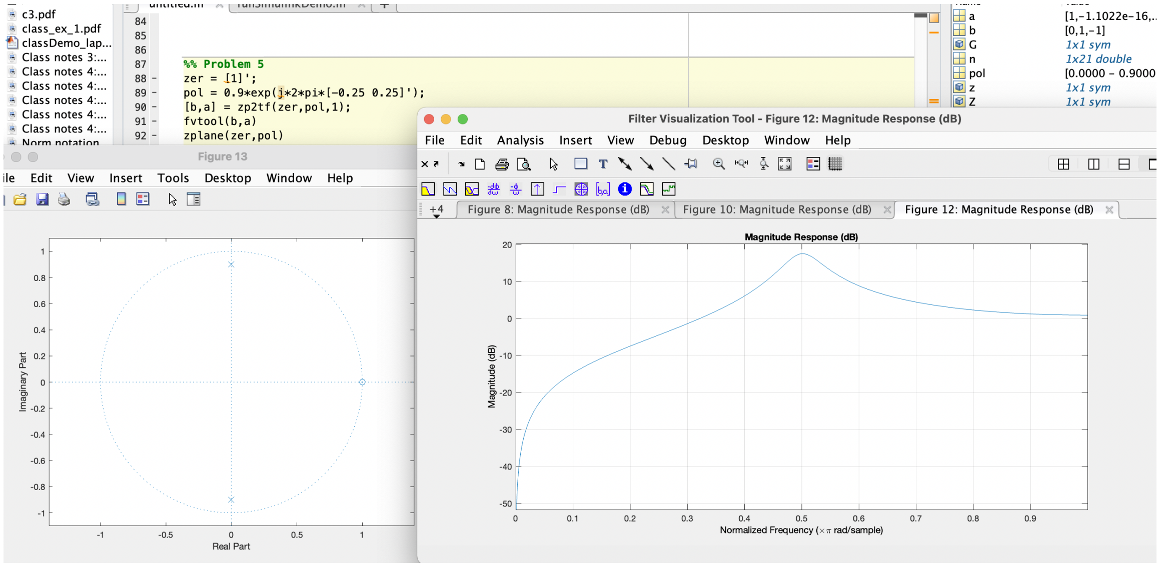


Figure 7: 2.a

3. d

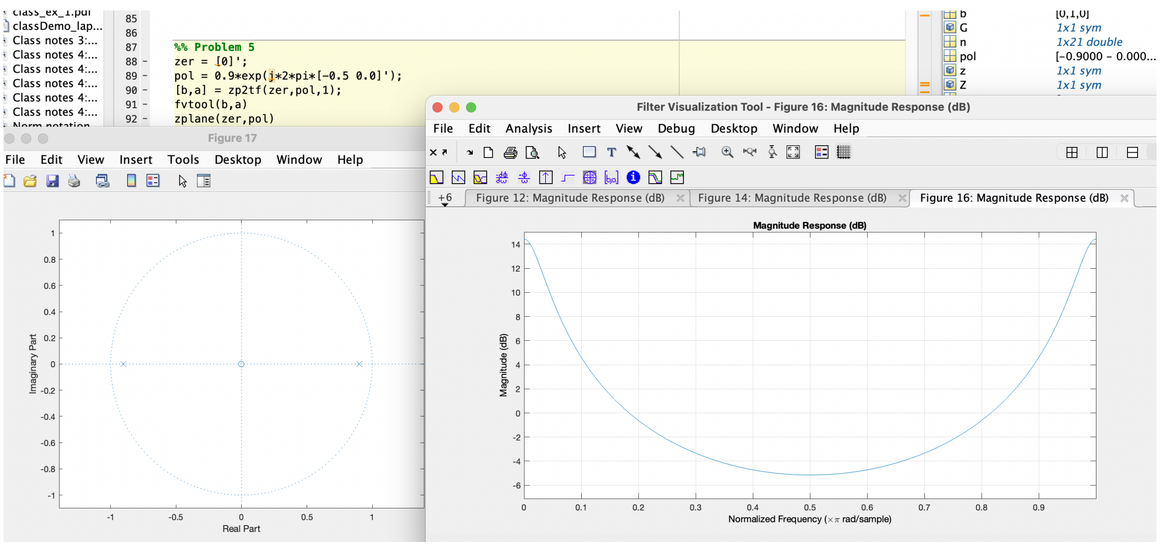


Figure 8: 3d

4. b

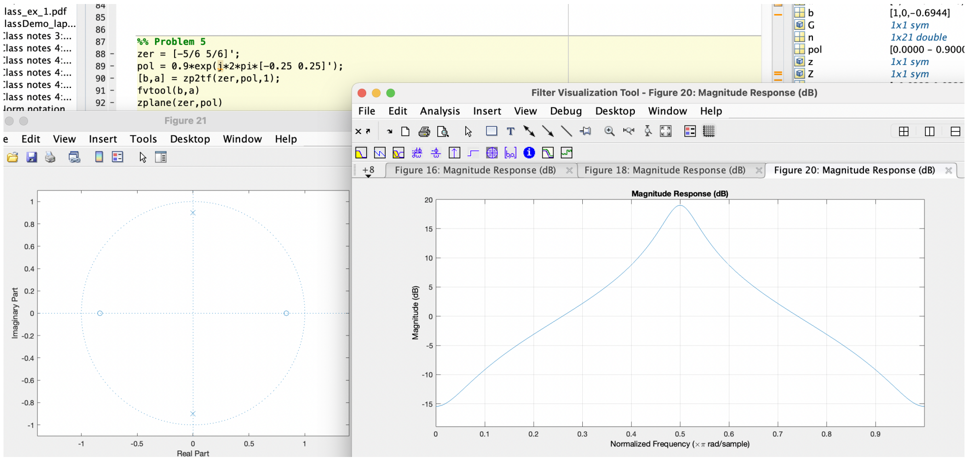


Figure 9: Caption

5. c

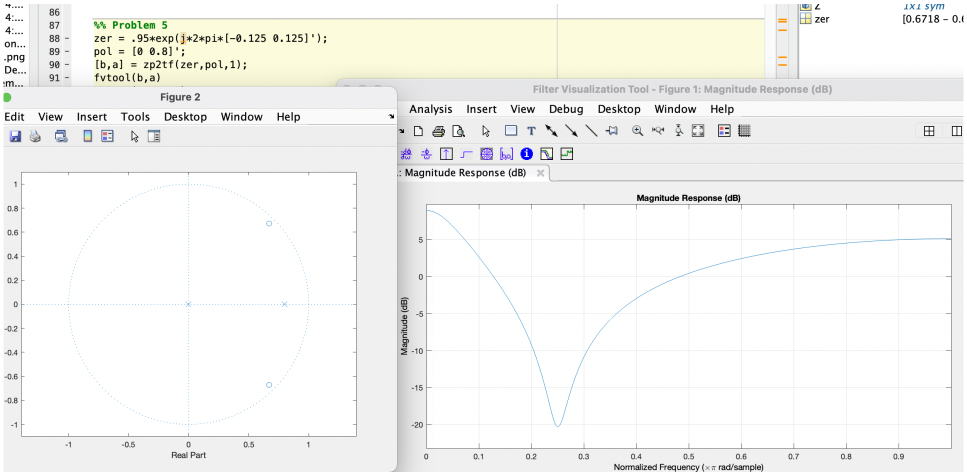


Figure 10: 5.c

Problem 6 (7 pts)

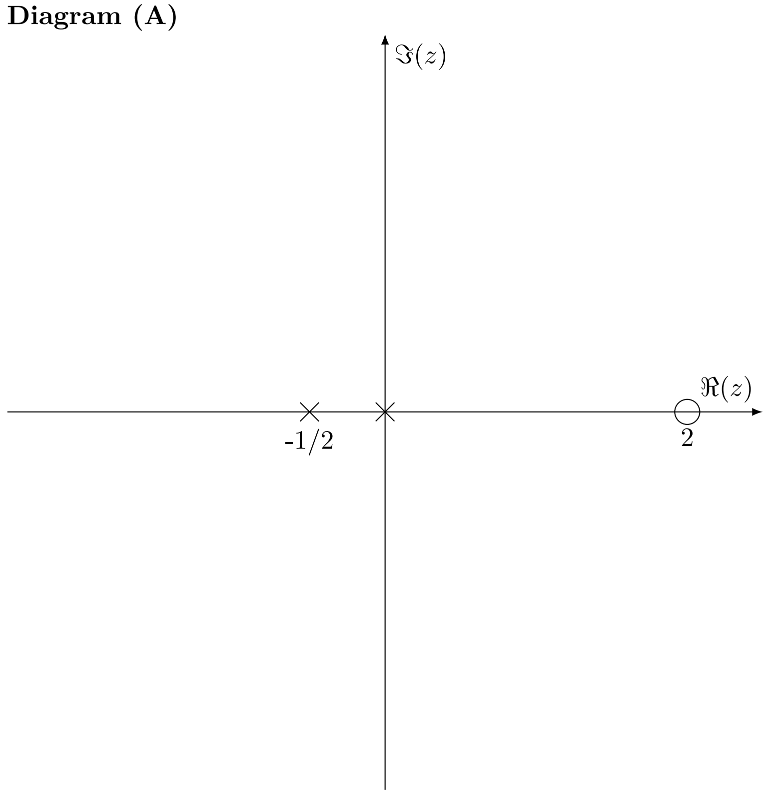
1. **What are the possible ROCs?** The possible regions of convergence are between the two poles, marked by the x, or outside the second pole at .5 radially outward toward infinity.

2. **Write the transfer function $H[z]$ of this system**

$$H[z] = \frac{z-2}{z^2+0.5z} \tag{14}$$

3. **Suppose we have a causal input signal ($x[n] = 0$ for all $n < 0$), what is the ROC in this case, and is it stable? Explain your reasoning.**
With the given causal system we know the system will be convergent outward from the .5 radius. This means the ROC of the system **will be stable** as it includes the unit circle 1.

Diagram (A)

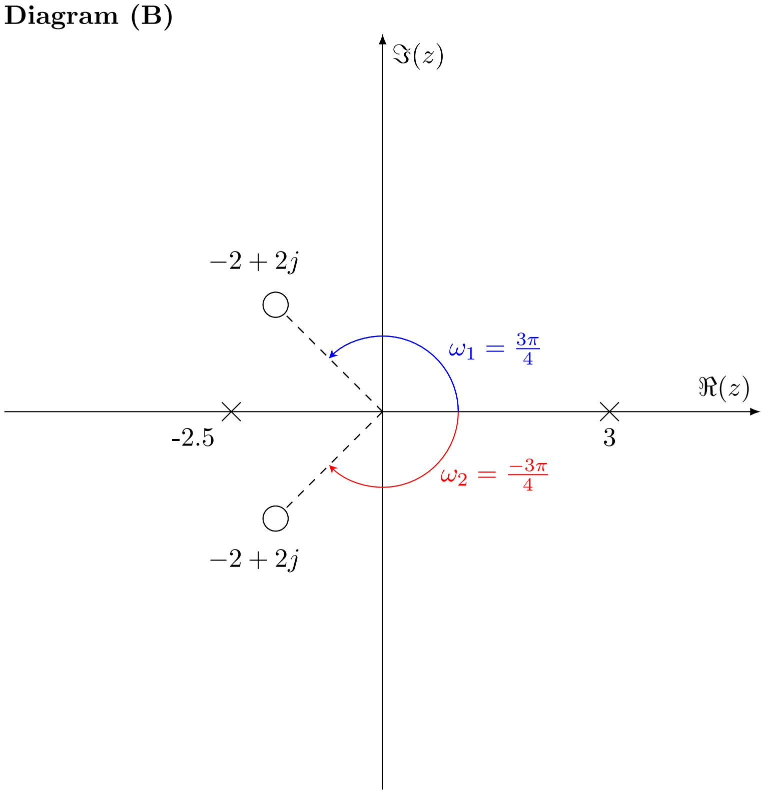


1. **What are the possible ROCs?** There are 3 possible regions of convergence
   (a) $|z| > 3$
   (b) $2.5 < |z| < 3$
   (c) $2.5 > |z|$
2. **Write the transfer function $H[z]$ of this system**

$$H[z] = \frac{z^2 + 4z + 8}{z^2 - 0.5z - 7.5} \tag{15}$$

3. **Suppose we have a causal input signal ($x[n] = 0$ for all $n < 0$), what is the ROC in this case, and is it stable? Explain your reasoning.**
   For the same reasons as part A, the system is going to emanate outward from the outer most pole. This means the system will not include the unit circle therefore it will not be a stable system.

Diagram (B)



```
\footnotesize
    %% Problem 2
    clc; clear all;
    syms n z a
    f = ((5/6)^n);
    u = heaviside(n-2);
    ztrans(f) % Discovered, z-tfm doesn't require heaviside. Time shift

    % Partial fraction decomposition
    xz = 25/(6*z*(6*z-5));
    partfrac(xz,z)

    %Pole zeroes, ROC: H(z)=B(z)/A(z)
    b = [0 0 25];
    a = [36 -30 0];
    H = zplane(b,a);
    pzmap(H)
    grid on


 %% Problem 2.1.2 using symsum
   clc;
    a = sym( 'a');
    n = sym( 'n');
    z = sym( 'z');
    assume(a > 0)
    assumeAlso(a < 1)
    assumeAlso(a < z)

    f = ((5/6)^n);
    Z = symsum(f*z^(-n),n,0,inf)
    Z = simplify(Z,'steps',20)

 %% Problem 2.2
   clc; clear all;
    a = sym( 'a');
    n = sym( 'n');
    z = sym( 'z');


    assume(a > 0)
    assumeAlso(a < 1)
    assumeAlso(a < z)

    Z = symsum((-(n)^3+3^(n))*z^(-n),n,1,3)

 %% Problem 3
    clc; clear all;
    a = sym( 'a');
    n = sym( 'n');
    z = sym( 'z');
```

```
    assume(a > 0)
    assumeAlso(a < 1)
    assumeAlso(a < z)

    Z = symsum((heaviside(n)-heaviside(n-1))*z^(-n),n,-inf,inf);
    Z = simplify(Z,'steps',100)

%% Problem 4.b
    clc; clear all;

    n = -1:1:20;
    a= (-0.8).^n;

    stem(n,a)


%% Problem 4.c

clc; clear all;
figure
n = linspace(0,20,21)';
Z = exp(-0.0526803.*n).*(1.05409.*sin((pi.*n)/2) - 0.5.*cos((pi.*n)/2));
stem(n,Z)


%% Problem 5
zer = .9*exp(1+2*pi*(j*[-.4 .4]'));
pol = [-2.5 3]';
[b,a] = zp2tf(zer,pol,1);
fvtool(b,a)
zplane(zer,pol)


%% Problem 6.1
    clc; clear all;
    b = [0 ;1 ; -2]';
    a = [1 ;.5 ; 0]';
    Z = zplane(b,a)
    G = tf(b,a)
    pzmap(Z)
    grid on

%%  Problem 6.2
    clc; clear all;
    b = [1 ; 4 ; 8]'; %numerator
    a = [1 ;-.5 ; -7.5]';%denominator
    Z = zplane(b,a)
    G = tf(b,a)
    pzmap(Z)
```

```
grid on
```