ECE 316: Signals and Systems II **Spring 2021**

Homework 4
Due: Friday May 7, 11:59PM PT

Instructor Name: Taisa Kushner

Do not be alarmed by the length of this assignment. There is a lot of explanatory text to assist you with coding, and the questions themselves should be very doable. The goal is to have you work with Simulink and become comfortable designing and testing system models for simple LTI systems, and knowing if your answer "makes sense" with what you have learned in class. In order to receive credit for these problems, you must provide the requested screenshots of your Simulink diagrams and output plots (for the code portions), explain your reasoning, and show all your work for the non-code tasks. Partial credit will only be awarded if your reasoning (steps you worked through) is correct, and problems with correct answers but no work shown will receive 0 points.

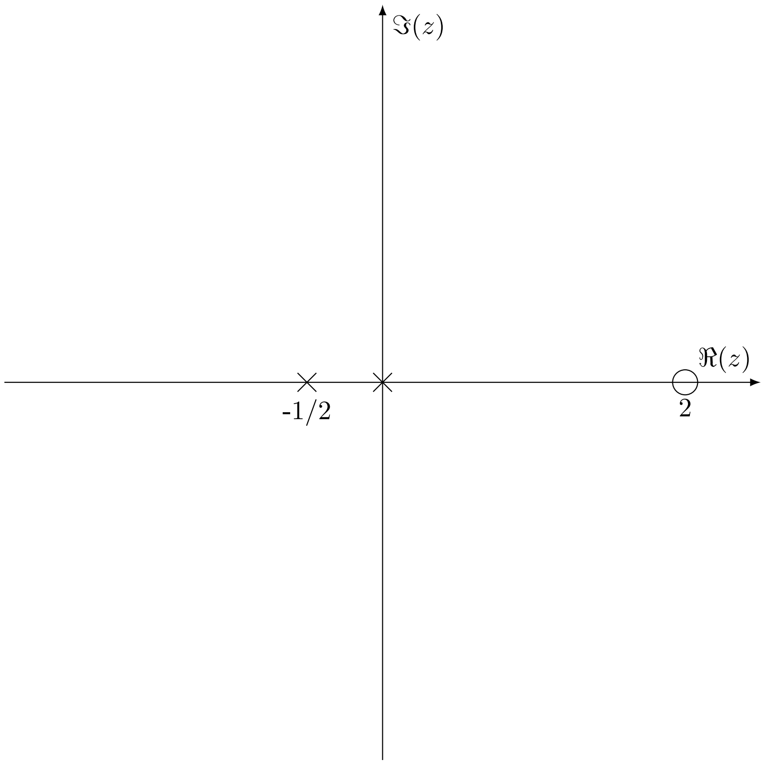
NOTE:

If you work with other students for this assignment, please write the names (first and last) of every student at the top of your assignment (below your name and the assignment title). As well as a couple sentences on how you worked together. Example follows:

> **Classmates worked with:**
>
> Ada Lovelace, Grace Hopper
>
> **Details:**
>
> I worked with Ada on Problems 2-5 we did problem 2 independently, but checked our answers together, we did the other problems together. We disagreed on part 3 of 5, but Ada helped me understand the reasoning. I worked with Grace on problem 8, we checked if our diagrams matched. I completed the rest on my own.

Problem 1 (10 pts)

For this problem, you will analyze the system described by the pole zero diagram from HW3 Problem 6A using Simulink. The diagram is reprinted below:



1. We define the *steady state gain* of a system to be the ratio of the output to the input of the system, $\frac{y(t)}{x(t)}$ at "steady state". **Steady state** means "what happens to the system when the output is no longer responding to an input". Note this means to reach steady state, a system should be **stable** and not constantly being poked and prodded with input. Being able to determine steady state behavior of a system is a very useful skill and you will now learn how to do this for LTI systems described by constant-coefficient differential (or difference) equations :)

   Conveniently, when working with LTI systems described by linear constant-coefficient differential equations (in continuous time)

$$a_0 \frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_n y(t) = b_0 \frac{d^m x}{dt^m} + b_1 \frac{d^{m-1} x}{dt^{m-1}} + \cdots + a_m x(t)$$

   or difference equations (in discrete time):

$$a_0 y[t-n] + a_1 y[t-(n-1)] + \cdots + a_n y[t] = b_0 x[t-m] + b_1 x[t-(m-1)] + \cdots + a_m x[t]$$

   we have found that we can convert our system to the frequency domain by taking the Laplace transform (for differential equations, continuous time) or Z-transform (for difference equations, discrete time) of the equations (with zero initial conditions), to get a system transfer function which is the ratio of the output to the input

$$H(s) = \frac{Y(s)}{X(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_n}{a_0 s^m + a_1 s^{m-1} + \cdots + a_m}$$

   or

$$H[z] = \frac{Y[z]}{X[z]} = \frac{b_0 z^n + b_1 z^{n-1} + \cdots + b_n}{a_0 z^m + a_1 z^{m-1} + \cdots + z_m}$$

From this form, we can determine the *steady state gain* of our system by observing the system response to the unit step input $x(t) = \delta(t) = e^{st}$ with $s = 0$, or $x[t] = \delta[n]$ which likewise amounts to setting $z = 1$. **We will now find the expected steady state behavior of the system described by the pole zero diagram above**.

(a) Write the transfer function for the pole-zero diagram above. Note, you did this in HW 3 already.

(b) Determine the expected *steady state gain* by setting $z = 1$ and computing the value of $H[z] = H[1]$. Record your answer.

2. Create a Simulink model for the system described by this pole-zero diagram using the three block model we worked with in class (source → transfer function → sink). Use the "Discrete Transfer Fcn" block for your transfer function, and a step function source block. Provide a screen shot of your final diagram.

3. Plot the response of the system to a step function input with step time = 1, initial value = 0, and final value = 1. Provide a plot of the output, running the model for long enough time to see the "steady state" (the output stops changing) and answer the following:

   (a) What value does the the system converge to? Note, you will have to add an "out1" block to your diagram in order to pull the output values into Matlab, we did this in class as well.

   (b) Does this match your expected *steady state* gain?

4. Now change your source to be a continuous "Sine wave" block, plot the output. Does your system reach a constant steady state? Does this result make sense to you given the definition of steady state and the sine wave input? Why or why not.

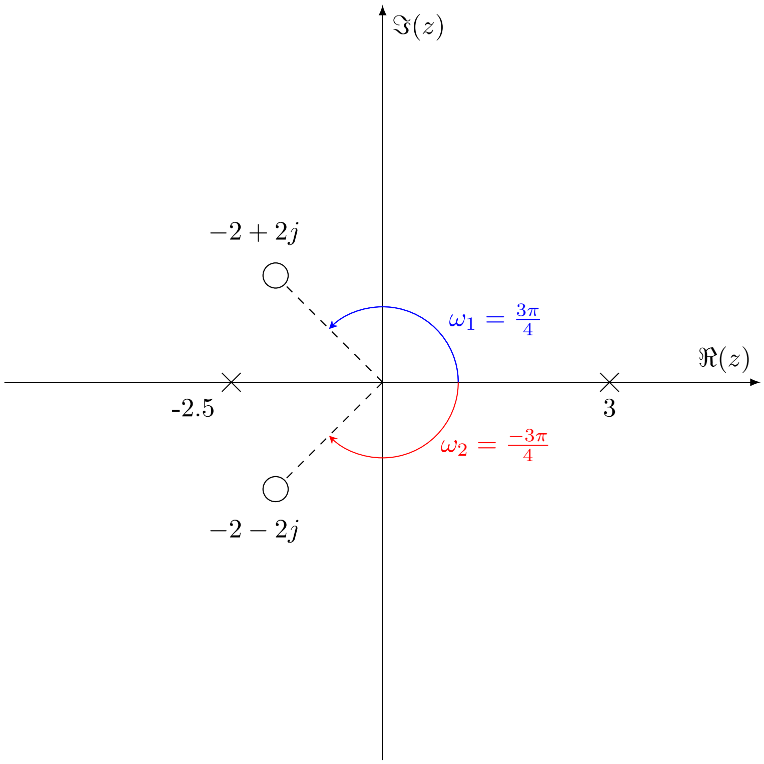
5. Now change your transfer function block to be a s-plane "Transfer Fcn" block. Keep the coefficients the same as you had in the Discrete Transfer Fcn block.

   (a) Plot the system response to the unit step input.

   (b) Does this system converge to a steady state? Does the result make sense to you given what you know about steady state requirements (system is stable, and no longer responding to input) and the stability of a system defined by a pole-zero diagram in the s-plane? Note you will have to re-analyze stability as if the pole-zero diagram axis were $Re(s)$ and $Im(s)$ instead of $Re[z]$ and $Im[z]$.

Problem 2 (4 pts)

We will now work with the pole-zero diagram from HW3 Problem 6B, shown below.



1. Create a Simulink model of the system described by this pole-zero diagram using the three-block model of (source → transfer function → sink) using the step function source, and the "Discrete Zero-Pole" transfer function. Note how the inputs to this function are the locations of your poles and zeros instead of the coefficients of the difference equation. When inputing poles and zeros, use i and not j to denote the imaginary unit. Include a screenshot of this model.

2. Does this model reach steady state? Does this behavior make sense to you? Explain why or why not. In addition, if you find the system reaches steady state: record the steady state gain value.

Problem 3 (4pts)

For this problem, we will keep the same base diagram from Problem 2, but change the locations of the poles and zeros. There are infinite correct answers for this question, the point is to create a transfer function that give you the intended behavior.

1. Change the poles such that you have an unstable system. Provide a screenshot of your diagram as well as the system response plot to the unit step function.

2. Change the poles such that you have a stable system. Provide a screenshot of your diagram as well as the system response plot to the unit step function.

Problem 4 (4pts)

Now we want to design a transfer function that gives us a specific steady state gain of 3. There are an infinite number of correct (and incorrect) answers to this question as well.

1. Knowing what you learned in Problem 1 about how to find steady state gain of a system in the z-plane (define transfer function as a ratio of polynomials, set $z = 1$ and determine the result), write down a

transfer function $H[z] = \frac{Y[z]}{X[z]}$ such that the steady state gain is 3, and the degree of the polynomial in the denominator is 4.

2. Create a Simulink block diagram of the model described by this transfer function. Use the unit step function as input, whichever transfer block you prefer (I would suggest the "Discrete Transfer Fcn" for ease), and remember to have a scope block to plot the output. Include a screenshot of this diagram as well as the steady state behavior of the system. Does your system converge to 3? If so, excellent! If not, think about why this may be, return to part (4.1) and try again.

Problem 5 (4pts)

Suppose we have a signal described by the following function

$$x(t) = 4\sin(18\pi t) + 3\cos(10\pi t)$$

1. Determine the frequencies present in the signal, and find the Nyquist sampling rate in number of samples per period.
2. Determine the fundamental frequency.
3. If we want to sample at a rate higher than Nyquist (pick number of samples per period = Nyquist + 1), and at a multiple of the fundamental frequency, how many samples per second should we take?

Problem 6 (2pts)

Suppose we have a bandlimited signal with max frequency $f_m = 75$Hz, what rate should we sample at to be able to reconstruct our original signal?

Problem 7 (2pts)

Suppose we have a signal such that the frequency range is $40 < |f| < 75$ (in Hz), what is the minimum sampling rate we should select to be able to reconstruct our original signal while avoiding over-sampling?

Problem 8 (4pts)

Explain why aliasing occurs, and why, when sampling continuous signals, we want to ensure we sample a bandlimited signal. In other words: why is aliasing undesirable, and what makes bandlimited signals useful?