Your name: Joshua Hoshiko
Assignment name: Homework 3
Date submitted: 9/16/19
Time spent on assignment: ~2 Hours

"How'd it go?" Overall, I think that the assignment went pretty well. It went pretty smooth for the most part!

Any remaining questions on the material? I don't think I have any that I can think of.

Who you collaborated with or got help from (if anyone), and what references you consulted beyond the text and course notes. This assignment was completed alone.

If this is an incomplete assignment, what is missing, or not working? Be specific. This assignment is complete.

Additional discussions specified for an individual assignment. None.

Anything else? No, but thank you!!

Joshua Hoshiko

Theory of Computation
Fall 2019, September 10
Dr. Gurka

Homework #3 – Pumping Lemma & NFA > DFA

Due: Tuesday, September 17, beginning of class, on paper; also post to Moodle, time-stamped before class. Both packets should include a cover letter (template on Moodle). No late assignments accepted; partial submissions will receive partial credit. Read specifications carefully. All solutions must be readable.

Part 1: Pumping lemma for regular languages

1. In-class exercise (no submission): Using the pumping lemma for regular languages, show that the following languages are not regular.

    a. L1 = {w#w}, Σ = {a, b, c}
    b. L2 = {$a^n b^n c^n$, n >= 0}, Σ = {a, b, c}

2. Homework. Using the pumping lemma for regular languages, show that the following languages are not regular.

    a. L3 = {$c^m b^n a^n$}, Σ = {a, b, c}
    b. L4 = {$0^n 1^m$, m = n+2}, Σ = {0, 1}

Part 2: Converting NFA to DFA

Convert the following NFA to DFA. Start with an NFA diagram, then show your first DFA diagram (before any minimizations). Your final DFA should have a complete formal definition plus the diagram. Remove any unreachable states; collapse equivalent states. Do not rename states (i.e., leave them with multiple, old-state names such as {1,3,4}). Total for each problem: NFA, unreduced DFA, reduced DFA (or, if not reduction is possible, say so), and the DFA formal definition. (Notes: an unreachable state is one that is not the start state and has no in-edges. Equivalent states are any that are both accept or both non-accept, and have exactly the same out edges; in-edges don't matter.) Σ = {a, b, c}

3. In-class exercises (no submission):
    a. L5 = {all strings exactly 2 characters long}
    b. L6 = {ab*(c U epsilon)}

4. Homework, to be submitted: convert the following NFA to DFA
    a. L7 = {end in 'b' or 'c'}
    b. L8 = {start with 'bb'}
    c. L9 = {[end with 'cc'] U [start with 'a']}
    d. pick a challenging NFA to convert from Sipser # 1.7 (state which)
    e. pick another challenging NFA to convert from Sipser # 1.7 (state which)

Homework #3

Joshua Hoshiko

2. a.) $L_3 = \{c^m b^n a^n\}$, $\Sigma = \{a, b, c\}$, $S = c^m b^p a^p$

Case 1: $\underset{x}{c^m}\ \underset{y}{b^p}\ \underset{z}{a^p}$ Invalid division, $|xy| > p$

Case 2: $\underset{x}{c}\ \underset{y}{b^{p-1}}\ \underset{z}{ba^p} \xrightarrow{\text{Pump}} \underset{x}{c}\ \underset{y}{b^{p-1}}\ \underset{y}{b^{p-1}}\ \underset{z}{ba^p} \rightarrow cb^{2p-1}a^p$

The generated string is not in L_3

Case 3: $\underset{x}{cb^p}\ \underset{y}{a}\ \underset{z}{a^{p-1}}$ Invalid division, $|xy| > p$

Case 4: $\underset{x}{\varepsilon}\ \underset{y}{c^m}\ \underset{z}{a^p b^p} \xrightarrow{\text{Pump}} \underset{x}{\varepsilon}\ \underset{y}{c^m}\ \underset{y}{c^m}\ \underset{z}{a^p b^p}$

Invalid division, $c^m \neq c^{2m}$

Based on the prior examples, all cases lead to contradictions and therefore L_3 is not regular

b.) $L_4 = \{0^n 1^m, m = n+2\}$, $\Sigma = \{0, 1\}$, $S = 0^p 1^{p+2}$

Case 1: $\underset{x}{\varepsilon}\ \underset{y}{0^p}\ \underset{z}{1^{p+2}} \xrightarrow{\text{Pump}} \underset{x}{\varepsilon}\ \underset{y}{0^p}\ \underset{y}{0^p}\ \underset{z}{1^{p+2}} \longrightarrow 0^{2p}1^{p+2}$ fails because the generated string is not in the language because $m = n+2$, $m = 2p$ and $n \neq 2p+2$

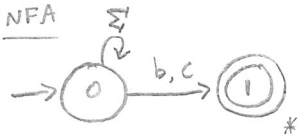
Case 2: $\underset{x}{0^p}\ \underset{y}{1^{p+2}}\ \underset{z}{\varepsilon}$ Invalid division, $|xy| > p$

Case 3: $\underset{x}{0^{p-2}}\ \underset{y}{0^2 1^2}\ \underset{z}{1^p}$ Invalid division, $|xy| > p$

Because all cases result in contradictions, L_4 is not regular

NFA → DFA

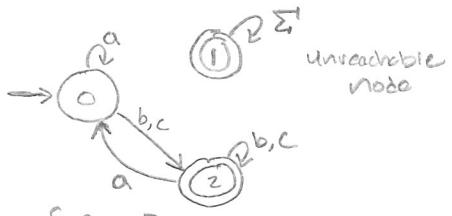
4. a) NFA



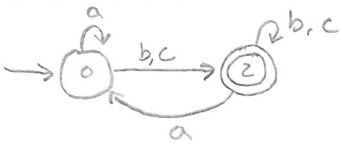
transition table

	a	b	c
0	0	0,1	0,1
1	—	—	—
*2=0,1	0	0,1	01

unreduced DFA



reduced DFA



δ =

	a	b	c
0	0	2	2
2	0	2	2

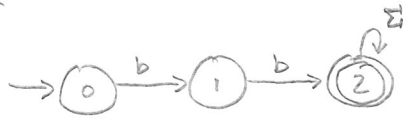
$Q = \{0, 2\}$

$S = 0$

$F = \{2\}$

$\Sigma = \{a, b, c\}$

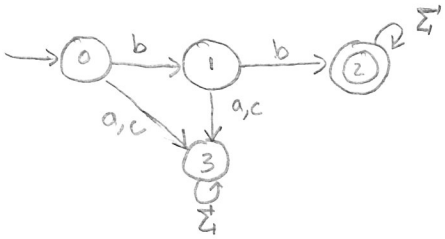
b.) NFA



transition table

	a	b	c
0	—	1	— =3
1	—	2	—
*2	2	2	2
3	3	3	3

unreduced DFA



reduced DFA

not required

δ =

	a	b	c
0	3	1	3
1	3	2	3
2	2	2	2
3	3	3	3

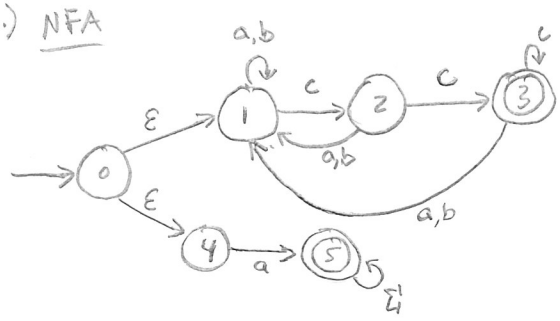
$Q = \{0, 1, 2, 3\}$

$S = 0$

$F = \{2\}$

$\Sigma = \{a, b, c\}$

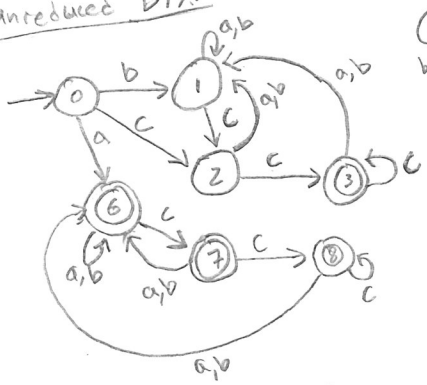
c.) NFA

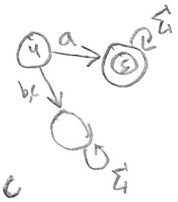


transition table

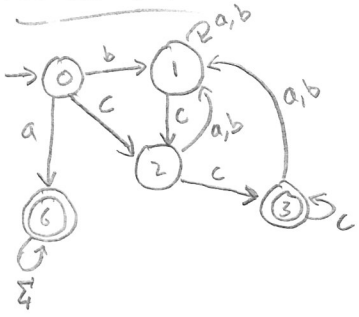
	a	b	c
0	1,5	1	2
1	1	1	2
2	1	1	3
*3	1	1	3
4	5	—	—
*5	5	5	5
*6=1,5	1,5	1,5	2,5
*7=2,5	1,5	1,5	3,5
*8=3,5	1,5	1,5	3,5

unreduced DFA





reduced DFA



δ =

	a	b	c
0	6	1	2
1	1	1	2
2	1	1	3
3	1	1	3
6	6	6	6

$Q = \{0, 1, 2, 3, 6\}$

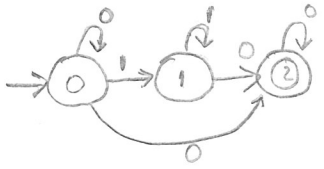
$S = 0$

$F = \{3, 6\}$

$\Sigma = \{a, b, c\}$

(4.) e.) (Page 84) $0^*1^*0^+$ with 3 States, $\Sigma = \{0,1\}$

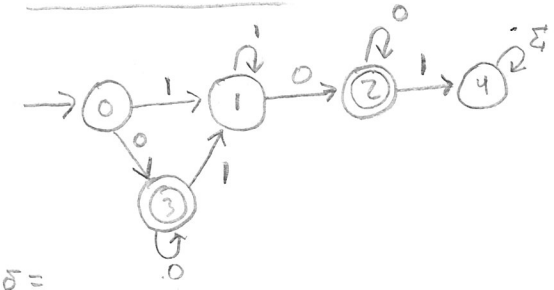
NFA $= 0^*1^*00^*$



transition table

	0	1
0	0,2	1
1	2	1
*2	2	— = 4
*3 = 0,2	0,2	1

non-reduced DFA



reduced DFA

(doesn't need to be reduced)

$\Sigma = \{0,1\}$

$Q = \{0,1,2,3,4\}$

$S = 0$

$F = \{2,3\}$

$\delta =$

	0	1
0	3	1
1	2	1
2	2	4
3	3	1
4	4	4

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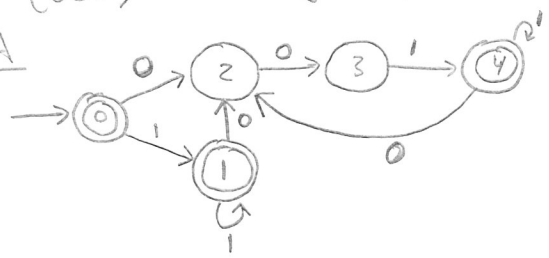
f.) $1^*(001^+)^* \rightarrow 1^*(0011^*)^*$

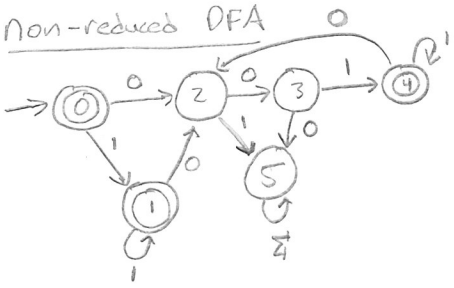
NFA



transition table

	0	1
*0	2	1
*1	2	1
2	3	— = 5
3	—	4
*4	2	4

non-reduced DFA



reduced DFA

(doesn't need to be reduced)

$\Sigma = \{0,1\}$

$Q = \{0,1,2,3,4,5\}$

$S = 0$

$F = \{0,1,4\}$

$\delta =$

	0	1
0	2	1
1	2	1
2	3	5
3	5	4
4	2	4
5	5	5