CS 4050 Test 3 Name Joshua Hoshiko

The questions are equally weighted. You may use any materials that you had before you started the test (no searching for additional materials online, and no communicating with anyone other than instructor)

1. Recall the transportation problem, which was a special form of LP instance that we solved by the simplex method for Exercise 24.

Consider this proposed algorithm for solving the transportation problem:

> Repeat:
>
> > find the cell with the smallest cost number (written in the upper left corner for each cell) that is unfilled
> >
> > write in that cell the largest amount you can, subject to not violating the constraints that the row and column totals are less than or equal to the given numbers
>
> until the amounts written fulfill all the row and column totals

For example, on this instance:

	1	2	3	4	5	
1	3	7	11	4	2	10
2	5	9	4	2	8	15
3	6	1	9	4	7	12
	8	6	10	7	6	

we would find the cell with cost 1 and fill it as much as possible:

	1	2	3	4	5	
1	3	7	11	4	2	10
2	5	9	4	2	8	15
3	6	1 6	9	4	7	12
	8	6	10	7	6	

Then we would use the cell with cost 2 (to break ties, use the uppermost row, and then the leftmost column, among cells with the same costs), obtaining:

	1	2	3	4	5	
1	3	7	11	4	2 6	10
2	5	9	4	2	8	15
3	6	1 6	9	4	7	12
	8	6	10	7	6	

Then we would fill the other cell with cost 2, obtaining:

	1	2	3	4	5	
1	^{3}	^{7}	^{11}	^{4}	^{2} 6	10
2	^{5}	^{9}	^{4}	^{2} 7	^{8}	15
3	^{6}	^{1} 6	^{9}	^{4}	^{7}	12
	8	6	10	7	6	

Next we would fill the cell with cost 3, obtaining:

	1	2	3	4	5	
1	^{3} 4	^{7}	^{11}	^{4}	^{2} 6	10
2	^{5}	^{9}	^{4}	^{2} 7	^{8}	15
3	^{6}	^{1} 6	^{9}	^{4}	^{7}	12
	8	6	10	7	6	

Then the first cell with cost 4 would have a 0 put in it, because column 4 is used up (and so is row 1, for that matter), and we would then fill the next cell with cost 4, obtaining:

	1	2	3	4	5	
1	^{3} 4	^{7} 0	^{11} 0	^{4} 0	^{2} 6	10
2	^{5} 0	^{9} 0	^{4} 8	^{2} 7	^{8} 0	15
3	^{6} 4	^{1} 6	^{9} 2	^{4} 0	^{7} 0	12
	8	6	10	7	6	

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a. [3 points] In the chart above (or written somewhere separately is that's more convenient), finish this example of the algorithm on the given instance.

b. [5 points] Now you have to decide whether this algorithm always produces the optimal cost, or whether it can fail. If you think that the algorithm is correct, just say so. If you think that it can fail, find the simplest (fewest number of cells) example you can for which this greedy algorithm fails—draw the chart, show how the greedy algorithm fills it in, and then show that there is a lower cost way of choosing the amounts for the cells.

greedy:

	1	2	
1	^{3} 1	^{3} 1	2
2	^{4} 0	^{7} 2	5
	1	3	

Cost = 3+3+14= 20

However, this chart results in a lower cost

	1	2	
1	^{3} 1	^{3} 2	2
2	^{4} 0	^{7} 1	5
	1	3	

Cost: 3+6+7= 16

2. The file `test3-2` contains the data for a tableau for an instance of LP with 9 variables. Your job on this problem is to use `ManualSimplex` on this data file and solve the instance.

For you answer you just need to state here clearly the answers to these questions:

What are the feasible choices for basic variables (list them below):

- I will upload the Chart Seperately, it is too large

feasable Combinations:

$X_1 X_2 X_6$

$X_1 X_4 X_6$

$X_1 X_5 X_6$

$X_1 X_6 X_7$

$X_2 X_3 X_6$

$X_3 X_4 X_6$

$X_3 X_5 X_6$

$X_3 X_6 X_7$

What is the optimal objective function value? 17.366412

What are the values of the basic variables for the optimal choice of basic variables that gives this optimal objective function value?

$X_5 = 2.89$

$X_6 = 1.24$

$X_1 = 1.45$

question 2

b-Variables	Feasable	
$X_1 X_2 X_3$	no	
$X_1 X_2 X_4$	no	
$X_1 X_2 X_5$	no	
$X_1 X_2 X_6$	y	$X_1 X_2 X_6$
$X_1 X_2 X_7$	n	
$X_1 X_3 X_4$	n	
$X_1 X_3 X_5$	n	
$X_1 X_3 X_6$	n	
$X_1 X_3 X_7$	n	
$X_1 X_4 X_5$	n	
$X_1 X_4 X_6$	y	$X_1 X_4 X_6$
$X_1 X_4 X_7$	n	
$X_1 X_5 X_6$	y	$X_1 X_5 X_6$
$X_1 X_5 X_7$	n	
$X_1 X_6 X_7$	y	$X_1 X_6 X_7$
$X_2 X_3 X_4$	n	
$X_2 X_3 X_5$	n	
$X_2 X_3 X_6$	y	$X_2 X_3 X_6$
$X_2 X_3 X_7$	n	
$X_2 X_4 X_5$	n	
$X_2 X_4 X_6$	n	
$X_2 X_4 X_7$	n	
$X_2 X_5 X_6$	n	
$X_2 X_5 X_7$	n	
$X_2 X_6 X_7$	n	
$X_3 X_4 X_5$	n	
$X_3 X_4 X_6$	y	$X_3 X_4 X_6$
$X_3 X_4 X_7$	n	
$X_3 X_5 X_6$	y	$X_3 X_5 X_6$
$X_3 X_5 X_7$ ← $X_3 X_6 X_7$ y	n	$X_3 X_6 X_7$
$X_4 X_5 X_6$	n	
$X_4 X_5 X_7$	n	
$X_4 X_6 X_7$	n	
$X_5 X_6 X_7$	n	

3. Consider this optimization problem:

$$\begin{aligned} \max \quad & 5x_1 + 4x_2 + 6x_3 \quad \text{subject to} \\ & -x_1 + 6x_2 + 2x_3 \geq 40 \\ & 3x_1 + 3x_2 - x_3 \leq 30 \\ & 2x_1 + 2x_2 + 7x_3 \leq 12 \\ & 5x_1 + 5x_2 + 6x_3 = 18 \\ & x \geq 0 \end{aligned}$$

Convert inequalities to equalities by adding slack and surplus variables, and add artificial variables to provide basic variables for constraints that don't have a handy slack variable to be basic, and write the Phase 1 problem to be solved by the simplex method:

$$\begin{aligned} & z - 5x_1 - 4x_2 - 6x_3 = 0 \\ & -x_1 + 6x_2 + 2x_3 - s_1 + a_1 = 40 \\ & 3x_1 + 3x_2 - x_3 + s_2 = 30 \\ & 2x_1 + 2x_2 + 7x_3 + s_3 = 12 \\ & 5x_1 + 5x_2 + 6x_3 = 18 \\ & x \geq 0 \\ & s \geq 0 \\ & a \geq 0 \end{aligned}$$

Write here (you may not need all the spots provided in the grid, or you might need more—it's just there for convenience) the tableau that you would use to solve this instance. Be sure to show column labels and what variables will be basic at the beginning of Phase 1:

	z	x_1	x_2	x_3	s_1	s_2	s_3	a_1	rhs
z	1	0	0	0	0	0	0	1	0
a_1	0	-1	6	2	-1	0	0	1	40
s_2	0	3	3	-1	0	1	0	0	30
s_3	0	2	2	7	0	0	1	0	12
-	0	5	5	6	0	0	0	0	18

4. The file `test3-4` contains data for a tableau for any instance of LP, ready to "price out" to start the Phase 1 simplex method.

Your job is to perform the Phase 1 simplex method, and possibly determine that the instance is infeasible (in which case you should say so below and stop).

If the instance turns out to be feasible, put in this Phase 2 objective function:

$$\max 7x_1 + 11x_2 + 8x_3$$

and perform the Phase 2 simplex method.

You may find out that the problem is unbounded (if so, say so below and stop), or you may continue until you find the optimal solution for the instance.

Write your conclusions here (say infeasible, unbounded, or state the optimal objective function value, which variables are basic for the optimal point, and what values those variables have at the optimal point):

The optimal objective function value: 252.5

$S_3 = 72$

$S_1 = 87.5$

$S_4 = 152.5$

$X_3 = 25$

$X_1 = 7.5$

5. (directions)

Demonstrate the branch and bound heuristic algorithm for the 0-1 knapsack instance with this data:

Item i	p_i	w_i	$\frac{p_i}{w_i}$
1	100	5	20
2	72	4	18
3	112	7	16
4	90	6	15
5	48	4	12

with the knapsack capacity being 15.

You must demonstrate the "best first" version of the algorithm that always explores the node with the best bound, where the bound is obtained by computing the profit that could be achieved, given the current choices at the node, if we were allowed to use fractional parts of the following item(s), and prunes nodes whenever their bound is less than a known achievable profit or their weight is too high.

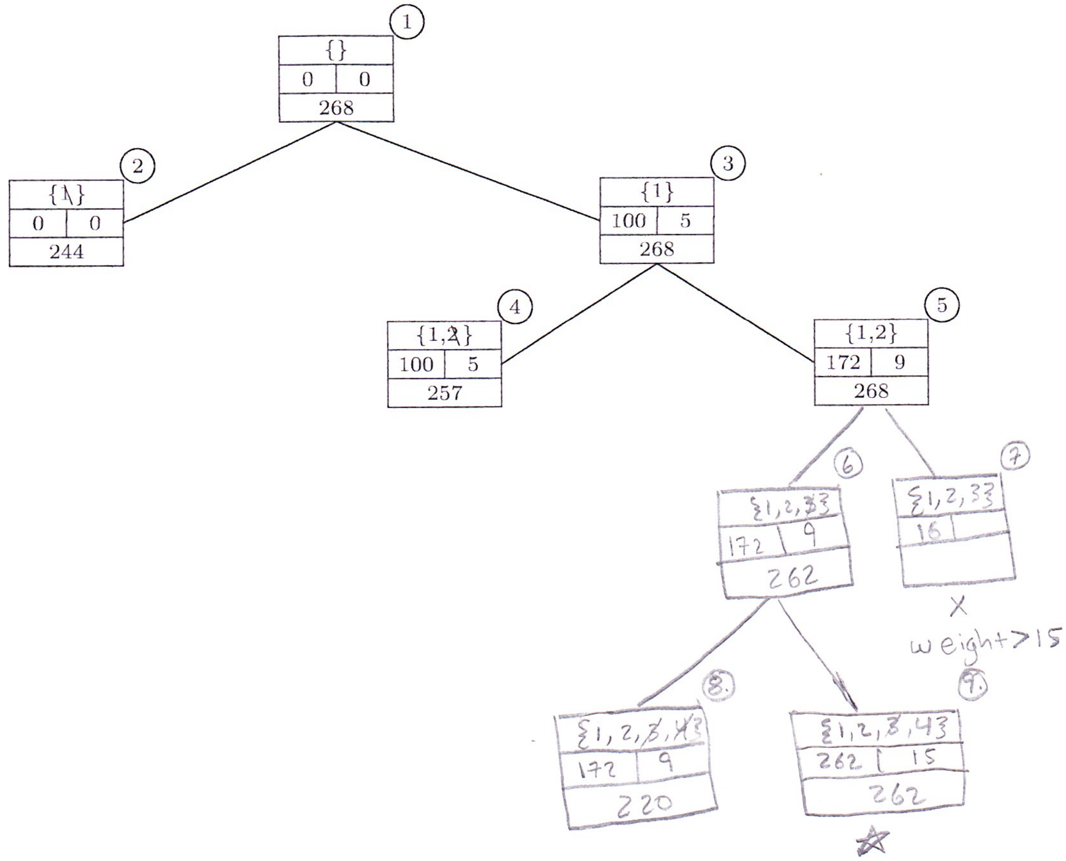
For each node that is drawn, use the format shown in the part that is already done on the next page, including numbering the nodes in the order they are added to the priority queue.

Write you answer on the next page. The first few nodes have already been done, to save you time and to show the desired format. This is a snapshot of the algorithm at the point where nodes 2, 4, and 5 are in the priority queue.

Whenever a node is pruned, write immediately below out why it has been pruned.

In general, show all your work and explain all your reasoning.

5. (answer)



Optimal Combination is 1, 2, 4,
with weight 15 and Profit 262

6. Write down in mathematical form each of the following constraints for ETSP. Recall that x_{jk} is the intensity of the connection between vertex j and vertex k, and that $j < k$.

Assume that $n = 7$ for all parts.

a. [2 points] Write down the constraint that says that the sum of the intensities of the edges connected to vertex 3 is 2.

$$X_{13} + X_{23} + X_{34} + X_{35} + X_{36} + X_{37} = 2$$

b. [2 points] Write down the constraint that says that the intensity of the edge between vertex 2 and vertex 5 is at most 1.

$$X_{25} \leq 1$$

c. [2 points] Write down the constraints that says that the total intensity of all edges connecting a vertex in $\{1, 5\}$ to a vertex in ~~$\{2, 3, 4, 6\}$~~ is at least 2 (this is the "cut" of 1 and 5):

$\{2,3,4,6,7\}$

$$X_{12} + X_{13} + X_{14} + X_{16} + X_{17} + X_{25} + X_{35} + X_{45} + X_{56} + X_{57} \geq 2$$

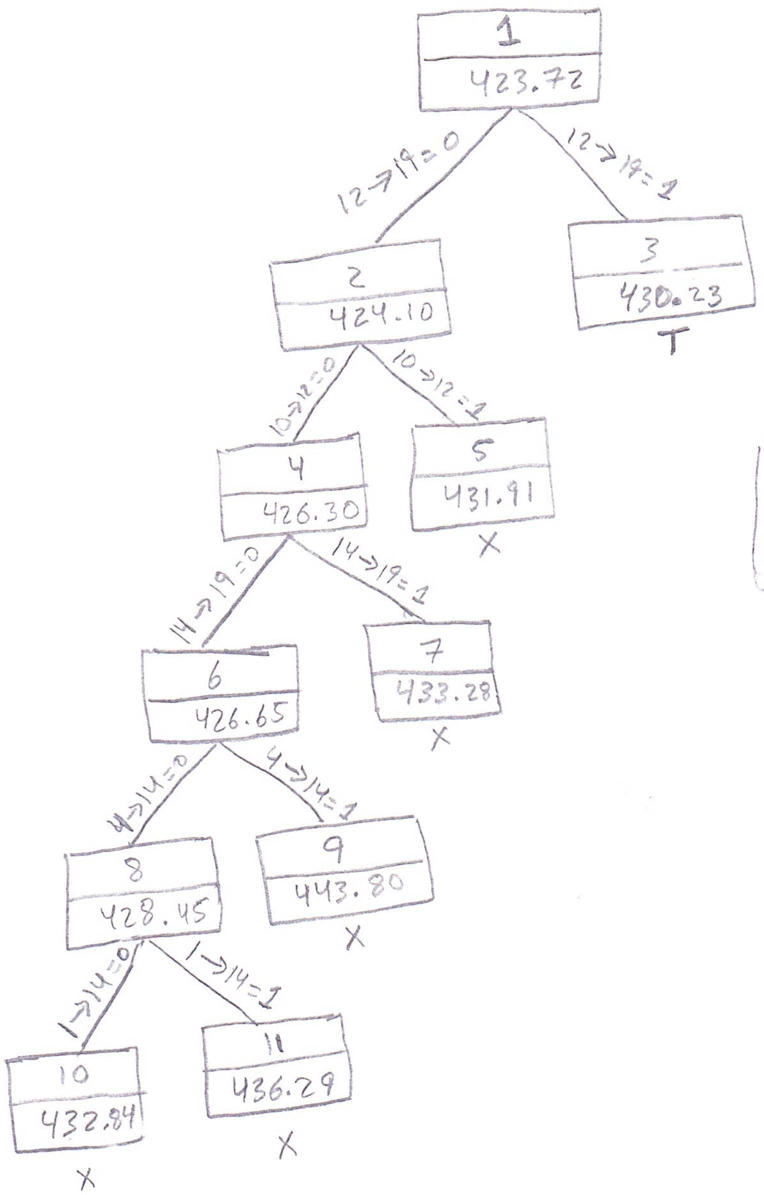
d. [2 points] Why would it be silly to add a cut constraint on just vertex 3?

each vertex Constraint already Specifies that the number of edges must be equal to 2. Adding a Constraint Saying ≥ 2 is unecessary because the previous Constraint is more restrictive.

7. Consider the ETSP instance with the 20 points given in the file `test3-7`. Your job on this problem is to use `AutoHeuristicTSP` (or `AutoTSPtext`) to solve this instance.

Draw the branch and bound tree showing the node number and score for each node (you can just draw the first, say, 10 nodes to show that you understand how it works, and then you can stop drawing). You don't have to write down the cuts you make (write the score after all cuts have been done), but you should clearly show on the edges what branching choices you are making.

Clearly state which node gives the optimal tour and state the total length of that tour.



Optimal tour is Node 3:
430.23