

Appendix S1. Linear regression models and non-linear population dynamics

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1 One species, logistic growth

Population growth over time in a single species is first modelled using a Beverton-Holt (discrete-time, logistic) model (Beverton and Holt (1957)), using an intra-specific competition coefficient for density-dependent growth (Hart and Marshall (2013)) - thus $\alpha = 1/K$.

$$N_{i,t+1} = \frac{r_i N_{i,t}}{1 + \alpha_{ii} N_{i,t}}$$

Note that in this model, the system is at equilibrium when $N_{i,t+1} = N_{i,t}$, and therefore:

$$N^* = N^* \frac{r_i}{1 + \alpha_{ii} N^*}$$

$$1 = \frac{r_i}{1 + \alpha_{ii} N^*}$$

$$N^* = \frac{r_i - 1}{\alpha_{ii}}$$

1.1 Population dynamics simulation

In the metacommunity simulation in the main text, a species resides in a site with an initial population size $N_{i,0} \sim \text{Pois}(10)$, a growth rate r_i that depends on the local environmental value E_k and the species trait x_i , and a fixed intra-specific competition coefficient of $\alpha_{ii} = 0.00125$. We simulate population growth here:

```
set.seed(42)
# Simulate initial species population growth
N1.0 <- rpois(1, 10)
r1.0 <- 1.67
alpha.11 <- 0.00125
# model function
disc_log <- function(r, N0, alpha) {
  Nt1 <- (r * N0) / (1 + alpha * N0)
```

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```

    return(Nt1)
}
# Simulation of model for t time steps
t <- 30
N <- rep(NA, t)
N[1] <- N1.0
for (i in 2:t) {
  N[i] <- disc_log(r = r1.0, N0 = N[i - 1], alpha = alpha.11)
}

```

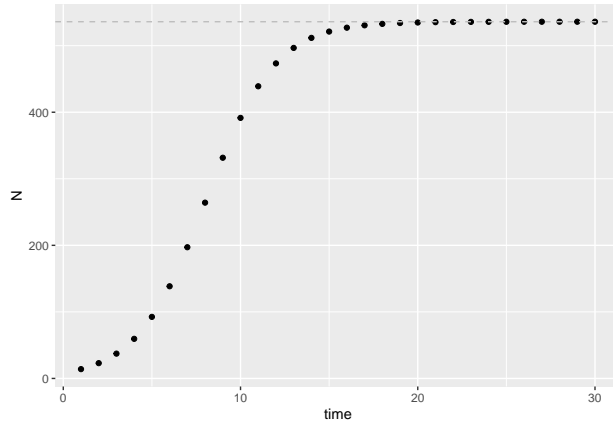


Figure 1: Population size N over time t for a discrete-time logistic growth model, with parameters $r_i = 1.67$, $N_{1,0} = 14$, and $\alpha_{11} = 0.00125$.

14 1.2 Linear statistical model

15 We fit the population time series data to a first-order auto-regressive model to predict N_{t+1} as a function of
 16 N_t , and compare that to a linear regression. We use \ln -N after Ives (1995), as also discussed in Certain et
 17 al. (2018) and Olivença et al. (2021).

$$N_{t+1} = \beta_0 + \beta_1 N_t + \epsilon_t$$

```

# Fit the model
m.1.ar <- arima(x = log(N), order = c(1, 0, 0), include.mean = T, method = "CSS")
m.1.lm <- lm(log(dat$N[2:t]) ~ log(dat$N[1:(t - 1)]))
# plotting the series along with the fitted values
m.1.ar.fit <- log(N) - residuals(m.1.ar)
m.1.lm.fit <- log(dat$N[2:t]) - m.1.lm$resid
dat$ar1.fit <- m.1.ar.fit
dat$lm.fit <- NA
dat$lm.fit[2:t] <- m.1.lm.fit

```

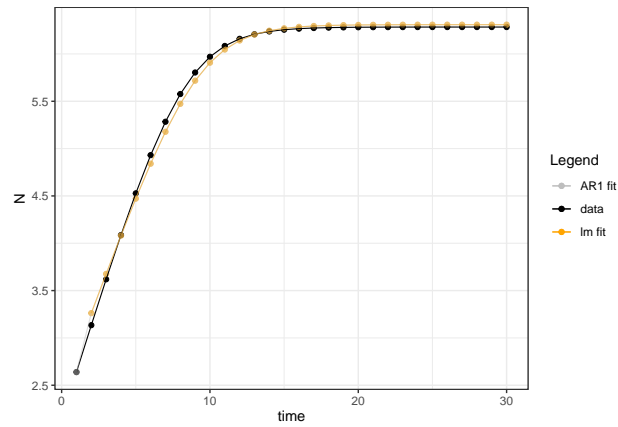


Figure 2: Population size over time (black line) with fitted values from a first-order autoregressive model (red dashed line).

- 18 The linear model is a good fit, and N_{t+1} and N_t are well-represented by a linear function:

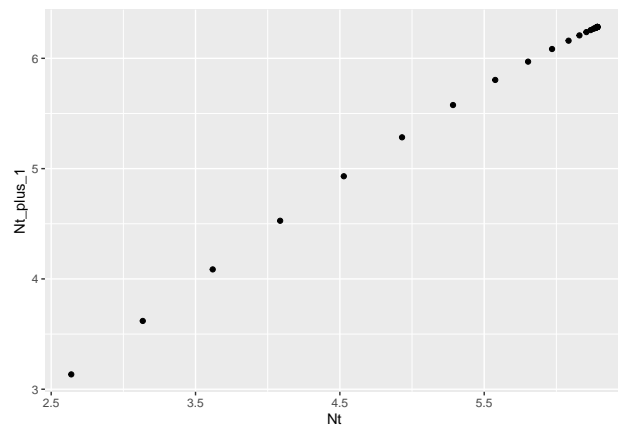


Figure 3: Population size (logarithm) at one time step N_{t+1} as a function of log-population size in the previous time step N_t .

- 19 We also examine density dependence by plotting $\Delta N = N_{t+1} - N_t$ vs. N_t :

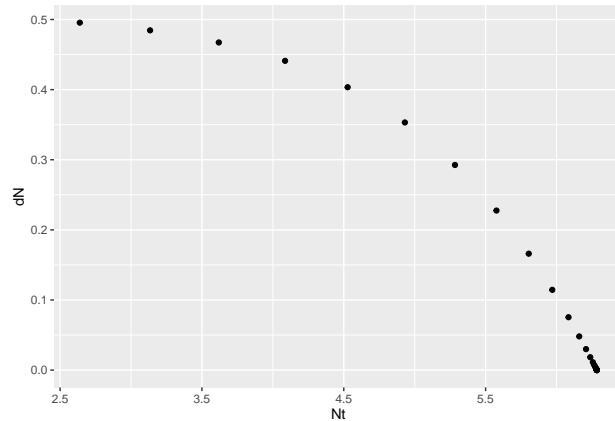


Figure 4: Change in population size from one time step to the next N_{t+1} as a function of N_{t+1}

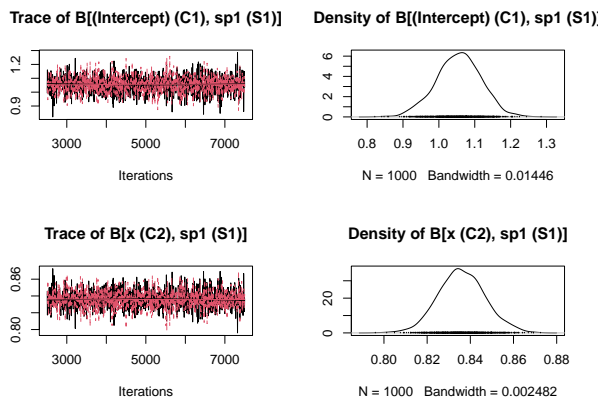
1.3 Bayesian linear statistical model: HMSC

We can estimate the same model parameters using HMSC:

```
# prepare data in HMSC format
Y <- as.matrix(log(dat$N[2:t]))
XData <- data.frame(x = log(dat$N[1:(t - 1)]))
m.1.hmsc <- Hmsc(Y = Y, XData = XData, XFormula = ~x)
# Bayesian model parameters
nChains <- 2
thin <- 5
samples <- 1000
transient <- 500 * thin
verbose <- 500 * thin
# sample MCMC
m.1.sample <- sampleMcmc(m.1.hmsc, thin = thin, sample = samples, transient = transient,
  nChains = nChains, verbose = verbose)
#> setting updater$GammaEta=FALSE due to absence of random effects included to the model
#> Computing chain 1
#> Chain 1, iteration 2500 of 7500 (transient)
#> Chain 1, iteration 5000 of 7500 (sampling)
#> Chain 1, iteration 7500 of 7500 (sampling)
#> Computing chain 2
#> Chain 2, iteration 2500 of 7500 (transient)
#> Chain 2, iteration 5000 of 7500 (sampling)
#> Chain 2, iteration 7500 of 7500 (sampling)

m.post.hmsc <- convertToCodaObject(m.1.sample)
summary(m.post.hmsc$Beta)
#>
#> Iterations = 2505:7500
#> Thinning interval = 5
#> Number of chains = 2
#> Sample size per chain = 1000
#>
#> 1. Empirical mean and standard deviation for each variable,
```

```
#>      plus standard error of the mean:
#>
#>               Mean      SD Naive SE Time-series SE
#> B[(Intercept) (C1), sp1 (S1)] 1.0550 0.06240 0.0013952      0.0013954
#> B[x (C2), sp1 (S1)]          0.8361 0.01083 0.0002422      0.0002415
#>
#> 2. Quantiles for each variable:
#>
#>           2.5%    25%    50%    75%    97.5%
#> B[(Intercept) (C1), sp1 (S1)] 0.9288 1.0132 1.0564 1.0974 1.1693
#> B[x (C2), sp1 (S1)]          0.8154 0.8287 0.8358 0.8431 0.8582
plot(m.post.hmsc$Beta)
```



22

23 These estimates match well with those from the AR1 and linear model:

```
# AR1 coefficients (recall that the intercept is the term below multiplied by 1
# - phi1)
m.1.ar$coef
#>      ar1 intercept
#> 0.8359839 6.4371388
m.1.ar$coef[2] * (1 - m.1.ar$coef[1])
#> intercept
#> 1.055794
# linear model
summary(m.1.lm)$coefficients[1:2, 1:2]
#>               Estimate Std. Error
#> (Intercept)      1.0557944 0.054425861
#> log(dat$N[1:(t - 1)]) 0.8359839 0.009441702
# Bayesian estimates
summary(m.post.hmsc$Beta)$statistics[1:2, 1:2]
#>               Mean      SD
#> B[(Intercept) (C1), sp1 (S1)] 1.0549596 0.06239605
#> B[x (C2), sp1 (S1)]          0.8360545 0.01082936
```

```
Gradient <- constructGradient(m.1.sample, focalVariable = "x", ngrid = 29)
predY <- predict(m.1.sample, Gradient = Gradient, expected = TRUE)
# preds <- computePredictedValues(m.1.sample)
plotGradient(m.1.sample, Gradient, pred = predY, showData = T, measure = "Y", main = "",
  xlab = "N_t", ylab = "predicted N_t+1")
```

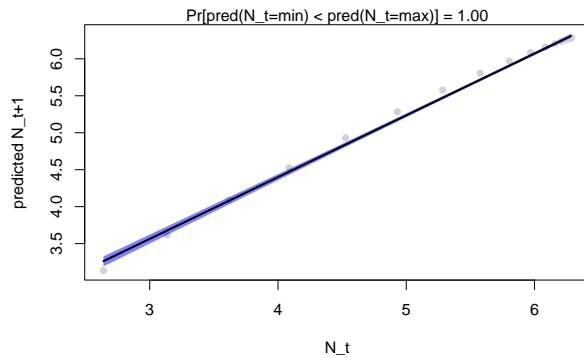
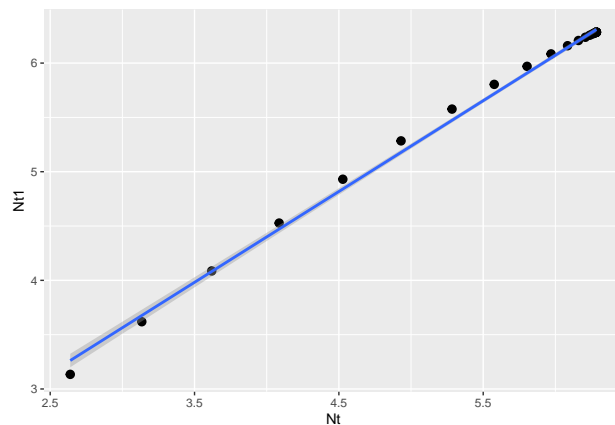


Figure 5: Observed (grey) and model-fit (blue) values for population size at time t (x-axis) and $t+1$ (y-axis).

```
lm_dat <- data.frame(cbind(log(dat$N[2:t]), log(dat$N[1:(t - 1)])))
colnames(lm_dat) <- c("Nt1", "Nt")
ggplot(lm_dat, aes(Nt, Nt1)) + stat_summary(fun.data = mean_cl_normal) + geom_smooth(method = "lm")
#> `geom_smooth()` using formula = 'y ~ x'
#> Warning: Removed 29 rows containing missing values (`geom_segment()`).
```



1.4 Conclusions

In this example, a first-order auto-regressive model works well, bypassing the need to estimate logistic growth parameters r_i and α_{ii} . The density-dependence dynamics ($\Delta N \sim f(N_t)$) show an overall declining trend over time. The Bayesian estimation implemented in HMSC gives good parameter estimates.

2 One species, logistic growth, environmental covariate

We now consider using a linear model to analyze population growth when the species growth rate is impacted by a single environmental covariate.

2.1 Growth depends on environment

First we add environment-dependent growth rate. The growth rate r_i becomes:

$$r_i = \hat{W}e^{-(E-x_{i,t})^2}$$

Here, \hat{W} is the maximal population growth rate (set to 1.67 as above), E is the local environmental trait optimum value, and $x_{i,t}$ is species i trait value at time t . We see that if $E = x_{i,t}$ then the growth rate is at the value $r = 1.67$. Here, we begin with $E = x_{i,t} = 0.8$, then simulate the environment E value fluctuating randomly over time, and finally use a linear model to fit E as a covariate.

```
# Simulate initial species population growth with environment fluctuations
N1.0 <- 10
r1.0 <- 1.67
alpha.11 <- 0.00125
E.0 <- 0.8
x1.0 <- 0.8
# model function
disc_log_E <- function(r, N0, alpha, E, x) {
  Nt1 <- ((r * exp(-(E - x)^2)) * N0)/(1 + alpha * N0)
  return(Nt1)
}
# Simulation of model for t time steps
t <- 40
N <- rep(NA, t)
N[1] <- N1.0
E <- rep(NA, t)
E[1] <- E.0
for (i in 2:t) {
  N[i] <- disc_log_E(r = r1.0, N0 = N[i - 1], alpha = alpha.11, E = E[i - 1], x = x1.0)
  E[i] <- E[i - 1] + rnorm(1, 0, 0.1)
}
```

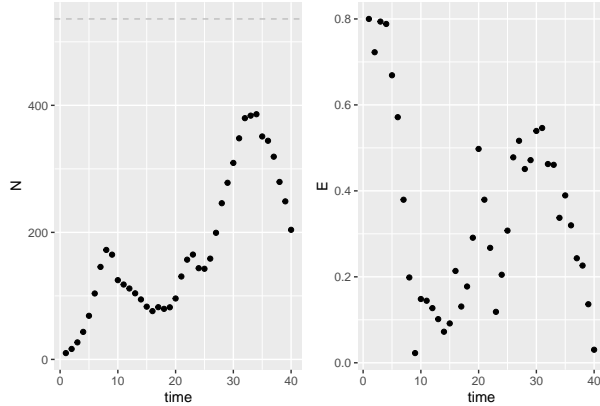


Figure 6: Population size N over time t for a discrete-time logistic growth model, with parameters $r_i = 1.67$, $N_{1,0} = 14$, and $\alpha_{11} = 0.00125$. Relationship between E and N_t is also shown.

2.2 Linear statistical model with environmental covariate

We now include environment E as a covariate in the linear model:

$$N_t = \beta_0 + \beta_1 N_{t-1} + \beta_2 E_{t-1} + \epsilon_t$$

```

# Fit the model
m.2.ar <- arima(x = log(N), order = c(1, 0, 0), include.mean = T, method = "CSS",
  xreg = E)
m.2.lm <- lm(log(dat$N[2:t]) ~ log(dat$N[1:(t - 1)]) + log(E[1:(t - 1)]))
# plotting the series along with the fitted values
m.2.ar.fit <- log(N) - residuals(m.2.ar)
m.2.lm.fit <- log(dat$N[2:t]) - m.2.lm$resid
dat$ar2.fit <- m.2.ar.fit
dat$lm2.fit <- NA
dat$lm2.fit[2:t] <- m.2.lm.fit

```

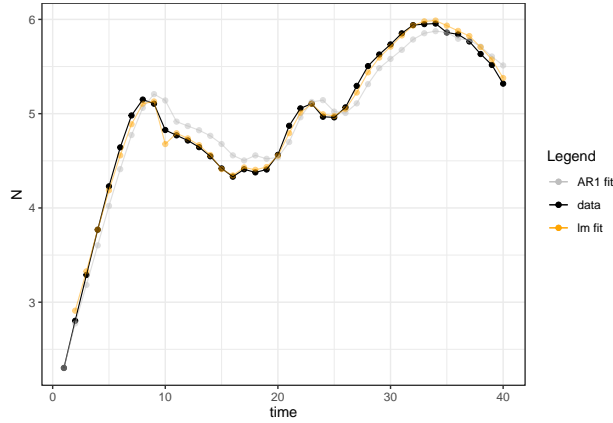


Figure 7: Population size over time (black line) with fitted values from a first-order autoregressive model (red dashed line).

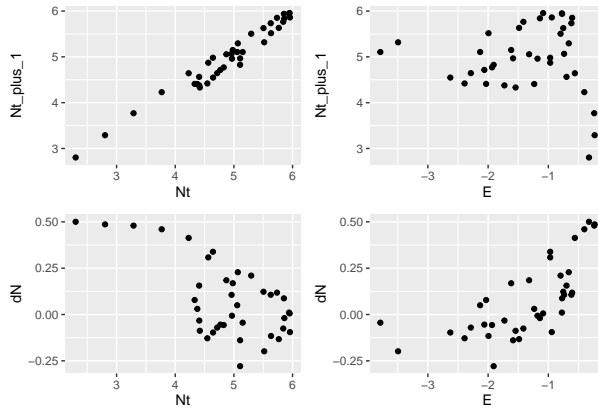


Figure 8: Population size (logarithm) at one time step N_{t+1} as a function of log-population size in the previous time step N_t .

40 The linear model is a good fit when including the environmental covariate. N_{t+1} and N_t can still be captured
 41 by a linear relationship. However we see that the relationship between N_{t+1} and E_t is non-linear. This tells
 42 us that the lm is good for predictions, but not for inference (for capturing well the relationship between
 43 the predictor and response variable). The use of linear relationships in JSDMs is discussed in (Ingram et
 44 al. 2020), and in many applications (e.g. (Erickson and Smith 2023)) quadratic terms are used, which

45 create bell-shaped response curves that may better match species with optimal niches (as opposed to linear,
 46 monotonically increasing relationships between population size and environmental predictors). We thus
 47 include a quadratic term for E_t to provide a better fit to the data.

```
df <- data.frame(cbind(log(dat$N[2:t]), log(dat$N[1:(t - 1)]), E[1:(t - 1)], E[1:(t - 1)]^2))
colnames(df) <- c("Nt1", "Nt", "E", "Esq")
m.2.lm <- lm(Nt1 ~ Nt + E + Esq, data = df)
```

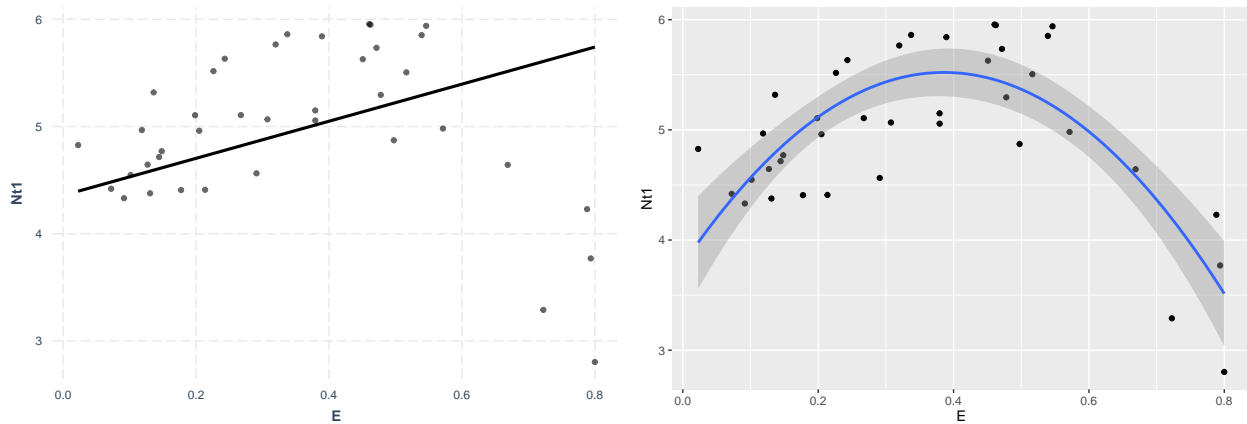


Figure 9: Population size (logarithm) at one time step N_{t+1} as a function of log-population size in the previous time step N_t .

48 2.3 Bayesian linear statistical model: HMSC

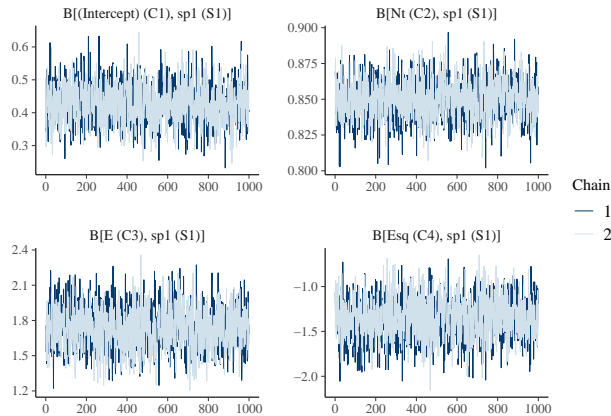
49 We can estimate the same model parameters using HMSC:

```
# prepare data in HMSC format
Y <- as.matrix(log(dat$N[2:t]))
XData <- df
m.2.hmsc <- Hmsc(Y = Y, XData = XData, XFormula = ~Nt + E + Esq)
# Bayesian model parameters
nChains <- 2
thin <- 5
samples <- 1000
transient <- 500 * thin
verbose <- 500 * thin
# sample MCMC
m.2.sample <- sampleMcmc(m.2.hmsc, thin = thin, sample = samples, transient = transient,
  nChains = nChains, verbose = verbose)
#> setting updater$GammaEta=FALSE due to absence of random effects included to the model
#> Computing chain 1
#> Chain 1, iteration 2500 of 7500 (transient)
#> Chain 1, iteration 5000 of 7500 (sampling)
#> Chain 1, iteration 7500 of 7500 (sampling)
#> Computing chain 2
#> Chain 2, iteration 2500 of 7500 (transient)
#> Chain 2, iteration 5000 of 7500 (sampling)
#> Chain 2, iteration 7500 of 7500 (sampling)
```

```

m2.post.hmsc <- convertToCodaObject(m.2.sample)
summary(m2.post.hmsc$Beta)
#>
#> Iterations = 2505:7500
#> Thinning interval = 5
#> Number of chains = 2
#> Sample size per chain = 1000
#>
#> 1. Empirical mean and standard deviation for each variable,
#>    plus standard error of the mean:
#>
#>               Mean      SD Naive SE Time-series SE
#> B[(Intercept) (C1), sp1 (S1)] 0.4287 0.05840 0.0013059    0.0013051
#> B[Nt (C2), sp1 (S1)]          0.8500 0.01357 0.0003034    0.0003136
#> B[E (C3), sp1 (S1)]           1.7365 0.17407 0.0038922    0.0039946
#> B[Esq (C4), sp1 (S1)]        -1.3544 0.22541 0.0050403    0.0051805
#>
#> 2. Quantiles for each variable:
#>
#>               2.5%      25%      50%      75%      97.5%
#> B[(Intercept) (C1), sp1 (S1)] 0.3125 0.3902 0.4285 0.467 0.5435
#> B[Nt (C2), sp1 (S1)]          0.8235 0.8413 0.8500 0.859 0.8768
#> B[E (C3), sp1 (S1)]           1.3967 1.6194 1.7361 1.853 2.0816
#> B[Esq (C4), sp1 (S1)]        -1.8157 -1.5001 -1.3518 -1.207 -0.9151
bayesplot::mcmc_trace(m2.post.hmsc$Beta)

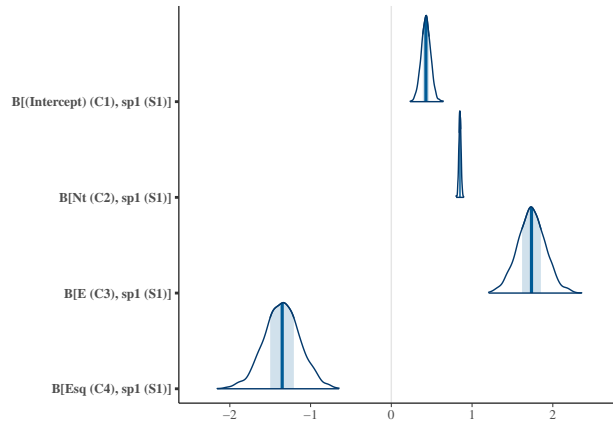
```



```

bayesplot::mcmc_areas(m2.post.hmsc$Beta, area_method = c("equal height"))

```



51

52 These estimates match well with those from the AR1 and linear model:

```
# AR1 coefficients (recall that the intercept is the term below multiplied by 1
# - phi1)
m.2.ar$coef
#>      ar1  intercept      E
#> 0.8471120 5.4224546 -0.1096173
m.2.ar$coef[2] * (1 - m.2.ar$coef[1])
#> intercept
#> 0.829028
# linear model
summary(m.2.lm)$coefficients[1:4, 1:2]
#>      Estimate Std. Error
#> (Intercept) 0.4286437 0.04917405
#> Nt          0.8502417 0.01151860
#> E           1.7296394 0.14595151
#> Esq        -1.3462240 0.18902070
# Bayesian estimates
summary(m2.post.hmsc$Beta)$statistics[1:4, 1:2]
#>      Mean      SD
#> B[(Intercept) (C1), sp1 (S1)] 0.4287409 0.05840051
#> B[Nt (C2), sp1 (S1)]          0.8500472 0.01356690
#> B[E (C3), sp1 (S1)]           1.7365191 0.17406609
#> B[Esq (C4), sp1 (S1)]        -1.3543664 0.22541119
```

53 We recall that the interpretation of the coefficients in an arimaX (arima with covariates) model is difficult.
 54 They do not give the impact on N_t per unit increase in X as in a regression. So we do not interpret the
 55 causation implied by the coefficient in the arimaX model. In the regression model, we can see that E has a
 56 positive impact on N_t .

```
Gradient <- constructGradient(m.2.sample, focalVariable = "E", non.focalVariables = list(Nt = list(2),
  Esq = list(2)), ngrid = 39)
# Esq is manually constructed as gradient-produced E^2
Gradient$XDataNew$Esq <- Gradient$XDataNew$E^2
predY <- predict(m.2.sample, XData = Gradient$XDataNew, expected = TRUE)
plotGradient(m.2.sample, Gradient, pred = predY, showData = T, measure = "Y", main = "",
  xlab = "E_t", ylab = "predicted N_t+1")
```

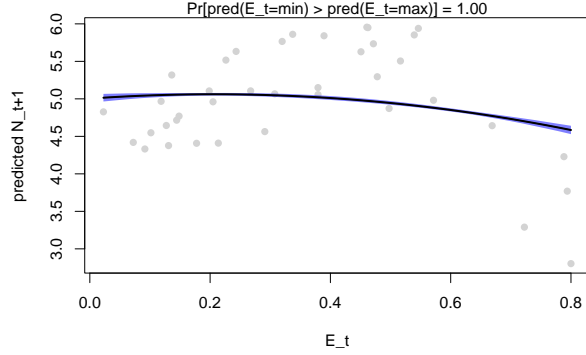


Figure 10: Observed (grey) and model-fit (blue) values for population size at time t (x-axis) and $t+1$ (y-axis).

2.4 Conclusions

In this example, the linear regression again works well to describe the impact of E_t for N_t when using the quadratic formulation. The arimaX model works well for fitting and subsequent prediction, but less well for inference about the impacts of E . From the quadratic regression terms for E , we correctly see that the population size is maximal at the species trait value and decreases away from that value. We will continue to use log-transformed abundance and now introduce quadratic terms for the environmental parameter.

3 Two species, logistic growth, competition

Here we investigate how a second species impacts the inference we can make from linear models.

3.1 Interspecific competition

The growth equation for each species now becomes:

$$N_{i,t+1} = \frac{r_i N_{i,t}}{1 + \alpha_{ii} N_{i,t} + \alpha_{ij} N_{j,t}}$$

We use a distinct growth rate for the 2nd species and introduce the interspecific interaction coefficient α_{ij} .

```
# Simulate initial species population growth with a second species present
N1.0 <- 10
N2.0 <- 10
r1.0 <- 1.67
r2.0 <- 1.7
alpha.11 <- 0.00125
alpha.22 <- 0.00125
alpha.12 <- 0.008
alpha.21 <- 0.008725
# model function
disc_LV_comp <- function(r1, r2, N1.0, N2.0, alpha.11, alpha.22, alpha.12, alpha.21) {
  Nt1 <- (r1 * N1.0) / (1 + alpha.11 * N1.0 + alpha.12 * N2.0)
```

```

    Nt2 <- (r2 * N2.0)/(1 + alpha.22 * N2.0 + alpha.21 * N1.0)
    return(c(Nt1, Nt2))
}
# Simulation of model for t time steps
t <- 40
N <- array(NA, dim = c(t, 2))
N <- as.data.frame(N)
colnames(N) <- c("N1", "N2")
N$N1[1] <- N1.0
N$N2[1] <- N2.0
for (i in 2:t) {
  res <- disc_LV_comp(r1 = r1.0, r2 = r2.0, N1.0 = N[i - 1, 1], N2.0 = N[i - 1,
    2], alpha.11 = alpha.11, alpha.22 = alpha.22, alpha.12 = alpha.12, alpha.21 = alpha.21)
  N$N1[i] <- res[1]
  N$N2[i] <- res[2]
}

```

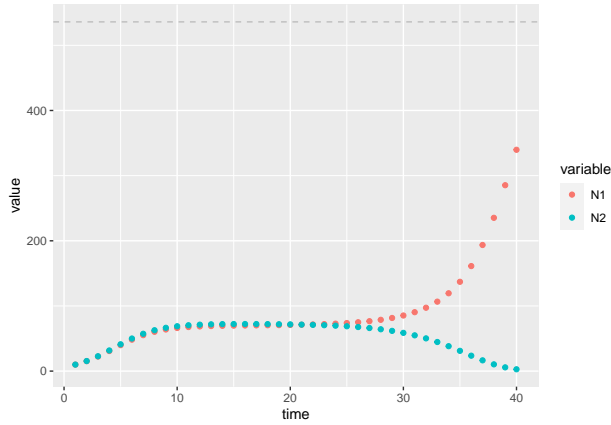


Figure 11: Population size N over time t for a discrete-time logistic growth model, with parameters $r_1 = 1.67$, $r_2 = 1.7$, $N_{i,0} = 10$, $\alpha_{ii} = 0.00125$, $\alpha_{12} = 0.008$, and $\alpha_{21} = 0.008725$.

3.2 Linear statistical model with covariate for both species

We fit each species' population time series to an arimaX and lm with population size of the others species as a covariate.

$$N_{1,t} = \beta_0 + \beta_1 N_{1,t-1} + \beta_2 N_{2,t-1} + \epsilon_t$$

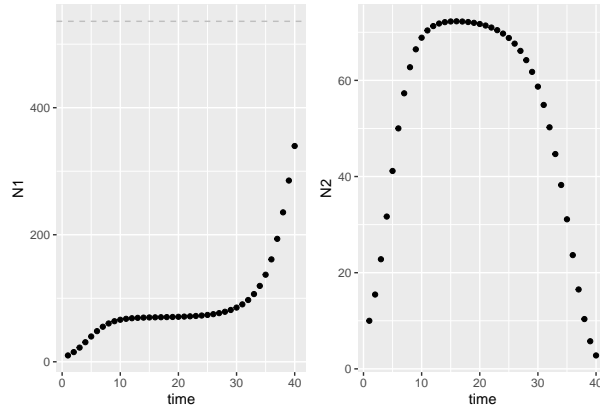


Figure 12: Population size N over time t for a discrete-time logistic growth model, with competition.

```
## Fit the model for Species 1
m.3.ar.n1 <- arima(x = log(N$N1), order = c(1, 0, 0), include.mean = T, method = "CSS",
  xreg = N$N2)
m.3.lm.n1 <- lm(log(dat$N1[2:t]) ~ log(dat$N1[1:(t - 1)]) + log(dat$N2[1:(t - 1)]))
# plotting the series along with the fitted values
m.3.ar.fit.n1 <- log(N$N1) - residuals(m.3.ar.n1)
m.3.lm.fit.n1 <- log(dat$N1[2:t]) - m.3.lm.n1$resid
dat$ar3.fit.n1 <- m.3.ar.fit.n1
dat$lm3.fit.n1 <- NA
dat$lm3.fit.n1[2:t] <- m.3.lm.fit.n1
## Species 2
m.3.ar.n2 <- arima(x = log(N$N2), order = c(1, 0, 0), include.mean = T, method = "CSS",
  xreg = N$N1)
m.3.lm.n2 <- lm(log(dat$N2[2:t]) ~ log(dat$N2[1:(t - 1)]) + log(dat$N1[1:(t - 1)]))
# plotting the series along with the fitted values
m.3.ar.fit.n2 <- log(N$N2) - residuals(m.3.ar.n2)
m.3.lm.fit.n2 <- log(dat$N2[2:t]) - m.3.lm.n2$resid
dat$ar3.fit.n2 <- m.3.ar.fit.n2
dat$lm3.fit.n2 <- NA
dat$lm3.fit.n2[2:t] <- m.3.lm.fit.n2
```

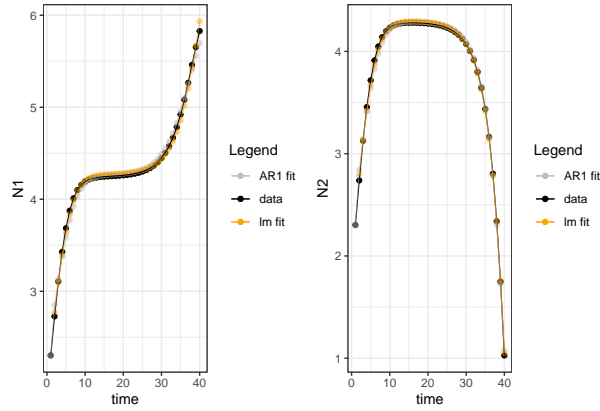


Figure 13: Population size over time (black line) with fitted values from a first-order autoregressive model (gray line) and from a linear model (orange line).

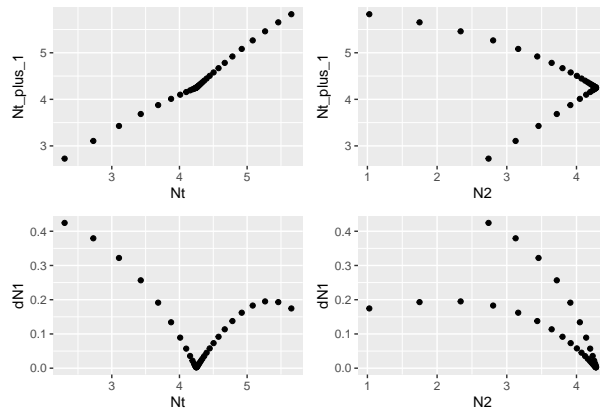


Figure 14: Population size (logarithm) at one time step N_{t+1} as a function of log-population size of Species 1 and Species 2 in the previous time step N_t . Also shown is the change in population size of Species 1 as a function of N_t for both species.

71 We can see that the population size of Species 2 does not have a linear relationship with that of Species 1.

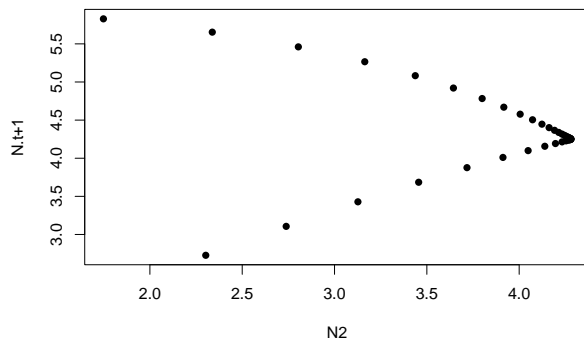
```
df <- data.frame(cbind(log(dat$N1[2:t]), log(dat$N1[1:(t - 1)]), log(dat$N2[1:(t - 1)])))
colnames(df) <- c("Nt1", "N1", "N2")
m.3.lm <- lm(Nt1 ~ I(1/N1) + I(1/N2), data = df)

cor(df$Nt1, predict(m.3.lm))
#> [1] 0.9845708
summary(m.3.lm)
#>
#> Call:
#> lm(formula = Nt1 ~ I(1/N1) + I(1/N2), data = df)
#>
#> Residuals:
#>      Min       1Q   Median       3Q      Max
```

```

#> -0.30952 -0.04467 -0.03442  0.03430  0.22447
#>
#> Coefficients:
#>             Estimate Std. Error t value Pr(>|t|)
#> (Intercept)   6.4056     0.1060   60.45 < 2e-16 ***
#> I(1/N1)      -12.2863     0.3774  -32.55 < 2e-16 ***
#> I(1/N2)       3.3309     0.2448   13.61 9.24e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#>
#> Residual standard error: 0.1063 on 36 degrees of freedom
#> Multiple R-squared:  0.9694, Adjusted R-squared:  0.9677
#> F-statistic: 569.8 on 2 and 36 DF,  p-value: < 2.2e-16
xx <- seq(0, 100, length = 1000)
prediction <- data.frame(N2 = xx, N1 = 100)
plot(df$N2, df$Nt1, xlab = "N2", ylab = "N.t+1", pch = 16)
lines(prediction$N2, predict(m.3.lm, prediction), lty = 2, col = "red", lwd = 3)

```



72

```

73 #> Using data df from global environment. This could cause incorrect results
74 #> if df has been altered since the model was fit. You can manually provide
75 #> the data to the "data =" argument.

```

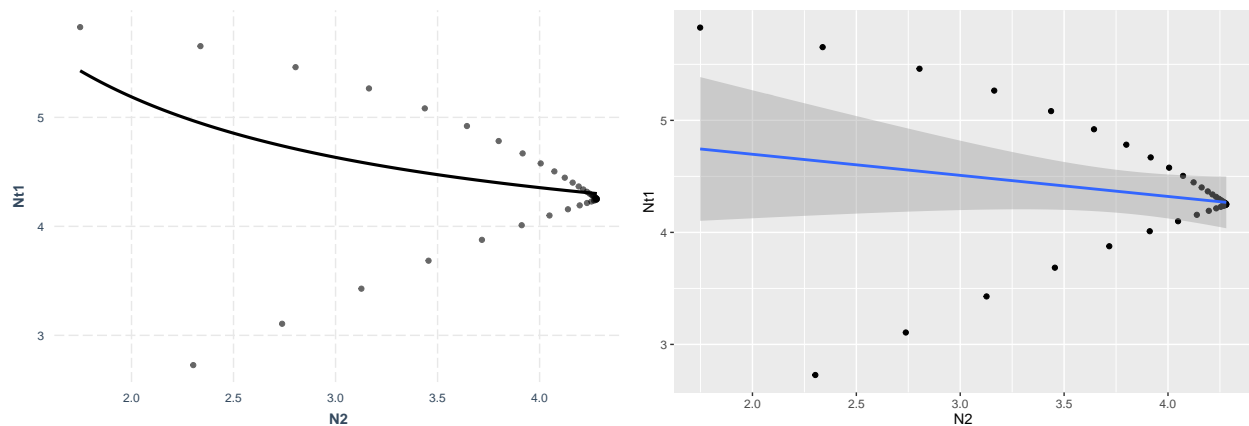


Figure 15: Population size (logarithm) at one time step N_{t+1} as a function of log-population size of the second species in the previous time step N_t . Fitted (line) and observed (points) values are shown.

We can already see that the linear model does not provide a useful fit to the data points (neither does taking the inverse function help). The regression parameters can predict well the population size of the other species, but they are not informative for inference. This has been observed previously (Certain et al. (2018)), and non-linear least squares models (among others) are more often used to estimate parameters for dynamics resulting from these kinds of species interaction models (Kloppers and Greeff (2013); Mühlbauer et al. (2020); Olivença et al. (2021)). The inability of the linear models to reproduce the interaction coefficients is more clearly demonstrated in Certain et al. (2018). However, they do indicate that the slope of the linear model can reflect the direction of effect for the interacting species.

Instead of further exploring techniques to fit to time-series derived from non-linear competition processes, we look more closely at how HMSC manages to fit data emerging from competition dynamics and make inference about species interactions. Instead of estimating full interaction coefficients, or assuming sparse interactions, they instead assume that a reduced number of linear combinations of species abundances that are most relevant to determining future growth rates for species in the community can bypass the ‘curse of dimensionality’ problem. This is well presented in Ovaskainen et al. (2017). We apply that approach here:

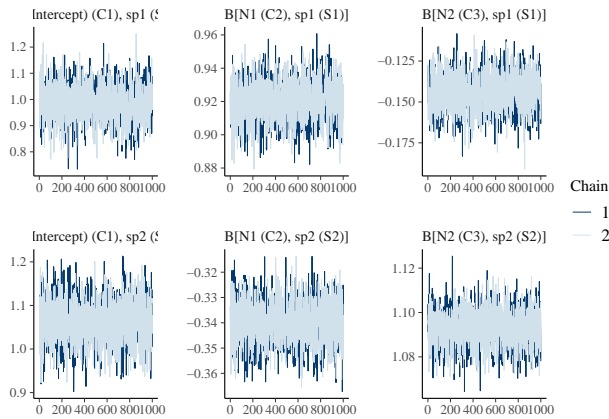
3.3 Bayesian linear statistical model: HMSC

We can fit the data to a linear model using HMSC, with latent variables to capture the species associations. These are implemented by including a random effect at the time level.

```
# prepare data in HMSC format
Y <- as.matrix(cbind(log(dat$N1[2:t]), log(dat$N2[2:t])))
XData <- df[, 2:3]
studyDesign = data.frame(sample = as.factor(1:(t - 1)))
rL = HmscRandomLevel(units = studyDesign$sample)
m.3.hmsc = Hmsc(Y = Y, XData = XData, studyDesign = studyDesign, ranLevels = list(sample = rL))
# Bayesian model parameters
nChains <- 2
thin <- 5
samples <- 1000
transient <- 500 * thin
verbose <- 500 * thin
# sample MCMC
m.3.sample <- sampleMcmc(m.3.hmsc, thin = thin, sample = samples, transient = transient,
  nChains = nChains, verbose = verbose)
#> Computing chain 1
#> Chain 1, iteration 2500 of 7500 (transient)
#> Chain 1, iteration 5000 of 7500 (sampling)
#> Chain 1, iteration 7500 of 7500 (sampling)
#> Computing chain 2
#> Chain 2, iteration 2500 of 7500 (transient)
#> Chain 2, iteration 5000 of 7500 (sampling)
#> Chain 2, iteration 7500 of 7500 (sampling)

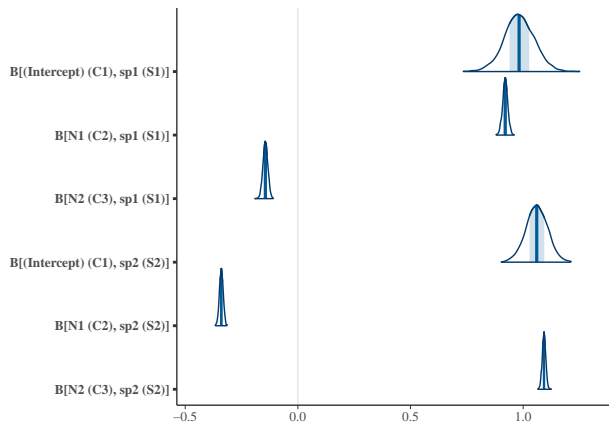
m3.post.hmsc <- convertToCodaObject(m.3.sample)
summary(m3.post.hmsc$Beta)
#>
#> Iterations = 2505:7500
#> Thinning interval = 5
#> Number of chains = 2
#> Sample size per chain = 1000
#>
```

```
#> 1. Empirical mean and standard deviation for each variable,
#>   plus standard error of the mean:
#>
#>
#>               Mean      SD Naive SE Time-series SE
#> B[(Intercept) (C1), sp1 (S1)] 0.9829 0.067231 0.0015033 0.0015031
#> B[N1 (C2), sp1 (S1)]          0.9202 0.011671 0.0002610 0.0002610
#> B[N2 (C3), sp1 (S1)]         -0.1442 0.011042 0.0002469 0.0002516
#> B[(Intercept) (C1), sp2 (S2)] 1.0606 0.048507 0.0010846 0.0010845
#> B[N1 (C2), sp2 (S2)]         -0.3398 0.008349 0.0001867 0.0001867
#> B[N2 (C3), sp2 (S2)]          1.0926 0.007894 0.0001765 0.0001827
#>
#> 2. Quantiles for each variable:
#>
#>               2.5%    25%    50%    75%    97.5%
#> B[(Intercept) (C1), sp1 (S1)] 0.8470 0.9394 0.9820 1.0269 1.1174
#> B[N1 (C2), sp1 (S1)]          0.8973 0.9129 0.9205 0.9278 0.9432
#> B[N2 (C3), sp1 (S1)]         -0.1653 -0.1514 -0.1442 -0.1369 -0.1226
#> B[(Intercept) (C1), sp2 (S2)] 0.9638 1.0285 1.0603 1.0938 1.1557
#> B[N1 (C2), sp2 (S2)]         -0.3563 -0.3455 -0.3397 -0.3339 -0.3235
#> B[N2 (C3), sp2 (S2)]          1.0772 1.0874 1.0926 1.0977 1.1080
bayesplot::mcmc_trace(m3.post.hmsc$Beta)
```



93

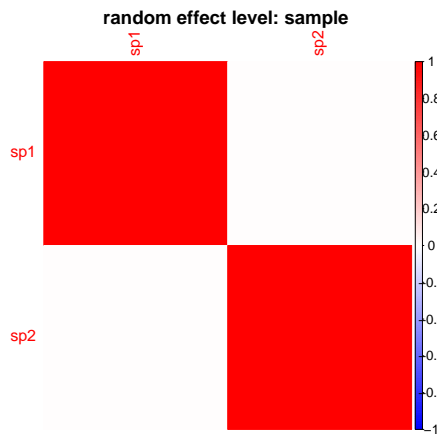
```
bayesplot::mcmc_areas(m3.post.hmsc$Beta, area_method = c("equal height"))
```



94

95 Now we look at the estimates for the species associations:

```
OmegaCor = computeAssociations(m.3.sample)
supportLevel = 0.95
toPlot = ((OmegaCor[[1]]$support > supportLevel) + (OmegaCor[[1]]$support < (1 -
  supportLevel)) > 0) * OmegaCor[[1]]$mean
corrplot::corrplot(toPlot, method = "color", col = colorRampPalette(c("blue", "white",
  "red"))(200), title = paste("random effect level:", m.3.sample$rLNames[1]), mar = c(0,
  0, 1, 0))
```



The model fixed effect estimates match well with those from the AR1 and linear model. However, the model up to this point includes N_{t1} and N_{t2} as fixed predictors. I change that here, as it better matches what can be done in large multi-species models, and the residual associations will better match the effects species have on one another.

```
# AR1 coefficients (recall that the intercept is the term below multiplied by 1
# - phi1)
m.3.ar.n1$coef
#>      ar1      intercept      N$N2
#> 0.79696422 5.69840619 -0.01881979
m.3.ar.n1$coef[2] * (1 - m.3.ar.n1$coef[1])
#> intercept
#> 1.15698
# linear model
summary(m.3.lm.n1)$coefficients[1:3, 1:2]
#>              Estimate Std. Error
#> (Intercept) 0.9839341 0.053084123
#> log(dat$N1[1:(t - 1)]) 0.9201864 0.009157214
#> log(dat$N2[1:(t - 1)]) -0.1443693 0.008777527
# Bayesian estimates
summary(m3.post.hmsc$Beta)$statistics[1:3, 1:2]
#>              Mean      SD
#> B[(Intercept) (C1), sp1 (S1)] 0.9829117 0.06723119
#> B[N1 (C2), sp1 (S1)] 0.9202360 0.01167056
#> B[N2 (C3), sp1 (S1)] -0.1441626 0.01104150
```

I try again with a next example where the only difference between species is the α_{ij} .

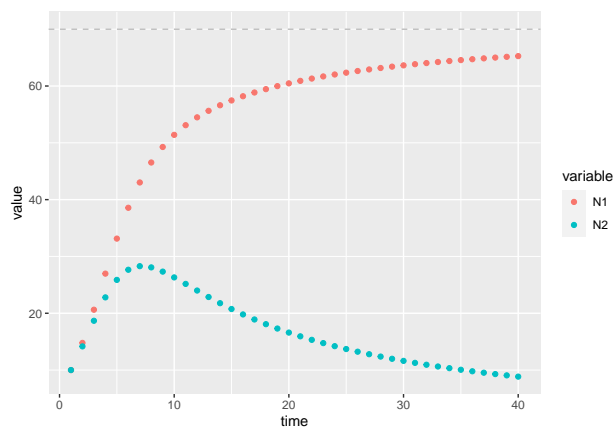
```
# Simulate initial species population growth with a second species present
N1.0 <- 10
N2.0 <- 10
```

```

r1.0 <- 1.7
r2.0 <- 1.7
alpha.11 <- 0.01
alpha.22 <- 0.01
alpha.12 <- 0.005
alpha.21 <- 0.01
# model function
disc_LV_comp <- function(r1, r2, N1.0, N2.0, alpha.11, alpha.22, alpha.12, alpha.21) {
  Nt1 <- (r1 * N1.0)/(1 + alpha.11 * N1.0 + alpha.12 * N2.0)
  Nt2 <- (r2 * N2.0)/(1 + alpha.22 * N2.0 + alpha.21 * N1.0)
  return(c(Nt1, Nt2))
}
# Simulation of model for t time steps
t <- 40
N <- array(NA, dim = c(t, 2))
N <- as.data.frame(N)
colnames(N) <- c("N1", "N2")
N$N1[1] <- N1.0
N$N2[1] <- N2.0
for (i in 2:t) {
  res <- disc_LV_comp(r1 = r1.0, r2 = r2.0, N1.0 = N[i - 1, 1], N2.0 = N[i - 1,
    2], alpha.11 = alpha.11, alpha.22 = alpha.22, alpha.12 = alpha.12, alpha.21 = alpha.21)
  N$N1[i] <- res[1]
  N$N2[i] <- res[2]
}

# Plot simulation: ggplot
N$time <- 1:t
dat <- melt(N, id.vars = "time")
ggplot2::ggplot(dat, aes(time, value, col = variable)) + geom_point() + geom_hline(yintercept = ((r1.0 *
  1)/alpha.11), linetype = "dashed", color = "gray")

```



```

# Plot simulation: ggplot
dat <- as.data.frame(cbind(N$N1, N$N2))
colnames(dat) <- c("N1", "N2")
dat$time <- 1:t
df <- data.frame(cbind(log(dat$N1[2:t]), log(dat$N2[2:t])))
colnames(df) <- c("Nt1", "Nt2")

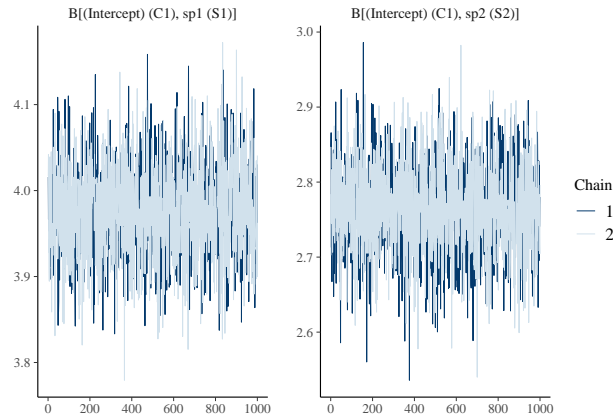
```

```

# prepare data in HMSC format
Y <- as.matrix(cbind(df$Nt1, df$Nt2))
XData <- data.frame(cbind(log(dat$N1[1:(t - 1)]), log(dat$N2[1:(t - 1)])))
studyDesign = data.frame(sample = as.factor(1:(t - 1)))
rL = HmscRandomLevel(units = studyDesign$sample)
m.4.hmsc = Hmsc(Y = Y, XData = XData, XFormula = ~1, studyDesign = studyDesign, ranLevels = list(sample))
# Bayesian model parameters
nChains <- 2
thin <- 5
samples <- 1000
transient <- 500 * thin
verbose <- 500 * thin
# sample MCMC
m.4.sample <- sampleMcmc(m.4.hmsc, thin = thin, sample = samples, transient = transient,
  nChains = nChains, verbose = verbose)
#> Computing chain 1
#> Chain 1, iteration 2500 of 7500 (transient)
#> Chain 1, iteration 5000 of 7500 (sampling)
#> Chain 1, iteration 7500 of 7500 (sampling)
#> Computing chain 2
#> Chain 2, iteration 2500 of 7500 (transient)
#> Chain 2, iteration 5000 of 7500 (sampling)
#> Chain 2, iteration 7500 of 7500 (sampling)

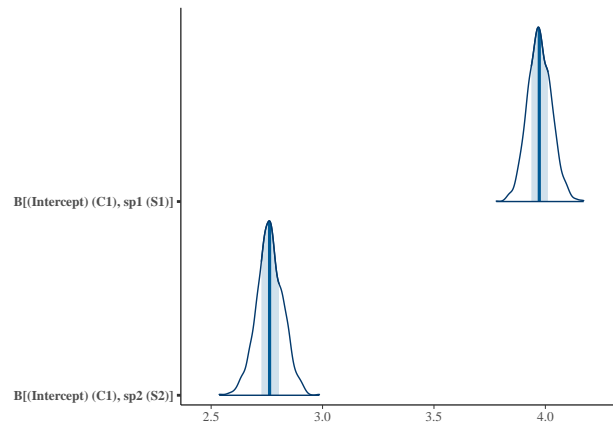
m4.post.hmsc <- convertToCodaObject(m.4.sample)
summary(m4.post.hmsc$Beta)
#>
#> Iterations = 2505:7500
#> Thinning interval = 5
#> Number of chains = 2
#> Sample size per chain = 1000
#>
#> 1. Empirical mean and standard deviation for each variable,
#>    plus standard error of the mean:
#>
#>
#>               Mean      SD Naive SE Time-series SE
#> B[(Intercept) (C1), sp1 (S1)] 3.974 0.05406 0.001209      0.001173
#> B[(Intercept) (C1), sp2 (S2)] 2.764 0.06002 0.001342      0.001388
#>
#> 2. Quantiles for each variable:
#>
#>
#>               2.5%   25%   50%   75% 97.5%
#> B[(Intercept) (C1), sp1 (S1)] 3.872 3.937 3.973 4.011 4.085
#> B[(Intercept) (C1), sp2 (S2)] 2.642 2.725 2.762 2.804 2.885
bayesplot::mcmc_trace(m4.post.hmsc$Beta)

```



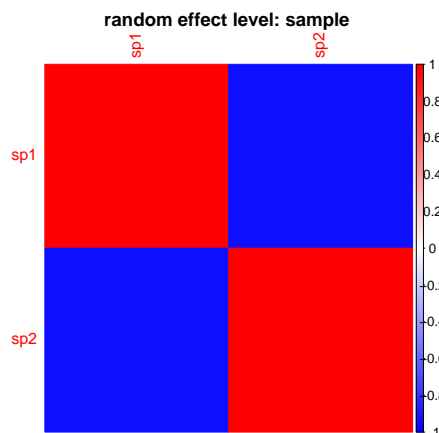
103

```
bayesplot::mcmc_areas(m4.post.hmsc$Beta, area_method = c("equal height"))
```



104

```
OmegaCor = computeAssociations(m.4.sample)
supportLevel = 0.95
toPlot = ((OmegaCor[[1]]$support > supportLevel) + (OmegaCor[[1]]$support < (1 -
  supportLevel)) > 0) * OmegaCor[[1]]$mean
corrplot::corrplot(toPlot, method = "color", col = colorRampPalette(c("blue", "white",
  "red"))(200), title = paste("random effect level:", m.4.sample$rLNames[1]), mar = c(0,
  0, 1, 0))
```

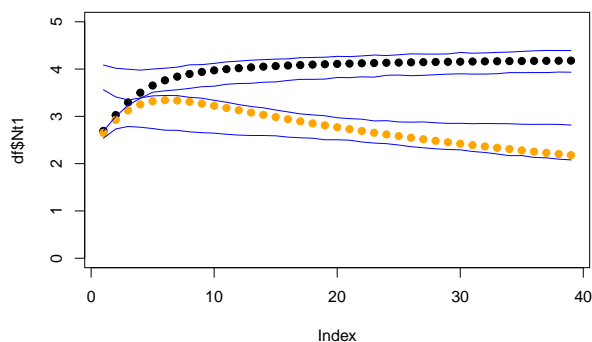


105

106 The HMSC model correctly shows the negative association between the species (shown here as correlations
 107 - JSDM CHapter 7 tells us the intr-specific correlation is always 1).

```
# explanatory power
preds = computePredictedValues(m.4.sample)
# evaluateModelFit(hM = m.4.sample, predY = preds) predictive power /
# cross-validation partition = createPartition(m.4.sample, nfolds = 2) preds =
# computePredictedValues(m.4.sample, partition = partition, nParallel =
# nChains) evaluateModelFit(hM = m.4.sample, predY = preds) xx preds =
# computePredictedValues(m.4.sample, partition=partition, partition.sp=c(1,2),
# mcmcStep=10, nParallel = nChains) evaluateModelFit(hM=m.4.sample,
# predY=preds) xx etaPost=getPostEstimate(m.4.sample, 'Eta')
# lambdaPost=getPostEstimate(m.4.sample, 'Lambda') biPlot(m.4.sample, etaPost =
# etaPost, lambdaPost = lambdaPost, factors = c(1,2),'X1')
```

```
df$N1_25 <- NA
df$N1_975 <- NA
df$N2_25 <- NA
df$N2_975 <- NA
for (i in 1:39) {
  red <- preds[i, , ]
  a <- apply(red, 1, quantile, probs = c(0.025, 0.975))
  df$N1_25[i] <- a[1, 1]
  df$N1_975[i] <- a[2, 1]
  df$N2_25[i] <- a[1, 2]
  df$N2_975[i] <- a[2, 2]
}
plot(df$Nt1, pch = 19, col = "black", ylim = c(0, 5))
points(df$Nt2, pch = 19, col = "orange")
lines(df$N1_25, col = "blue")
lines(df$N1_975, col = "blue")
lines(df$N2_25, col = "blue")
lines(df$N2_975, col = "blue")
```



108

109 The residual associations should not be interpreted as interaction coefficients, but rather used for prediction.
 110 We can see here they are effective for this (shown above is the observed and model-predicted 95% CI).

4 Species growth, competition, with environmental change

The goal of the process was to use HMSC's latent variable approach to study the results of non-linear processes in ecology using a linear model. We can infer the direction and magnitude of the species interaction. We now consider an environmental covariate that changes over time and impacts each species growth. We will use only HMSC from this point forward, as the species interactions represent a departure from what a linear or AR model can manage.

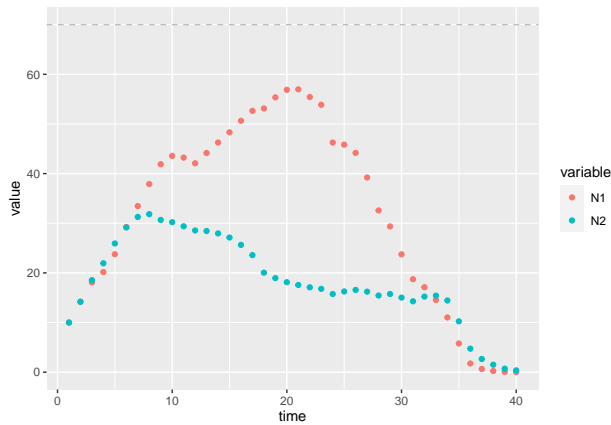
In this example, the local environmental optimum trait value $E = 0.8$, and Species 1 $x_1 = 0.6$, while $x_2 = 0.8$.

```
# Simulate initial species population growth with environment fluctuations
N1.0 <- 10
N2.0 <- 10
r1.0 <- 1.7
r2.0 <- 1.7
alpha.11 <- 0.01
alpha.22 <- 0.01
alpha.12 <- 0.005
alpha.21 <- 0.01
E.0 <- 0.8
x1.0 <- 0.6
x2.0 <- 0.8

# model function
disc_LV_E <- function(r1, r2, N1.0, N2.0, alpha.11, alpha.22, alpha.12, alpha.21,
  E, x1, x2) {
  Nt1 <- ((r1 * exp(-(E - x1)^2)) * N1.0)/(1 + alpha.11 * N1.0 + alpha.12 * N2.0)
  Nt2 <- ((r2 * exp(-(E - x2)^2)) * N2.0)/(1 + alpha.22 * N2.0 + alpha.21 * N1.0)
  return(c(Nt1, Nt2))
}

# Simulation of model for t time steps
t <- 40
N <- array(NA, dim = c(t, 2))
N <- as.data.frame(N)
colnames(N) <- c("N1", "N2")
N$N1[1] <- N1.0
N$N2[1] <- N2.0
E <- rep(NA, t)
E[1] <- E.0
for (i in 2:t) {
  res <- disc_LV_E(r1 = r1.0, r2 = r2.0, N1.0 = N[i - 1, 1], N2.0 = N[i - 1, 2],
    alpha.11 = alpha.11, alpha.22 = alpha.22, alpha.12 = alpha.12, alpha.21 = alpha.21,
    E = E[i - 1], x1 = x1.0, x2 = x2.0)
  N$N1[i] <- res[1]
  N$N2[i] <- res[2]
  E[i] <- E[i - 1] + rnorm(1, 0, 0.1)
}

# Plot simulation: ggplot
N$time <- 1:t
dat <- melt(N, id.vars = "time")
ggplot2::ggplot(dat, aes(time, value, col = variable)) + geom_point() + geom_hline(yintercept = ((r1.0 *
  1)/alpha.11), linetype = "dashed", color = "gray")
```

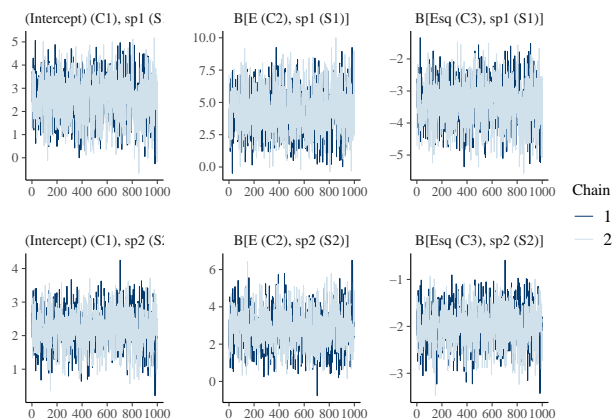



118

```
# Plot simulation: ggplot
dat <- as.data.frame(cbind(log(N$N1), log(N$N2)))
colnames(dat) <- c("N1", "N2")
dat$time <- 1:t
df <- data.frame(cbind(dat$N1[2:t], dat$N2[2:t]))
colnames(df) <- c("Nt1", "Nt2")
# prepare data in HMSC format
Y <- as.matrix(cbind(df$Nt1, df$Nt2))
XData <- data.frame(cbind(dat$N1[1:(t - 1)], dat$N2[1:(t - 1)]), E[1:(t - 1)], E[1:(t - 1)]^2)
colnames(XData) <- c("n1", "n2", "E", "Esq")
studyDesign = data.frame(sample = as.factor(1:(t - 1)))
rL = HmscRandomLevel(units = studyDesign$sample)
m.5.hmsc = Hmsc(Y = Y, XData = XData, XFormula = ~E + Esq, studyDesign = studyDesign,
  ranLevels = list(sample = rL))
# Bayesian model parameters
nChains <- 2
thin <- 5
samples <- 1000
transient <- 500 * thin
verbose <- 500 * thin
# sample MCMC
m.5.sample <- sampleMcmc(m.5.hmsc, thin = thin, sample = samples, transient = transient,
  nChains = nChains, verbose = verbose)
#> Computing chain 1
#> Chain 1, iteration 2500 of 7500 (transient)
#> Chain 1, iteration 5000 of 7500 (sampling)
#> Chain 1, iteration 7500 of 7500 (sampling)
#> Computing chain 2
#> Chain 2, iteration 2500 of 7500 (transient)
#> Chain 2, iteration 5000 of 7500 (sampling)
#> Chain 2, iteration 7500 of 7500 (sampling)

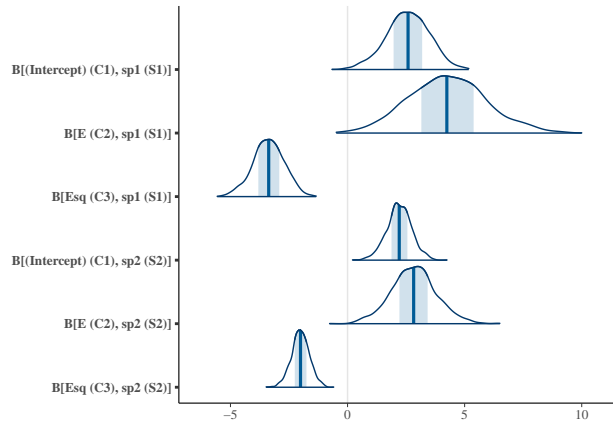
m5.post.hmsc <- convertToCodaObject(m.5.sample)
summary(m5.post.hmsc$Beta)
#>
#> Iterations = 2505:7500
#> Thinning interval = 5
#> Number of chains = 2
```

```
#> Sample size per chain = 1000
#>
#> 1. Empirical mean and standard deviation for each variable,
#>    plus standard error of the mean:
#>
#>                                Mean      SD Naive SE Time-series SE
#> B[(Intercept) (C1), sp1 (S1)] 2.572 0.9131 0.020417      0.02342
#> B[E (C2), sp1 (S1)]          4.274 1.6630 0.037186      0.04303
#> B[Esq (C3), sp1 (S1)]        -3.374 0.6726 0.015039      0.01740
#> B[(Intercept) (C1), sp2 (S2)] 2.204 0.5331 0.011920      0.01379
#> B[E (C2), sp2 (S2)]          2.832 0.9647 0.021571      0.02548
#> B[Esq (C3), sp2 (S2)]        -2.010 0.3893 0.008705      0.01037
#>
#> 2. Quantiles for each variable:
#>
#>                                2.5%    25%    50%    75%   97.5%
#> B[(Intercept) (C1), sp1 (S1)] 0.6559  1.966  2.584  3.186  4.300
#> B[E (C2), sp1 (S1)]          1.1272  3.151  4.240  5.384  7.713
#> B[Esq (C3), sp1 (S1)]        -4.7501 -3.810 -3.367 -2.915 -2.088
#> B[(Intercept) (C1), sp2 (S2)] 1.1184  1.876  2.208  2.555  3.278
#> B[E (C2), sp2 (S2)]          0.8746  2.216  2.823  3.419  4.800
#> B[Esq (C3), sp2 (S2)]        -2.8112 -2.254 -2.009 -1.756 -1.235
bayesplot::mcmc_trace(m5.post.hmsc$Beta)
```



119

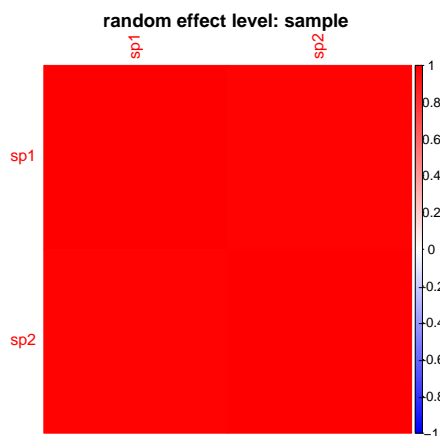
```
bayesplot::mcmc_areas(m5.post.hmsc$Beta, area_method = c("equal height"))
```



120

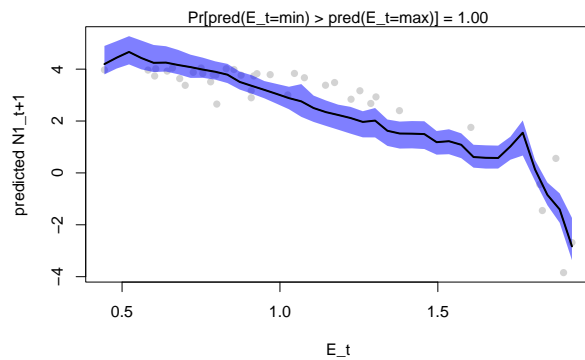
```
# Bayesian estimates
summary(m5.post.hmsc$Beta)$statistics[1:4, 1:2]
#>
#>           Mean      SD
#> B[(Intercept) (C1), sp1 (S1)]  2.571888 0.9130903
#> B[E (C2), sp1 (S1)]           4.274430 1.6630073
#> B[Esq (C3), sp1 (S1)]         -3.373885 0.6725570
#> B[(Intercept) (C1), sp2 (S2)]  2.203589 0.5330636
```

```
OmegaCor = computeAssociations(m.5.sample)
supportLevel = 0.7
toPlot = ((OmegaCor[[1]]$support > supportLevel) + (OmegaCor[[1]]$support < (1 -
  supportLevel)) > 0) * OmegaCor[[1]]$mean
corrplot::corrplot(toPlot, method = "color", col = colorRampPalette(c("blue", "white",
  "red"))(200), title = paste("random effect level:", m.5.sample$rLNames[1]), mar = c(0,
  0, 1, 0))
```



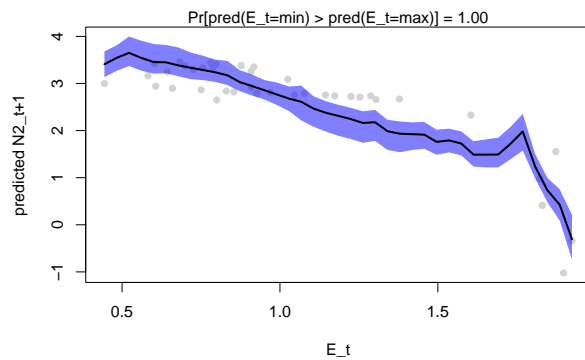
121

```
Gradient <- constructGradient(m.5.sample, focalVariable = "E", non.focalVariables = list(Esq = list(2))
  ngrid = 39)
predY <- predict(m.5.sample, XData = Gradient$XDataNew, expected = TRUE)
a <- plotGradient(m.5.sample, Gradient, pred = predY, showData = T, measure = "Y",
  index = 1, main = "", xlab = "E_t", ylab = "predicted N1_t+1")
```



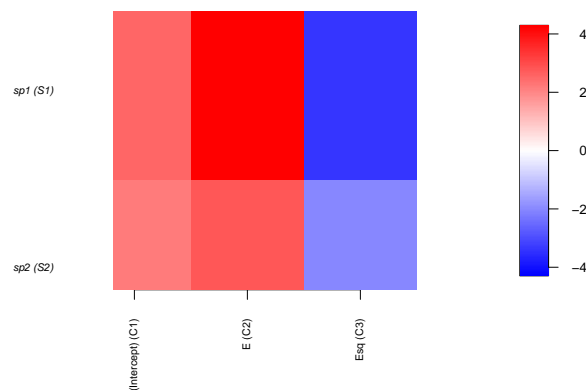
122

```
b <- plotGradient(m.5.sample, Gradient, pred = predY, showData = T, measure = "Y",
  index = 2, main = "", xlab = "E_t", ylab = "predicted N2_t+1")
```



123

```
postBeta = getPostEstimate(m.5.sample, parName = "Beta")
plotBeta(m.5.sample, post = postBeta, param = "Mean", supportLevel = 0.7)
```

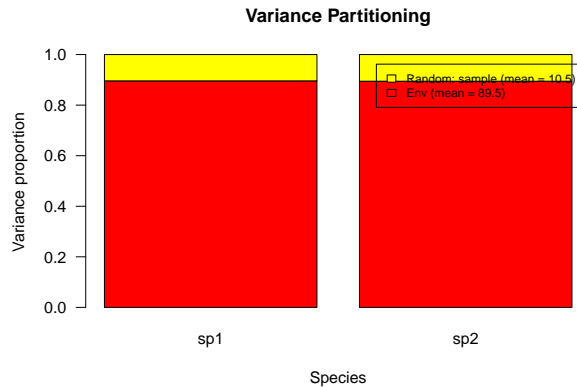


124

125 We can see that the Environment has an overall neg-
 126 ative impact for both species as E drifts randomly towards more positive values (where Species 2 is closer
 127 because it has a higher trait value). We also see that the residual species association effects are strongly
 positive.

128 We use variation partitioning to estimate the relative importance of environment and species associations for
 129 species abundances.

```
VP <- computeVariancePartitioning(m.5.sample, group = c(1, 1, 1), groupnames = "Env")
plotVariancePartitioning(m.5.sample, VP, args.legend = list(cex = 0.75, bg = "transparent"))
```



130

131 We evaluate this by greatly increasing the effects if interspecific competition (while decreasing the strength
 132 of environmental variation).

```
# Simulate initial species population growth with environment fluctuations
N1.0 <- 10
N2.0 <- 10
r1.0 <- 1.7
r2.0 <- 1.7
alpha.11 <- 0.01
alpha.22 <- 0.01
alpha.12 <- 0.015
alpha.21 <- 0.02
E.0 <- 0.8
x1.0 <- 0.8
x2.0 <- 0.8

# model function
disc_LV_E <- function(r1, r2, N1.0, N2.0, alpha.11, alpha.22, alpha.12, alpha.21,
  E, x1, x2) {
  Nt1 <- ((r1 * exp(-(E - x1)^2)) * N1.0)/(1 + alpha.11 * N1.0 + alpha.12 * N2.0)
  Nt2 <- ((r2 * exp(-(E - x2)^2)) * N2.0)/(1 + alpha.22 * N2.0 + alpha.21 * N1.0)
  return(c(Nt1, Nt2))
}

# Simulation of model for t time steps
t <- 40
N <- array(NA, dim = c(t, 2))
N <- as.data.frame(N)
colnames(N) <- c("N1", "N2")
N$N1[1] <- N1.0
N$N2[1] <- N2.0
E <- rep(NA, t)
E[1] <- E.0
for (i in 2:t) {
```

```

res <- disc_LV_E(r1 = r1.0, r2 = r2.0, N1.0 = N[i - 1, 1], N2.0 = N[i - 1, 2],
  alpha.11 = alpha.11, alpha.22 = alpha.22, alpha.12 = alpha.12, alpha.21 = alpha.21,
  E = E[i - 1], x1 = x1.0, x2 = x2.0)
N$N1[i] <- res[1]
N$N2[i] <- res[2]
E[i] <- E[i - 1] + rnorm(1, 0, 0.001)
}

```

```

# Plot simulation: ggplot

```

```

N$time <- 1:t

```

```

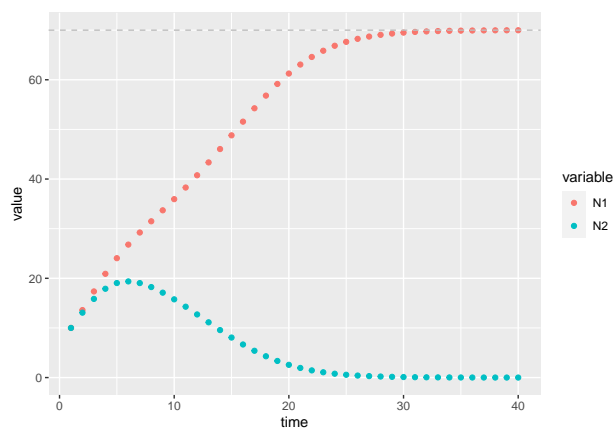
dat <- melt(N, id.vars = "time")

```

```

ggplot2::ggplot(dat, aes(time, value, col = variable)) + geom_point() + geom_hline(yintercept = ((r1.0 +
1)/alpha.11), linetype = "dashed", color = "gray")

```



133

```

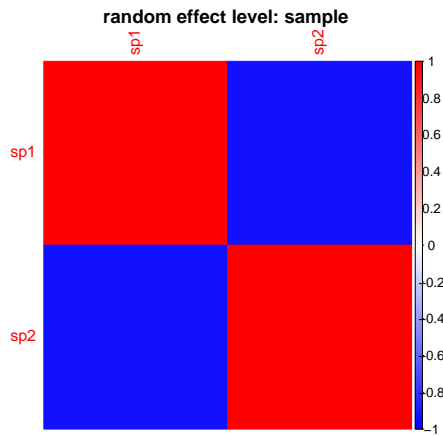
134 #> Computing chain 1
135 #> Chain 1, iteration 2500 of 10500 (sampling)
136 #> Chain 1, iteration 5000 of 10500 (sampling)
137 #> Chain 1, iteration 7500 of 10500 (sampling)
138 #> Chain 1, iteration 10000 of 10500 (sampling)
139 #> Computing chain 2
140 #> Chain 2, iteration 2500 of 10500 (sampling)
141 #> Chain 2, iteration 5000 of 10500 (sampling)
142 #> Chain 2, iteration 7500 of 10500 (sampling)
143 #> Chain 2, iteration 10000 of 10500 (sampling)

```

```

OmegaCor = computeAssociations(m.6.sample)
supportLevel = 0.7
toPlot = ((OmegaCor[[1]]$support > supportLevel) + (OmegaCor[[1]]$support < (1 -
  supportLevel)) > 0) * OmegaCor[[1]]$mean
corrplot::corrplot(toPlot, method = "color", col = colorRampPalette(c("blue", "white",
  "red"))(200), title = paste("random effect level:", m.6.sample$rLNames[1]), mar = c(0,
  0, 1, 0))

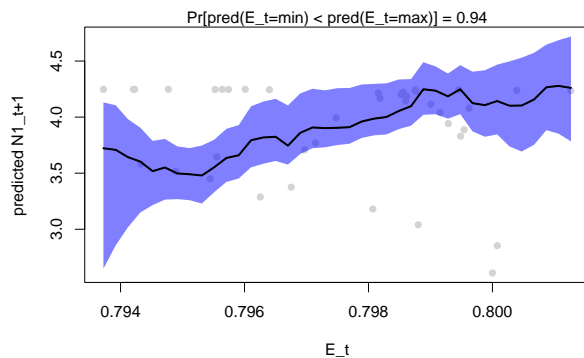
```



144

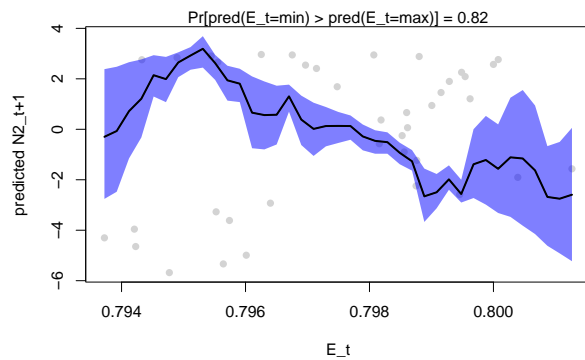
```
OmegaCor
#> [[1]]
#> [[1]]$mean
#>           sp1           sp2
#> sp1  1.0000000 -0.9201566
#> sp2 -0.9201566  1.0000000
#>
#> [[1]]$support
#>           sp1 sp2
#> sp1      1   0
#> sp2      0   1
```

```
Gradient <- constructGradient(m.6.sample, focalVariable = "E", non.focalVariables = list(Esq = list(2))
  ngrid = 39)
predY <- predict(m.6.sample, XData = Gradient$XDataNew, expected = TRUE)
a <- plotGradient(m.6.sample, Gradient, pred = predY, showData = T, measure = "Y",
  index = 1, main = "", xlab = "E_t", ylab = "predicted N1_t+1")
```



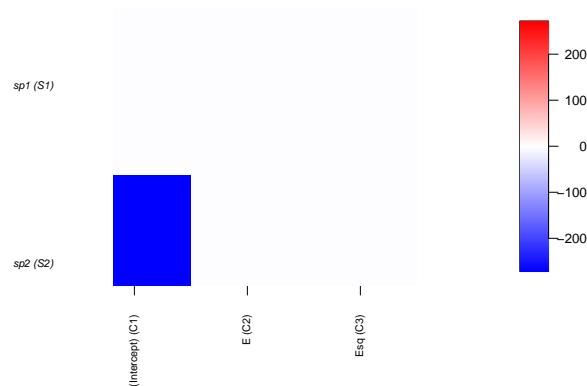
145

```
b <- plotGradient(m.6.sample, Gradient, pred = predY, showData = T, measure = "Y",
  index = 2, main = "", xlab = "E_t", ylab = "predicted N2_t+1")
```



146

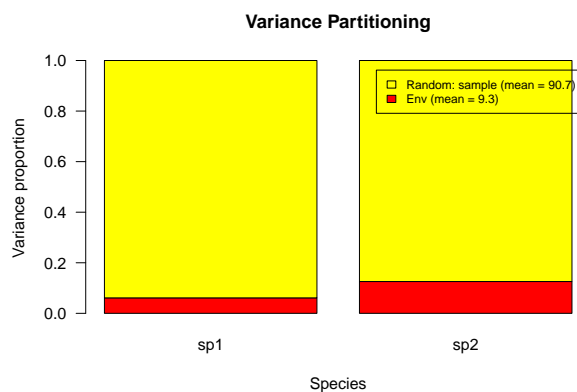
```
postBeta = getPostEstimate(m.6.sample, parName = "Beta")
plotBeta(m.6.sample, post = postBeta, param = "Mean", supportLevel = 0.7)
```



147

148 With the reduced impact of the environment, we now
 149 see the strong negative associations between the two competing species. We also correctly see no impact of
 environment for species abundances.

```
VP <- computeVariancePartitioning(m.6.sample, group = c(1, 1, 1), groupnames = "Env")
plotVariancePartitioning(m.6.sample, VP, args.legend = list(cex = 0.75, bg = "transparent"))
```



150

5 Species growth, competition, environmental change, and trait evolution

We now consider that trait evolution can occur, with an impact on the species growth rate / fitness in the context of the environment.

5.1 Trait evolution

Previously the population growth rate was fixed as $r_i = \hat{W}e^{-(E-x_{i,t})^2}$. However, we now introduce the quantitative genetic model of evolutionary rescue (Gomulkiewicz and Holt 1995).

The growth equation for each species now becomes:

$$N_{i,t+1} = \frac{\hat{W}e^{-\frac{[(\frac{w+(1-h^2)P}{P+w})(E-x_{i,t})]^2}{2(P+w)}}}{1 + \alpha_{ii}N_{i,t} + \alpha_{ij}N_{j,t}}N_{i,t}$$

where \hat{W} is calculated as $\hat{W} = W_{max}\sqrt{\frac{w}{P+w}}$, W_{max} is the species' maximum per-capita growth rate, w is the width of the Gaussian fitness function (which determines the strength of selection, as increasing values indicate a weaker reduction in fitness with distance from optimum trait value), P is the width of the distribution of the phenotype x , and h^2 is the heritability of the trait x . For the simulation we use $W_{max} = 2$, $P = 1$, and $w = 10$.

The change in the average trait value each time step is given by:

$$d_{i,t+1} = kd_{i,t}$$

where $k = \frac{w+(1-h^2)P}{w+P}$ and $d_{i,t} = E_t - x_{i,t}$.

5.2 Population dynamics simulation

We use the same examples as above, with weaker then stronger strength of interspecific interactions.

```
# Case 1: Weaker interactions, stronger environment Simulate initial species
# population growth with environment fluctuations
N1.0 <- 10
N2.0 <- 10
alpha.11 <- 0.01
alpha.22 <- 0.01
alpha.12 <- 0.005
alpha.21 <- 0.01
E.0 <- 0.8
x1.0 <- 0.6
x2.0 <- 0.8
P <- 1
w <- 10
Wmax <- 2
h2 <- 0.1
k <- (w + (1 - h2) * P)/(P + w)

# model function
disc_LV_evol <- function(N1.0, N2.0, alpha.11, alpha.22, alpha.12, alpha.21, E, x1.0,
```

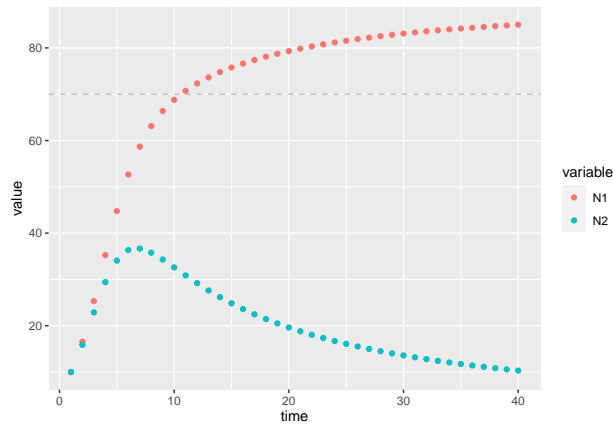
```

x2.0, P, w, Wmax, h2) {
  What <- Wmax * sqrt(w/(P + w))
  r1 <- What * exp(-(((w + (1 - h2) * P)/(P + w)) * (E - x1.0))^2)/(2 * (P + w)))
  Nt1 <- (r1 * N1.0)/(1 + alpha.11 * N1.0 + alpha.12 * N2.0)
  r2 <- What * exp(-(((w + (1 - h2) * P)/(P + w)) * (E - x2.0))^2)/(2 * (P + w)))
  Nt2 <- (r2 * N2.0)/(1 + alpha.22 * N2.0 + alpha.21 * N1.0)
  return(c(Nt1, Nt2, r1, r2))
}

# Simulation of model for t time steps
t <- 40
N <- array(NA, dim = c(t, 2))
N <- as.data.frame(N)
colnames(N) <- c("N1", "N2")
x <- array(NA, dim = c(t, 2))
x <- as.data.frame(x)
colnames(x) <- c("x1", "x2")
r <- array(NA, dim = c(t, 2))
r <- as.data.frame(r)
colnames(r) <- c("r1", "r2")
N$N1[1] <- N1.0
N$N2[1] <- N2.0
x$x1[1] <- x1.0
x$x2[1] <- x2.0
r$r1[1] <- exp(-(((w + (1 - h2) * P)/(P + w)) * (E.0 - x1.0))^2)/(2 * (P + w)))
r$r2[1] <- exp(-(((w + (1 - h2) * P)/(P + w)) * (E.0 - x2.0))^2)/(2 * (P + w)))
E <- rep(NA, t)
E[1] <- E.0
for (i in 2:t) {
  res <- disc_LV_evol(N1.0 = N[i - 1, 1], N2.0 = N[i - 1, 2], alpha.11 = alpha.11,
    alpha.22 = alpha.22, alpha.12 = alpha.12, alpha.21 = alpha.21, E = E[i -
    1], x1 = x[i - 1, 1], x2 = x[i - 1, 2], P = P, w = w, Wmax = Wmax, h2 = h2)
  N$N1[i] <- res[1]
  N$N2[i] <- res[2]
  r$r1[i] <- res[3]
  r$r2[i] <- res[4]
  # trait change
  d1 <- E[i - 1] - x[i - 1, 1]
  d2 <- E[i - 1] - x[i - 1, 2]
  d1.1 <- k * d1
  d2.1 <- k * d2
  x$x1[i] <- E[i - 1] - d1.1
  x$x2[i] <- E[i - 1] - d2.1
  # environmental change
  E[i] <- E[i - 1] + rnorm(1, 0, 0.01)
}

# Plot simulation: ggplot
N$time <- 1:t
dat <- melt(N, id.vars = "time")
ggplot2::ggplot(dat, aes(time, value, col = variable)) + geom_point() + geom_hline(yintercept = ((r1.0 *
  1)/alpha.11), linetype = "dashed", color = "gray")

```



168

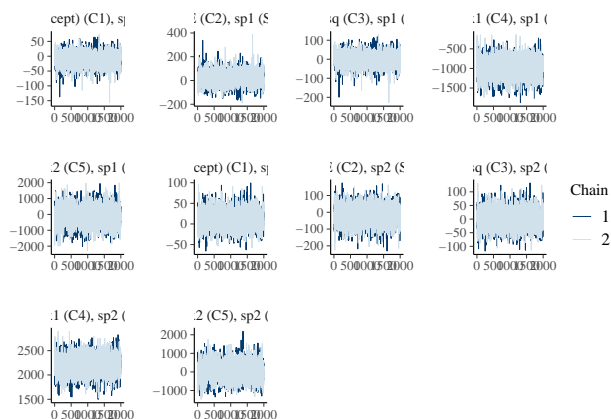
169 The overall research goal is to determine the effect of trait evolution for species abundances. We use the
 170 absolute value of the change in trait value from one time point to the next (the total amount of trait change,
 171 regardless of direction) as a fixed effect in the HMSC.

```
# Plot simulation: ggplot
dat <- as.data.frame(cbind(log(N$N1), log(N$N2), x$x1, x$x2))
colnames(dat) <- c("N1", "N2", "x1", "x2")
dat$time <- 1:t
df <- data.frame(cbind(dat$N1[2:t], dat$N2[2:t], dat$x1[2:t], dat$x2[2:t]))
colnames(df) <- c("Nt1", "Nt2", "xt1", "xt2")
# prepare data in HMSC format
Y <- as.matrix(cbind(df$Nt1, df$Nt2))
XData <- data.frame(cbind(dat$N1[1:(t - 1)], dat$N2[1:(t - 1)]), E[1:(t - 1)], E[1:(t - 1)]^2, abs(dat$x1[2:t] - dat$x1[1:(t - 1)]), abs(dat$x2[2:t] - dat$x2[1:(t - 1)]))
colnames(XData) <- c("n1", "n2", "E", "Esq", "dx1", "dx2")
studyDesign = data.frame(sample = as.factor(1:(t - 1)))
rL = HmscRandomLevel(units = studyDesign$sample)
m.7.hmsc = Hmsc(Y = Y, XData = XData, XFormula = ~E + Esq + dx1 + dx2, studyDesign = studyDesign,
  ranLevels = list(sample = rL))
# Bayesian model parameters
nChains <- 2
thin <- 5
samples <- 2000
transient <- 100 * thin
verbose <- 500 * thin
# sample MCMC
m.7.sample <- sampleMcmc(m.7.hmsc, thin = thin, sample = samples, transient = transient,
  nChains = nChains, verbose = verbose)
#> Computing chain 1
#> Chain 1, iteration 2500 of 10500 (sampling)
#> Chain 1, iteration 5000 of 10500 (sampling)
#> Chain 1, iteration 7500 of 10500 (sampling)
#> Chain 1, iteration 10000 of 10500 (sampling)
#> Computing chain 2
#> Chain 2, iteration 2500 of 10500 (sampling)
#> Chain 2, iteration 5000 of 10500 (sampling)
#> Chain 2, iteration 7500 of 10500 (sampling)
#> Chain 2, iteration 10000 of 10500 (sampling)
```

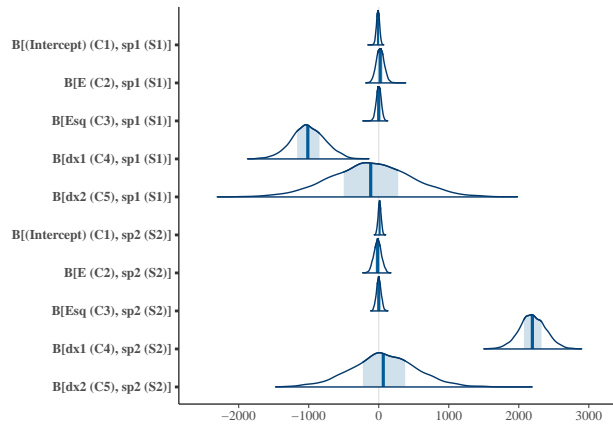
```

m7.post.hmsc <- convertToCodaObject(m.7.sample)
summary(m7.post.hmsc$Beta)
#>
#> Iterations = 505:10500
#> Thinning interval = 5
#> Number of chains = 2
#> Sample size per chain = 2000
#>
#> 1. Empirical mean and standard deviation for each variable,
#>    plus standard error of the mean:
#>
#>
#>               Mean      SD Naive SE Time-series SE
#> B[(Intercept) (C1), sp1 (S1)] -11.0305 23.18 0.3665 0.3675
#> B[E (C2), sp1 (S1)] 21.5170 57.41 0.9077 0.9098
#> B[Esq (C3), sp1 (S1)] -0.9265 35.68 0.5642 0.5652
#> B[dx1 (C4), sp1 (S1)] -1006.3502 239.24 3.7827 3.6098
#> B[dx2 (C5), sp1 (S1)] -112.0149 582.94 9.2171 8.8384
#> B[(Intercept) (C1), sp2 (S2)] 12.4114 21.93 0.3467 0.3668
#> B[E (C2), sp2 (S2)] -16.2922 54.40 0.8601 0.9069
#> B[Esq (C3), sp2 (S2)] 0.2300 33.83 0.5349 0.5622
#> B[dx1 (C4), sp2 (S2)] 2196.8954 190.73 3.0156 2.9408
#> B[dx2 (C5), sp2 (S2)] 75.0324 469.34 7.4209 7.0985
#>
#> 2. Quantiles for each variable:
#>
#>
#>               2.5%      25%      50%      75%      97.5%
#> B[(Intercept) (C1), sp1 (S1)] -56.42 -25.929 -10.7491 4.685 32.51
#> B[E (C2), sp1 (S1)] -87.48 -17.276 20.8826 58.607 135.83
#> B[Esq (C3), sp1 (S1)] -71.47 -23.962 -0.4462 22.965 66.66
#> B[dx1 (C4), sp1 (S1)] -1480.08 -1166.247 -1013.1698 -846.504 -534.31
#> B[dx2 (C5), sp1 (S1)] -1264.83 -500.459 -114.8955 277.368 1024.06
#> B[(Intercept) (C1), sp2 (S2)] -30.42 -2.131 12.1906 26.674 55.16
#> B[E (C2), sp2 (S2)] -122.98 -51.706 -15.4321 19.822 90.98
#> B[Esq (C3), sp2 (S2)] -67.22 -22.090 -0.3323 22.531 66.52
#> B[dx1 (C4), sp2 (S2)] 1820.29 2072.995 2193.8622 2324.701 2578.50
#> B[dx2 (C5), sp2 (S2)] -861.36 -226.971 64.8218 377.947 1014.43
bayesplot::mcmc_trace(m7.post.hmsc$Beta)

```



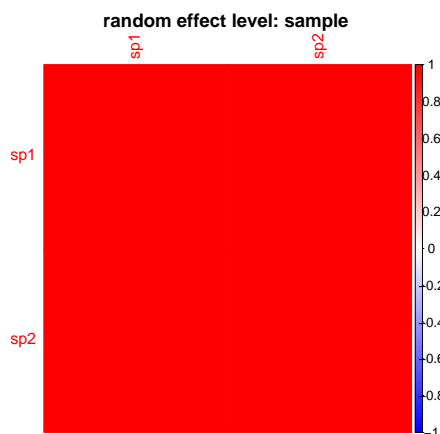
```
bayesplot::mcmc_areas(m7.post.hmsc$Beta, area_method = c("equal height"))
```



173

```
# Bayesian estimates
summary(m7.post.hmsc$Beta)$statistics[1:10, 1:2]
#>               Mean      SD
#> B[(Intercept) (C1), sp1 (S1)] -11.0305027 23.17686
#> B[E (C2), sp1 (S1)]          21.5170323 57.40594
#> B[Esq (C3), sp1 (S1)]        -0.9265490 35.68207
#> B[dx1 (C4), sp1 (S1)]       -1006.3501626 239.23588
#> B[dx2 (C5), sp1 (S1)]       -112.0149112 582.94321
#> B[(Intercept) (C1), sp2 (S2)]  12.4113535 21.92925
#> B[E (C2), sp2 (S2)]         -16.2922082 54.39857
#> B[Esq (C3), sp2 (S2)]         0.2299636 33.83029
#> B[dx1 (C4), sp2 (S2)]       2196.8953753 190.72582
#> B[dx2 (C5), sp2 (S2)]         75.0323912 469.33756
```

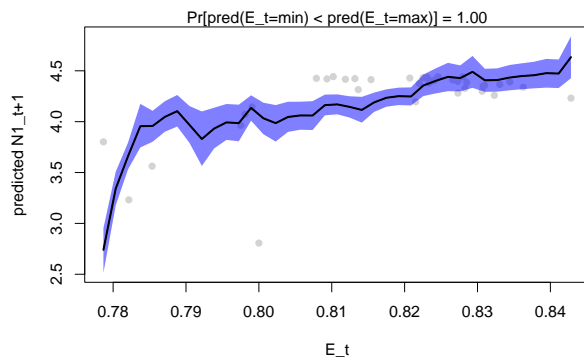
```
OmegaCor = computeAssociations(m.7.sample)
supportLevel = 0.7
toPlot = ((OmegaCor[[1]]$support > supportLevel) + (OmegaCor[[1]]$support < (1 -
  supportLevel)) > 0) * OmegaCor[[1]]$mean
corrplot::corrplot(toPlot, method = "color", col = colorRampPalette(c("blue", "white",
  "red"))(200), title = paste("random effect level:", m.7.sample$rLNames[1]), mar = c(0,
  0, 1, 0))
```



174

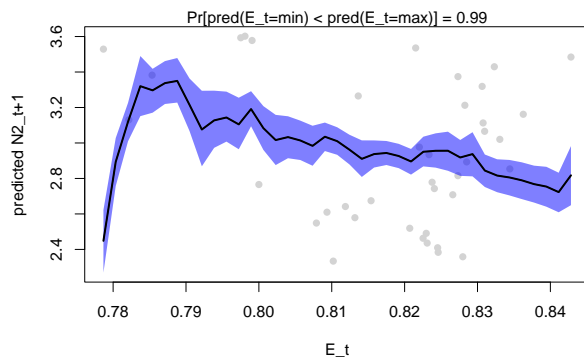
```
OmegaCor
#> [[1]]
#> [[1]]$mean
#>          sp1          sp2
#> sp1 1.0000000 0.9947954
#> sp2 0.9947954 1.0000000
#>
#> [[1]]$support
#>          sp1 sp2
#> sp1      1   1
#> sp2      1   1
```

```
Gradient <- constructGradient(m.7.sample, focalVariable = "E", non.focalVariables = list(Esq = list(2))
  ngrid = 39)
predY <- predict(m.7.sample, XData = Gradient$XDataNew, expected = TRUE)
a <- plotGradient(m.7.sample, Gradient, pred = predY, showData = T, measure = "Y",
  index = 1, main = "", xlab = "E_t", ylab = "predicted N1_t+1")
```



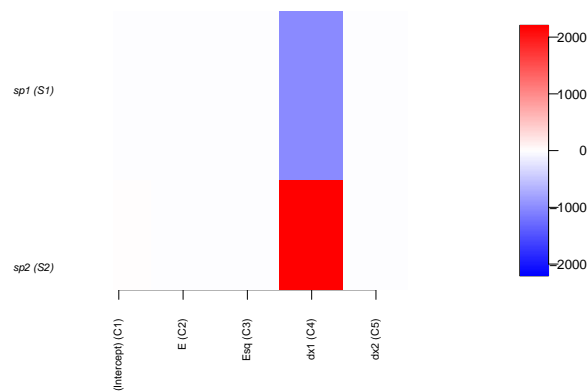
175

```
b <- plotGradient(m.7.sample, Gradient, pred = predY, showData = T, measure = "Y",
  index = 2, main = "", xlab = "E_t", ylab = "predicted N2_t+1")
```



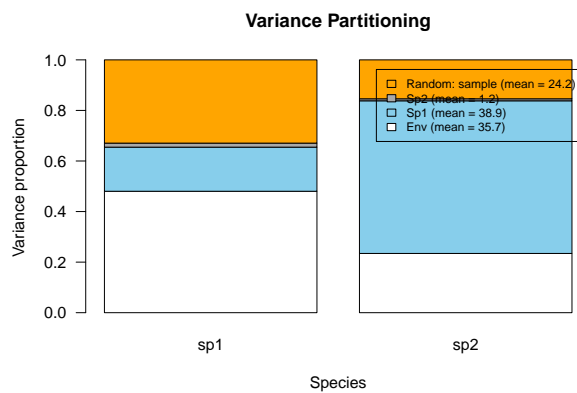
176

```
postBeta = getPostEstimate(m.7.sample, parName = "Beta")
plotBeta(m.7.sample, post = postBeta, param = "Mean", supportLevel = 0.7)
```



177

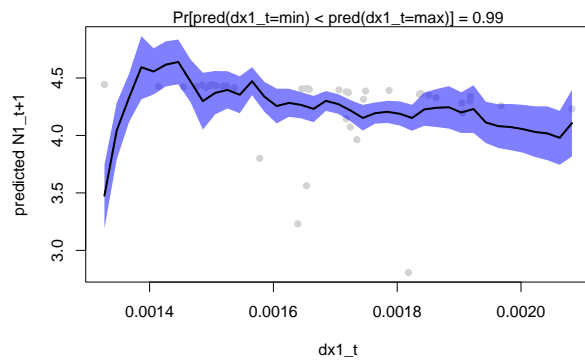
```
VP <- computeVariancePartitioning(m.7.sample, group = c(1, 1, 1, 2, 3), groupnames = c("Env",
"Sp1", "Sp2"))
plotVariancePartitioning(m.7.sample, VP, cols = c("white", "skyblue", "darkgrey",
"orange"), args.legend = list(cex = 0.75, bg = "transparent"))
```



178

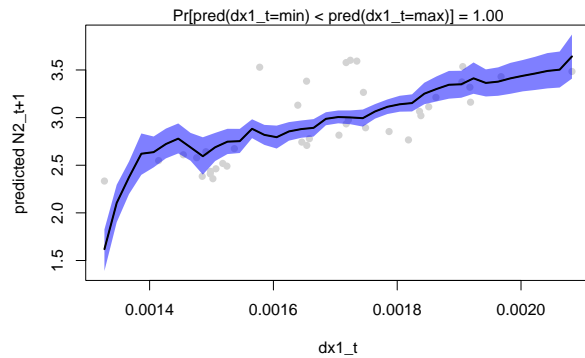
179 We can also look at a gradient plot for the effect of evolution in Species 1 for abundance in Species 1 and Species 2.

```
Gradient <- constructGradient(m.7.sample, focalVariable = "dx1", non.focalVariables = list(E = list(2),
Esq = list(2), dx2 = list(2)), ngrid = 39)
Gradient$XDataNew$Esq <- Gradient$XDataNew$E^2
predY <- predict(m.7.sample, XData = Gradient$XDataNew, expected = TRUE)
a <- plotGradient(m.7.sample, Gradient, pred = predY, showData = T, measure = "Y",
index = 1, main = "", xlab = "dx1_t", ylab = "predicted N1_t+1")
```



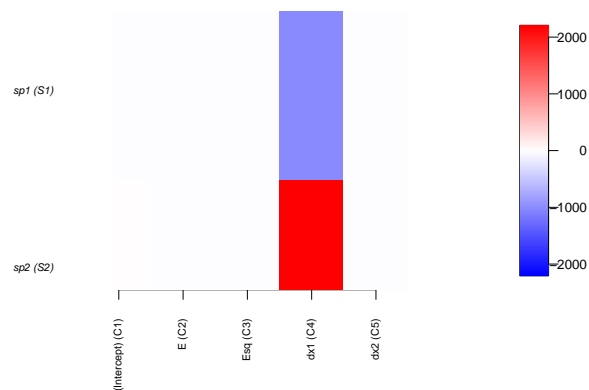
180

```
b <- plotGradient(m.7.sample, Gradient, pred = predY, showData = T, measure = "Y",
  index = 2, main = "", xlab = "dx1_t", ylab = "predicted N2_t+1")
```



181

```
postBeta = getPostEstimate(m.7.sample, parName = "Beta")
plotBeta(m.7.sample, post = postBeta, param = "Mean", supportLevel = 0.7)
```



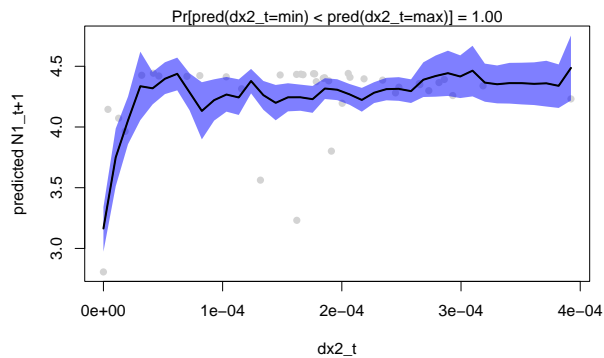
182

183 We repeat this plot for the effect of evolution in Species 2 for abundance in Species 1 and Species 2.


```

Gradient <- constructGradient(m.7.sample, focalVariable = "dx2", non.focalVariables = list(E = list(2),
  Esq = list(2), dx1 = list(2)), ngrid = 39)
Gradient$XDataNew$Esq <- Gradient$XDataNew$E^2
predY <- predict(m.7.sample, XData = Gradient$XDataNew, expected = TRUE)
a <- plotGradient(m.7.sample, Gradient, pred = predY, showData = T, measure = "Y",
  index = 1, main = "", xlab = "dx2_t", ylab = "predicted N1_t+1")

```

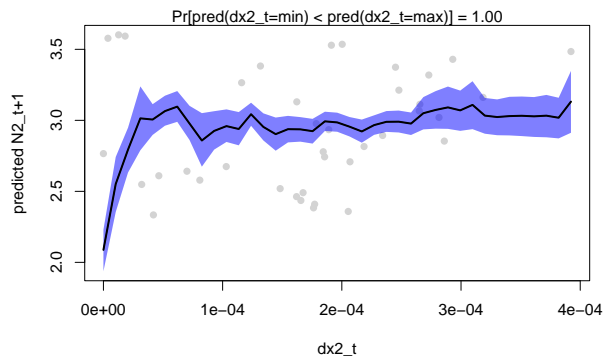


184

```

b <- plotGradient(m.7.sample, Gradient, pred = predY, showData = T, measure = "Y",
  index = 2, main = "", xlab = "dx2_t", ylab = "predicted N2_t+1")

```



185

186 THESE SHOULD NOT BE THE SAME

187 We repeat the analysis for

188

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