

# Solutions: Exercise 2.2 - Survey of ecological models, Part 2

Jelena H. Pantel

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## Instructions

Create either an R script (.R file) or R Markdown document (.Rmd) to save all of your work for today.

## Exercise 1. Work with simulation of discrete-time logistic growth

In class, I discussed how making simulations of the models we learn in class can help us understand how our system works. Let's take a closer look at the *discrete-time model of logistic population growth*. The model formula is:

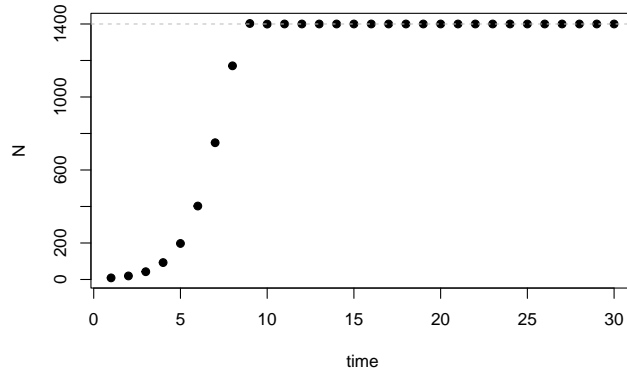
$$n_{t+1} = n_t + rn_t(1 - \frac{n_t}{K})$$

And the simulation works as follow:

```
# Parameter values to use for simulation
r <- 1.21
K <- 1400
NO <- 4
t <- 30
# model function
disc_log <- function(r, NO, K) {
  Nt1 <- NO + r * NO * (1 - NO/K)
  return(Nt1)
}
# Simulation of model for t time steps
N <- rep(NA, t)
for (i in 1:t) {
  N[i] <- disc_log(r, NO, K)
  NO <- N[i]
}
# Plot simulation: ggplot
dat <- as.data.frame(N)
dat$time <- as.numeric(rownames(dat))
ggplot2::ggplot(dat, aes(time, N)) + geom_point() + geom_hline(yintercept = K,
  linetype = "dashed", color = "gray")
```

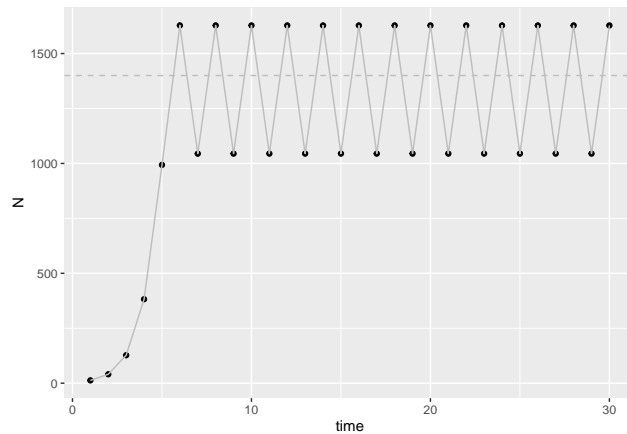


```
# Plot simulation: base R
plot(N, xlab = "time", ylab = "N", pch = 19, col = "black")
abline(h = K, col = "grey", lty = "dashed")
```

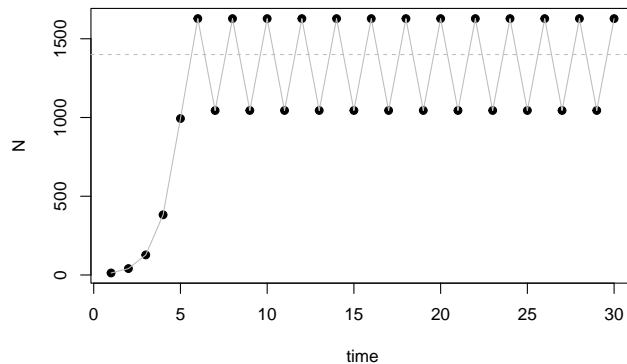


**Question 1.** Please change the simulation to run with a growth rate of 2.2, and show me a plot of the population dynamics over time.

```
# Parameter values to use for simulation
r <- 2.2
K <- 1400
N0 <- 4
t <- 30
# Simulation of model for t time steps
N <- rep(NA, t)
for (i in 1:t) {
  N[i] <- disc_log(r, N0, K)
  N0 <- N[i]
}
# Plot simulation: ggplot
dat <- as.data.frame(N)
dat$time <- as.numeric(rownames(dat))
ggplot2::ggplot(dat, aes(time, N)) + geom_point() + geom_line(color = "grey") +
  geom_hline(yintercept = K, linetype = "dashed", color = "grey")
```



```
# Plot simulation: base R
plot(N, xlab = "time", ylab = "N", pch = 19, col = "black")
lines(N, col = "grey")
abline(h = K, col = "grey", lty = "dashed")
```



**Question 2.** A. Use R's index syntax `var[i]` - where *var* is the name of your variable and *[i]* is the *i*th position of that variable - to show me the value of population size *N* at time 5, 6, and 7.

```
N[5:7]
```

```
## [1] 993.4435 1628.1292 1044.4632
```

B. Use the `'disc_log'` function to calculate the value of *N* at time 6 using the value of *N* at time 5 for *N*<sub>0</sub>

```
disc_log(r, N[5], K)
```

```
## [1] 1628.129
```

C. Use the `'disc_log'` function to calculate the value of *N* at time 7 using the value of *N* at time 6 for *N*<sub>0</sub>

```
disc_log(r, N[6], K)
```

```
## [1] 1044.463
```

D. Use the formula above for discrete-time logistic growth to calculate population size at time 7 using the value of *N* at time 6 for *N*<sub>0</sub>

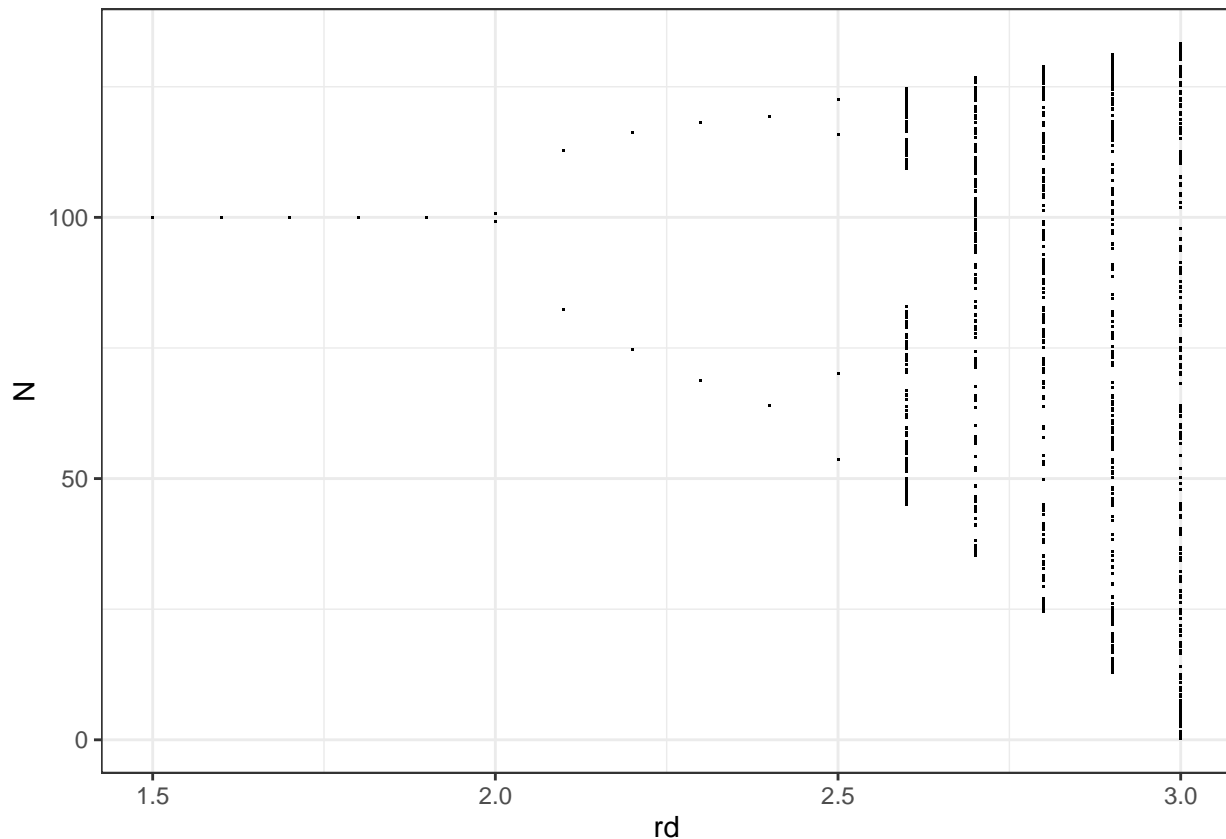
```
N[6] + r * N[6] * (1 - (N[6]/K))
```

```
## [1] 1044.463
```

E. Why is the population not going to its carrying capacity  $K$ ? How many *periodic attractors* (values of  $N$  for which the system is stably drawn towards) are there in this system? In other words, how many values does  $N$  cycle between when  $r = 2.2$ , and what are those values of  $N$ ?

```
# 2 stable attractors 1044 and 1628
```

**Question 3.** Look at the *bifurcation plot* for this discrete-time logistic growth system:



We see that as  $r$  increases the number of attractors continues to double, growing geometrically. Eventually, we reach a point when there becomes an infinite number of unique points, that are determined by  $r$ . This completely deterministic, non-repeating pattern in  $N$  is a property of *chaos*. *Chaos* is not a random phenomenon; rather it is the result of deterministic mechanisms generating non-repeating patterns.

A. How many periodic attractors are there at  $r = 2, 2.1, 2.2, 2.3, 2.4$ , and  $2.5$ ?

```
# We read this directly from the graph: 2, 2, 2, 2, 2, 4
```

B. How many periodic attractors are there at  $r = 2.55$ ? I share the code below to produce a finer scale of  $r$  values:

```

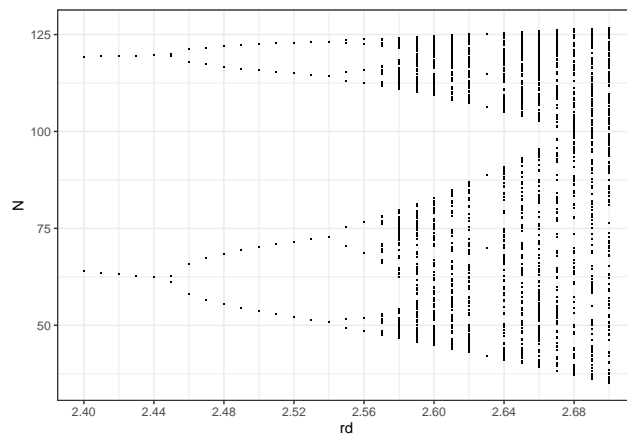
# We read this directly from the graph: 8

dlogistic <- function(t, y, p) {
  N <- y[1]
  with(as.list(p), {
    N.diff <- rd * N * (1 - alpha * N)
    return(list(N.diff))
  })
}

rd.s <- seq(2.4, 2.7, by = 0.01) # the actual rd values
t <- 0:1000 # how long to run each population
N0 <- c(N = 99) # N-zero
## do all the dynamical simulations putting each run into a
## column of a matrix.
Ns <- sapply(rd.s, function(r) {
  p <- c(rd = r, alpha = 0.01)
  outd <- deSolve::ode(y = N0, times = t, func = dlogistic,
    parms = p, method = "euler")[, 2]
})
out <- data.frame(t = t, Ns) # put times to the front of the matrix
names(out) <- c("t", paste("rd=", rd.s, sep = "")) # relabel columns
out.last <- subset(out, t > 0.8 * max(t)) # keep only the last 20%
## put the data in the 'long format'
out.l <- pivot_longer(out.last, cols = -1, names_to = "r.d",
  values_to = "N")
out.l <- dplyr::arrange(out.l, r.d, t) # re-order the data
## extract text out of the 'r.d' label that is the numeric
## value
text.values <- substr(out.l$r.d, regexpr("=", out.l$r.d) + 1,
  100)
## convert the characters of '1.5' to the number 1.5
out.l$rd <- as.numeric(text.values)

## plot the stable limits to show the bifurcations
ggplot(out.l, aes(rd, N)) + geom_point(pch = ".") + theme_bw() +
  scale_x_continuous(breaks = seq(2.4, 2.7, by = 0.04))

```

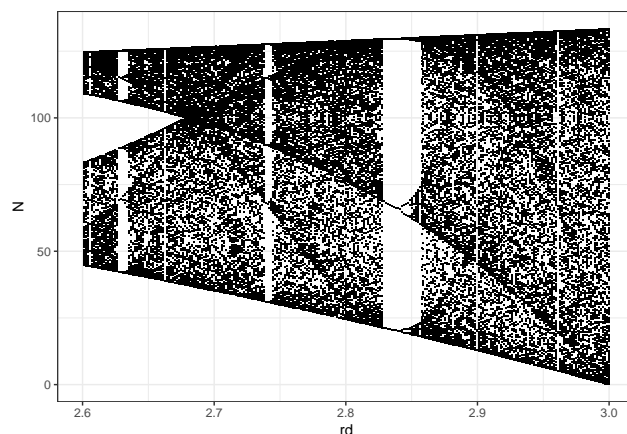


C. Let's look closer at the chaotic portion of the graph, with a much finer range of  $r$  values. Convince yourself about chaos - that a TINY change in the parameter value  $r$  can lead to very different observed

dynamics of  $N$ . You can read a bit more about chaos at this link: (<https://geoffboeing.com/2015/03/chaos-theory-logistic-map/>)

```
num.rd <- 1001 # the number of rd's you want
rd.s <- seq(1.5, 3, length = num.rd) # the actual rd values
t <- 0:1000 # how long to run each population
N0 <- c(N = 99) # N-zero
## do all the dynamical simulations putting each run into a
## column of a matrix.
Ns <- sapply(rd.s, function(r) {
  p <- c(rd = r, alpha = 0.01)
  outd <- deSolve::ode(y = N0, times = t, func = dlogistic,
    parms = p, method = "euler")[, 2]
})
out <- data.frame(t = t, Ns) # put times to the front of the matrix
names(out) <- c("t", paste("rd=", rd.s, sep = "")) # relabel columns
out.last <- subset(out, t > 0.8 * max(t)) # keep only the last 20%
## put the data in the 'long format'
out.l <- pivot_longer(out.last, cols = -1, names_to = "r.d",
  values_to = "N")
out.l <- dplyr::arrange(out.l, r.d, t) # re-order the data
## extract text out of the 'r.d' label that is the numeric
## value
text.values <- substr(out.l$r.d, regexpr("=", out.l$r.d) + 1,
  100)
## convert the characters of '1.5' to the number 1.5
out.l$rd <- as.numeric(text.values)

out.l.chaotic <- subset(out.l, rd > 2.6)
ggplot(out.l.chaotic, aes(rd, N)) + geom_point(pch = ".") + theme_bw()
```



**Question 4.** Plot dynamics in the discrete-time logistic growth system.

In class, I showed a plot of population dynamics then ran the simulation for 8 different values of the population growth rate  $r$  :

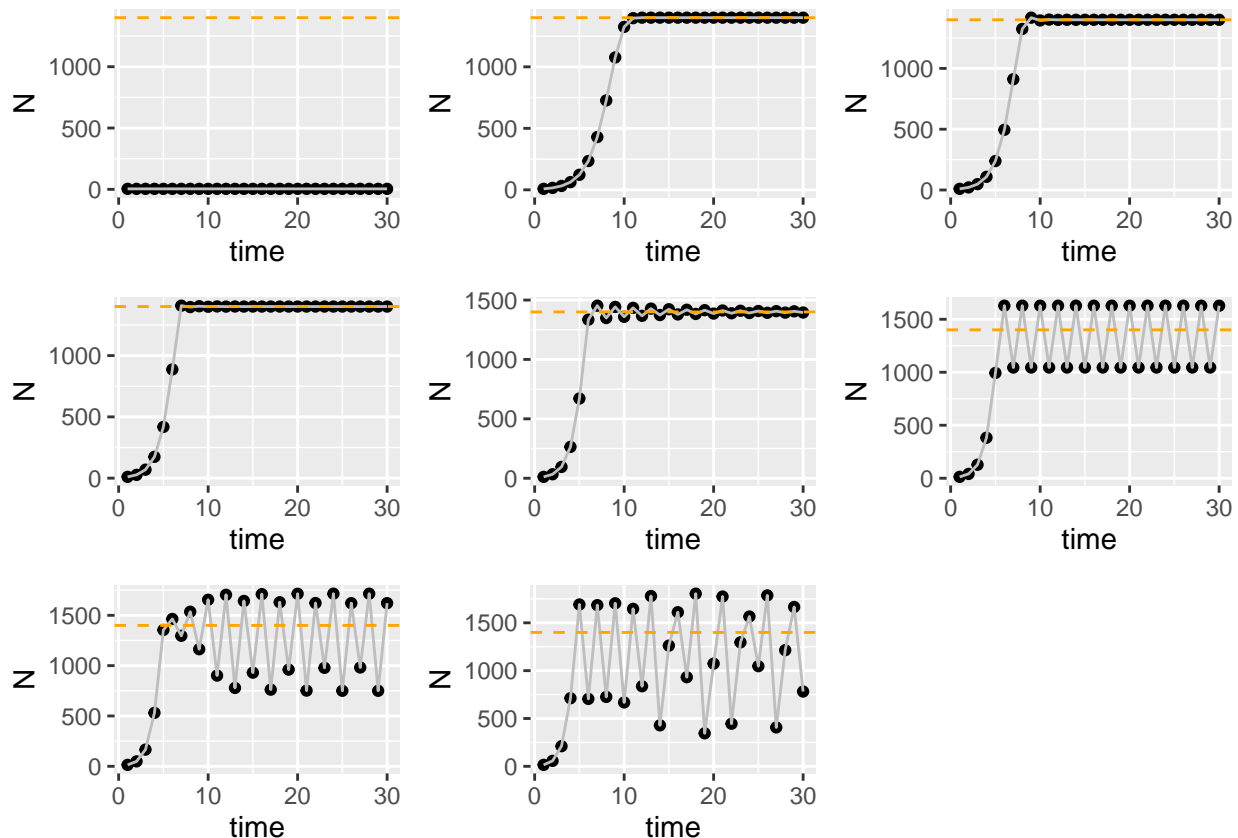
```
# Parameter values to use for simulation
r_range <- c(0, 1, 1.3, 1.6, 1.9, 2.2, 2.5, 2.8)
K <- 1400
N0 <- 4
```

```

t <- 30
# Simulation of model for t time steps
N.g <- numeric()
plist <- list()
for (i in 1:length(r_range)) {
  N <- rep(NA, t)
  for (k in 1:t) {
    N[k] <- disc_log(r_range[i], N0, K)
    N0 <- N[k]
  }
  dat <- as.data.frame(N)
  dat$time <- as.numeric(rownames(dat))
  dat$r <- rep(r_range[i], t)
  N.g <- rbind(N.g, dat)
  N0 <- 4
  p <- ggplot(dat, aes(time, N)) + geom_point() + geom_line(color = "gray") +
    geom_hline(yintercept = K, linetype = "dashed", color = "orange")
  plist[[i]] <- p
}

gridExtra::grid.arrange(grobs = plist, nrow = round(length(r_range)/3))

```



Please use this plotting code to show dynamics for  $r = 2, 2.2, 2.4, 2.5, 2.55,$  and  $2.8$ , for 50 time steps. Please use RMarkdown to write an informative caption for this figure, explaining what the graphs show. You can use this link: (<https://uoepsy.github.io/scs/rmd-bootcamp/06-figs.html#captions>)

```

# Parameter values to use for simulation
r_range <- c(2, 2.2, 2.4, 2.5, 2.55, 2.8)
K <- 1400
NO <- 4
t <- 50
# Simulation of model for t time steps
N.g <- numeric()
plist <- list()
for (i in 1:length(r_range)) {
  N <- rep(NA, t)
  for (k in 1:t) {
    N[k] <- disc_log(r_range[i], NO, K)
    NO <- N[k]
  }
  dat <- as.data.frame(N)
  dat$time <- as.numeric(rownames(dat))
  dat$r <- rep(r_range[i], t)
  N.g <- rbind(N.g, dat)
  NO <- 4
  p <- ggplot(dat, aes(time, N)) + geom_point() + geom_line(color = "gray") +
    geom_hline(yintercept = K, linetype = "dashed", color = "orange")
  plist[[i]] <- p
}

gridExtra::grid.arrange(grobs = plist, nrow = 3)

```

## Exercise 2. Create simulation of continuous-time Lotka-Volterra competition model

Here is code for a continuous-time logistic growth model:

The formula:

$$\frac{dn}{dt} = rn(1 - \frac{n}{K})$$

The code:

```

# Parameter values to use for simulation
parameters <- c(r = 1.21, K = 1400)
state <- c(N = 4)
times <- seq(0, 30, by = 0.01)
# model function
cont_log <- function(t, state, parameters) {
  with(as.list(c(state, parameters)), {
    dN <- r * N * (1 - N/K)
    return(list(dN))
  })
}
# Simulation of model for t time steps
out <- deSolve::ode(y = state, times = times, func = cont_log,
  parms = parameters)
# Plot simulation: ggplot

```



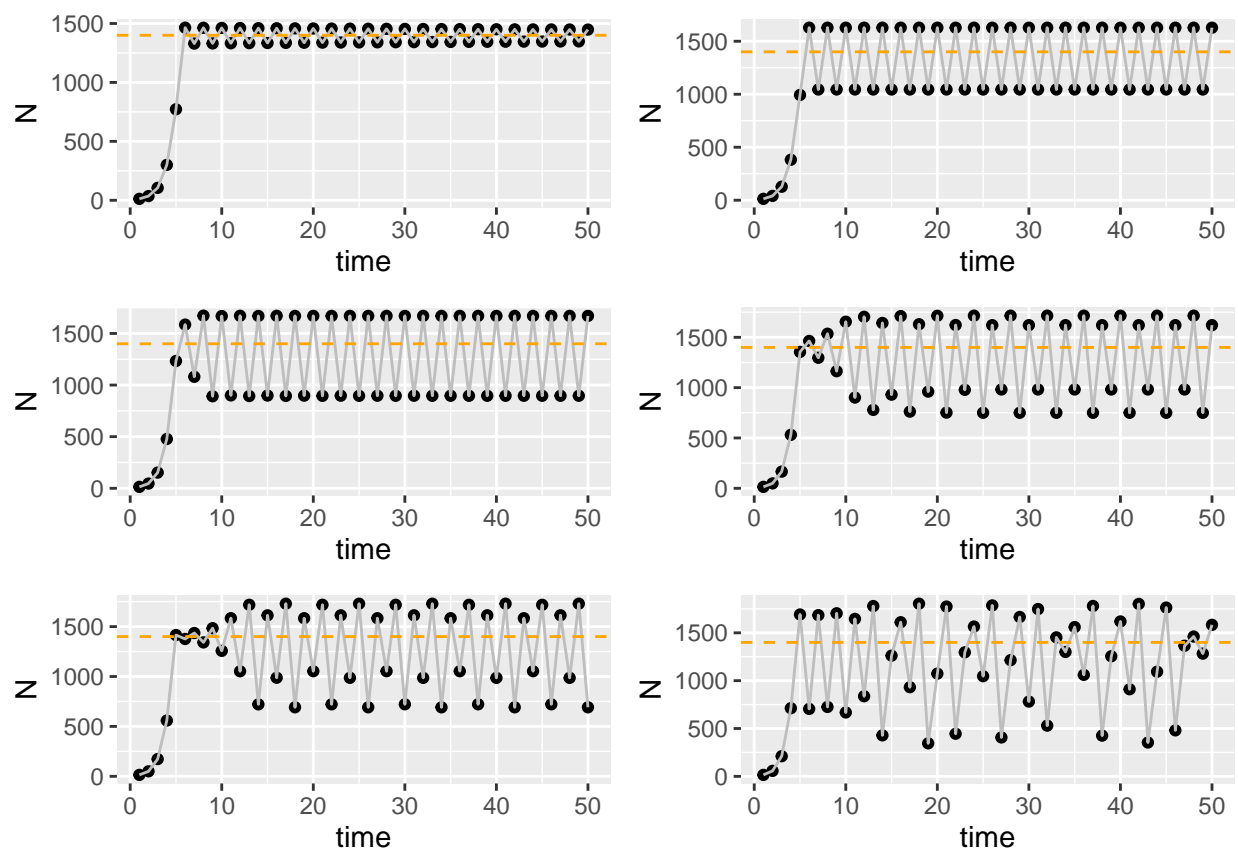
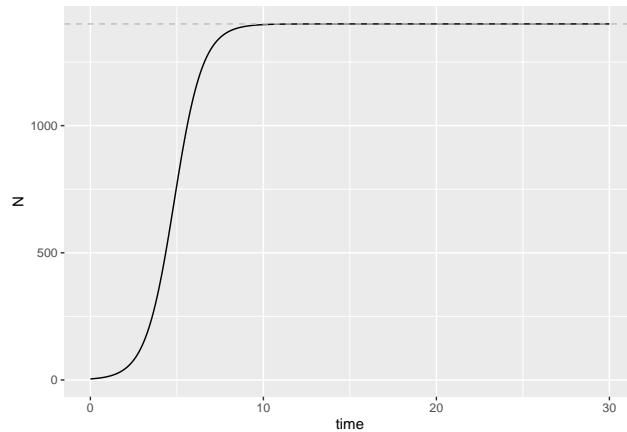
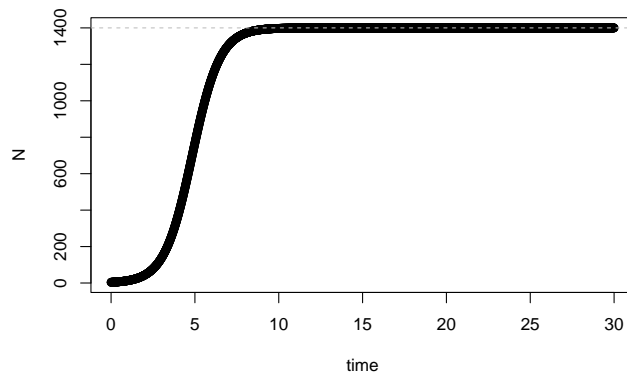


Figure 1: A figure of population size over time using the discrete-time logistic growth model

```
out.g <- as.data.frame(out)
ggplot2::ggplot(out.g, aes(time, N)) + geom_line() + geom_hline(yintercept = K,
  linetype = "dashed", color = "gray")
```



```
# Plot simulation: base R
plot(out.g$time, out.g$N, xlab = "time", ylab = "N", pch = 19,
  col = "black")
abline(h = K, col = "grey", lty = "dashed")
```



A. Please modify this to create a simulation of the continuous time Lotka-Volterra competition equations, seen as equations 3.15 in Otto & Day Chapter 3.

The formula:

$$\begin{aligned}\frac{dn_1}{dt} &= r_1 n_1 \left(1 - \frac{n_1 t + \alpha_{12} n_2}{K_1}\right) \\ \frac{dn_2}{dt} &= r_2 n_2 \left(1 - \frac{n_2 t + \alpha_{21} n_1}{K_2}\right)\end{aligned}$$

The parameters you will need are:

`r1`, `r2`, `K1`, `K2`, `alpha_12`, `alpha_21`, `n1_0`, `n2_0`

Use these values:

`r1 = 1.3`, `r2 = 1.5`, `K1 = 1,400`, `K2 = 1,000`, `alpha_12 = 0.4`, `alpha_21 = 0.6`, `N1_0 = 5`, `N2_0 = 5`

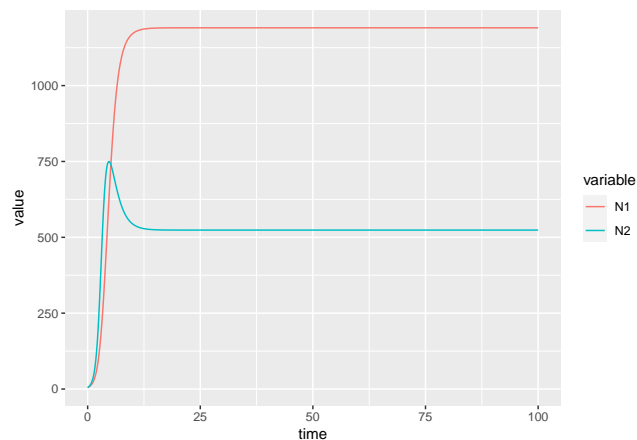
*Note: be very careful with your parentheses*

Here I show with prompts what code you need to fill in. Then I show my code for running the continuous simulation and plotting.

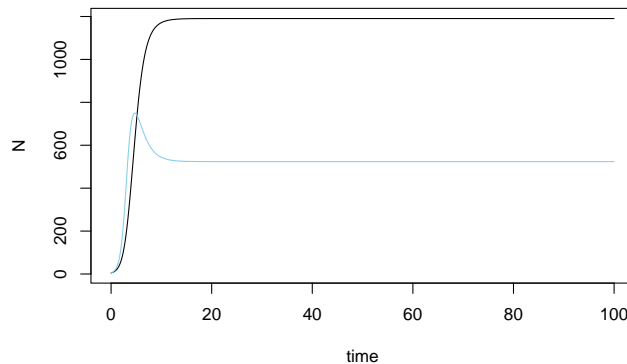
```
# Parameter values to use for simulation
parameters <- 1 ## fill in param values where the 1 is
state <- 1 ## fill in initial values for state variables where the 1 is
times <- 1 ## fill in values for time intervals where the 1 is
# model function
cont_LV <- function(t, state, parameters) {
  with(as.list(c(state, parameters)), {
    ## fill in equation for N1 fill in equation for N2
    return(list(c(dN1, dN2)))
  })
}
```

```
# Parameter values to use for simulation
parameters <- c(r1 = 1.3, r2 = 1.7, K1 = 1400, K2 = 1000, alpha_12 = 0.4,
  alpha_21 = 0.6)
state <- c(N1 = 5, N2 = 5)
times <- seq(0, 100, by = 0.1)
# model function
cont_LV <- function(t, state, parameters) {
  with(as.list(c(state, parameters)), {
    dN1 <- r1 * N1 * (1 - ((N1 + alpha_12 * N2)/K1))
    dN2 <- r2 * N2 * (1 - ((N2 + alpha_12 * N1)/K2))
    return(list(c(dN1, dN2)))
  })
}
```

```
# Simulation of model for t time steps
out <- deSolve::ode(y = state, times = times, func = cont_LV,
  parms = parameters)
# Plot simulation: ggplot
out.g <- as.data.frame(out)
out.melt <- reshape2::melt(out.g, id.var = "time")
ggplot2::ggplot(out.melt, aes(x = time, y = value, col = variable)) +
  geom_line()
```



```
# Plot simulation: base R
plot(out.g$time, out.g$N1, xlab = "time", ylab = "N", col = "black",
     type = "l")
lines(out.g$time, out.g$N2, col = "skyblue")
```



## OPTIONAL BONUS CONTENT

Amazing animated graph by classmate Tina Ross! Animation of discrete-time logistic growth graphs across values of  $r$

```
# set my parameters
rs <- c(2, 2.2, 2.4, 2.5, 2.55, 2.8)
t <- 50
N0 <- 4
K <- 1400

grid.log <- as.data.frame(matrix(NA, nrow = t + 1, ncol = length(rs) +
  1))
colnames(grid.log) <- c("t", paste("r", rs, sep = ""))
grid.log$t <- c(0:t)
ggrid <- list()

for (i in 1:length(rs)) {
  r <- rs[i]
  N <- c(N0, rep(NA, t))
  for (j in 1:t) {
    N[j + 1] <- disc_log(r, N[j], K)
  }
  grid.log[, i + 1] <- N
  ggrid[[i]] <- ggplot(grid.log, aes(x = t, y = .data[[names(grid.log)[i +
    1]])) + geom_line(col = "turquoise") + geom_point(size = 1,
    shape = 19, col = "turquoise4") + geom_hline(yintercept = K,
    linetype = "dashed", color = "grey60") + theme_light() +
    labs(title = (paste("r =", rs[i])), y = "N", x = "t (days)") +
    theme(axis.title.y = element_text(angle = 0, vjust = 0.5))
}
remove(i, j)

grid.long <- melt(grid.log, id.vars = "t", variable.name = "r",
  value.name = "N")
```

```

grid.long$r <- as.numeric(substring(grid.long$r, 2))
aux <- data.frame(x = c(0:t), y = rep(K, t + 1))

gganimate::animate(ggplot(grid.long, aes(x = t, y = N)) + geom_line(col = "turquoise") +
  geom_line(data = aux, aes(x = x, y = y), linetype = 2, color = "grey60") +
  geom_point(size = 2, shape = 19, col = "turquoise4") + scale_x_continuous(expand = c(0,
0)) + theme_light() + theme(axis.title.y = element_text(angle = 0,
vjust = 0.5)) + transition_states(r, transition_length = 1,
state_length = 1) + labs(title = "r = {closest_state}", y = "N",
x = "t (days)") + ease_aes("sine-in-out"), renderer = gifski_renderer())

```