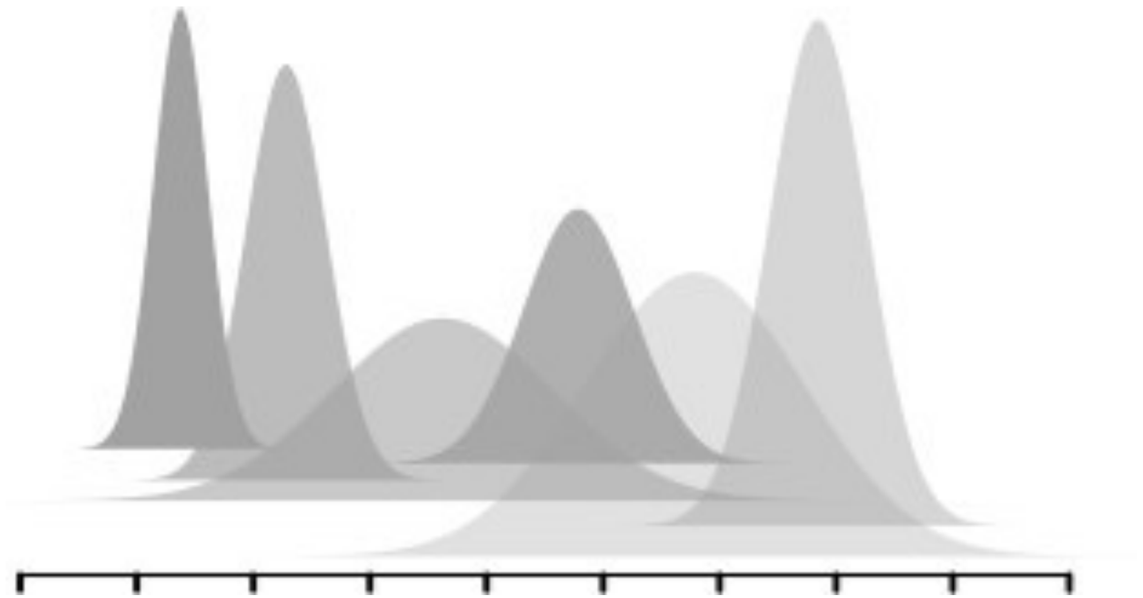
4.1b How To Use Ecological Models



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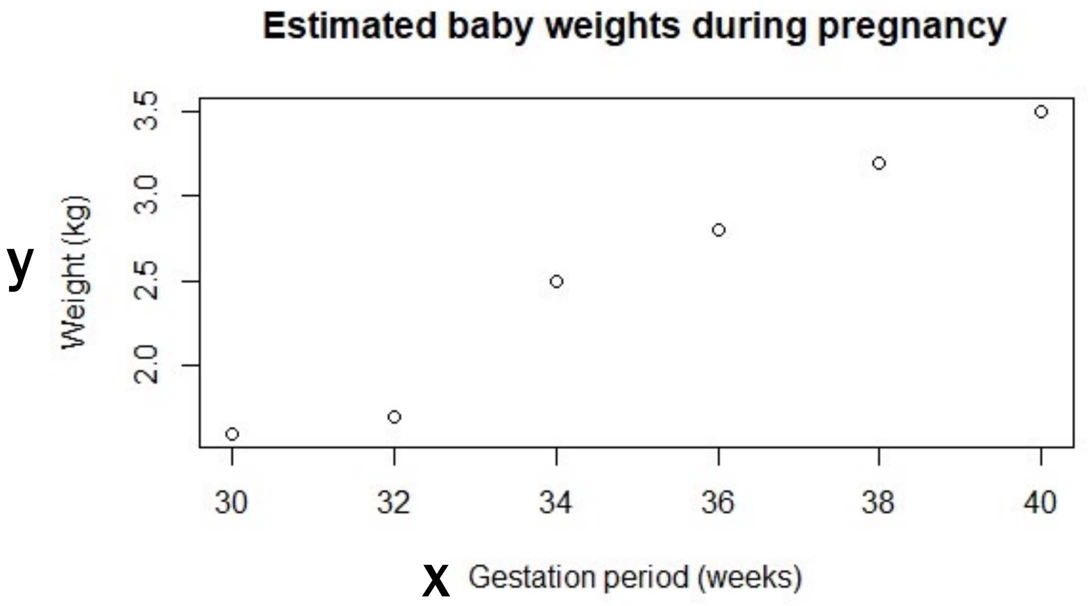
How to use a mathematical model?

Step 1. Evaluate potential model equations

$$y = mx + b$$
$$weight = m(gestation\ period) + b$$



Step 2. State clearly the model parameters you wish to estimate

$$weight = m(gestation\ period) + b$$

How to use a mathematical model?

gestation	weight
30	1.6
32	1.7
34	2.5
36	2.8
38	3.2
40	3.5

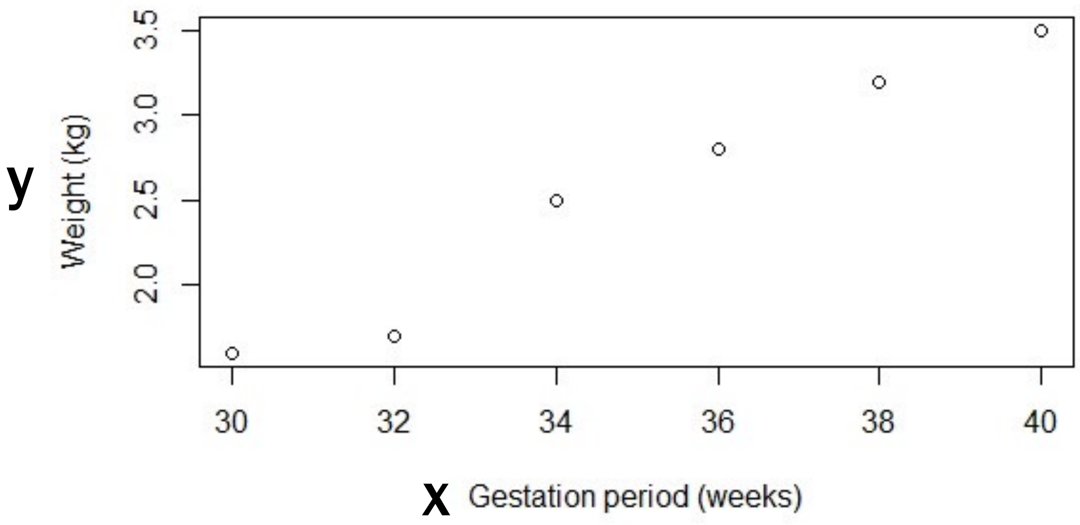
```
Coefficients:
(Intercept)    gestation
    -4.6000       0.2043

> confint(model, level = 0.95)
                 2.5 %     97.5 %
(Intercept) -6.3862379 -2.8137621
gestation    0.1534916  0.2550798
```

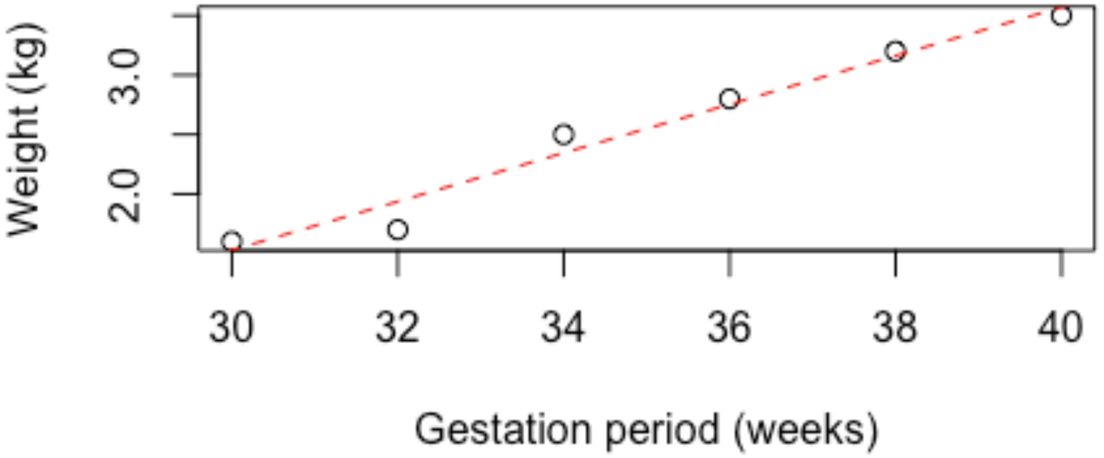
Example 1. Linear causal model (linear regression)

$$y = mx + b$$
$$weight = m(gestation\ period) + b$$

Estimated baby weights during pregnancy



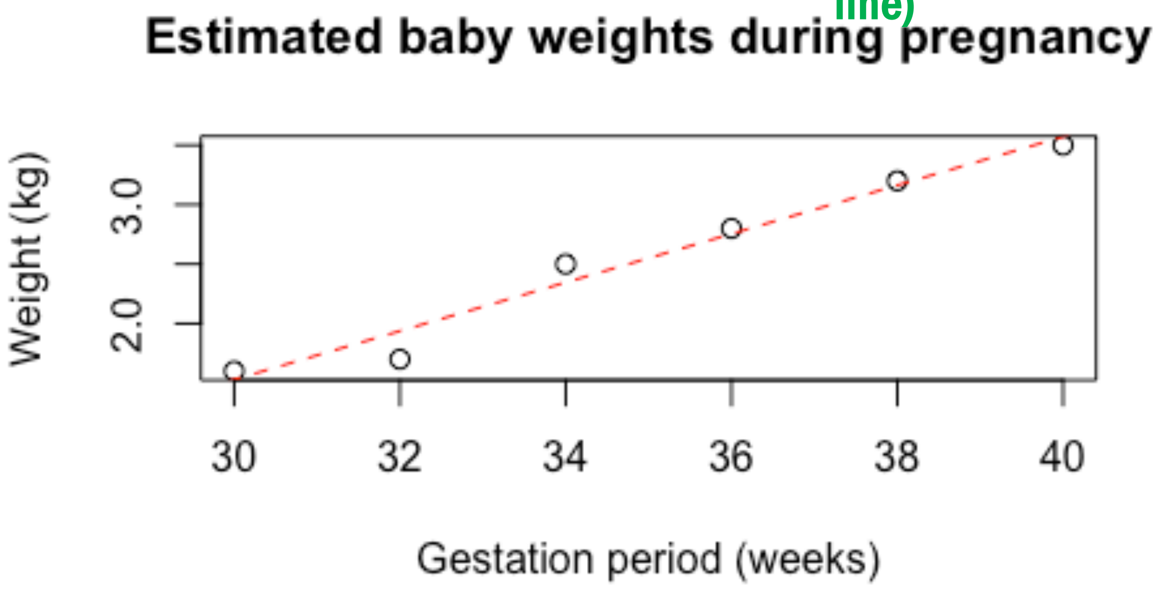
Estimated baby weights during pregnancy

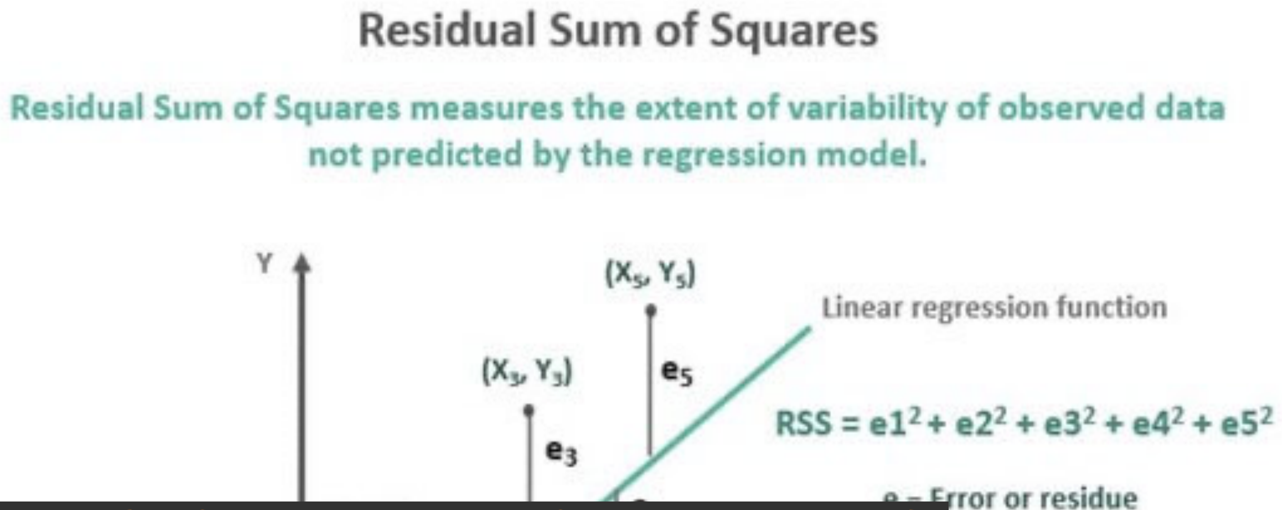


How to use a mathematical model?

$$weight = m(gestation\ period) + b$$

Least squares regression (line that minmizes sum of squared distances from observed points to model-fit line)





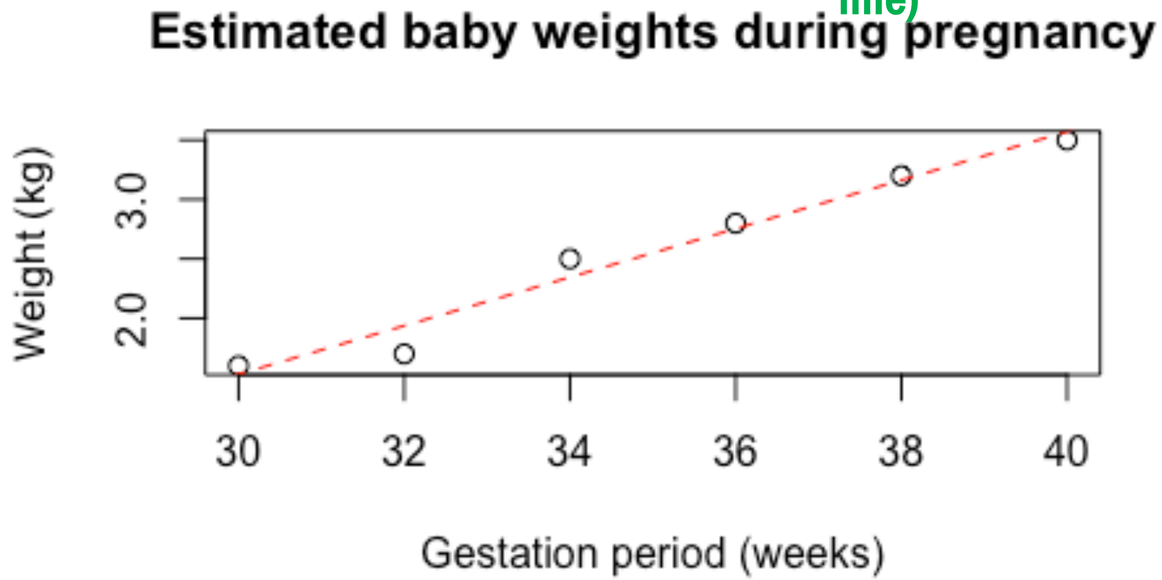

```
  predicted observed predicted_minus_observed
1  1.528571      1.6              -0.07142857
2  1.937143      1.7               0.23714286
3  2.345714      2.5              -0.15428571
4  2.754286      2.8              -0.04571429
5  3.162857      3.2              -0.03714286
6  3.571429      3.5               0.07142857
```

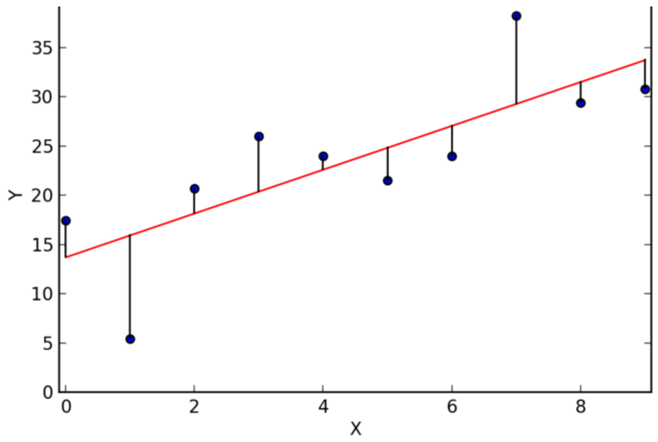
```
> model = lm(weight ~ gestation,data=baby)
> sum(dat$predicted_minus_observed^2)
[1] 0.09371429
> deviance(model)
[1] 0.09371429
```

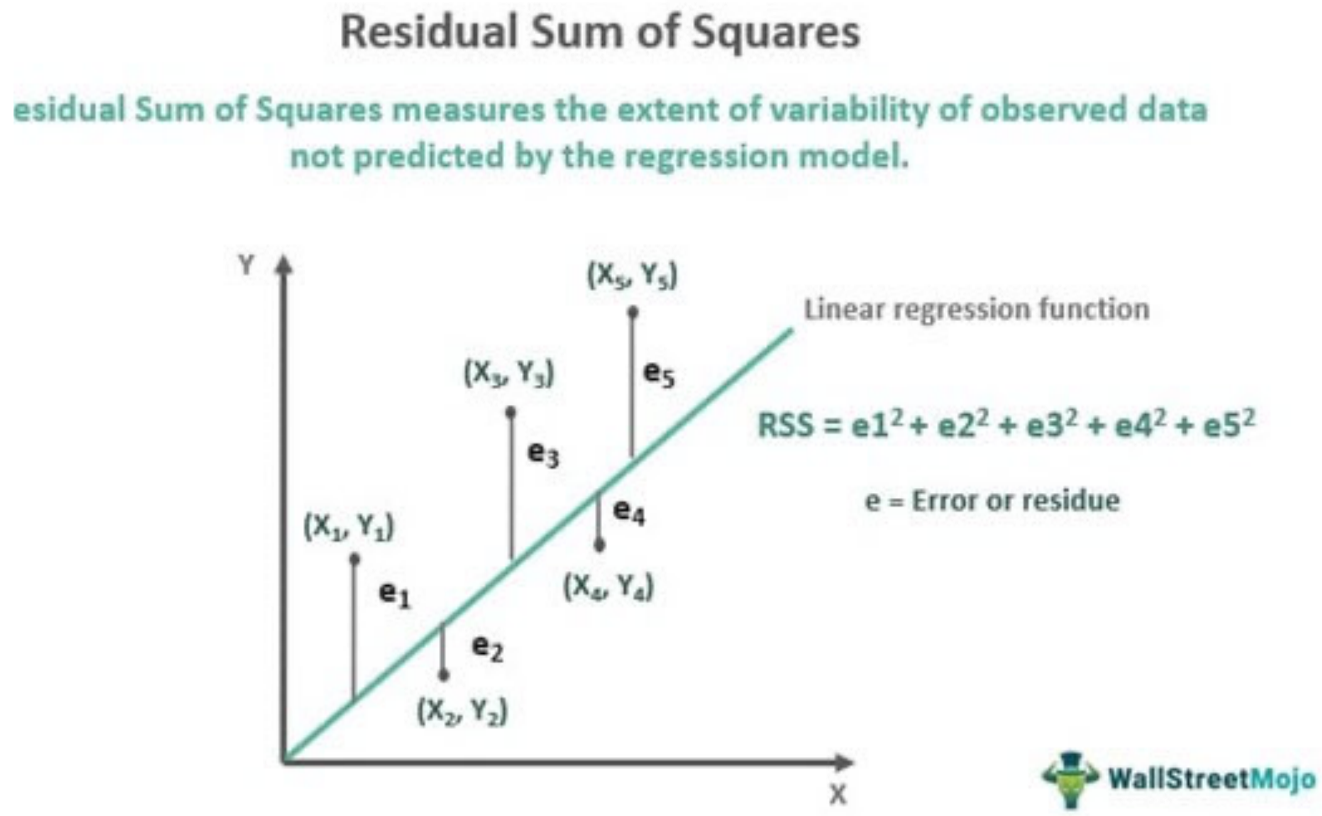
How to use a mathematical model?

$$weight = m(gestation\ period) + b$$

Least squares regression (line that minmizes sum of squared distances from observed points to model-fit line)





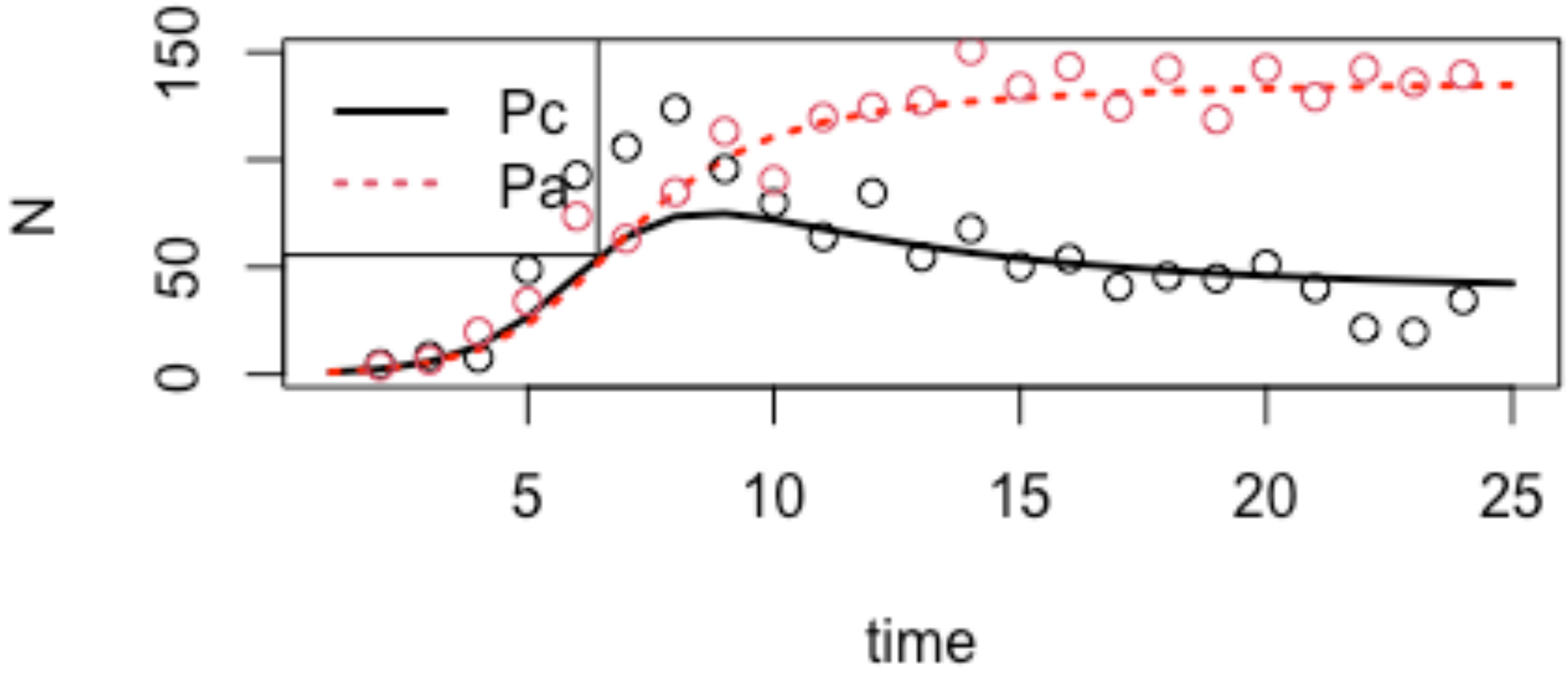


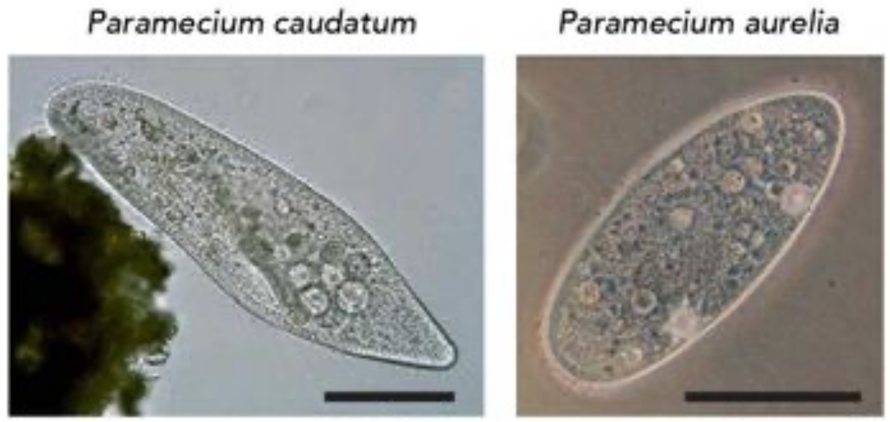
How to use a mathematical model?

Lotka Volterra model

$$\frac{dn_i}{dt} = rn_i\left(1 - \frac{n_i + \alpha_{ij}n_j}{K_i}\right) \qquad \frac{dn_j}{dt} = rn_j\left(1 - \frac{n_j + \alpha_{ji}n_i}{K_j}\right)$$

Non-linear least squares regression – find parameter values that minimize sum of squared distances from observed points to model-fit line





How to use a mathematical model?

Lotka Volterra model

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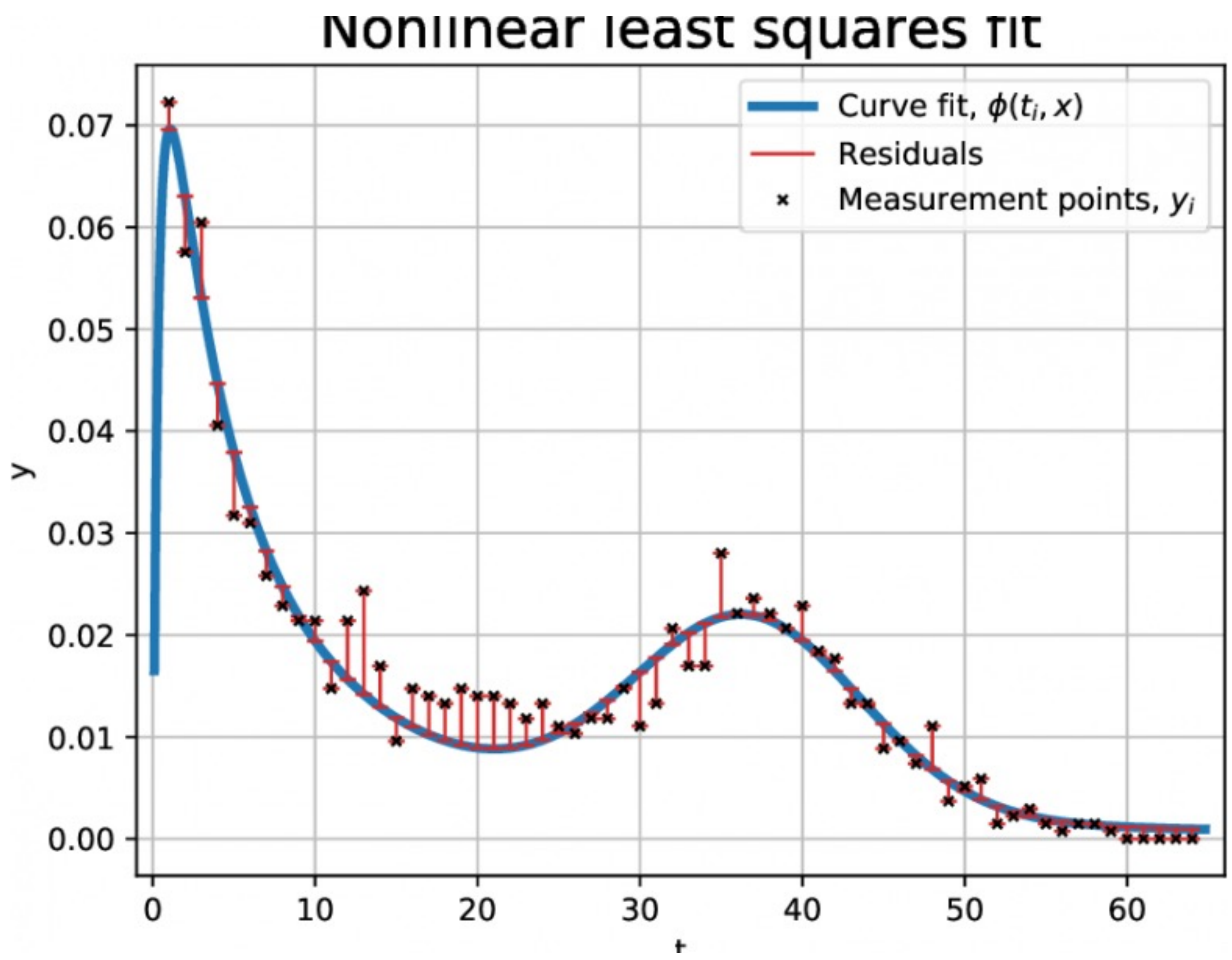
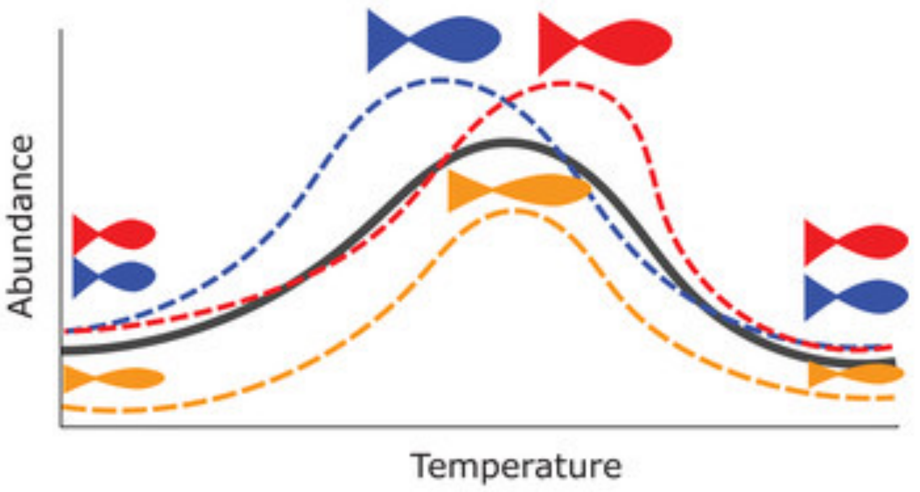
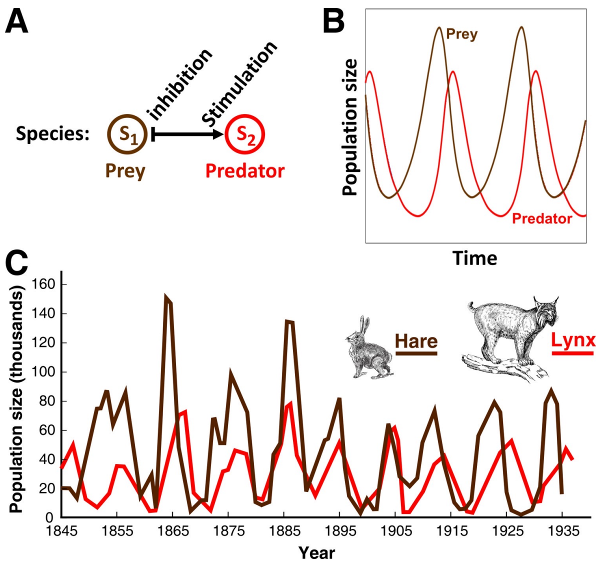
Non-linear least squares regression – find parameter values that minimize sum of squared distances from observed points to model-fit line

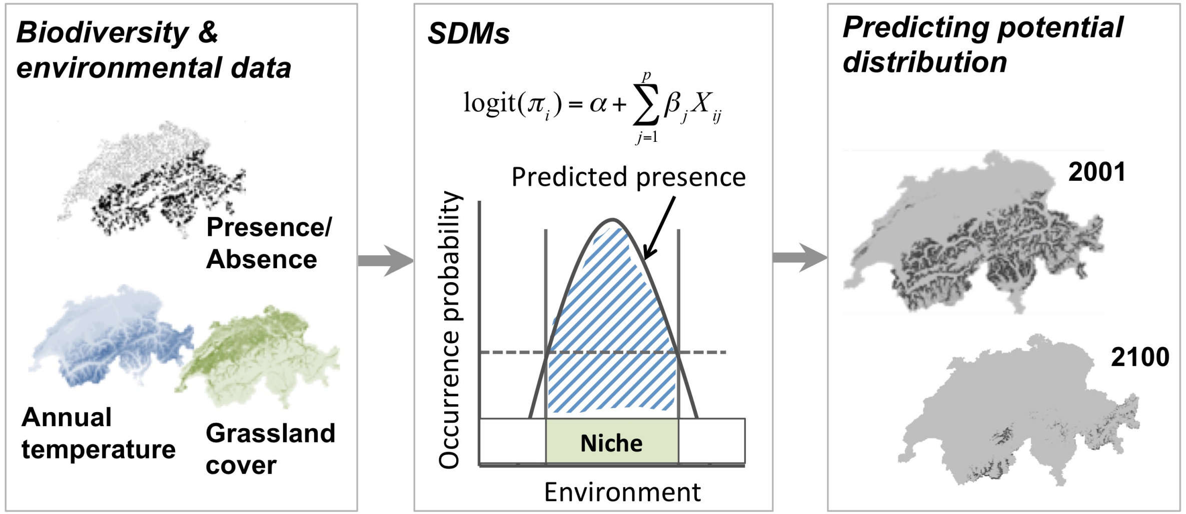


Fitting data to models: probability distributions and statistical models





Key link between data and models – estimating model parameter values given the observed data



What is a statistical model?

Definition

- a mathematical model that embodies a set of statistical assumptions concerning the generation of data sampled from a larger population
- A statistical model represents, often in considerably idealized form, the data-generating process

Bighorn sheep, Ovis canadensis

Réale & Festa-Bianchet, 2000

trait	mean	variance (σ^2)
Longevity (L,year)	7.06	19.08
Lifetime fecundity (F,no. offspring produced)	5.33	14.88
Adult body mass (m,kg)	71.1	20

$$y_i \sim N(\mu, \sigma)$$

Longevity of sheep *i*

is drawn from / is distributed as

a normal distribution

with mean μ

and standard deviation $\sigma = \sqrt{\sigma^2}$

$$\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

Calculate variance for: 3, 6, 9

$$L_i \sim N(7.06, \sqrt{19.08}) \qquad m_i \sim N(71.1, \sqrt{20})$$

$$F_i \sim N(5.33, \sqrt{14.88})$$

What is a statistical model?

Definition

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$$m_i \sim N(71.1, \sqrt{20})$$

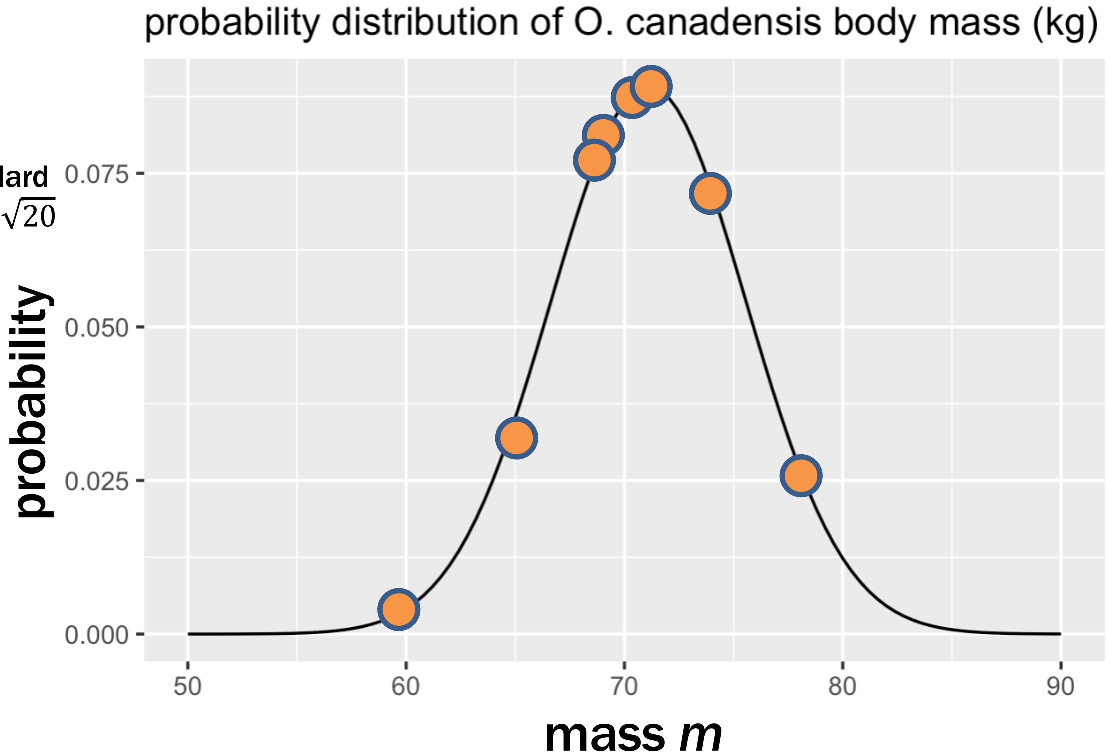
mass of sheep *i*

is drawn from / is distributed as

a normal distribution

with mean 71.1

and standard deviation $\sqrt{20}$



Probability distribution: Normal (Gaussian)

$$m_i \sim N(71.1, \sqrt{20})$$

mass of sheep *i*

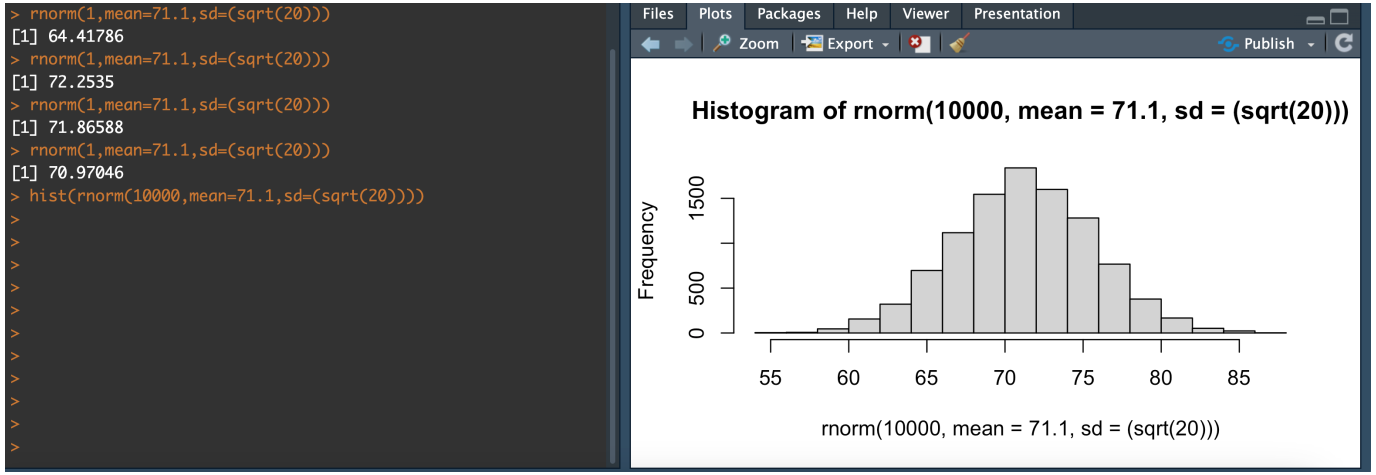
is drawn from / is distributed as

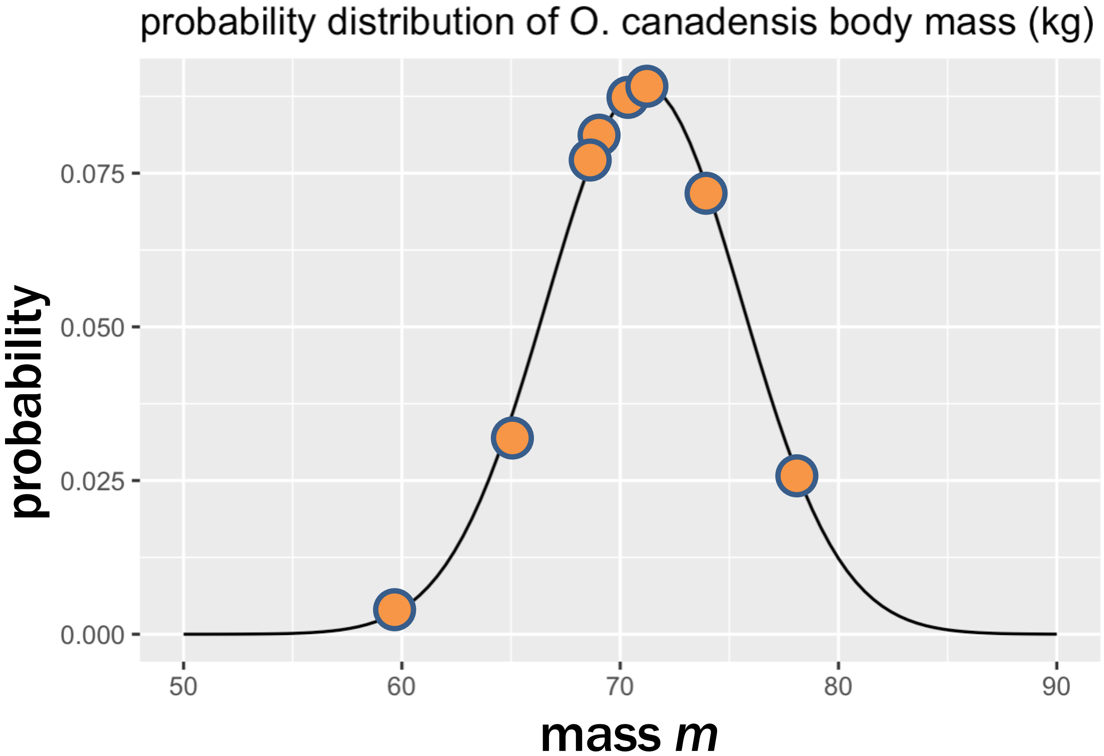
a normal distribution

with mean 71.1

and standard deviation $\sqrt{20}$

```
> dnorm(74,mean=71.1,sd=(sqrt(20)))
[1] 0.07229107
> dnorm(90,mean=71.1,sd=(sqrt(20)))
[1] 1.180421e-05
> dnorm(71,mean=71.1,sd=(sqrt(20)))
[1] 0.08918391
>
```





What is a statistical model?

Definition

- a mathematical model that embodies a set of statistical assumptions concerning the generation of data sampled from a larger population
- A statistical model represents, often in considerably idealized form, the data-generating process $y_i \sim Bernoulli(p)$

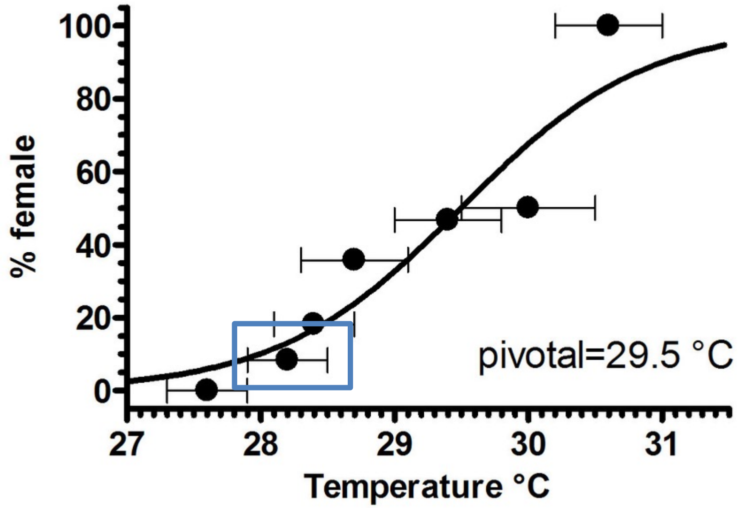
number of females for population *i*

is drawn from

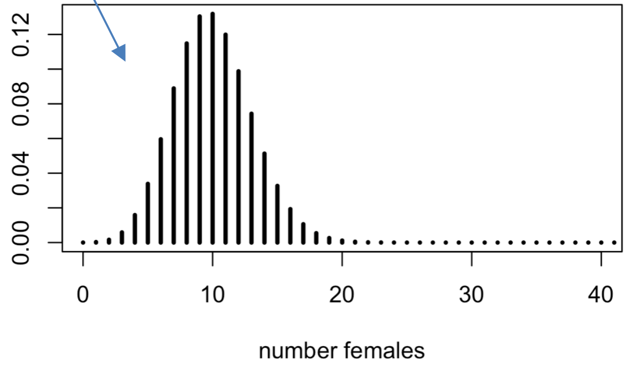
a Bernoulli distribution

with proportion *p*

$f_i \sim Bernoulli(0.1)$



Number of females out of 100 eggs



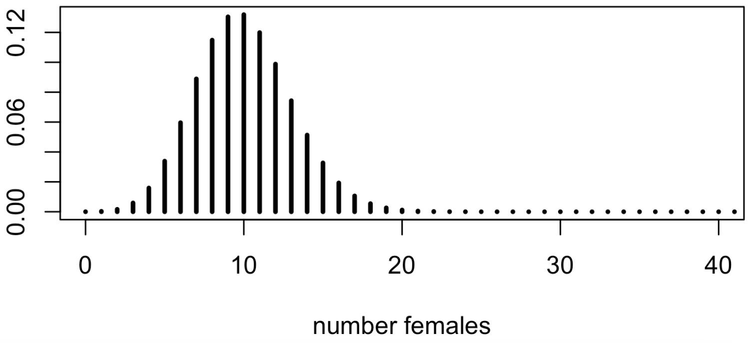
Godfrey & Mrosovsky, 2006

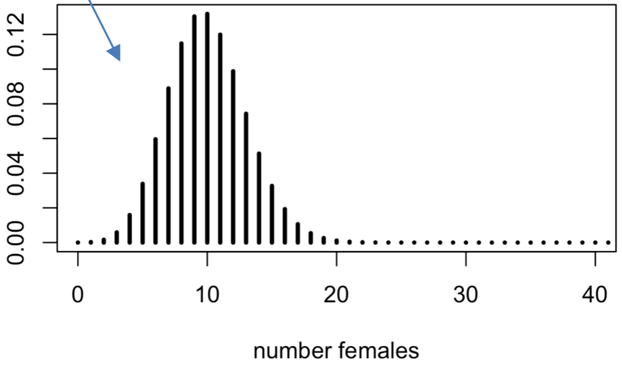
Probability distribution: Bernoulli (n=1), Binomial (n > 1)

$$f_i \sim Bernoulli(0.1)$$

Number of females out of 100 eggs

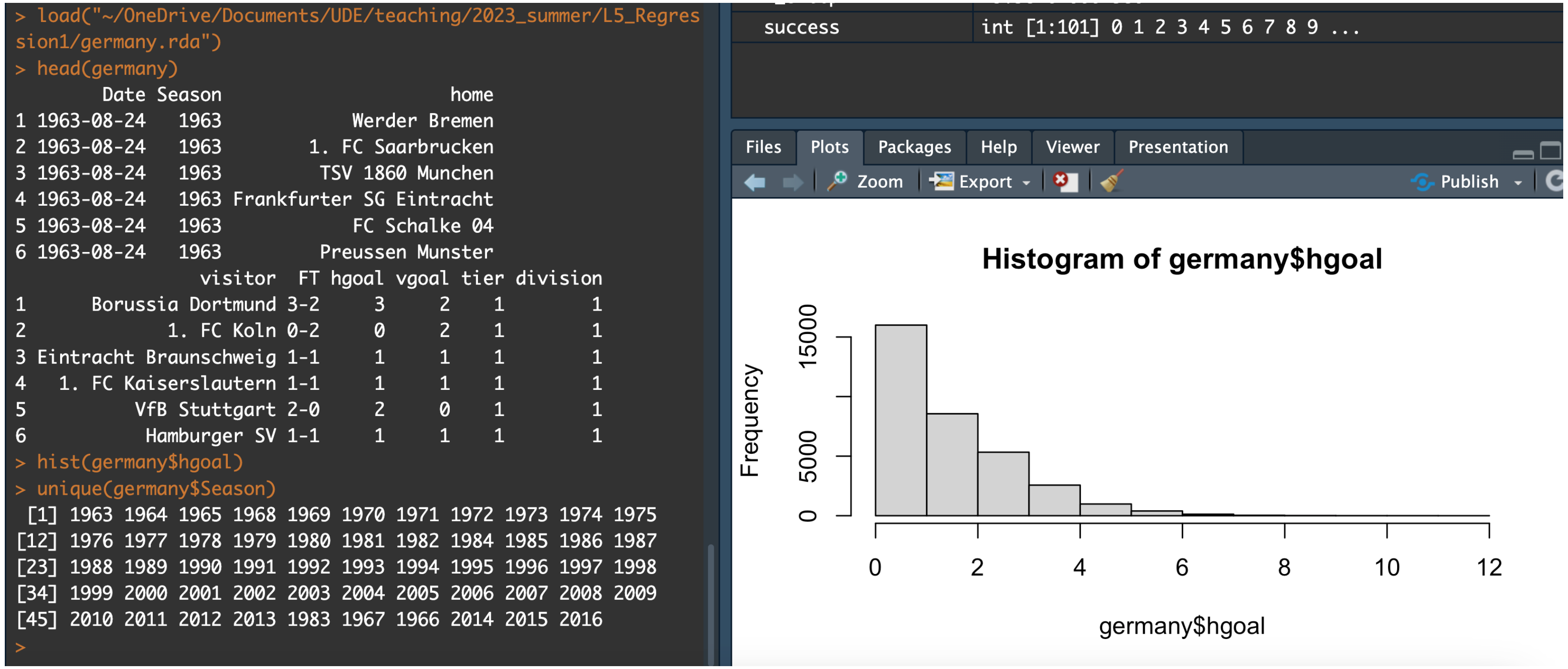
```
> rbinom(1,size=100,prob=0.1)
[1] 12
> rbinom(1,size=100,prob=0.1)
[1] 10
> rbinom(1,size=100,prob=0.1)
[1] 5
> rbinom(1,size=100,prob=0.1)
[1] 15
> rbinom(1,size=100,prob=0.1)
[1] 7
> dbinom(10,size=100,prob=0.1)
[1] 0.1318653
> dbinom(40,size=100,prob=0.1)
[1] 2.470212e-15
>
```



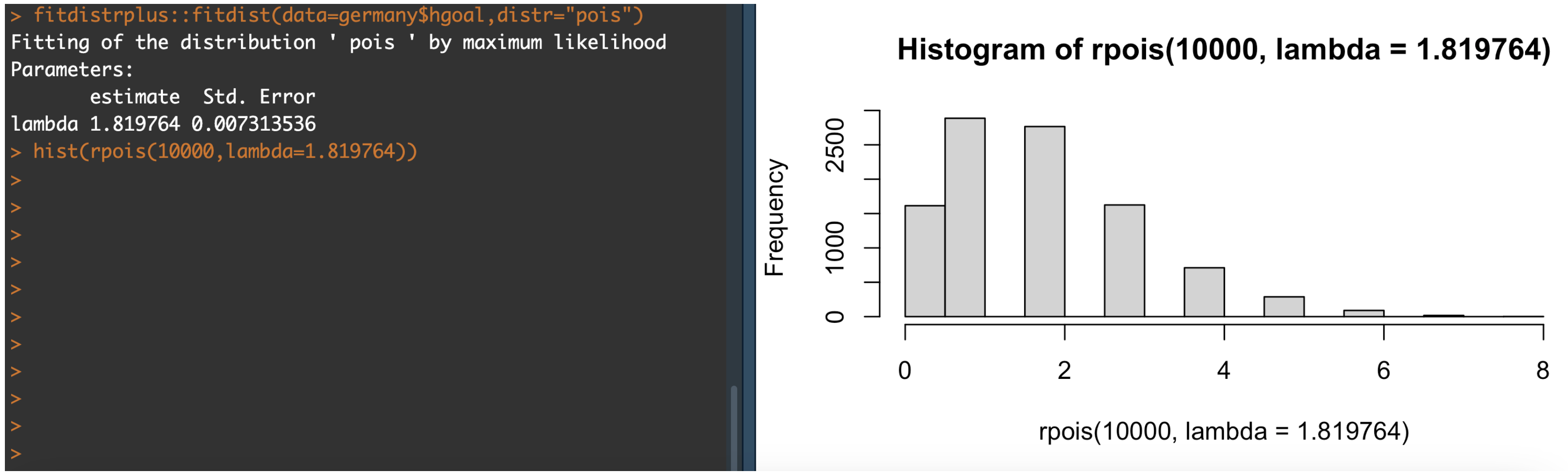


Godfrey & Mrosovsky, 2006

Probability distribution: Poisson



$g_i \sim Poisson(\lambda)$



$g_i \sim Poisson(\lambda = 1.819)$

Probability distributions: diverse families