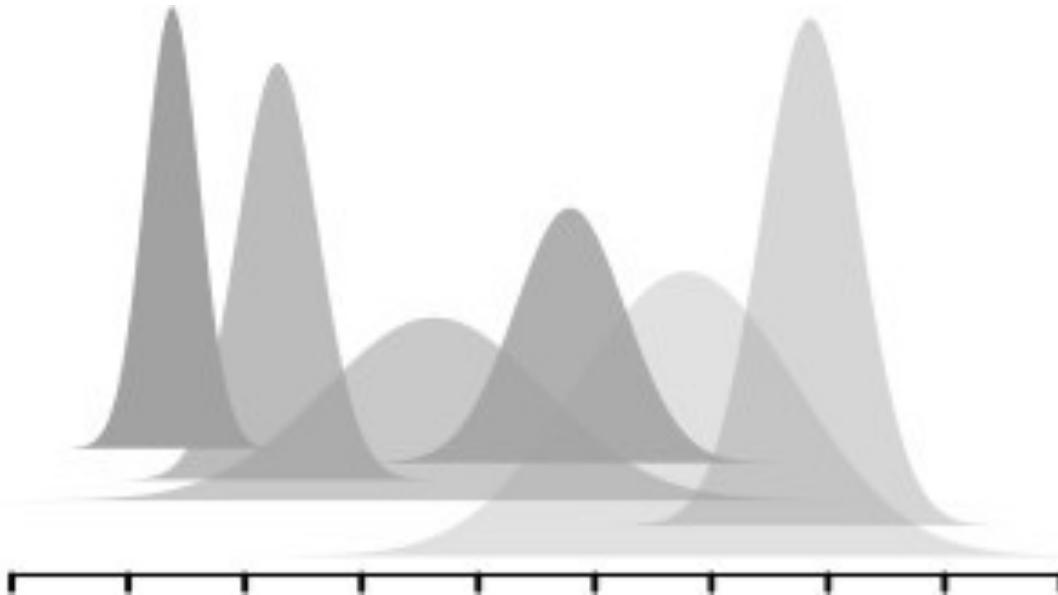


# 5.2 Fitting empirical data to models



Jelena H. Pantel

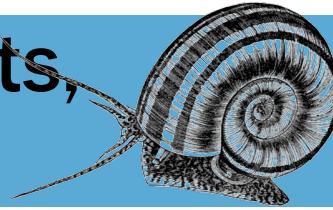
Faculty of Biology

University of Duisburg-Essen

jelena.pantel@uni-due.de

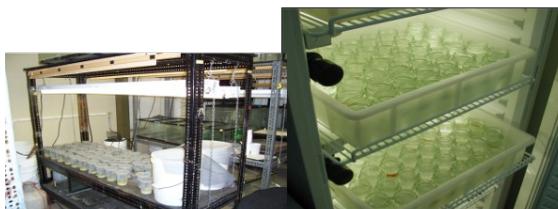


# Research approach – combine surveys, experiments, and models

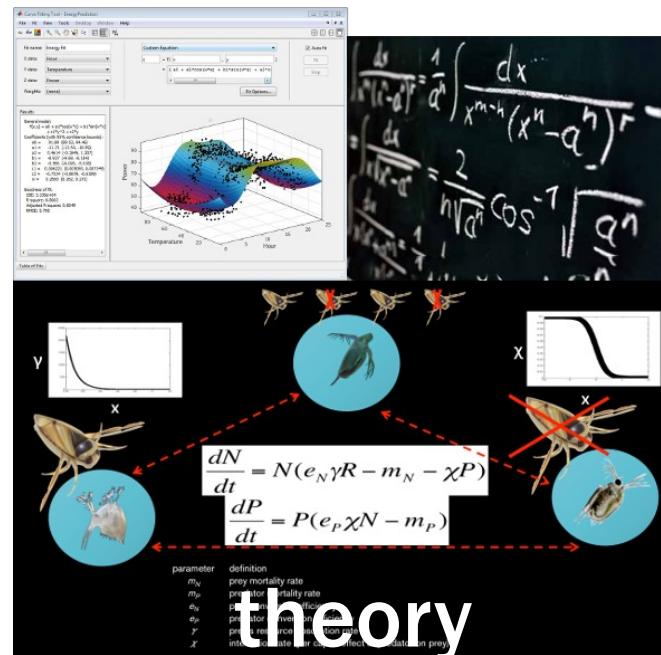


## Research goals

- Develop accurate models to explain and predict biodiversity
- Understand the consequences of evolution for communities, metacommunities, and ecosystems
- Determine how eco-evolutionary dynamics influence the response of biodiversity to environmental change

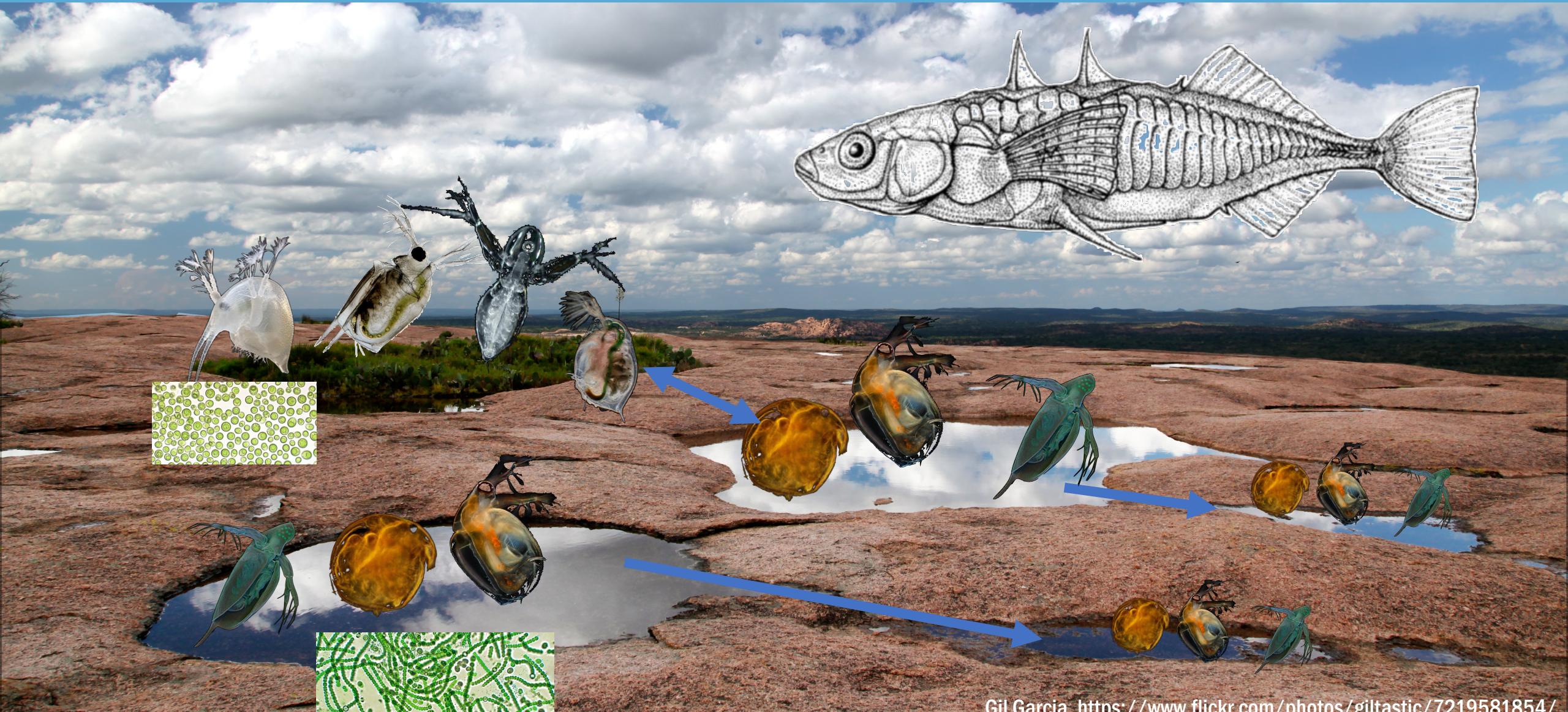


experiment



observation

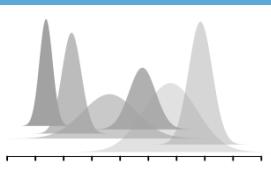
# Eco-evolutionary dynamics – overlapping ecological and evolutionary processes that are not independent of one another



# Eco-evolutionary dynamics – overlapping ecological and evolutionary processes that are not independent of one another



Gil Garcia, <https://www.flickr.com/photos/giltastic/7219581854/>



# Research approach – eco-evolutionary hypothesis testing to fit data to models



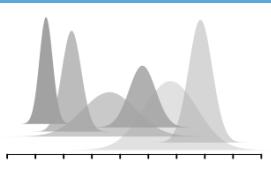
Candidate model

Ecoevo  
 $H_0, H_A$

Simulations

Compare to  
observed data

1. Identify candidate models for processes that structure observed data
2. Generate alternative eco-evolutionary hypotheses
3. Simulate possible observed data under each hypothesis
4. Compare observed data to alternative hypothesis simulations



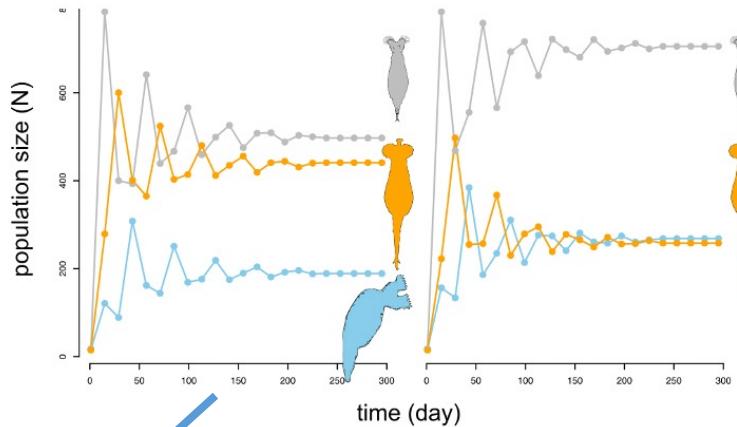
# Research approach – eco-evolutionary hypothesis testing to fit data to models



Candidate model

## 1. Identify candidate models for processes that structure observed data

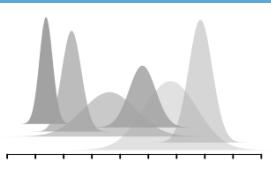
### Ex 1) Coexistence / competition for a few species



growth rate (as a function of a trait  $x$ )

$$\frac{N_{i,t+1}}{N_{i,t}} = \frac{\lambda_i(x)}{1 + \alpha_{ii}N_{i,t} + \alpha_{ij}N_{j,t}}$$

Intraspecific competition rate    Interspecific competition rate



# Research approach – eco-evolutionary hypothesis testing to fit data to models

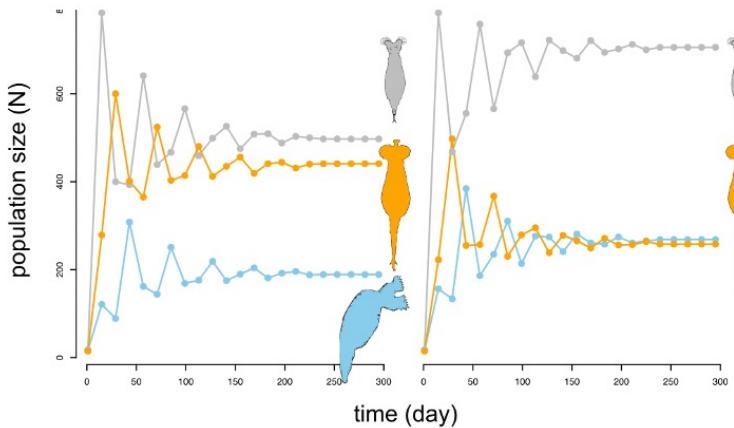


Ecoevo  
 $H_0, H_A$

Simulations

2. Generate alternative eco-evolutionary hypotheses
3. Simulate possible observed data under each hypothesis

## Ex 1) Coexistence / competition for a few species



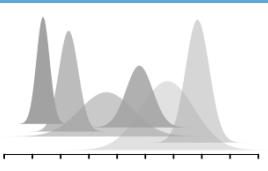
$H_{\text{eco}}$

Trait  $x$  does not evolve

$$\frac{N_{i,t+1}}{N_{i,t}} = \frac{\lambda_i(x)}{1 + \alpha_{ii}N_{i,t} + \alpha_{ij}N_{j,t}}$$

$H_{\text{eco-evo}}$

Trait  $x$  can evolve



# Research approach – eco-evolutionary hypothesis testing to fit data to models

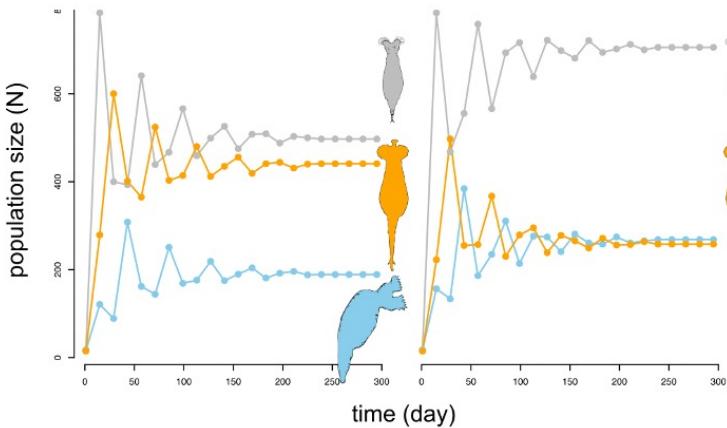


Ecoevo  
 $H_0, H_A$

Simulations

2. Generate alternative eco-evolutionary hypotheses
3. Simulate possible observed data under each hypothesis

## Ex 1) Coexistence / competition for a few species



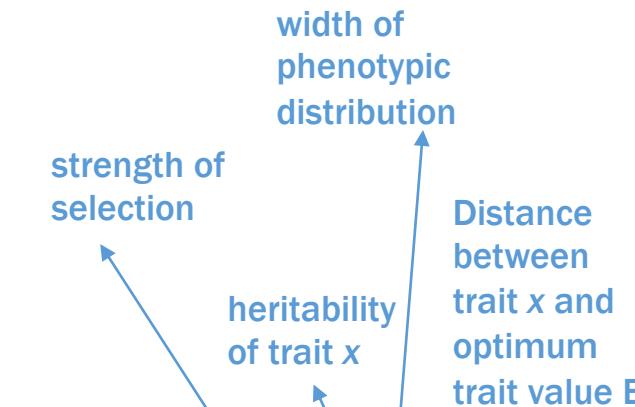
$H_{\text{eco}}$   
Trait  $x$  does not evolve

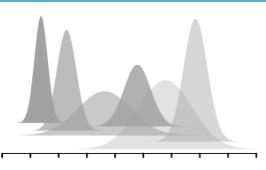
$H_{\text{eco-evo}}$   
Trait  $x$  can evolve

$$\frac{N_{i,t+1}}{N_{i,t}} = \frac{\lambda_i(x)}{1 + \alpha_{ii}N_{i,t} + \alpha_{ij}N_{j,t}}$$

$$\lambda_t = \bar{W}_t = \widehat{We} \frac{-\left[\left(\frac{(w+(1-h^2)P)}{P+w}\right)(E-x_t)\right]^2}{2(P+w)}$$

Growth rate → fitness





# Research approach – eco-evolutionary hypothesis testing to fit data to models

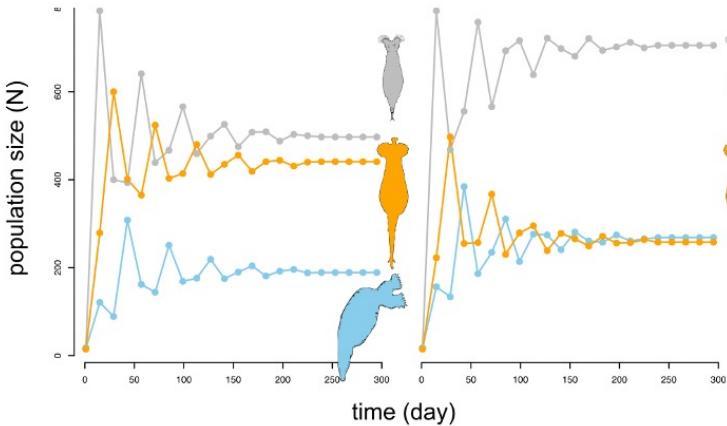


Ecoevo  
 $H_0, H_A$

Simulations

2. Generate alternative eco-evolutionary hypotheses
3. Simulate possible observed data under each hypothesis

## Ex 1) Coexistence / competition for a few species



$H_{\text{eco}}$   
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$$\frac{N_{i,t+1}}{N_{i,t}} = \frac{\lambda_i(x)}{1 + \alpha_{ii}N_{i,t} + \alpha_{ij}N_{j,t}}$$

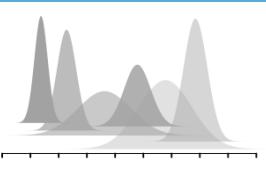
$H_{\text{eco-evo}}$   
Trait  $x$  can evolve

Growth rate  
→ fitness

$$\lambda_t = \bar{W}_t = \hat{W} e^{-\left[\left(\frac{(w+(1-h^2)P}{P+w}\right)(E-x_t)\right]^2}$$

$$N_{i,t+1} = \frac{\hat{W} e^{-\left[\left(\frac{(w+(1-h^2)P}{P+w}\right)(E-x_t)\right]^2}}{1 + \alpha_{ii}N_{i,t} + \sum_j \alpha_{ij}N_{j,t}} N_{i,t}$$

width of phenotypic distribution  
strength of selection  
heritability of trait  $x$   
Distance between trait  $x$  and optimum trait value  $E$



# Research approach – eco-evolutionary hypothesis testing to fit data to models

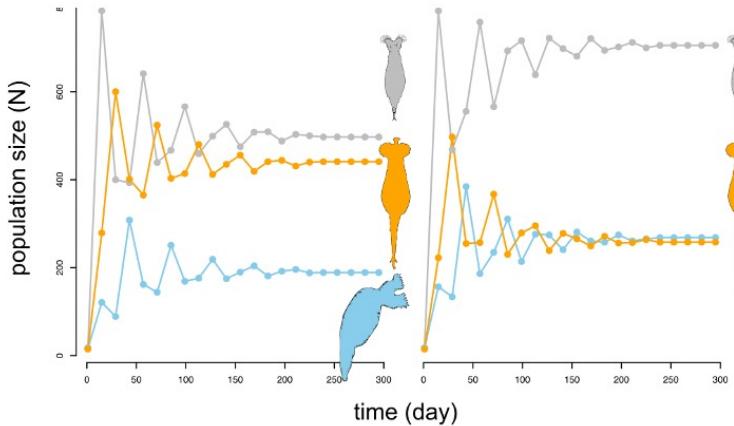


Ecoevo  
 $H_0, H_A$

Simulations

2. Generate alternative eco-evolutionary hypotheses
3. Simulate possible observed data under each hypothesis

## Ex 1) Coexistence / competition for a few species



$H_{\text{eco}}$

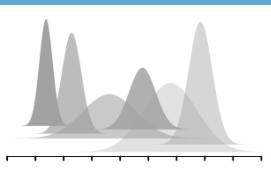
Trait  $x$  does not evolve

$$\frac{N_{i,t+1}}{N_{i,t}} = \frac{\lambda_i(x)}{1 + \alpha_{ii}N_{i,t} + \alpha_{ij}N_{j,t}}$$

$H_{\text{eco-evo}}$

Trait  $x$  can evolve

$$N_{i,t+1} = \frac{\hat{W} e^{\frac{-[(\frac{w+(1-h^2)P}{P+w})(E-x_t)]^2}{2(P+w)}}}{1 + \alpha_{ii}N_{i,t} + \sum_j \alpha_{ij}N_{j,t}} N_{i,t}$$



# Research approach – eco-evolutionary hypothesis testing to fit data to models



Simulations

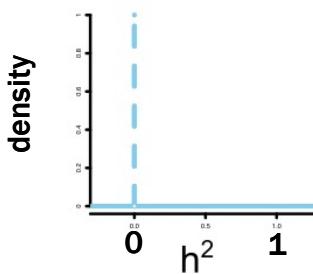
3. Simulate possible observed data under each hypothesis

Compare to observed data

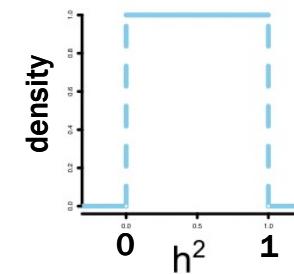
4. Compare observed data to alternative hypothesis simulations

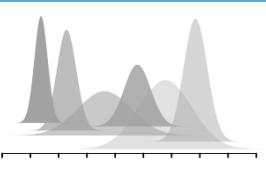
sample a candidate parameter vector  $\theta^*$  from a proposed prior distribution, 100,000 simulations per model

$H_1$ : No evolution ( $h^2=0$ )



$H_2$ : Evolution ( $h^2>0$ )





# Research approach – eco-evolutionary hypothesis testing to fit data to models



Simulations

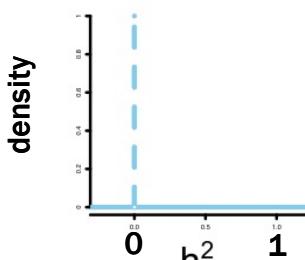
Compare to observed data

3. Simulate possible observed data under each hypothesis

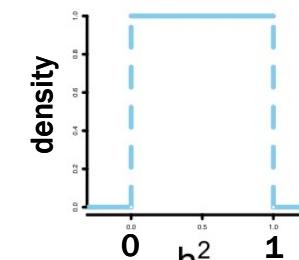
4. Compare observed data to alternative hypothesis simulations

sample a candidate parameter vector  $\theta^*$  from a proposed prior distribution, 100,000 simulations per model

$H_1$ : No evolution ( $h^2=0$ )

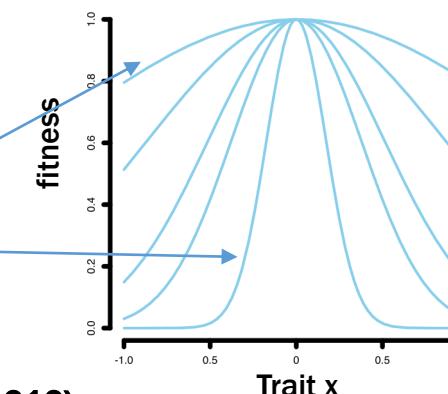
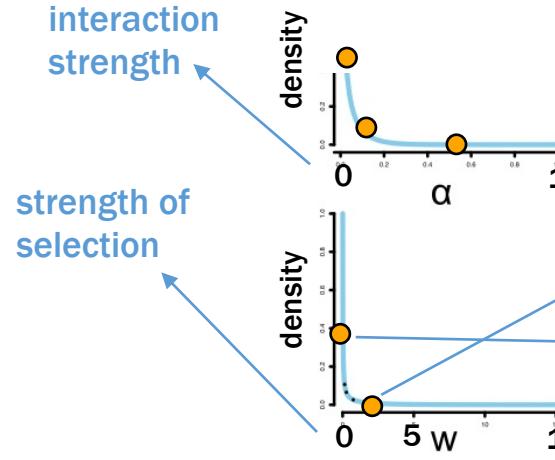


$H_2$ : Evolution ( $h^2>0$ )



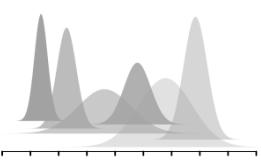
interaction strength

strength of selection



Approximate Bayesian Computing (eg Beaumont 2010)

Model parameters:  $h^2, \alpha, w, \widehat{W}, P$



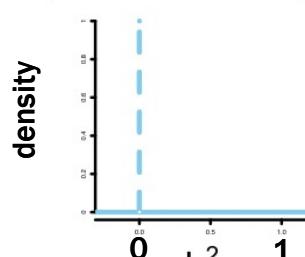
# Research approach - eco-evolutionary hypothesis testing to fit data to models



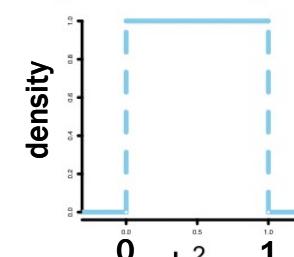
Simulations

Compare to observed data

$H_1$ : No evolution ( $h^2=0$ )

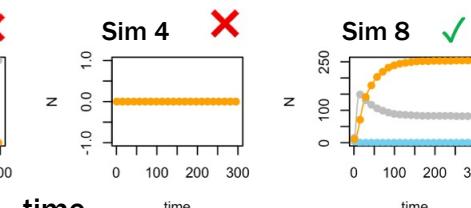
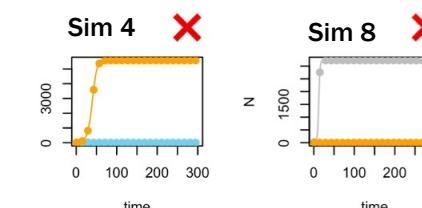
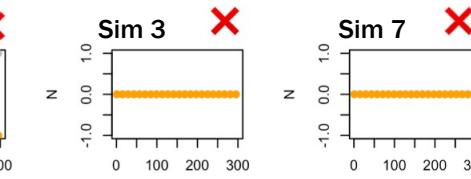
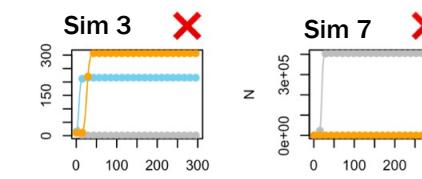
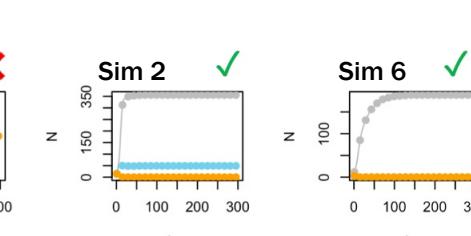
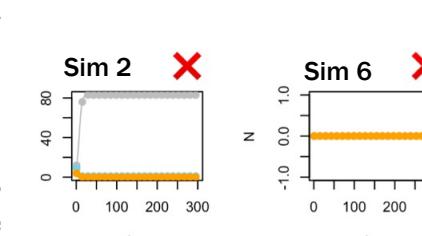
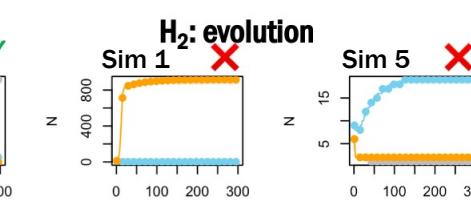
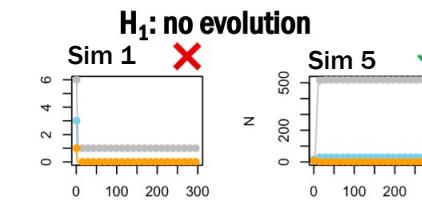
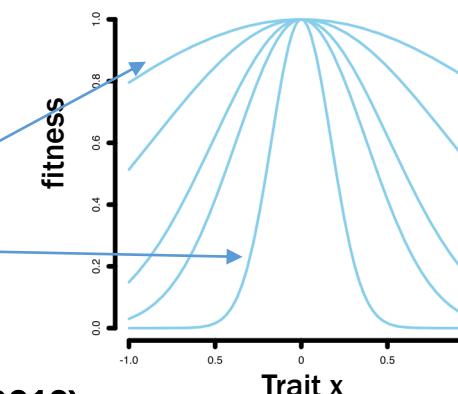
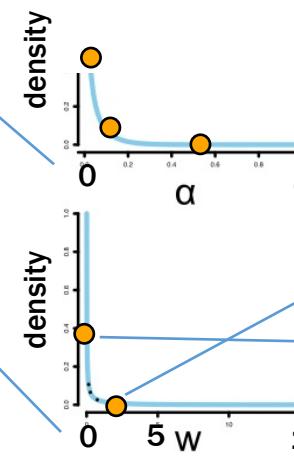


$H_2$ : Evolution ( $h^2>0$ )



interaction strength

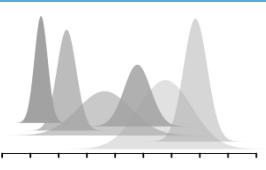
strength of selection



compare the simulated dataset  $x^*$  to the observed data  $x_0$ , using a distance function  $d$  and a tolerance  $\varepsilon$ : if  $d(x_0, x^*) \leq \varepsilon$ , accept  $\theta^*$

Approximate Bayesian Computing (eg Beaumont 2010)

Model parameters:  $h^2, \alpha, w, \widehat{W}, P$



# Research approach - eco-evolutionary hypothesis testing to fit data to models



Compare to observed data

## 4. Compare observed data to alternative hypothesis simulations

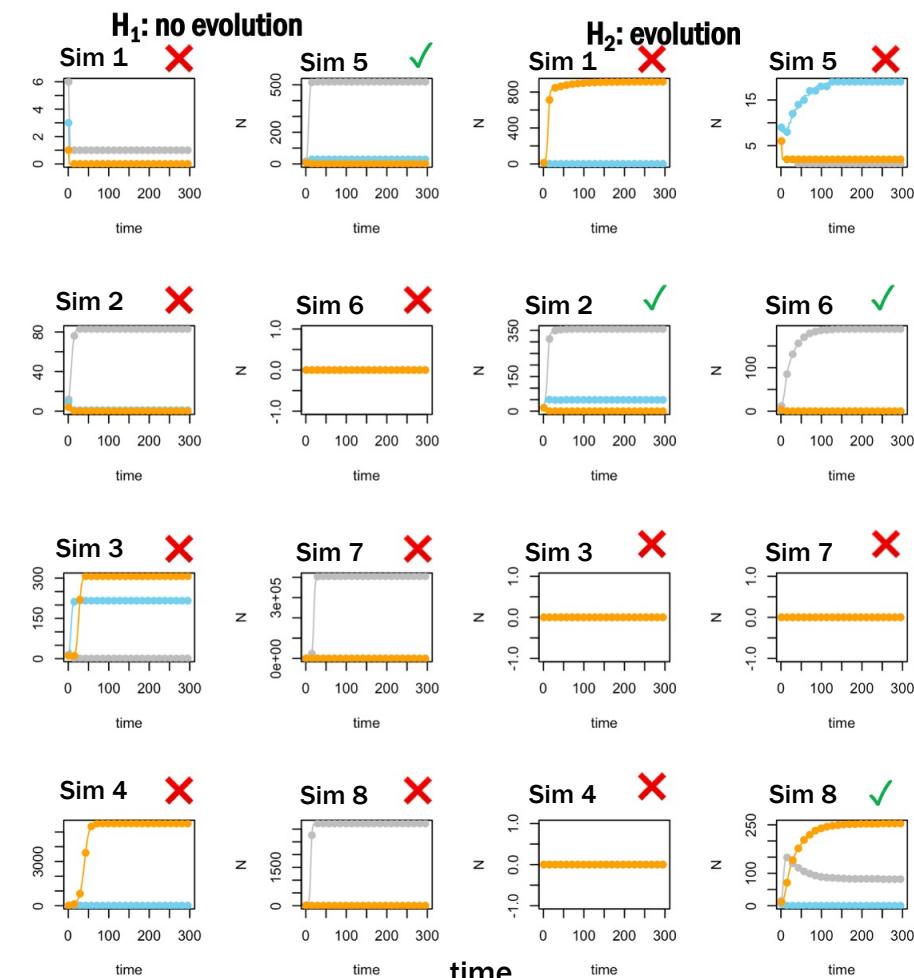
### (i) Estimate posterior probability of alternative model hypotheses

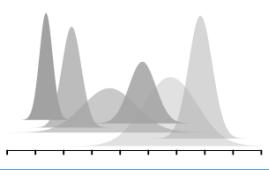
$H_{\text{eco}}$

$$\frac{n_{i,t+1}}{n_{i,t}} = \frac{\lambda_i}{1 + \alpha_{ii}N_{i,t} + \alpha_{ij}N_{j,t}}$$

$H_{\text{eco-evo}}$

$$N_{i,t+1} = \hat{W}e^{\frac{-[(\frac{w+(1-h^2)P}{P+w})(E-x_t)]^2}{2(P+w)}} N_{i,t}$$





# Research approach – eco-evolutionary hypothesis testing to fit data to models



## Compare to observed data

#### **4. Compare observed data to alternative hypothesis simulations**

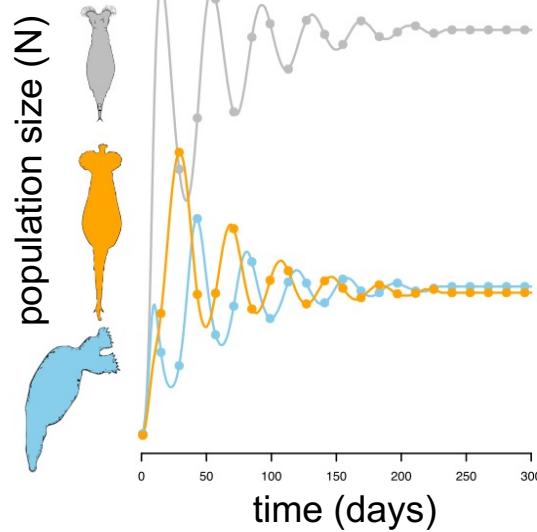
(i) Estimate posterior probability of alternative model hypotheses

$$\frac{n_{i,t+1}}{n_{i,t}} = \frac{\lambda_i}{1 + \alpha_{ii}N_{i,t} + \alpha_{ij}N_{j,t}} \quad N_{i,t+1}$$

$$H_{\text{eco-evo}} \quad N_{i,t+1} = \frac{\hat{We}^{\frac{-[(\frac{w+(1-h^2)P}{P+w})(E-x_t)]^2}{2(P+w)}} N_{i,t}}{1 + \alpha_{ii}N_{i,t} + \sum_j \alpha_{ij}N_{j,t}}$$

## Case 1. With simulated model + data

With  $x_0$  and  $x_{300}$  trait values



## Posterior model probabilities for Observed Data

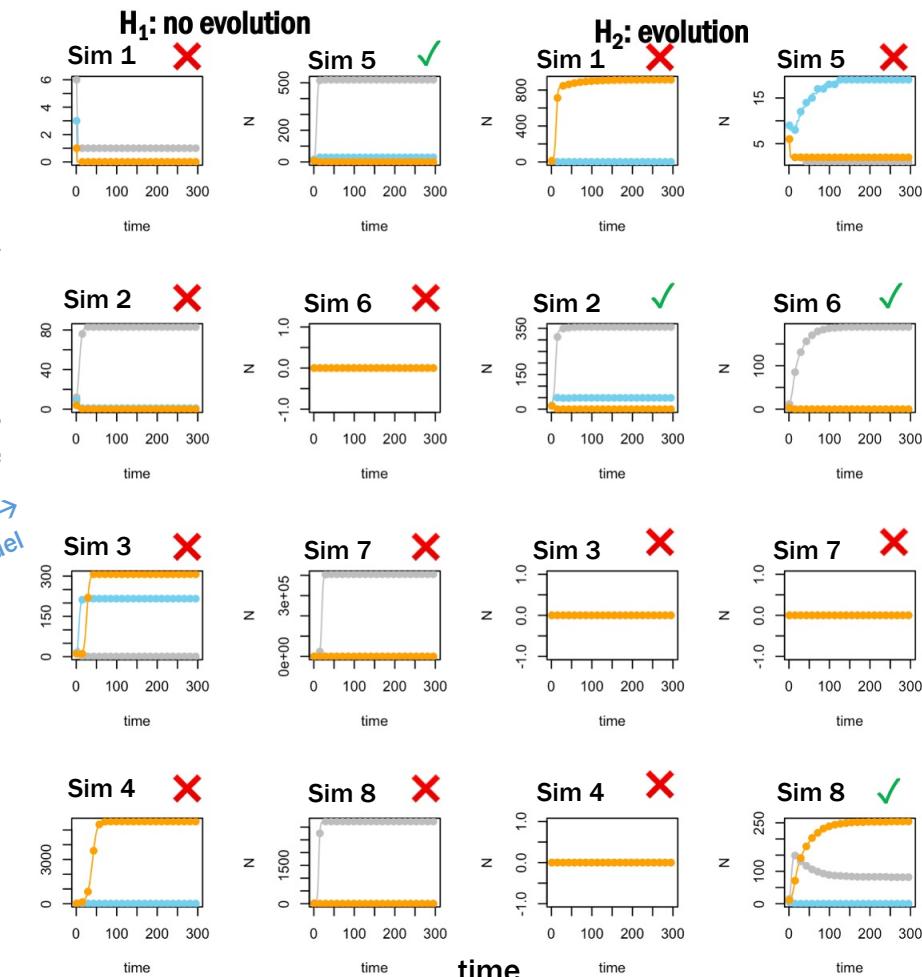
$$H_{\text{eco}} = 0.0001$$

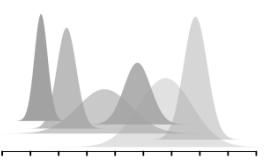
$H_{\text{eco-geo}} = 0.9999$

Approximate Bayesian Computing +  
Neural network (for model classification)  
trained on 100,000 simulations per mode

Based on % Accepted vs.  
Rejected per  $H_A$  model

$N(\text{popul})$





# Research approach - eco-evolutionary hypothesis testing to fit data to models



Compare to observed data

## 4. Compare observed data to alternative hypothesis simulations

### (i) Estimate posterior probability of alternative model hypotheses

$H_{\text{eco}}$

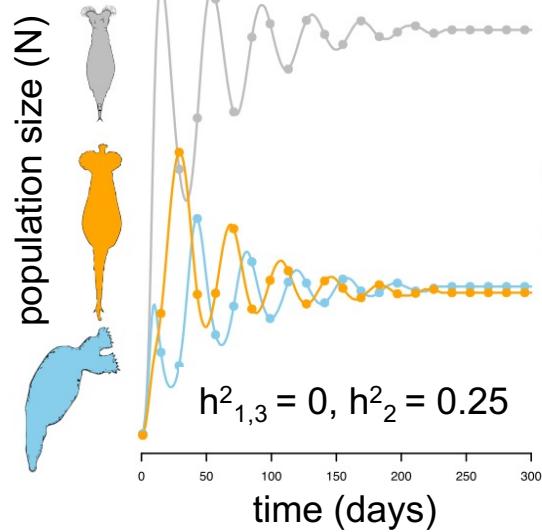
$$\frac{n_{i,t+1}}{n_{i,t}} = \frac{\lambda_i}{1 + \alpha_{ii}N_{i,t} + \alpha_{ij}N_{j,t}}$$

$H_{\text{eco-evo}}$

$$N_{i,t+1} = \hat{W}e^{\frac{-[(\frac{w+(1-h^2)P}{P+w})(E-x_t)]^2}{2(P+w)}} N_{i,t}$$

Case 1. With simulated model + data

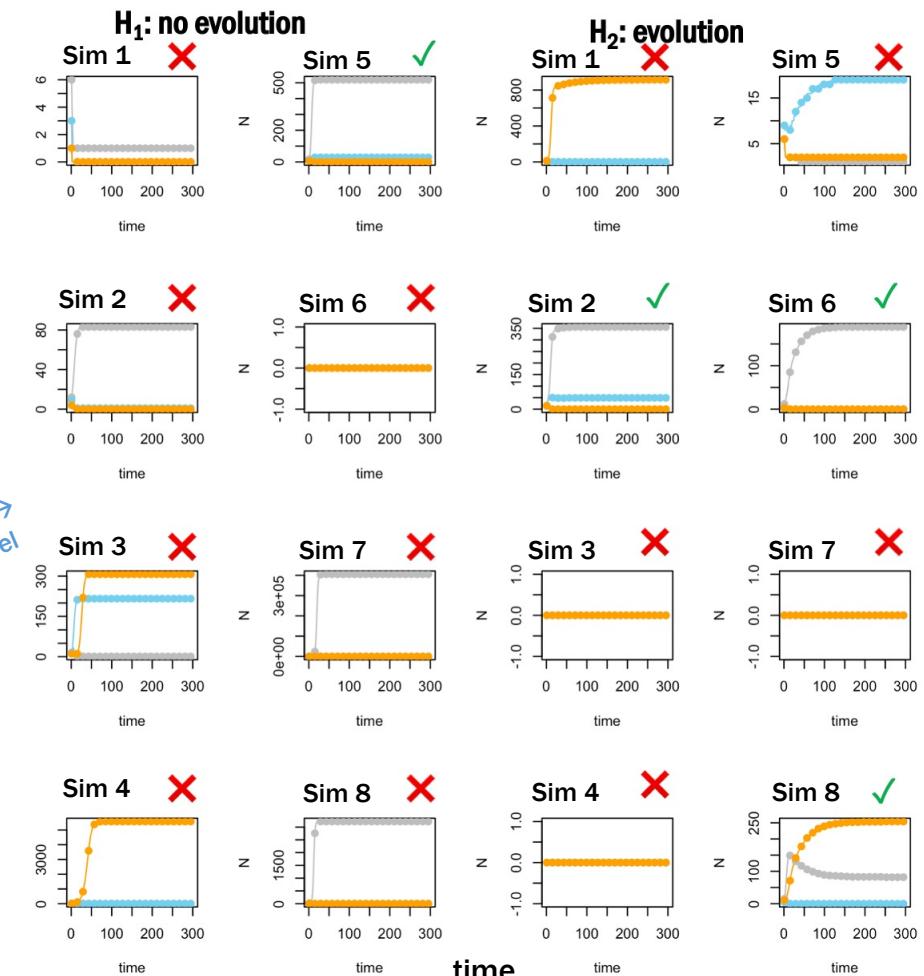
With  $x_0$  and  $x_{300}$  trait values

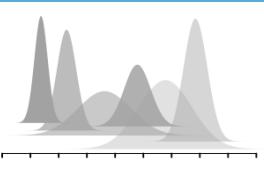


Posterior model probabilities  
for Observed Data

$$H_{\text{eco}} = 0.0001 \quad \text{X} \quad H_{\text{eco-evo}} = 0.9999 \quad \checkmark$$

Based on % Accepted vs.  
Rejected per  $H_A$  model  
Approximate Bayesian Computing +  
Neural network (for model classification) →  
trained on 100,000 simulations per model





# Research approach - eco-evolutionary hypothesis testing to fit data to models



Compare to observed data

## 4. Compare observed data to alternative hypothesis simulations

### (i) Estimate posterior probability of alternative model hypotheses

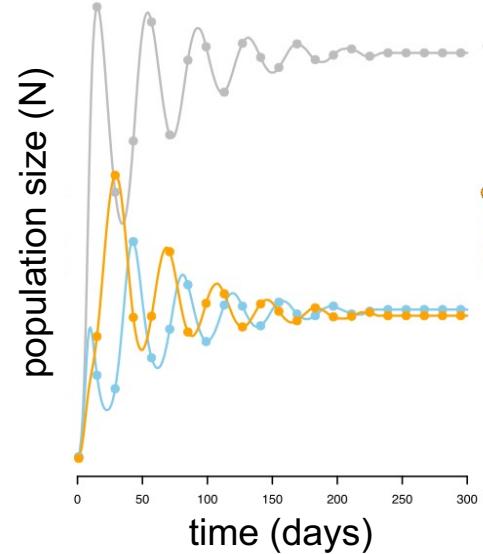
$$\frac{n_{i,t+1}}{n_{i,t}} = \frac{\lambda_i}{1 + \alpha_{ii}N_{i,t} + \alpha_{ij}N_{j,t}}$$

$$H_{\text{eco-evo}} = \frac{-[(\frac{w+(1-h^2)P}{P+w})(E-x_t)]^2}{2(P+w)} N_{i,t}$$

Case 2. With simulated model + data

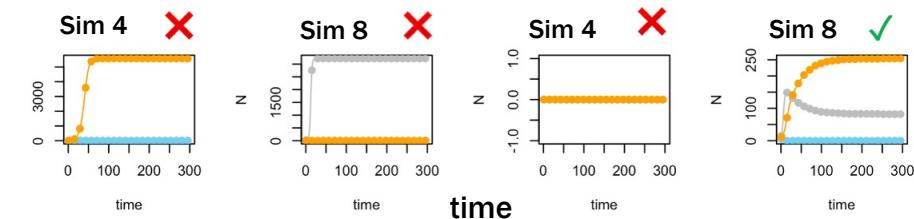
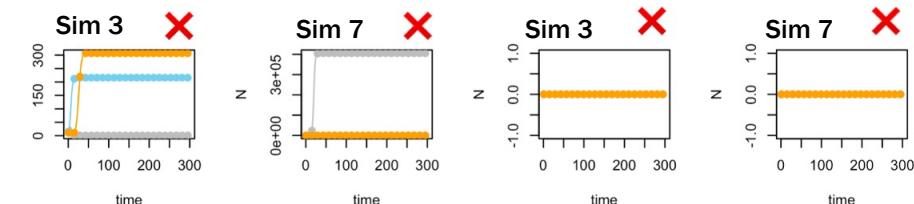
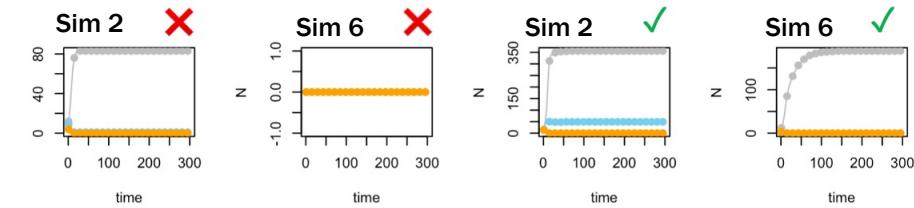
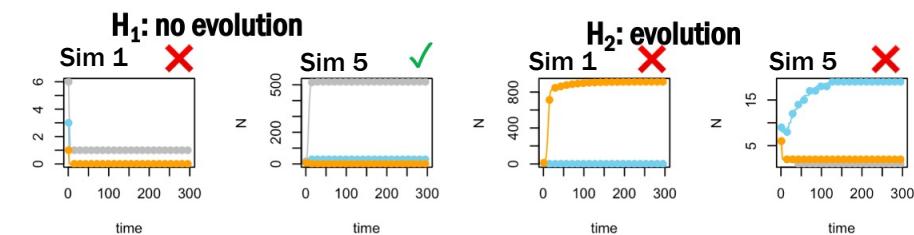
With  $x_{300}$  trait values only (!!)

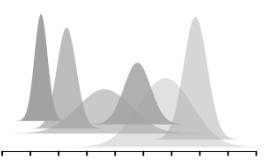
Posterior model probabilities for Observed Data



Based on % Accepted vs. Rejected per  $H_A$  model

N (population size)





# Research approach – eco-evolutionary hypothesis testing to fit data to models



Compare to observed data

## 4. Compare observed data to alternative hypothesis simulations

### (i) Estimate posterior probability of alternative model hypotheses

$$\frac{n_{i,t+1}}{n_{i,t}} = \frac{\lambda_i}{1 + \alpha_{ii}N_{i,t} + \alpha_{ij}N_{j,t}}$$

$$H_{\text{eco-evo}} = \frac{-[(\frac{w+(1-h^2)P}{P+w})(E-x_t)]^2}{2(P+w)} N_{i,t}$$

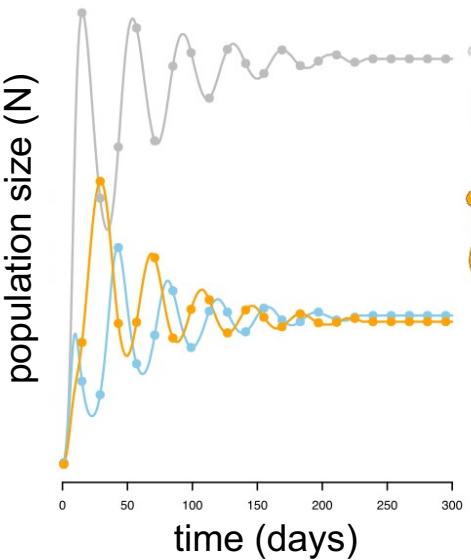
Case 2. With simulated model + data

With  $x_{300}$  trait values only (!!)

Posterior model probabilities  
for Observed Data

$$H_{\text{eco}} = 0.4676$$

$$H_{\text{eco-evo}} = 0.5324$$



Based on % Accepted vs.  
Rejected per  $H_A$  model

Case 3. With simulated model + data

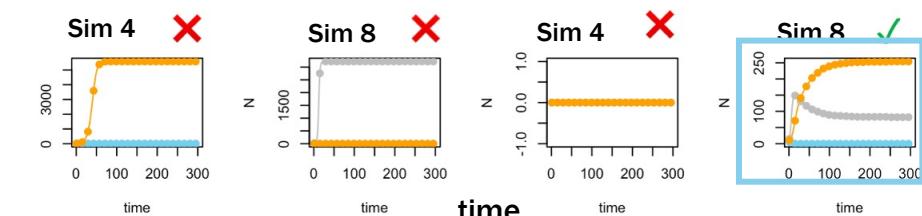
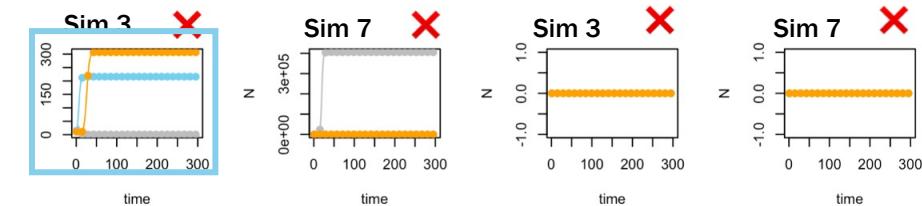
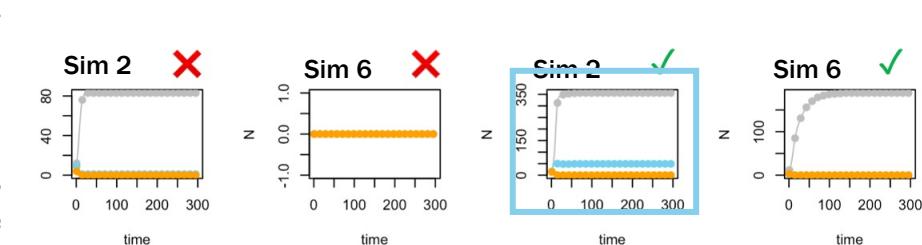
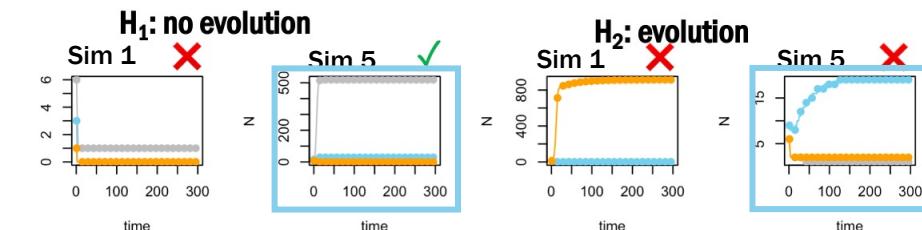
With  $x_{300}$  trait values only (!!)

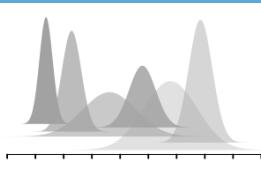
Restricted to 3-species  
persisting simulations only

Posterior model probabilities  
for Observed Data

$$H_{\text{eco}} = 0.0783$$

$$H_{\text{eco-evo}} = 0.9217$$





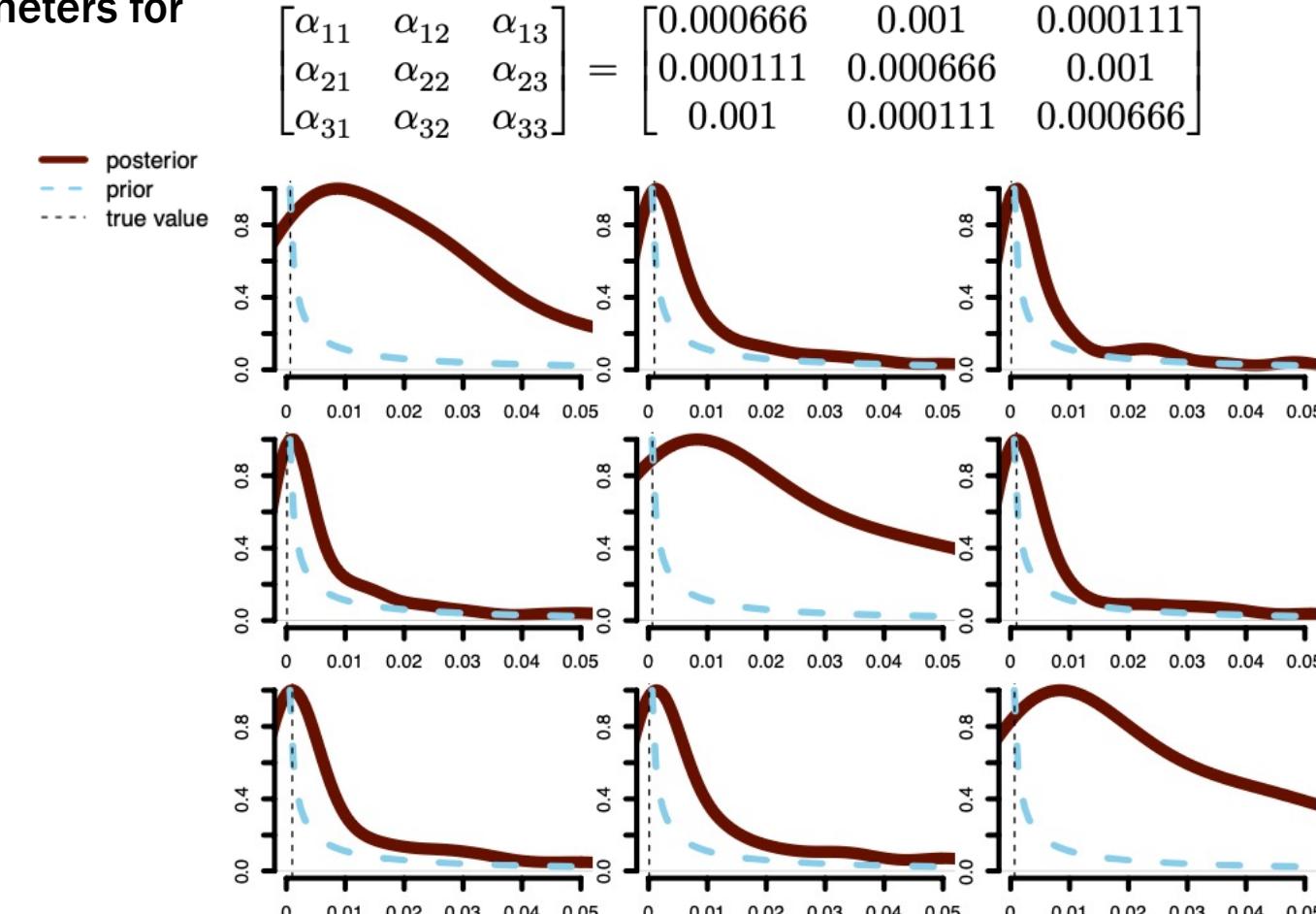
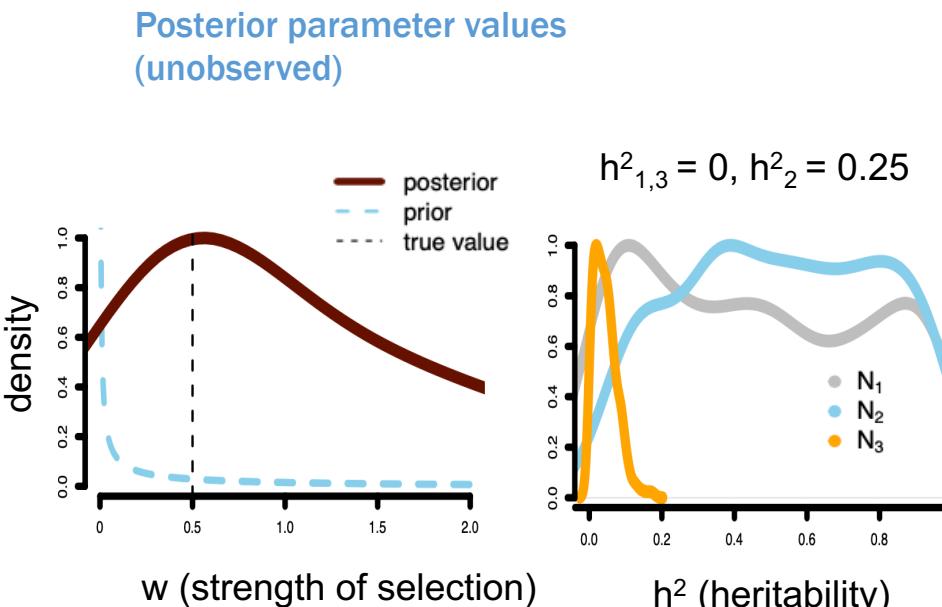
# Research approach – eco-evolutionary hypothesis testing to fit data to models

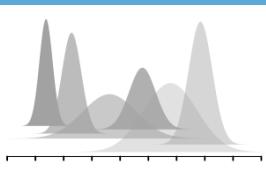


Compare to observed data

## 4. Compare observed data to alternative hypothesis simulations

- (i) Estimate posterior probability of alternative model hypotheses
- (ii) Use best-fit (or consensus) models to estimate parameters for observed data, make predictions for future cases





# Research approach – eco-evolutionary hypothesis testing to fit data to models



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## 4. Compare observed data to alternative hypothesis simulations

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