

$$\frac{dN_1}{dt} = r_1 N_1 \left(1 - \frac{N_1 + \alpha_{12} N_2}{K_1} \right)$$

$$\frac{dN_2}{dt} = r_2 N_2 \left(1 - \frac{N_2 + \alpha_{21} N_1}{K_2} \right)$$

① Set $\frac{dN_1}{dt} = 0$, solve for N_1^*

$$0 = r_1 N_1 \left(1 - \frac{N_1 + \alpha_{12} N_2}{K_1} \right) \Rightarrow 0 = \cancel{r_1} \cancel{N_1} \Rightarrow \boxed{(1a) N_1^* = 0}$$

$$1 - \frac{N_1 + \alpha_{12} N_2}{K_1} \quad 1 - \frac{N_1 + \alpha_{12} N_2}{K_1}$$

$$\Rightarrow \frac{0}{r_1 N_1} = \cancel{1} \left(1 - \frac{N_1 + \alpha_{12} N_2}{K_1} \right) \Rightarrow 0 = 1 - \frac{N_1 + \alpha_{12} N_2}{K_1} \Rightarrow \frac{N_1 + \alpha_{12} N_2}{K_1} = 1$$

$$\Rightarrow N_1 + \alpha_{12} N_2 = K_1 \Rightarrow \boxed{(1b) N_1^* = K_1 - \alpha_{12} N_2}$$

② Set $\frac{dN_2}{dt} = 0$, get N_2^*

$$0 = r_2 N_2 \left(1 - \frac{N_2 + \alpha_{21} N_1}{K_2} \right) \Rightarrow 0 = \cancel{r_2} \cancel{N_2} \Rightarrow \boxed{(2a) N_2^* = 0}$$

$$\Rightarrow 0 = 1 - \frac{N_2 + \alpha_{21} N_1}{K_2} \Rightarrow \frac{N_2 + \alpha_{21} N_1}{K_2} = 1 \Rightarrow N_2 + \alpha_{21} N_1 = K_2 \Rightarrow \boxed{(2b) N_2^* = K_2 - \alpha_{21} N_1}$$