Logistic growth model (continuous time)

What is it?

A model of population size over time when growth is limited. Some factors that might limit growth include XX, YY, or ZZ.

Equation

$$\frac{dN}{dt} - rN\left(1 - \frac{N}{K}\right)$$

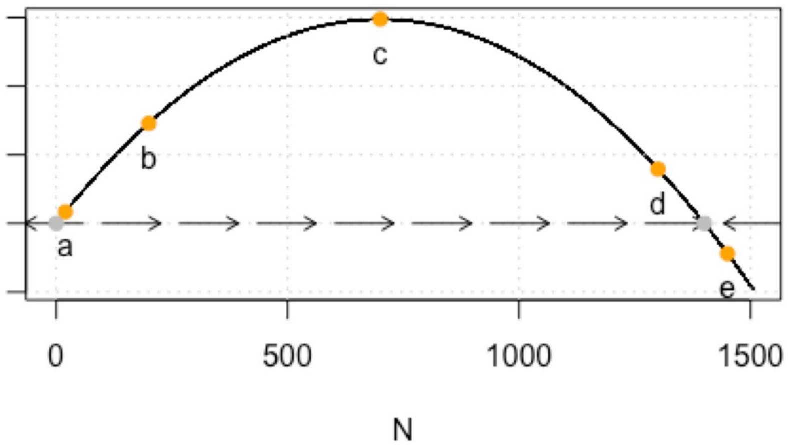

Figure 2. Phase plane diagram showing local stability analysis for the 2 equilibria of the model. The phase plane diagram plots the derivative of N (dn) on the y-axis and N on the x-axis, and indicates whether dN is increasing or decreasing for different N values. Points where arrows face toward each other indicate stability and away from each other denote instability. For logistic growth, $N^*=0$ is *unstable* , and $N^*=K$ is *stable*.

Parameter	Description	range	Biologically realistic range
r	Intrinsic rate of population growth	[-∞, ∞]	Depends on resources, but in practice [-10,10]
K	Carrying capacity	[0, ∞]	Depends on resources, species (quite large e.g. for bacteria), and area considered
State variable	**Description**	**range**	**Biologically realistic range**
N	Population size	[0, ∞]	Similar to *K*

Possible observed dynamics (N_0=45)

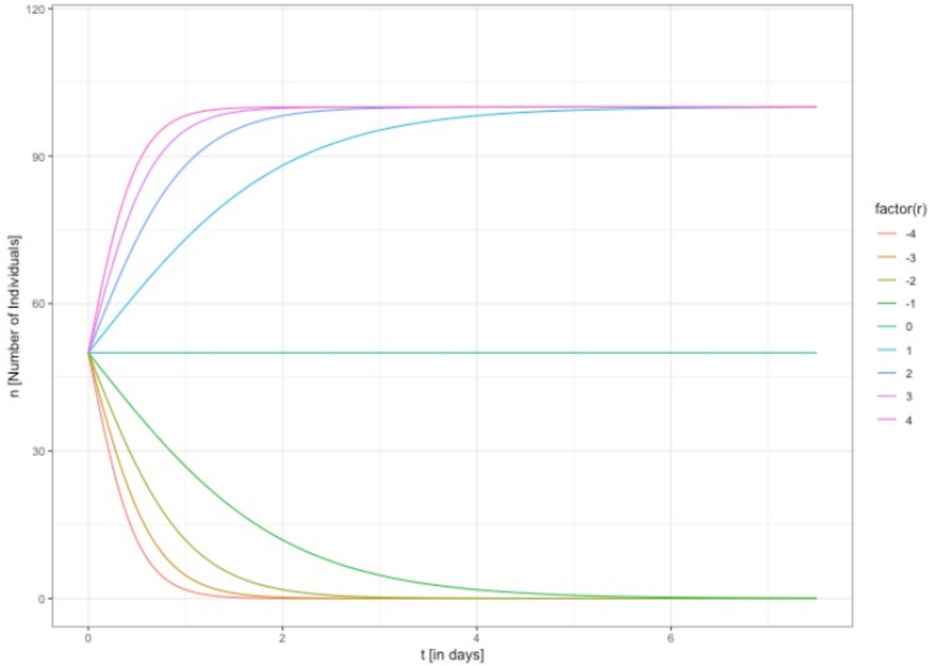

Figure 1. Plot of population size (x-axis) over time (y-axis) for different values of *r*, using a common initial population size (N0 = 45).

Equilibrium and Stability

Equilibrium analysis

When is a system at equilibrium?

In population ecology, a system is at equilibrium when the population size doesn't change over time and becomes constant.

To analyze a system regarding its equilibrium, $\frac{dn}{dt}$ is replaced with the equilibrium value (0). The equation is then solved for each variable:

$$rn(t)\left(1 - \frac{n(t)}{K}\right) = \frac{dn}{dt}$$

$$rn(t) = 0$$
$$\boldsymbol{r^* = 0}$$
$$\boldsymbol{n(t)^* = 0}$$

$$\left(1 - \frac{n(t)}{K}\right) = 0$$
$$-\frac{n(t)}{K} = -1$$
$$-K = -n(t)$$
$$\boldsymbol{K^* = n(t)}$$

For the logistic growth model, there are three cases in which the system reaches an equilibrium:

I. the intrinsic growth rate r equals zero.

II. The population size $n(t)$ equals zero.

III. The carrying capacity K equals the population size.