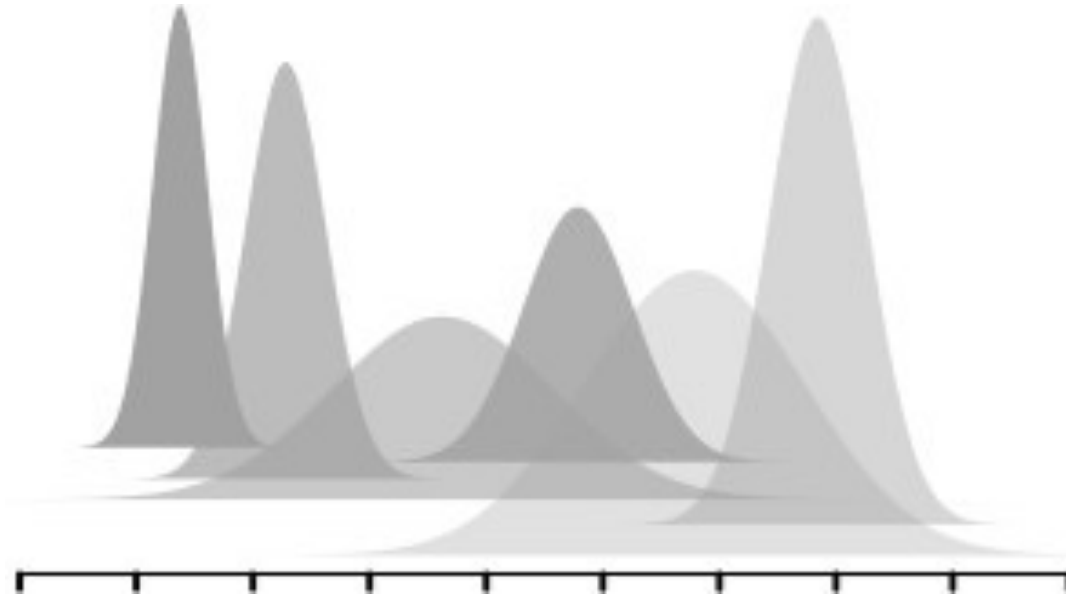


# 2.1 Population & Community Ecological Models



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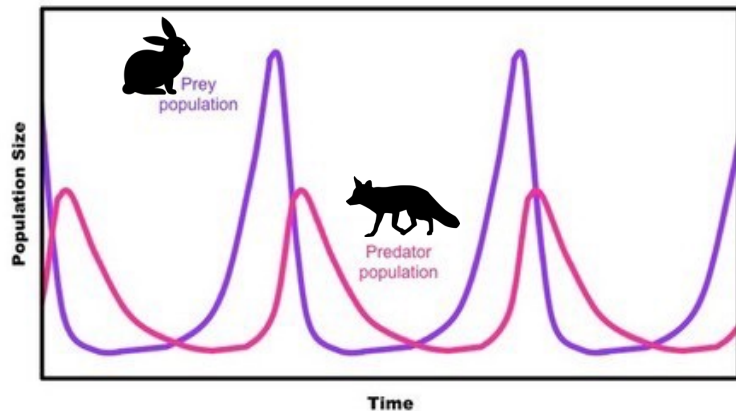
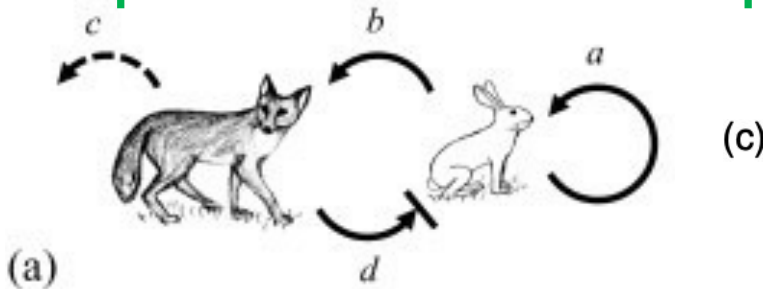
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# What is a model?

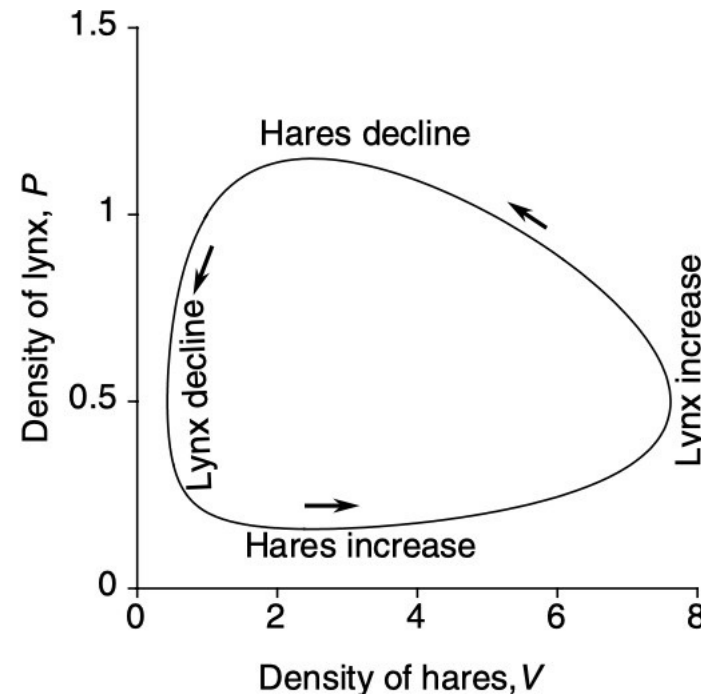
“A *model* is a representation of a particular thing, idea, or condition.”

“The *modeling process* is the series of steps taken to convert an idea first into a conceptual model and then into a quantitative model”

## Step 1. Formulate a conceptual model



## Step 2. Formulate a quantitative model



$$\frac{dV}{dt} = aV - bVP$$
$$\frac{dP}{dt} = -cP + dVP$$

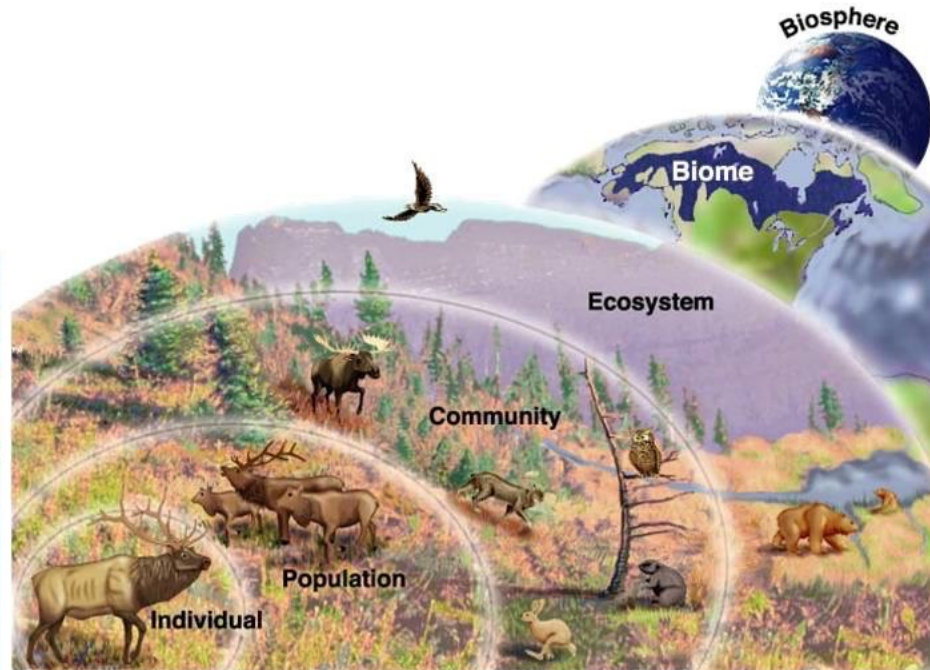
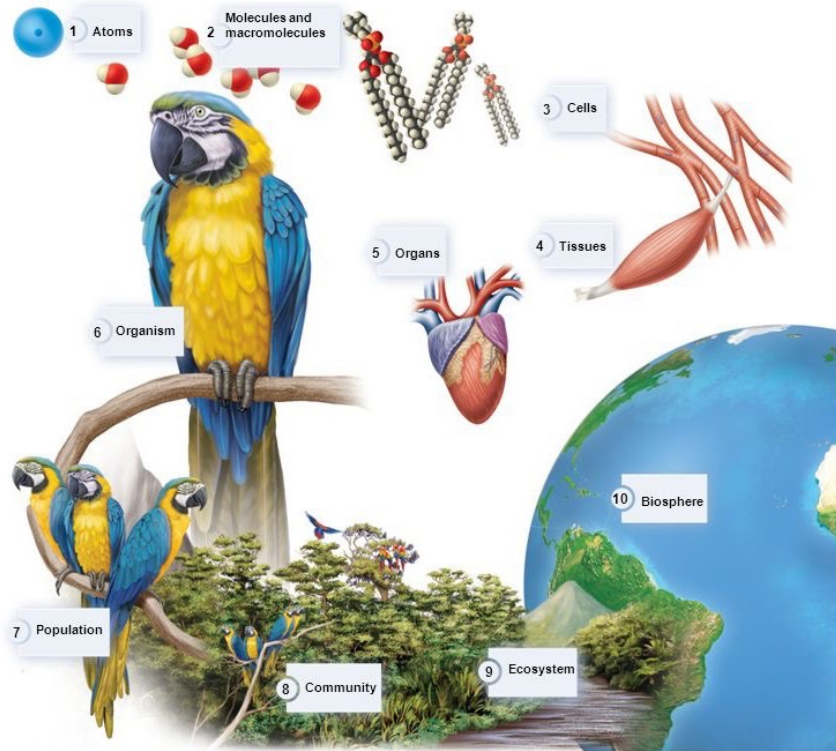
## Step 3. Learn about study system through analysis of model behavior

# What is ecology?

The study of interactions between organisms and their environment, and with one another

The science that investigates the abundance and distribution of organisms

## Levels of Biological Organization



## Step 3. Learn about study system through analysis of model behavior

# Population ecology

### Exponential growth

$$\frac{dn}{dt} = rn$$

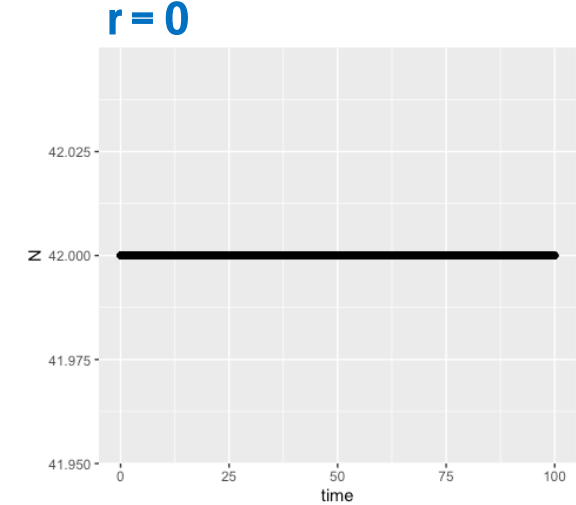
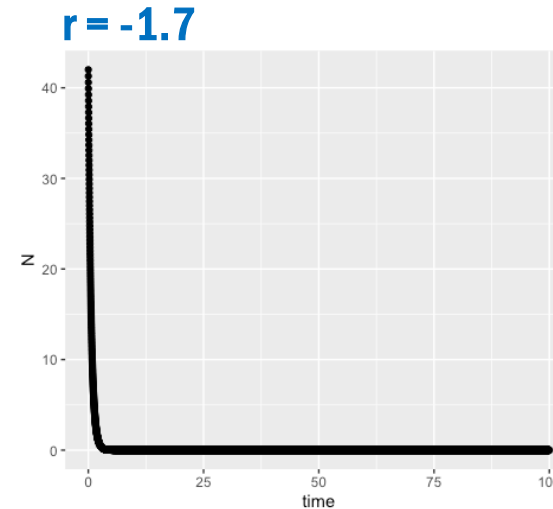
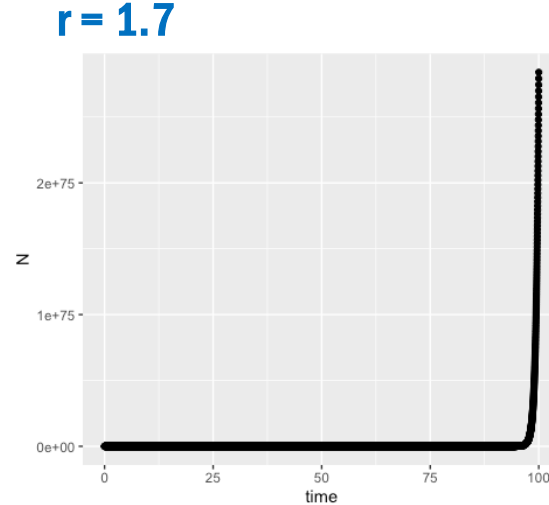
Graphical techniques: develop a feeling for your model

Expected dynamics → depends on  $r$  and  $n_0$  (initial population size)

When  $r > 0$ ?

When  $r < 0$ ?

When  $r = 0$ ?



parameter	
$r$	population growth rate

## Step 3. Learn about study system through analysis of model behavior

# Population ecology

## Exponential growth

$$\frac{dn}{dt} = rn$$

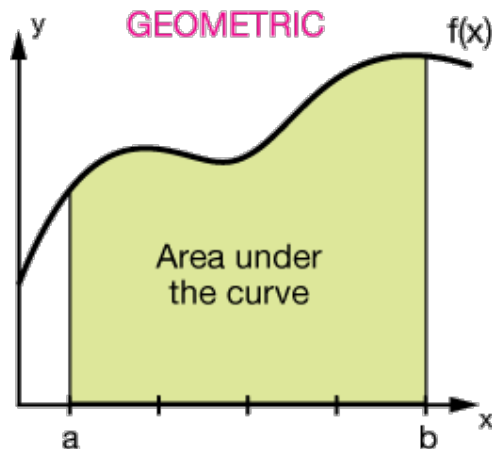
## Graphical techniques: develop a feeling for your model

How did I develop a graph of population size over time for this equation?

I solved for values of  $n(t)$  at different points in time.

Let's solve the equation to get a formula for this: we'll need to integrate the formula

$$\frac{dn}{dt} = rn \rightarrow \frac{dn}{n} = r dt \rightarrow \int_{n_0}^{n(t)} \frac{dn}{n} = r \int_0^t dt \rightarrow \ln \frac{n(t)}{n_0} = rt \rightarrow n(t) = n_0 e^{rt}$$



## ANALYTIC

$$A = \int_a^b f(x) dx$$

The definite integral of  $f(x)$  between  $x=a$  &  $x=b$

## Table of Integrals

### BASIC FORMS

- (1)  $\int x^n dx = \frac{1}{n+1} x^{n+1}$
- (2)  $\int \frac{1}{x} dx = \ln x$
- (3)  $\int u dv = uv - \int v du$
- (4)  $\int u(x)v'(x) dx = u(x)v(x) - \int v(x)u'(x) dx$

### RATIONAL FUNCTIONS

- (5)  $\int \frac{1}{ax+b} dx = \frac{1}{a} \ln(ax+b)$
- (6)  $\int \frac{1}{(x+a)^2} dx = \frac{-1}{x+a}$

### INTEGRALS WITH ROOTS

- (18)  $\int \sqrt{x-a} dx = \frac{2}{3} (x-a)^{3/2}$
- (19)  $\int \frac{1}{\sqrt{x \pm a}} dx = 2\sqrt{x \pm a}$
- (20)  $\int \frac{1}{\sqrt{a-x}} dx = 2\sqrt{a-x}$
- (21)  $\int x\sqrt{x-a} dx = \frac{2}{3} a(x-a)^{3/2} + \frac{2}{5} (x-a)^{5/2}$
- (22)  $\int \sqrt{ax+b} dx = \left( \frac{2b}{3a} + \frac{2x}{3} \right) \sqrt{b+ax}$
- (23)  $\int (ax+b)^{3/2} dx = \sqrt{b+ax} \left( \frac{2b^2}{5a} + \frac{4bx}{5} + \frac{2ax^2}{5} \right)$
- (24)  $\int \frac{x}{\sqrt{x-a}} dx = \frac{2}{3} (x-a)^{3/2} + \frac{2}{5} (x-a)^{5/2}$

## Step 3. Learn about study system through analysis of model behavior

# Population ecology

### Exponential growth

$$\frac{dn}{dt} = rn$$

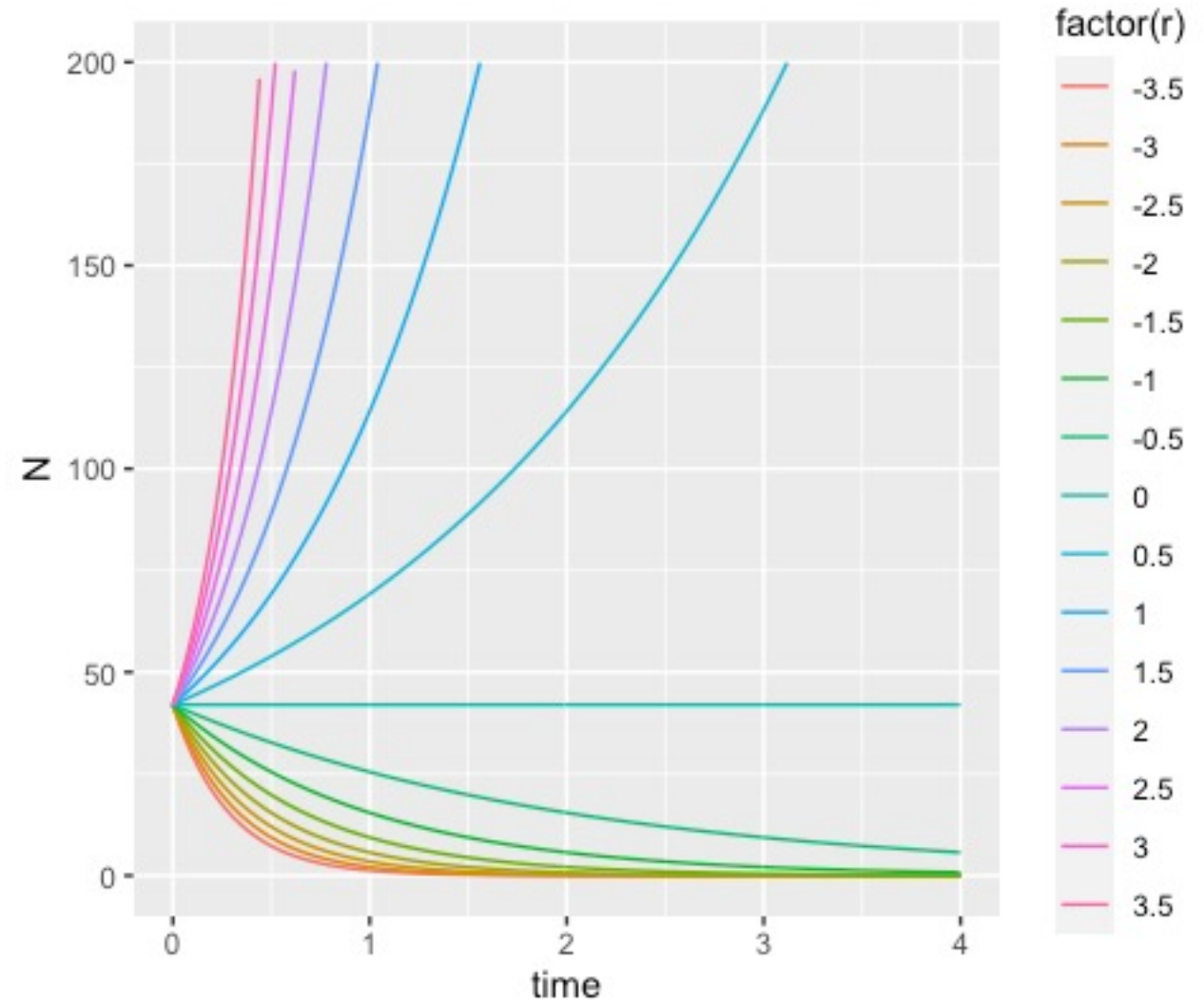
Graphical techniques: develop a feeling for your model

### Expected dynamics

When  $r > 0$ ?

When  $r < 0$ ?

When  $r = 0$ ?



parameter	
r	population growth rate



# Step 3. Learn about study system through analysis of model behavior

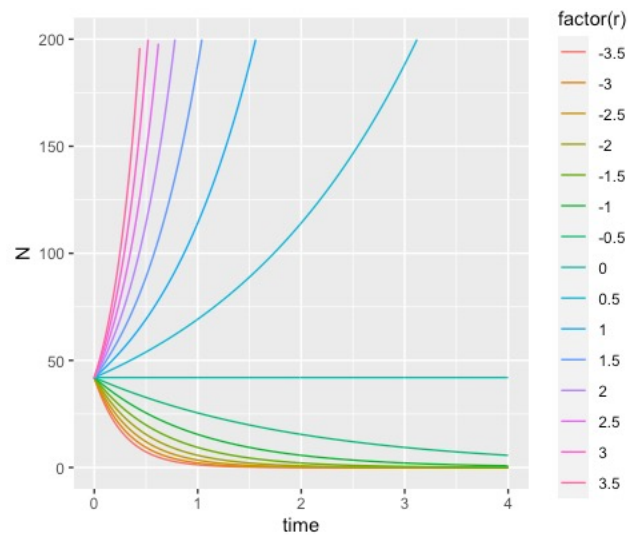
## Population ecology

### Exponential growth

$$\frac{dn}{dt} = rn$$

Graphical techniques: develop a feeling for your model Expected dynamics

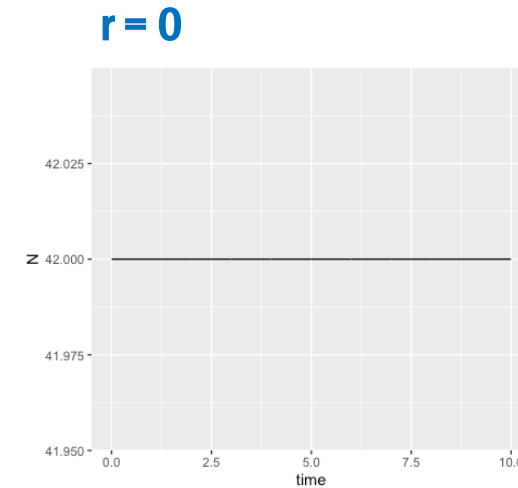
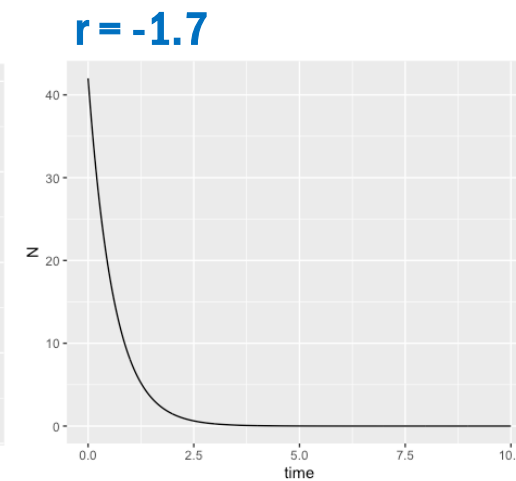
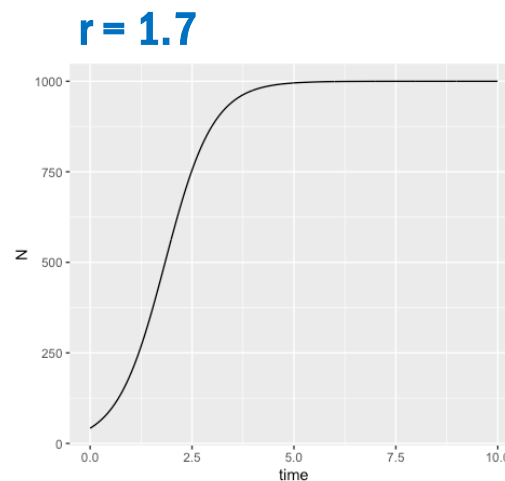
### Expected dynamics



### Logistic growth

$$\frac{dn}{dt} = rn \left(1 - \frac{n}{K}\right)$$

Expected dynamics



parameter	
$r$	population growth rate

parameter	
$r$	population growth rate
$K$	Carrying capacity

# Step 3. Learn about study system through analysis of model behavior

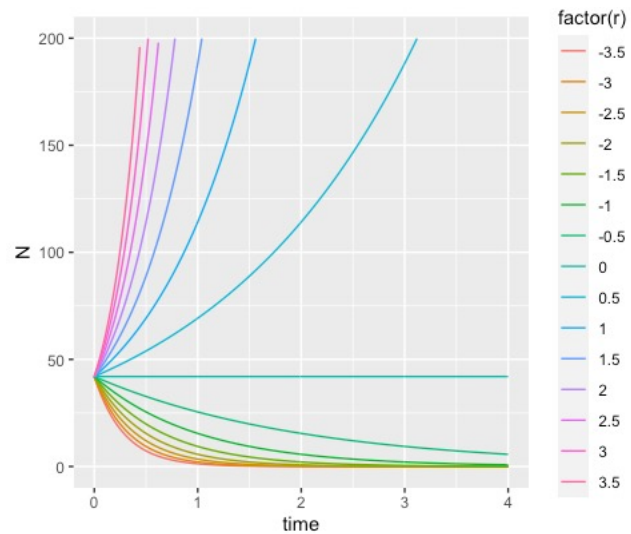
## Population ecology

### Exponential growth

$$\frac{dn}{dt} = rn \rightarrow n(t) = n_0 e^{rt}$$

Graphical techniques: develop a feeling for your model

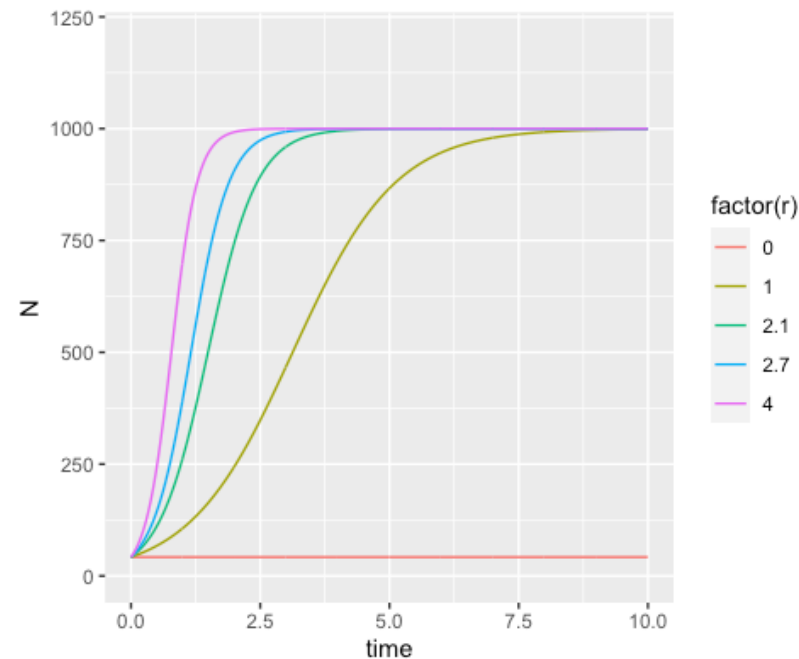
### Expected dynamics



### Logistic growth

$$\frac{dn}{dt} = rn \left(1 - \frac{n}{K}\right) \rightarrow n(t) = \frac{Kn_0 e^{rt}}{K + n_0(e^{rt} - 1)}$$

Expected dynamics



parameter	
r	population growth rate

parameter	
r	population growth rate
K	Carrying capacity