Solutions: Exercise 2.2 - Survey of ecological models, Part 2

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Instructions

Create either an R script (.R file) or R Markdown document (.Rmd) to save all of your work for today.

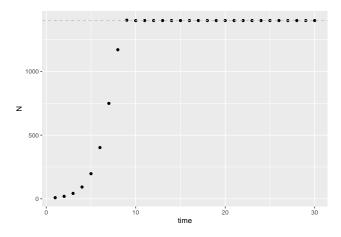
Exercise 1. Work with simulation of discrete-time logistic growth

In class, I discussed how making simulations of the models we learn in class can help us understand how our system works. Let's take a closer look at the discrete-time model of logistic population growth. The model formula is:

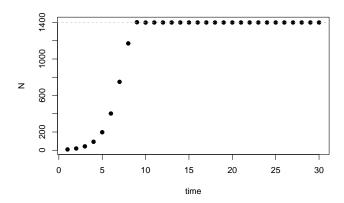
$$n_{t+1} = n_t + rn_t(1 - \frac{n_t}{K})$$

And the simulation works as follow:

```
# Parameter values to use for simulation
r < -1.21
K <- 1400
NO <- 4
t <- 30
# model function
disc_log <- function(r, NO, K) {</pre>
    Nt1 \leftarrow NO + r * NO * (1 - NO/K)
    return(Nt1)
}
# Simulation of model for t time steps
N \leftarrow rep(NA, t)
for (i in 1:t) {
    N[i] <- disc_log(r, NO, K)</pre>
    NO <- N[i]
}
# Plot simulation: ggplot
dat <- as.data.frame(N)</pre>
dat$time <- as.numeric(rownames(dat))</pre>
ggplot2::ggplot(dat, aes(time, N)) + geom_point() + geom_hline(yintercept = K,
    linetype = "dashed", color = "gray")
```

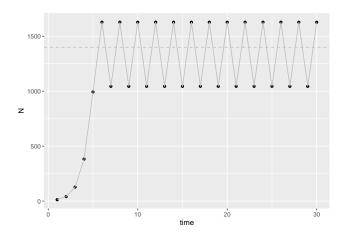


```
# Plot simulation: base R
plot(N, xlab = "time", ylab = "N", pch = 19, col = "black")
abline(h = K, col = "grey", lty = "dashed")
```

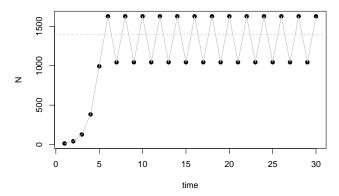


Question 1. Please change the simulation to run with a growth rate of 2.2, and show me a plot of the population dynamics over time.

```
# Parameter values to use for simulation
r < -2.2
K <- 1400
NO <- 4
t <- 30
\# Simulation of model for t time steps
N <- rep(NA, t)
for (i in 1:t) {
    N[i] <- disc_log(r, NO, K)
    NO <- N[i]
}
# Plot simulation: ggplot
dat <- as.data.frame(N)</pre>
dat$time <- as.numeric(rownames(dat))</pre>
ggplot2::ggplot(dat, aes(time, N)) + geom_point() + geom_line(color = "grey") +
    geom_hline(yintercept = K, linetype = "dashed", color = "grey")
```



```
# Plot simulation: base R
plot(N, xlab = "time", ylab = "N", pch = 19, col = "black")
lines(N, col = "grey")
abline(h = K, col = "grey", lty = "dashed")
```



Question 2. A. Use R's index syntax var[i] - where var is the name of your variable and [i] is the ith position of that variable - to show me the value of population size N at time 5, 6, and 7.

N[5:7]

[1] 993.4435 1628.1292 1044.4632

B. Use the 'disc_log' function to calculate the value of N at time 6 using the value of N at time 5 for N0

```
disc_log(r, N[5], K)
```

[1] 1628.129

C. Use the 'disc log' function to calculate the value of N at time 7 using the value of N at time 6 for N0

```
disc_log(r, N[6], K)
```

[1] 1044.463

D. Use the formula above for discrete-time logistic growth to calculate population size at time 7 using the value of N at time 6 for N0

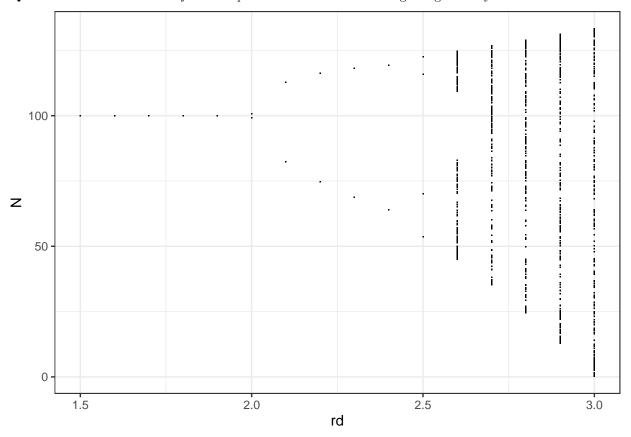
```
N[6] + r * N[6] * (1 - (N[6]/K))
```

[1] 1044.463

E. Why is the population not going to its carrying capacity K? How many periodic attractors (values of N for which the system is stably drawn towards) are there in this system? In other words, how many values does N cycle between when r = 2.2, and what are those values of N?

2 stable attractors 1044 and 1628

Question 3. Look at the bifurcation plot for this discrete-time logistic growth system:



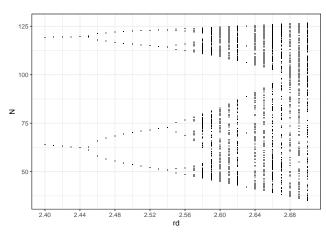
We see that as r increases the number of attractors continues to double, growing geometrically. Eventually, we reach a point when there becomes an infinite number of unique points, that are determined by r. This completely deterministic, non-repeating pattern in N is a property of chaos. Chaos is not a random phenomenon; rather it is the result of deterministic mechanisms generating non-repeating patterns.

A. How many periodic attractors are there at r = 2, 2.1, 2.2, 2.3, 2.4, and 2.5?

```
# We read this directly from the graph: 2, 2, 2, 2, 4
```

B. How many periodic attractors are there at r = 2.55? I share the code below to produce a finer scale of r values:

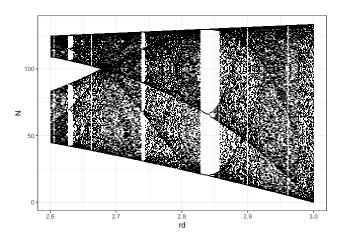
```
# We read this directly from the graph: 8
dlogistic <- function(t, y, p) {</pre>
    N \leftarrow y[1]
    with(as.list(p), {
        N.diff \leftarrow rd * N * (1 - alpha * N)
        return(list(N.diff))
    })
}
rd.s \leftarrow seq(2.4, 2.7, by = 0.01) # the actual rd values
t <- 0:1000 # how long to run each population
NO \leftarrow c(N = 99) \# N-zero
## do all the dynamical simulations putting each run into a
## column of a matrix.
Ns <- sapply(rd.s, function(r) {
    p \leftarrow c(rd = r, alpha = 0.01)
    outd <- deSolve::ode(y = NO, times = t, func = dlogistic,
        parms = p, method = "euler")[, 2]
})
out <- data.frame(t = t, Ns) # put times to the front of the matrix
names(out) <- c("t", paste("rd=", rd.s, sep = "")) # relabel columns</pre>
out.last <- subset(out, t > 0.8 * max(t)) # keep only the last 20%
## put the data in the 'long format'
out.1 <- pivot_longer(out.last, cols = -1, names_to = "r.d",
    values_to = "N")
out.1 <- dplyr::arrange(out.1, r.d, t) # re-order the data
## extract text out of the 'r.d' label that is the numeric
text.values <- substr(out.l$r.d, regexpr("=", out.l$r.d) + 1,
    100)
## convert the characters of '1.5' to the number 1.5
out.l$rd <- as.numeric(text.values)</pre>
## plot the stable limits to show the bifurcations
ggplot(out.1, aes(rd, N)) + geom_point(pch = ".") + theme_bw() +
    scale_x_continuous(breaks = seq(2.4, 2.7, by = 0.04))
```



C. Let's look closer at the chaotic portion of the graph, with a much finer range of r values. Convince yourself about chaos - that a TINY change in the parameter value r can lead to very different observed

dynamics of N. You can read a bit more about chaos at this link: (https://geoffboeing.com/2015/03/chaostheory-logistic-map/)

```
num.rd <- 1001 # the number of rd's you want
rd.s <- seq(1.5, 3, length = num.rd) # the actual rd values
t <- 0:1000 # how long to run each population
NO <- c(N = 99) # N-zero
## do all the dynamical simulations putting each run into a
## column of a matrix.
Ns <- sapply(rd.s, function(r) {</pre>
   p \leftarrow c(rd = r, alpha = 0.01)
   outd <- deSolve::ode(y = NO, times = t, func = dlogistic,
        parms = p, method = "euler")[, 2]
})
out <- data.frame(t = t, Ns) # put times to the front of the matrix
names(out) <- c("t", paste("rd=", rd.s, sep = "")) # relabel columns</pre>
out.last <- subset(out, t > 0.8 * max(t)) # keep only the last 20%
## put the data in the 'long format'
out.1 <- pivot_longer(out.last, cols = -1, names_to = "r.d",
   values to = "N")
out.l <- dplyr::arrange(out.l, r.d, t) # re-order the data
## extract text out of the 'r.d' label that is the numeric
## value
text.values <- substr(out.l\rd, regexpr("=", out.l\rd) + 1,
    100)
## convert the characters of '1.5' to the number 1.5
out.l$rd <- as.numeric(text.values)</pre>
out.1.chaotic <- subset(out.1, rd > 2.6)
ggplot(out.l.chaotic, aes(rd, N)) + geom_point(pch = ".") + theme_bw()
```



Question 4. Plot dynamics in the discrete-time logistic growth system.

In class, I showed a plot of population dynamics then ran the simulation for 8 different values of the population growth rate r:

```
# Parameter values to use for simulation
r_range <- c(0, 1, 1.3, 1.6, 1.9, 2.2, 2.5, 2.8)
K <- 1400
NO <- 4</pre>
```

```
t <- 30
# Simulation of model for t time steps
N.g <- numeric()</pre>
plist <- list()</pre>
for (i in 1:length(r_range)) {
    N \leftarrow rep(NA, t)
    for (k in 1:t) {
         N[k] <- disc_log(r_range[i], NO, K)</pre>
         NO \leftarrow N[k]
    }
    dat <- as.data.frame(N)</pre>
    dat$time <- as.numeric(rownames(dat))</pre>
    dat$r <- rep(r_range[i], t)</pre>
    N.g <- rbind(N.g, dat)
    NO <- 4
    p <- ggplot(dat, aes(time, N)) + geom_point() + geom_line(color = "gray") +</pre>
         geom_hline(yintercept = K, linetype = "dashed", color = "orange")
    plist[[i]] <- p</pre>
}
gridExtra::grid.arrange(grobs = plist, nrow = round(length(r_range)/3))
                                      1000
   1000 -
                                                                          1000
                                                                       Z
Z
                                   Z
    500 -
                                       500
                                                                           500
      0 -
                                          0
                                                                             0
                .
10
                       20
                                                   10
                                                           <u>.</u>
                                                                  30
                                                                                      10
                                                                                              20
        0
                               30
                                                                                                     30
                                            0
                  time
                                                      time
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                                      1500 -
                                                                          1500 -
   1000 -
                                      1000 -
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Z
                                                                       Z
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    500
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10
                       20
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                                                                               0
                                                                                      10
                                                                                              20
                                                                                                     30
        0
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                  time
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                                                                                         time
   1500
                                      1500 -
z 1000-
                                   z 1000 -
    500
                                       500
      0
                                          0
                10
                       20
                               30
                                                   10
                                                           20
                                                                  30
                  time
                                                      time
```

Please use this plotting code to show dynamics for r=2, 2.2, 2.4, 2.5, 2.55, and 2.8, for 50 time steps. Please use RMarkdown to write an informative caption for this figure, explaining what the graphs show. You can use this link: (https://uoepsy.github.io/scs/rmd-bootcamp/06-figs.html#captions)

```
# Parameter values to use for simulation
r_range \leftarrow c(2, 2.2, 2.4, 2.5, 2.55, 2.8)
K <- 1400
NO <- 4
t <- 50
# Simulation of model for t time steps
N.g <- numeric()
plist <- list()</pre>
for (i in 1:length(r_range)) {
    N \leftarrow rep(NA, t)
    for (k in 1:t) {
        N[k] <- disc_log(r_range[i], NO, K)</pre>
        NO <- N[k]
    }
    dat <- as.data.frame(N)</pre>
    dat$time <- as.numeric(rownames(dat))</pre>
    dat$r <- rep(r_range[i], t)</pre>
    N.g <- rbind(N.g, dat)</pre>
    NO <- 4
    p <- ggplot(dat, aes(time, N)) + geom_point() + geom_line(color = "gray") +</pre>
         geom_hline(yintercept = K, linetype = "dashed", color = "orange")
    plist[[i]] <- p</pre>
}
gridExtra::grid.arrange(grobs = plist, nrow = 3)
```

Exercise 2. Create simulation of continuous-time Lotka-Volterra competition model

Here is code for a continuous-time logistic growth model:

The formula:

$$\frac{dn}{dt} = rn(1 - \frac{n}{K})$$

The code:

```
# Parameter values to use for simulation
parameters <- c(r = 1.21, K = 1400)
state <- c(N = 4)
times <- seq(0, 30, by = 0.01)
# model function
cont_log <- function(t, state, parameters) {
    with(as.list(c(state, parameters)), {
        dN <- r * N * (1 - N/K)
        return(list(dN))
    })
}
# Simulation of model for t time steps
out <- deSolve::ode(y = state, times = times, func = cont_log,
    parms = parameters)
# Plot simulation: ggplot</pre>
```

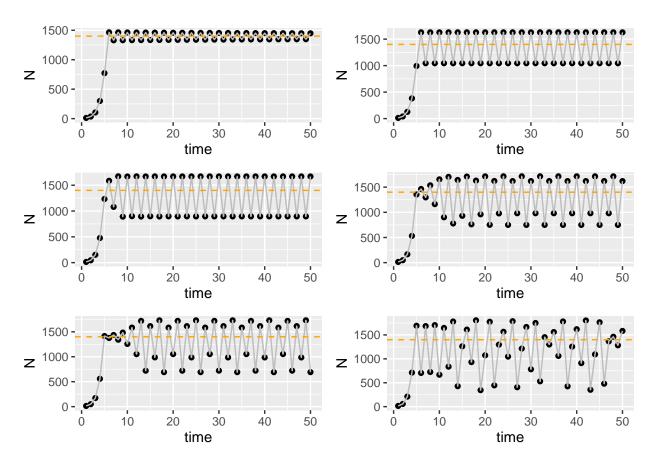
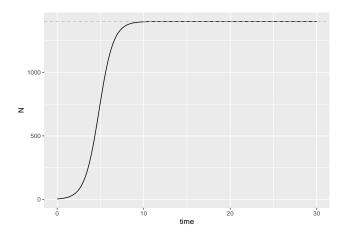
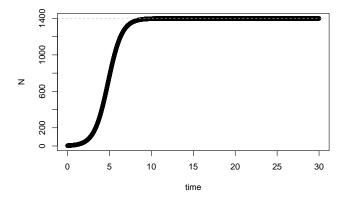


Figure 1: A figure of population size over time using the discrete-time logistic growth model





A. Please modify this to create a simulation of the continuous time Lotka-Volterra competition equations, seen as equations 3.15 in Otto & Day Chapter 3.

The formula:

$$\frac{dn_1}{dt} = r_1 n_1 \left(1 - \frac{n_1 t + \alpha_{12} n_2}{K_1}\right)$$
$$\frac{dn_2}{dt} = r_2 n_2 \left(1 - \frac{n_2 t + \alpha_{21} n_1}{K_2}\right)$$

The parameters you will need are:

r1, r2, K1, K1, alpha_12, alpha_21, n1_0, n2_0

Use these values:

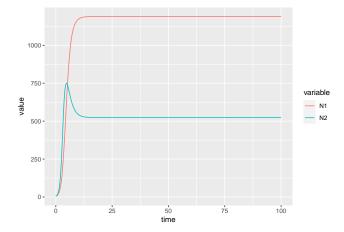
$$r1 = 1.3$$
, $r2 = 1.5$, $K1 = 1,400$, $K2 = 1,000$, $alpha_12 = 0.4$, $alpha_21 = 0.6$, $N1_0 = 5$, $N2_0 = 5$

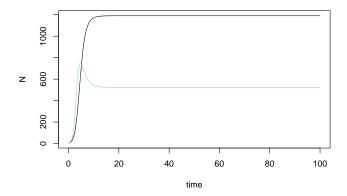
Note: be very careful with your parentheses

Here I show with prompts what code you need to fill in. Then I show my code for running the continuous simulation and plotting.

```
# Parameter values to use for simulation
parameters <- 1  ## fill in param values where the 1 is
state <- 1  ## fill in initial values for state variables where the 1 is
times <- 1  ## fill in values for time intervals where the 1 is
# model function
cont_LV <- function(t, state, parameters) {
    with(as.list(c(state, parameters)), {
        ## fill in equation for N1 fill in equation for N2
        return(list(c(dN1, dN2)))
    })
}</pre>
```

```
# Parameter values to use for simulation
parameters <- c(r1 = 1.3, r2 = 1.7, K1 = 1400, K2 = 1000, alpha_12 = 0.4,
        alpha_21 = 0.6)
state <- c(N1 = 5, N2 = 5)
times <- seq(0, 100, by = 0.1)
# model function
cont_LV <- function(t, state, parameters) {
    with(as.list(c(state, parameters)), {
        dN1 <- r1 * N1 * (1 - ((N1 + alpha_12 * N2)/K1))
        dN2 <- r2 * N2 * (1 - ((N2 + alpha_12 * N1)/K2))
        return(list(c(dN1, dN2)))
    })
}</pre>
```





OPTIONAL BONUS CONTENT

Amazing animated graph by classmate Tina Ross! Animation of discrete-time logistic growth graphs across values of r

```
# set my parameters
rs \leftarrow c(2, 2.2, 2.4, 2.5, 2.55, 2.8)
t <- 50
NO <- 4
K <- 1400
grid.log <- as.data.frame(matrix(NA, nrow = t + 1, ncol = length(rs) +</pre>
    1))
colnames(grid.log) <- c("t", paste("r", rs, sep = ""))</pre>
grid.log$t <- c(0:t)
ggrid <- list()</pre>
for (i in 1:length(rs)) {
    r <- rs[i]
    N \leftarrow c(N0, rep(NA, t))
    for (j in 1:t) {
        N[j + 1] \leftarrow disc_log(r, N[j], K)
    grid.log[, i + 1] <- N
    ggrid[[i]] <- ggplot(grid.log, aes(x = t, y = .data[[names(grid.log)[i +</pre>
        1]]])) + geom_line(col = "turquoise") + geom_point(size = 1,
        shape = 19, col = "turquoise4") + geom_hline(yintercept = K,
        linetype = "dashed", color = "grey60") + theme light() +
        labs(title = (paste("r =", rs[i])), y = "N", x = "t (days)") +
        theme(axis.title.y = element_text(angle = 0, vjust = 0.5))
}
remove(i, j)
grid.long <- melt(grid.log, id.vars = "t", variable.name = "r",</pre>
   value.name = "N")
```