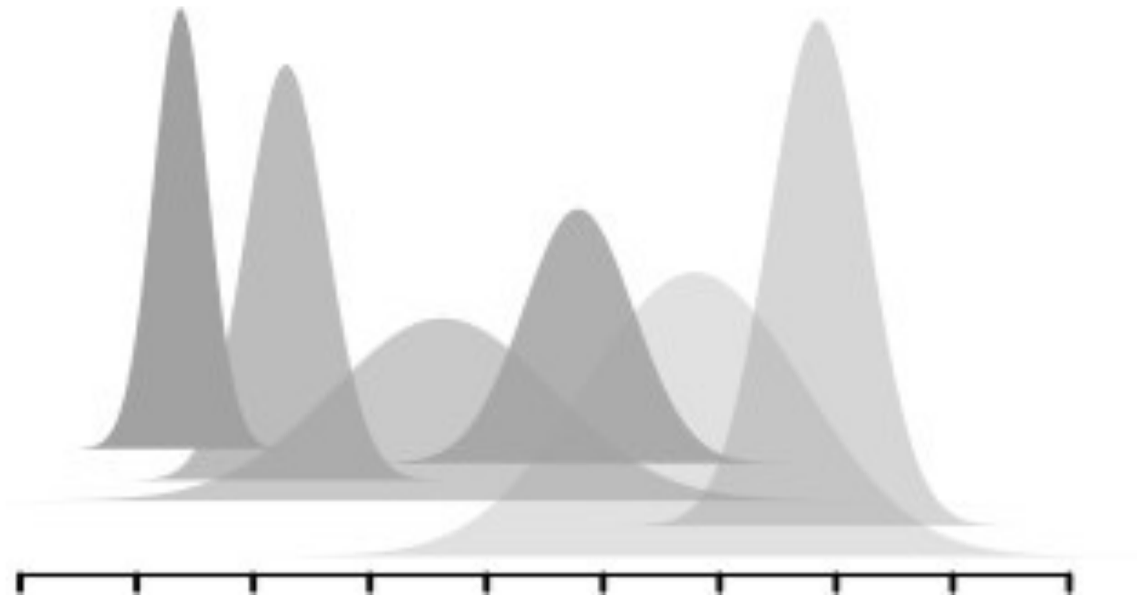
5.1 Fitting empirical data to models



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What is theory?

An explanation of an ecological phenomenon, that explains how an ecological process works or why an ecological pattern is observed

An idea becomes scientifically useful when expressed as a testable *theory*, often in the form of a *mathematical model*

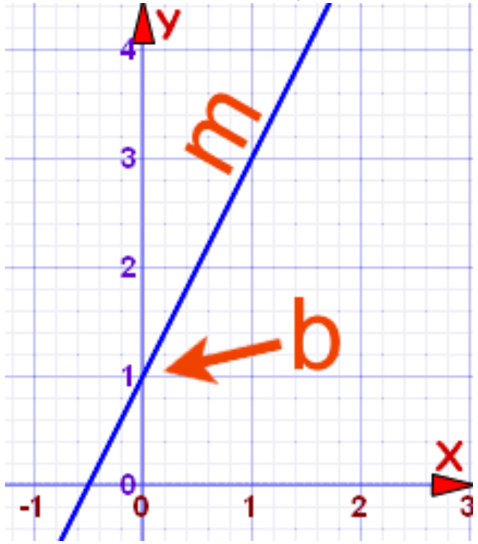
A *mathematical model* is an equation or a set of equations that describes how different aspects of a system relate to one another

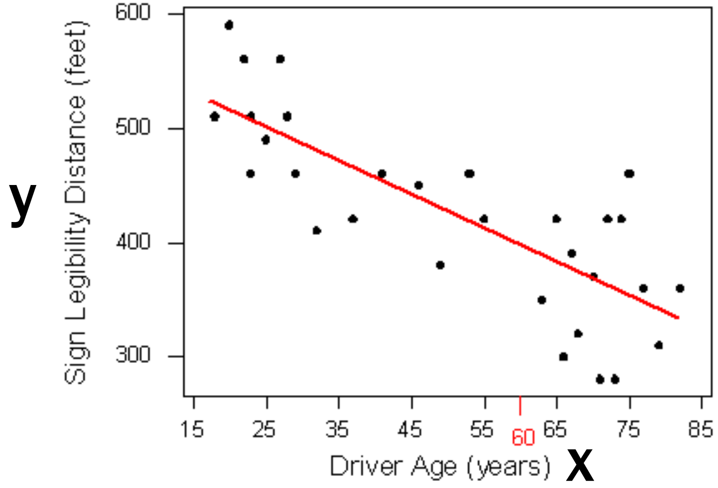
Example 1. Formula for a line → Linear model

$$y = mx + b$$

m - slope

b - y-intercept (where line crosses y-axis.... value of y when x=0)





Grainger et al, 2022

How to use a mathematical model?

Approach 3: Use the mathematical equations

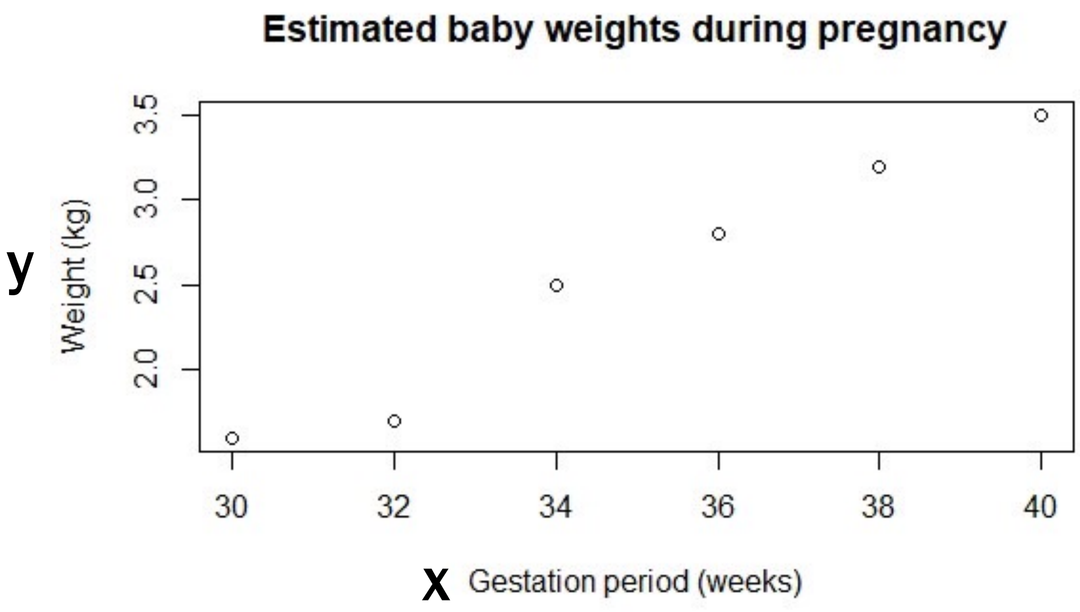
model fitting (aka “fitting models to data”) - collect measurements of the response and predictor variables in the model, and then use statistical techniques to estimate the values of free parameters (those whose values are unknown) that best match the relationship observed in the data

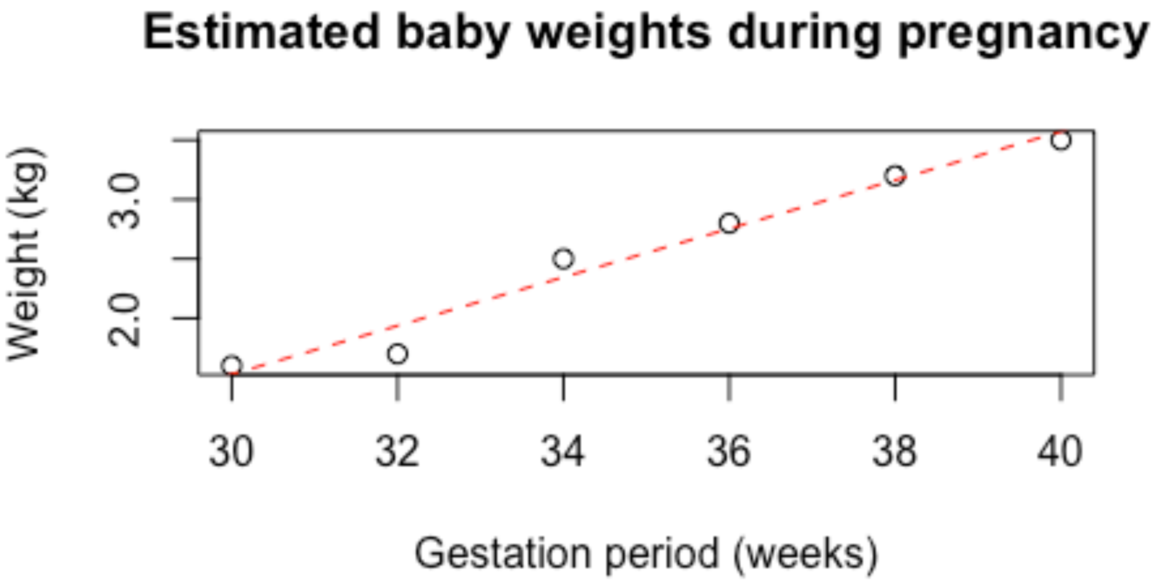
```
Coefficients:
(Intercept)    gestation
    -4.6000       0.2043

> confint(model, level = 0.95)
                 2.5 %     97.5 %
(Intercept) -6.3862379 -2.8137621
gestation    0.1534916  0.2550798
```

Example 1. Linear causal model (linear regression)

$$y = mx + b$$
$$weight = m(gestation\ period) + b$$



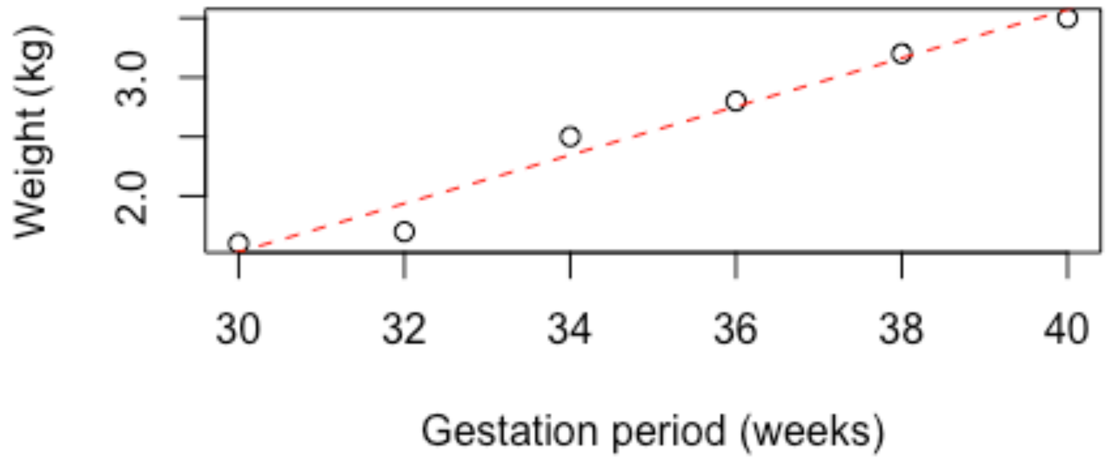


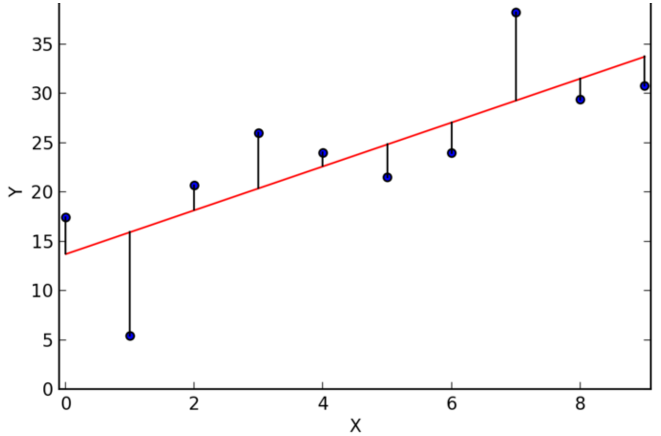
How to use a mathematical model?

$$weight = m(gestation\ period) + b$$

Least squares regression (line that minmizes sum of squared distances from observed points to model-fit line)

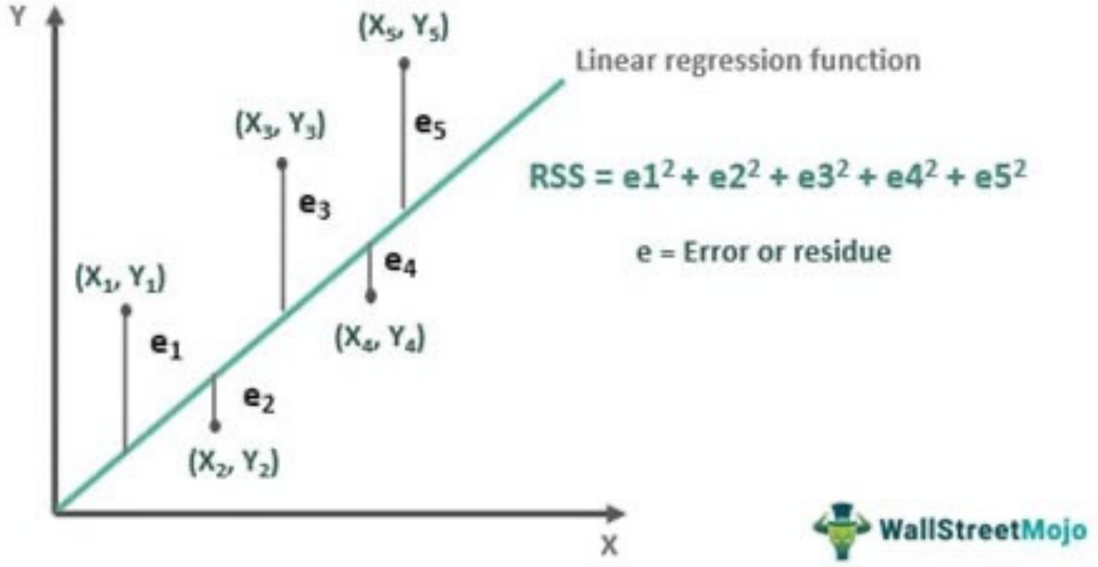
Estimated baby weights during pregnancy





Residual Sum of Squares

esidual Sum of Squares measures the extent of variability of observed data not predicted by the regression model.



Population ecology

Exponential growth

$$\frac{dn}{dt} = rn_t$$

Logistic growth

$$\frac{dn}{dt} = rn_t\left(1 - \frac{n_t}{K}\right)$$

Community ecology

Lotka Volterra model

$$\frac{dn_i}{dt} = rn_i\left(1 - \frac{n_i + \alpha_{ij}n_j}{K_i}\right) \qquad \frac{dn_j}{dt} = rn_j\left(1 - \frac{n_j + \alpha_{ji}n_i}{K_j}\right)$$

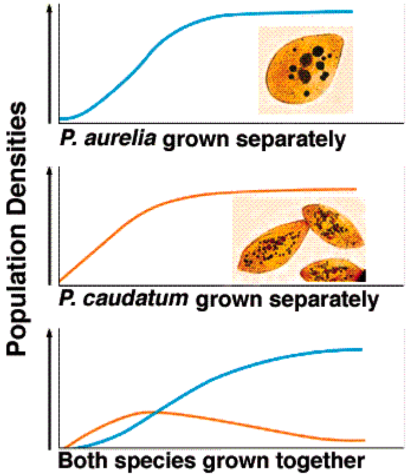
Alfred J. Lotka (1880-1949)
Chemist, ecologist, mathematician
Ukrainian immigrant to the USA

Vito Volterra (1860-1940)
Mathematical Physicist
Italian, refugee of fascist Italy

Testing the consequences of species interactions: Georgii Frantsevich Gause (b. 1910)

Paramecium caudatum

Paramecium aurelia

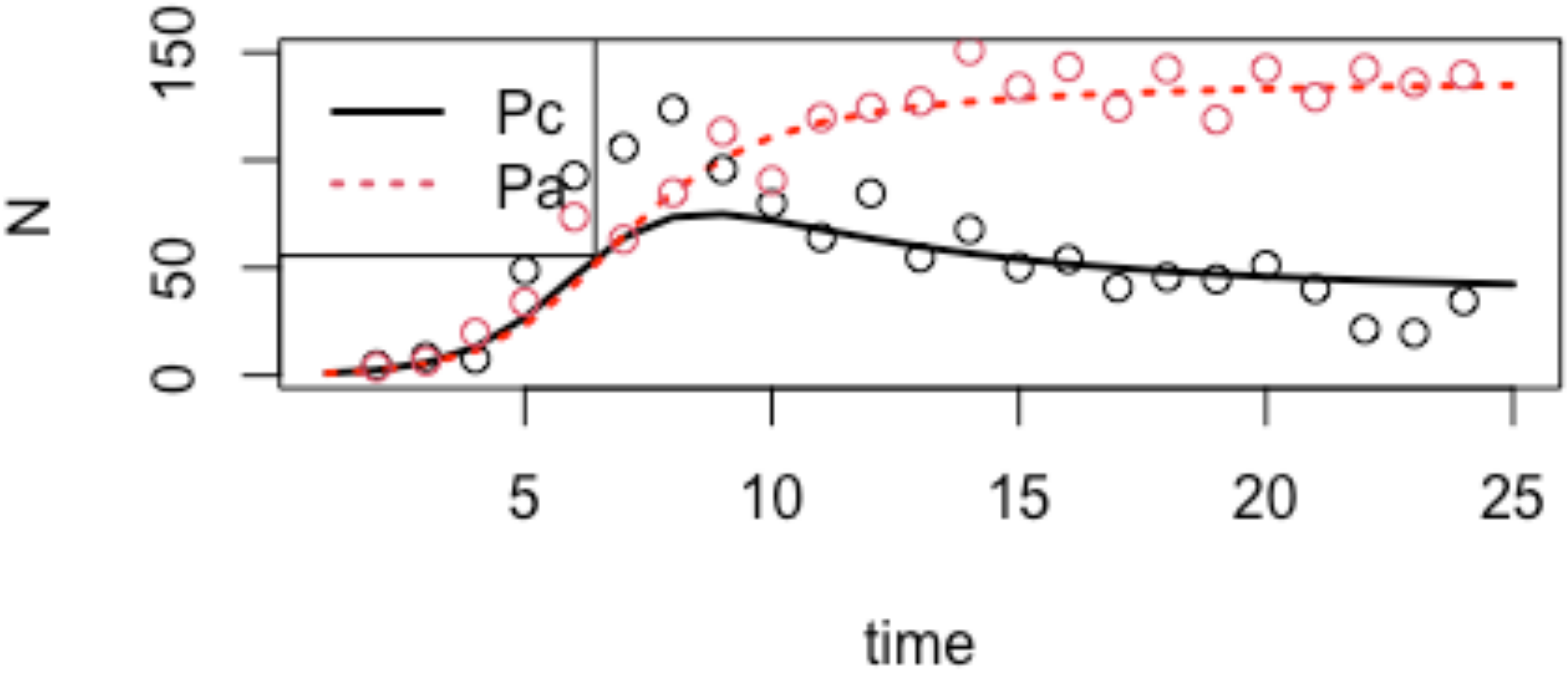


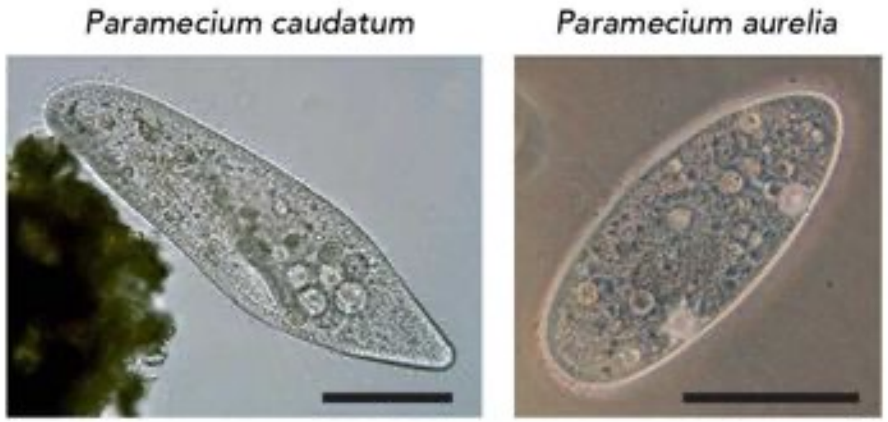
How to use a mathematical model?

Lotka Volterra model

$$\frac{dn_i}{dt} = rn_i\left(1 - \frac{n_i + \alpha_{ij}n_j}{K_i}\right) \qquad \frac{dn_j}{dt} = rn_j\left(1 - \frac{n_j + \alpha_{ji}n_i}{K_j}\right)$$

Non-linear least squares regression – find parameter values that minimize sum of squared distances from observed points to model-fit line





Population ecology

Exponential growth

$$\frac{dn}{dt} = rn_t$$

Logistic growth

$$\frac{dn}{dt} = rn_t\left(1 - \frac{n_t}{K}\right)$$

Stage-structured model

$$N_{t+1} = AN_t$$

$$\begin{bmatrix} N_1 \\ N_2 \end{bmatrix}_{t+1} = \begin{bmatrix} F_1 & F_2 \\ P_{1\to 2} & P_{2\to 2} \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}_t$$

Metapopulation model

$$\frac{dp}{dt} = cp(1 - p) - ep$$

Community ecology

Lotka Volterra model

$$\frac{dn_i}{dt} = rn_i\left(1 - \frac{n_i + \alpha_{ij}n_j}{K_i}\right) \qquad \frac{dn_j}{dt} = rn_j\left(1 - \frac{n_j + \alpha_{ji}n_i}{K_j}\right)$$

Rosenzweig-MacArthur consumer-resource model

$$\frac{dR}{dt} = rR\left(1 - \frac{R}{K}\right) - \frac{aR}{1 + ahR}C$$

$$\frac{dC}{dt} = e\frac{aR}{1 + ahR}C - mC$$

Individual ecology

Toxicokinetic-toxicodynamic model