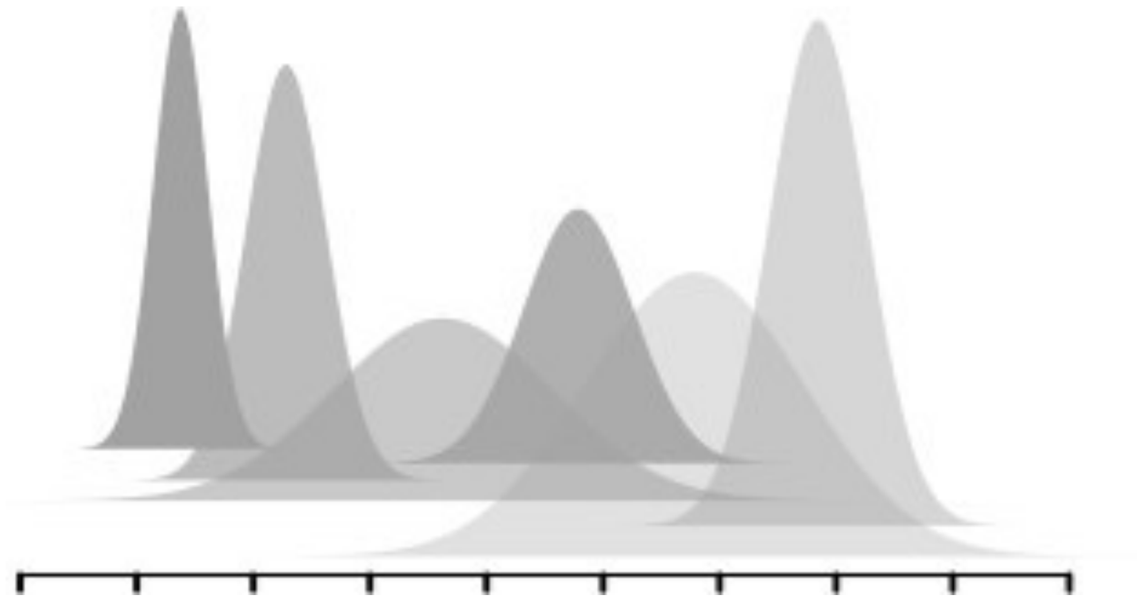
3.1 Population & Community Ecological Models



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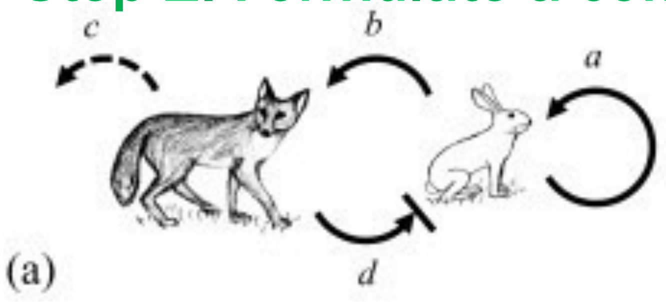
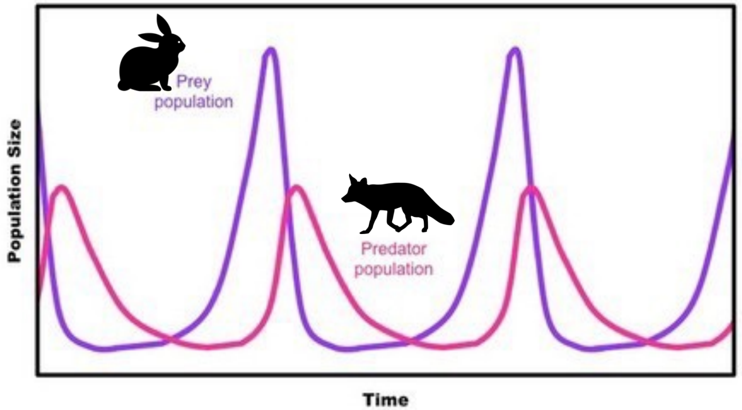
What is a model?

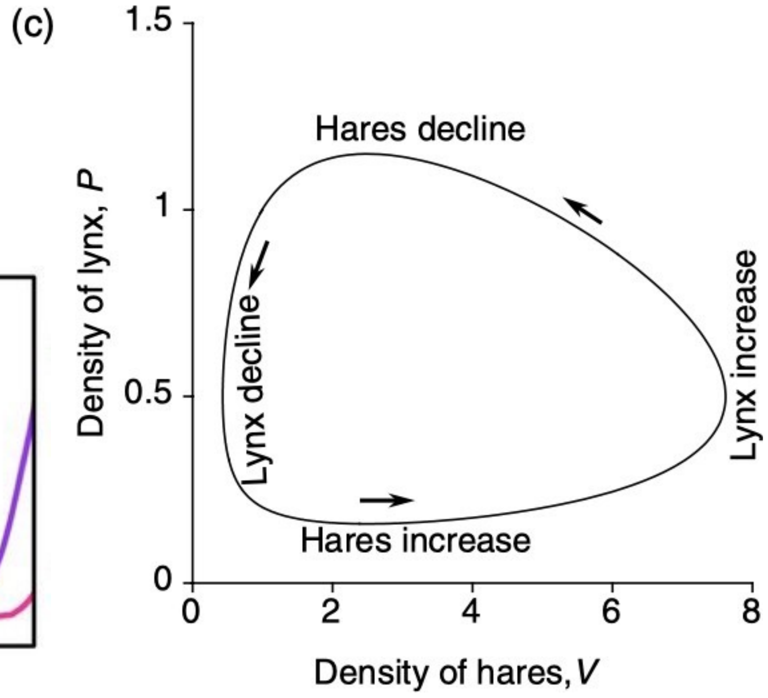
"A *model* is a representation of a particular thing, idea, or condition."

"The *modeling process* is the series of steps taken to convert an idea first into a conceptual model and then into a quantitative model"

Step 1. Formulate a conceptual model







Step 2. Formulate a quantitative model

$$\frac{\mathrm{d}V}{\mathrm{d}t} = aV - bVP$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = -cP + dVP$$

Step 3. Learn about study system through analysis of model behavior

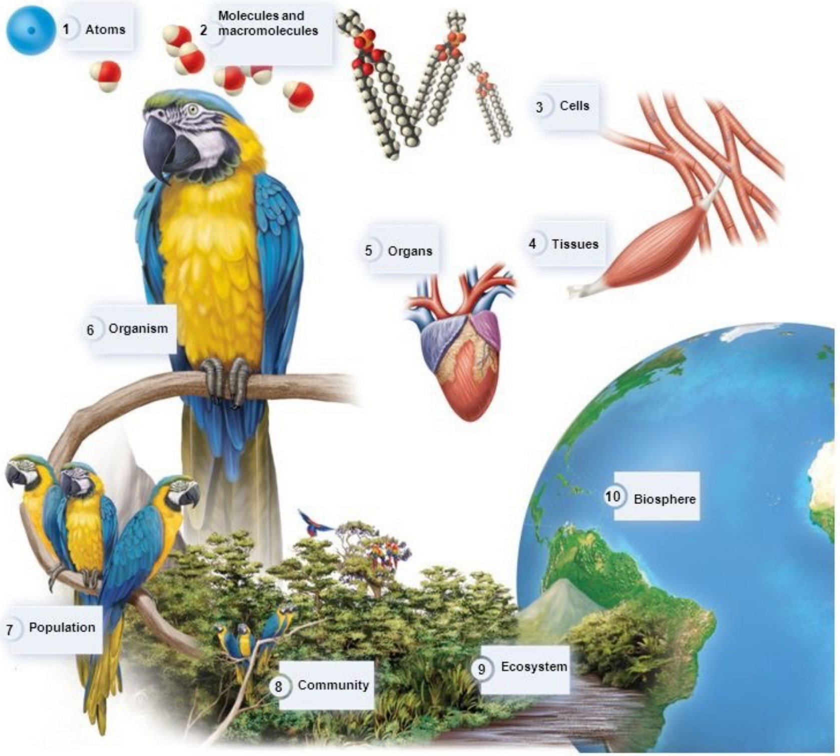
Kinoshita, 2013, Intro to Nonequilibrium Phenomena

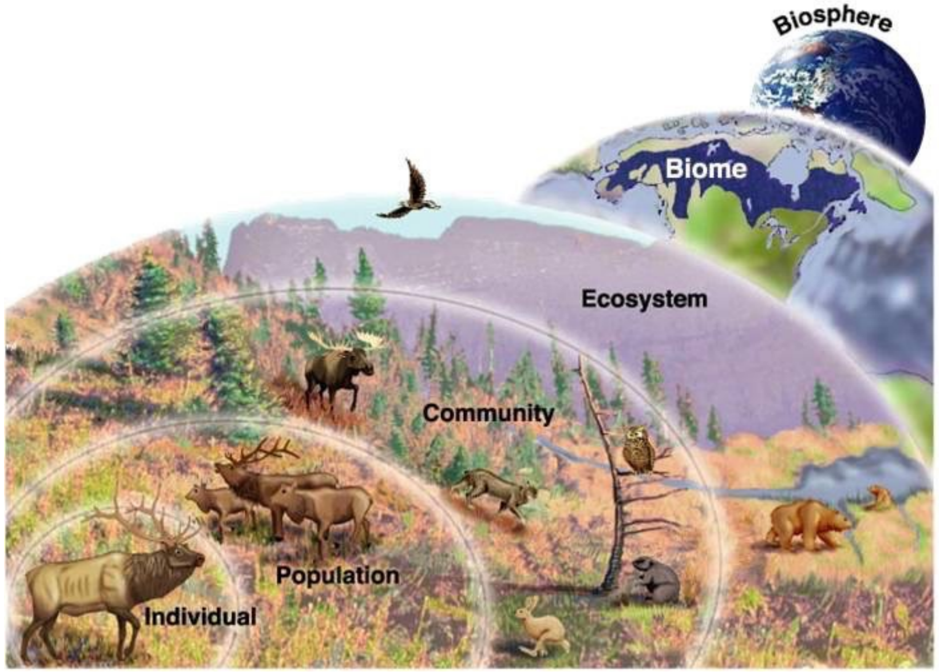
What is ecology?

The study of interactions between organisms and their environment, and with one another

The science that investigates the abundance and distribution of organisms

Levels of Biological Organization





Step 3. Learn about study system through analysis of model behavior

Population ecology

Exponential growth

$$\frac{dn}{dt} = rn_t$$

Graphical techniques: develop a feeling for your model

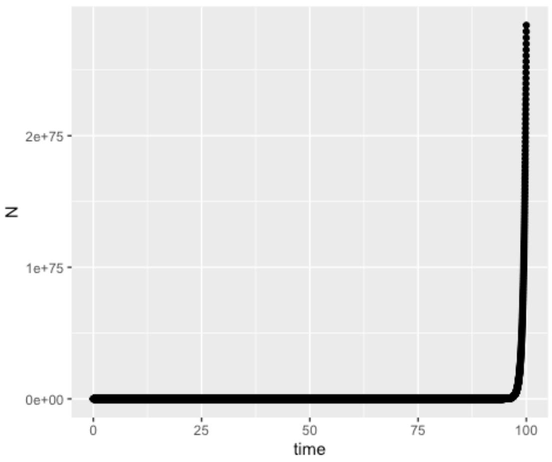
Expected dynamics

When r > 0?

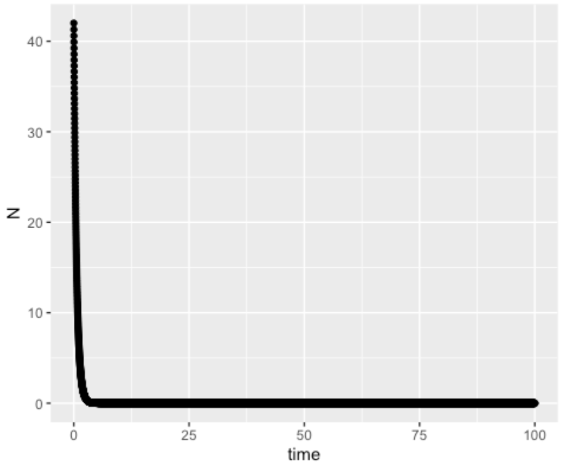
When r < 0?

When r = 0?

r = 1.7



r = -1.7



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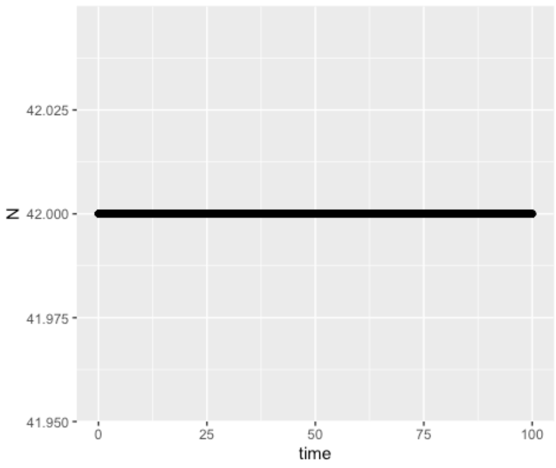

parameter	
r	population growth rate

Step 3. Learn about study system through analysis of model behavior

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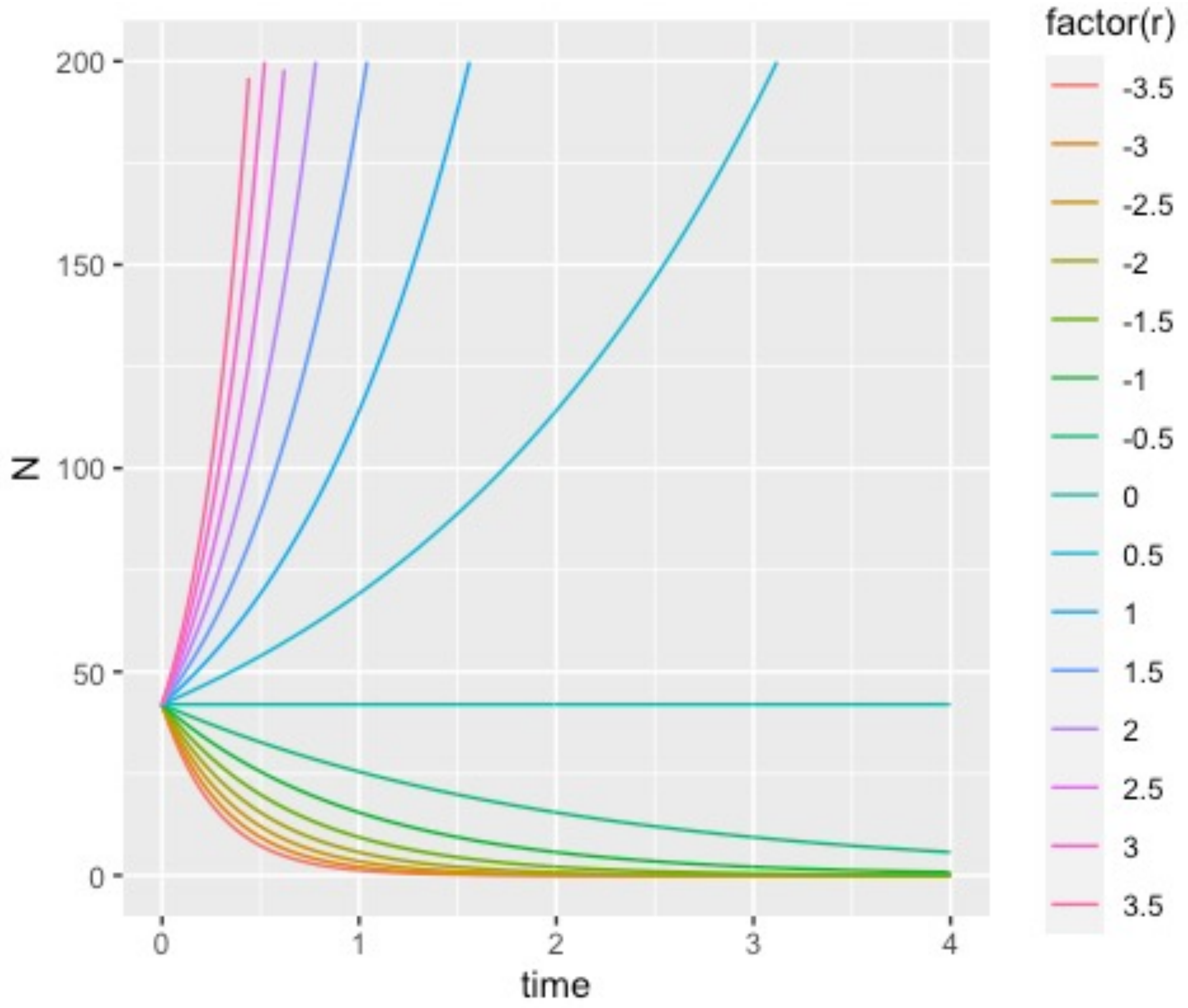

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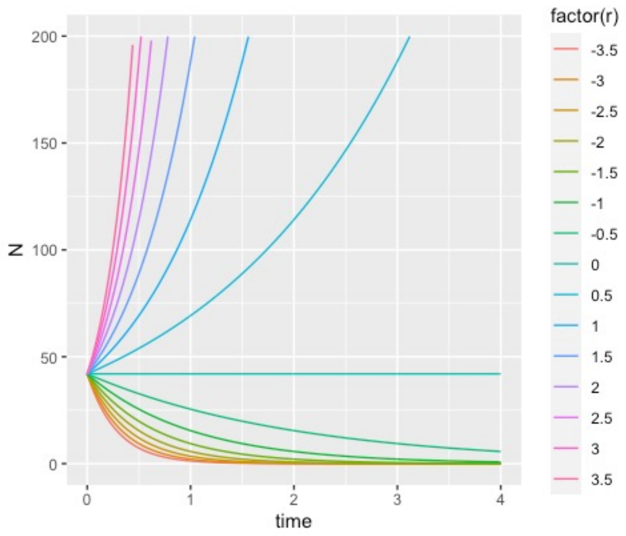

parameter	
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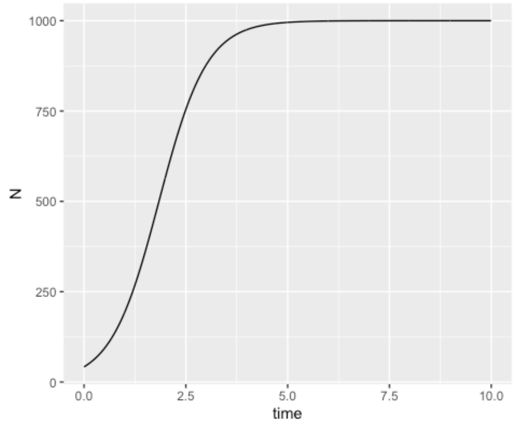
Logistic growth

$$\frac{dn}{dt} = rn_t\left(1 - \frac{n_t}{K}\right)$$

Expected dynamics

r = 1.7



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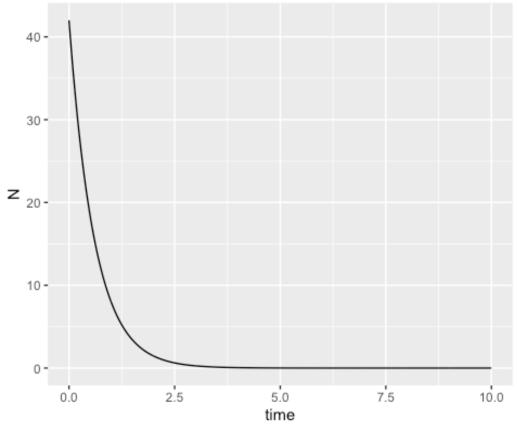
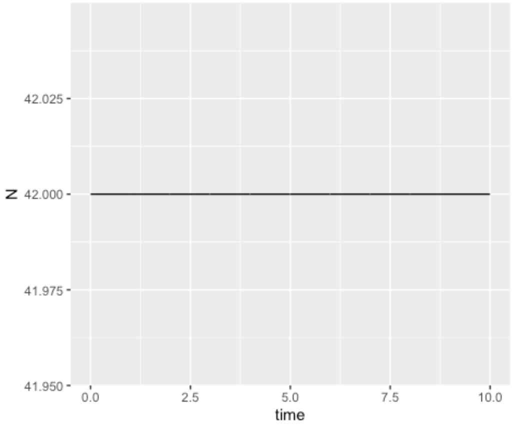


r = 0



parameter	
r	population growth rate
K	Carrying capacity

Step 3. Learn about study system through analysis of model behavior

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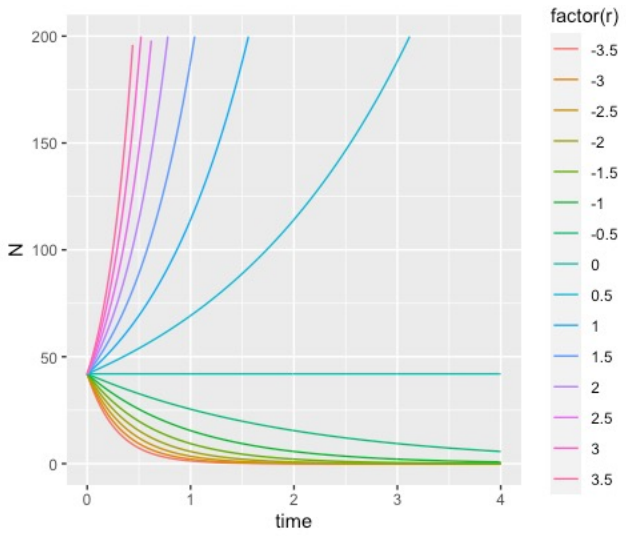
Exponential growth

$$\frac{dn}{dt} = rn_t$$

Graphical techniques: develop a feeling for your model

Expected dynamics



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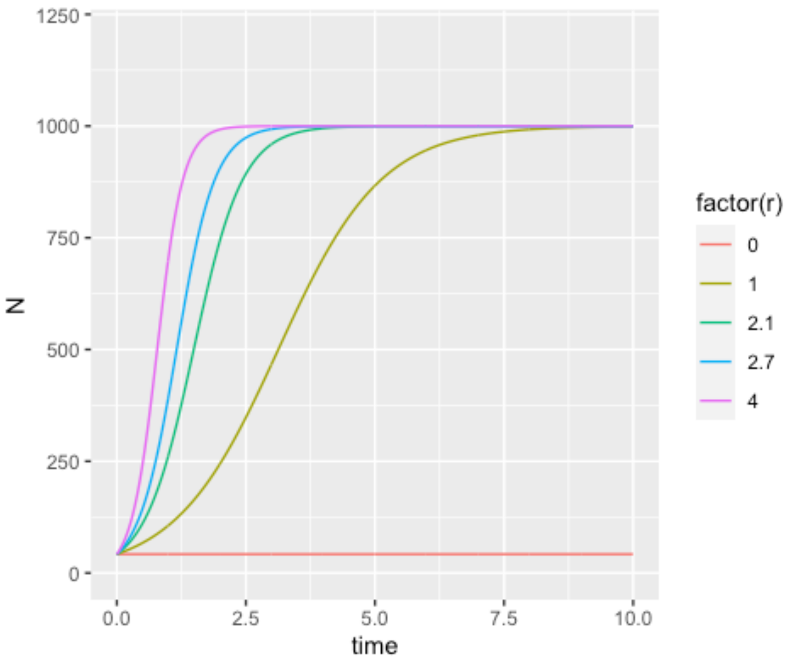

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Step 3. Learn about study system through analysis of model behavior

Population ecology

Exponential growth

$$\frac{dn}{dt} = rn_t$$

Graphical techniques: develop a feeling for your model

Equilibria and stability analyses

Equilibrium

- a system at *equilibrium* does not change over time
- A particular value of a variable is called an *equilibrium value* if, when the variable is *started* at this value, the system never changes
- At equilibrium in a continuous-time model, dn/dt must equal 0 for each variable

Logistic growth

$$\frac{dn}{dt} = rn_t\left(1 - \frac{n_t}{K}\right)$$

How to solve for equilibrium values

- Replace desired value with «the equilibrium value»

$$\frac{dn}{dt} = rn_t \longrightarrow \frac{dn}{dt} = r^* n_t$$

- We would like to know r*, «the value of r at which the state variable / system (n) is no longer changing»

$$\frac{dn}{dt} = 0 \longrightarrow 0 = r^* n_t$$

- Solve for the equilibrium value of r (r*)

$$0 = r^* n_t \longrightarrow \frac{0}{n_t} = \frac{r^*}{n_t} \longrightarrow 0 = r^*$$

- The system is at equilibrium (n is not changing, dn/dt = 0) when r* = 0

Population ecology

Exponential growth

$$\frac{dn}{dt} = rn_t$$

Logistic growth

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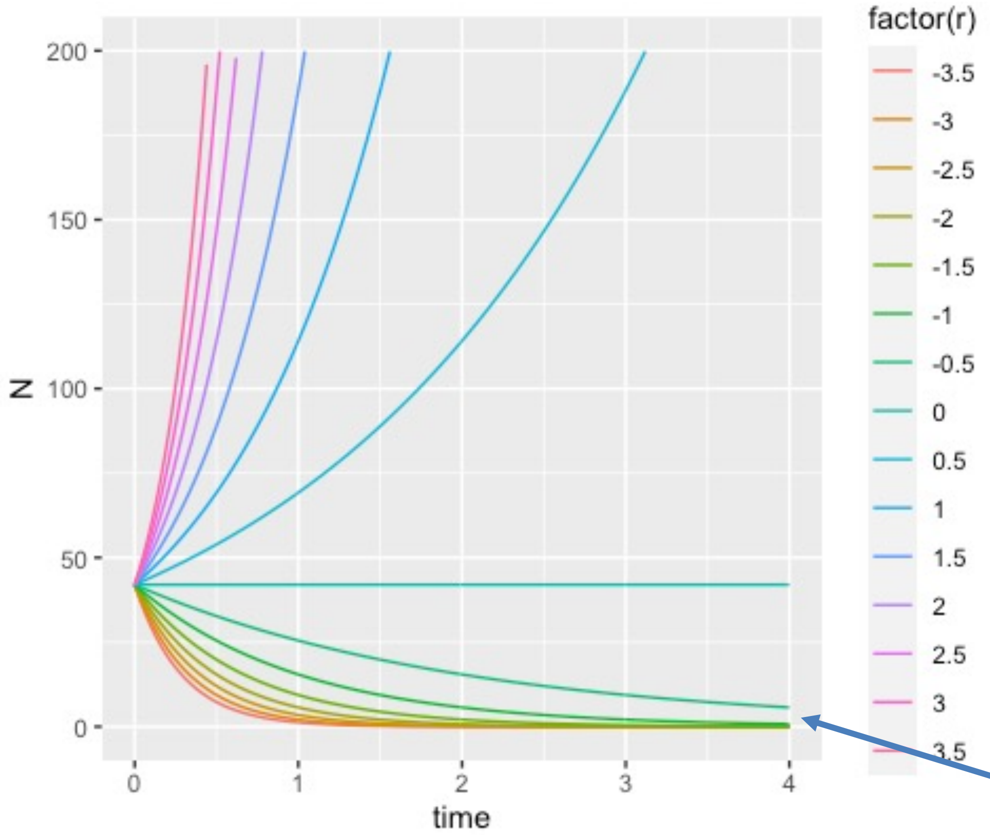
Graphical techniques: develop a feeling for your model

Equilibria and stability analyses

Equilibrium



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- Solve for the equilibrium value of n_t (n_t*)

Step 3. Learn about study system through analysis of model behavior

Population ecology

Exponential growth

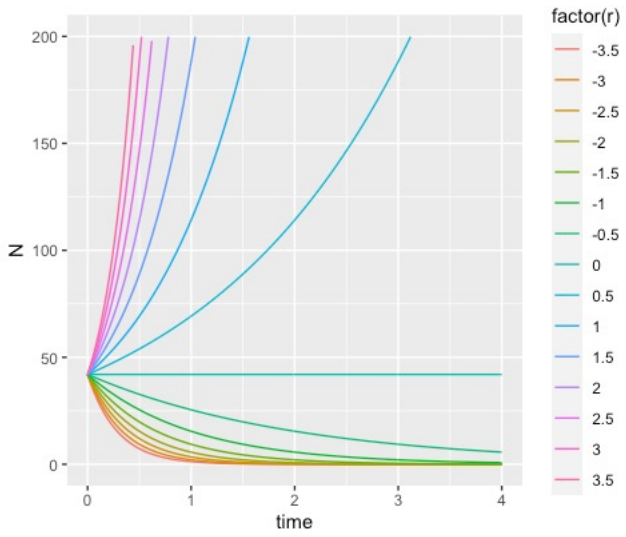
$$\frac{dn}{dt} = rn_t$$

Graphical techniques: develop a feeling for your model

Equilibria and stability analyses

Equilibrium

$$\frac{dn}{dt} = 0 \longrightarrow \begin{matrix} r^* = 0 \\ n_t^* = 0 \end{matrix}$$



Logistic growth

$$\frac{dn}{dt} = rn_t\left(1 - \frac{n_t}{K}\right)$$

$$0 = rn_t\left(1 - \frac{n_t}{K}\right) \longrightarrow \begin{matrix} r^* = 0 \\ n_t^* = 0 \\ K^* = ? \end{matrix}$$

$$\left(1 - \frac{n_t}{K}\right) = 0 \longrightarrow 1 = \frac{n_t}{K} \longrightarrow K = n_t$$

Population ecology

Exponential growth

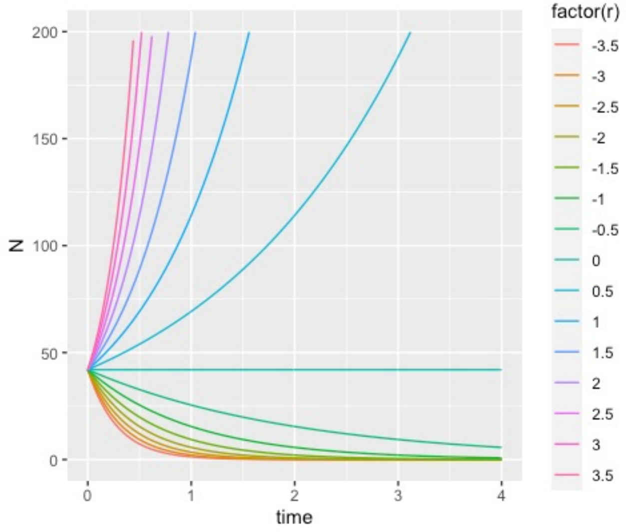
$$\frac{dn}{dt} = rn_t$$

Graphical techniques: develop a feeling for your model

Equilibria and stability analyses

Equilibrium

$$\frac{dn}{dt} = 0 \longrightarrow r^* = 0,\quad n_t^* = 0$$



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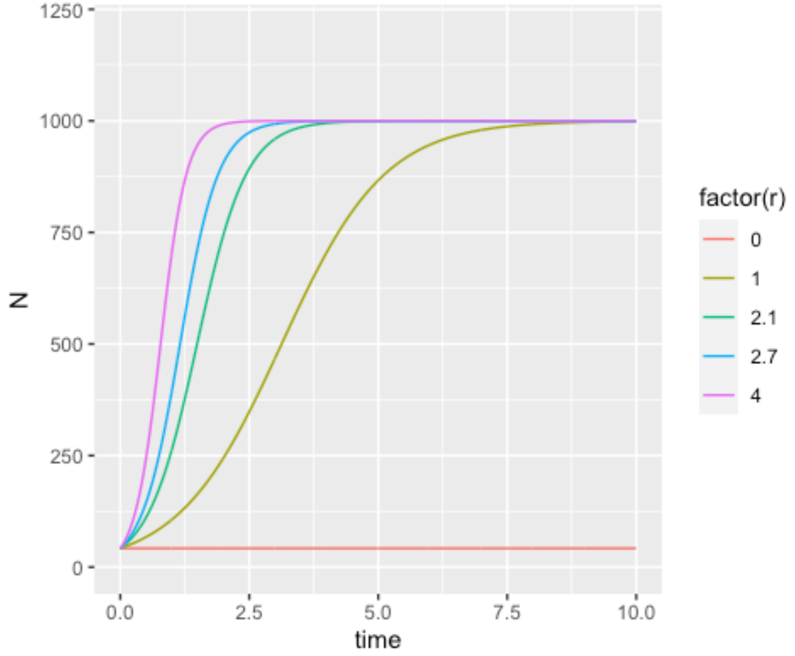

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Community ecology

Lotka Volterra model

$$\frac{dn_i}{dt} = rn_i\left(1 - \frac{n_i + \alpha_{ij}n_j}{K_i}\right) \qquad \frac{dn_j}{dt} = rn_j\left(1 - \frac{n_j + \alpha_{ji}n_i}{K_j}\right)$$

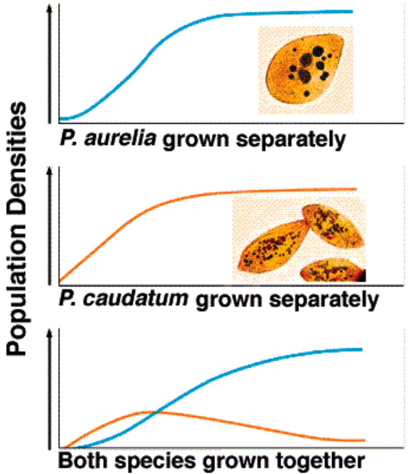
Alfred J. Lotka (1880-1949)
Chemist, ecologist, mathematician
Ukrainian immigrant to the USA

Vito Volterra (1860-1940)
Mathematical Physicist
Italian, refugee of fascist Italy

Testing the consequences of species interactions: Georgii Frantsevich Gause (b. 1910)

Paramecium caudatum

Paramecium aurelia



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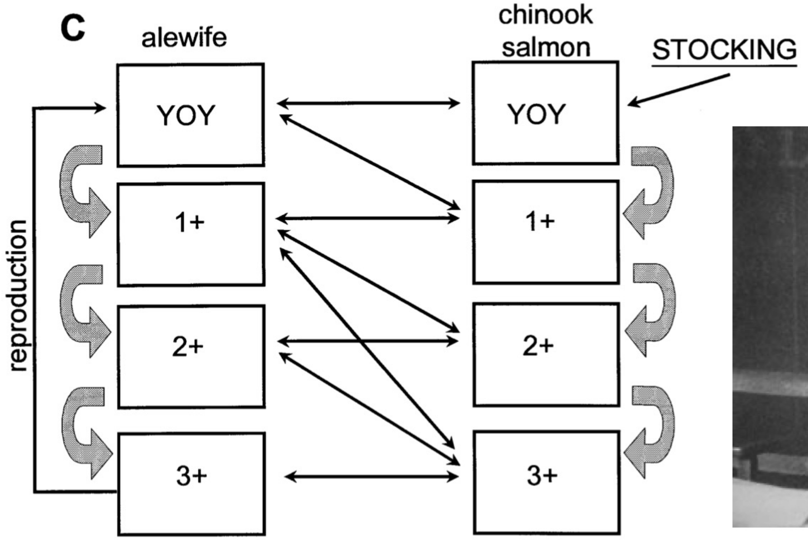
Stage-structured model



Jackson et al. 2000

Patrick H. Leslie

On the use of Matrices in Certain Population Mathematics, Biometrika , Vol. 32., pp. 183-212

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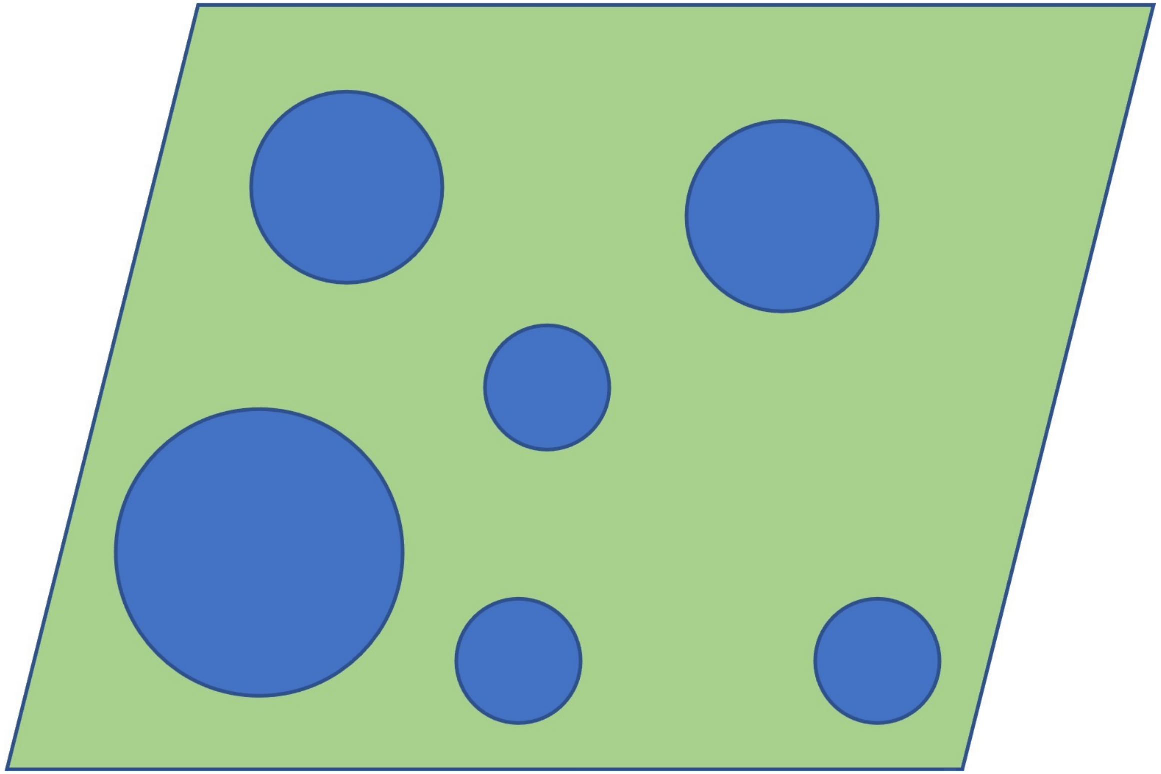
Metapopulation model



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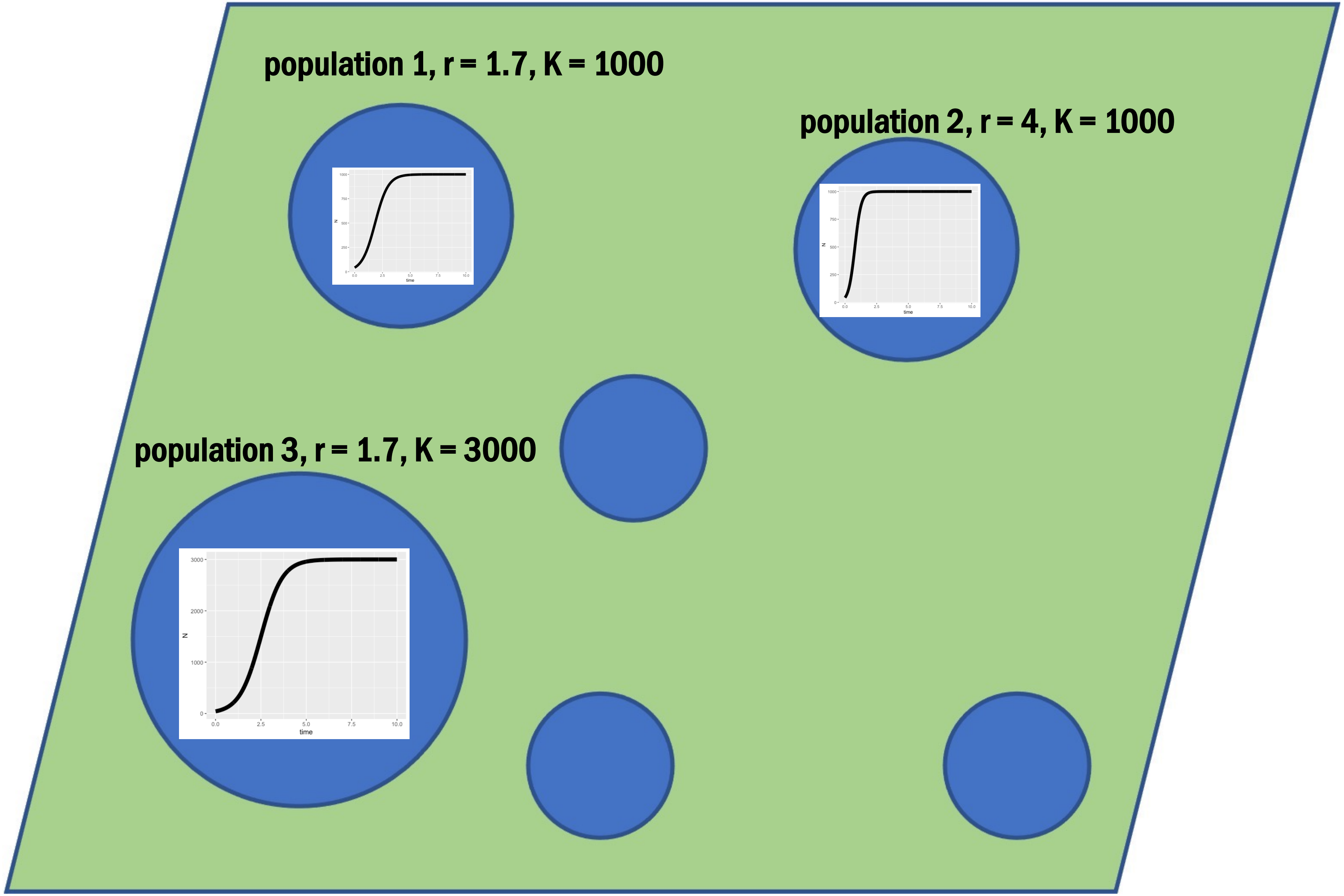
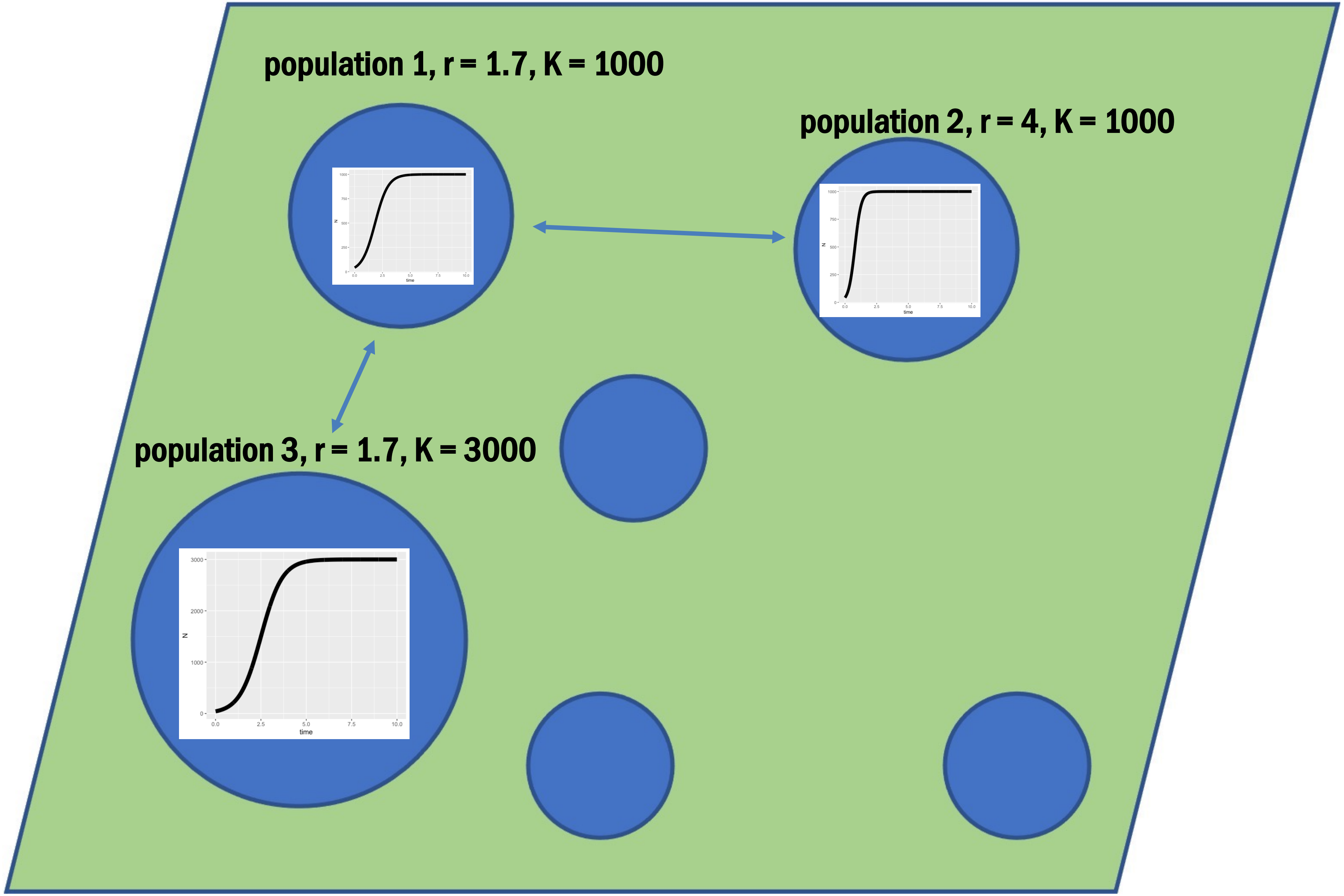
Population model

Assumes all population dynamics are independent



Metapopulation model

Assumes population dynamics are linked, as species can move among patches



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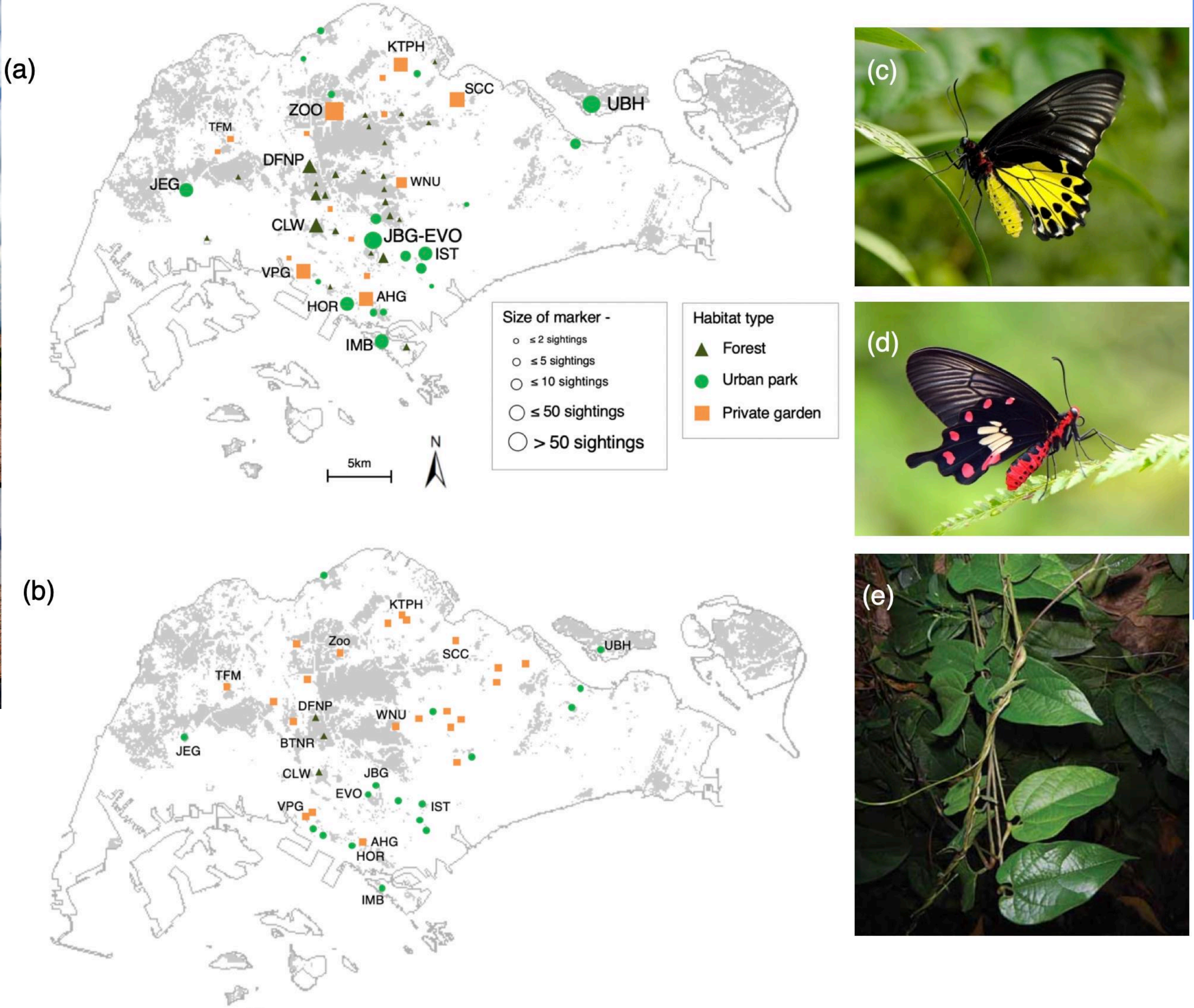
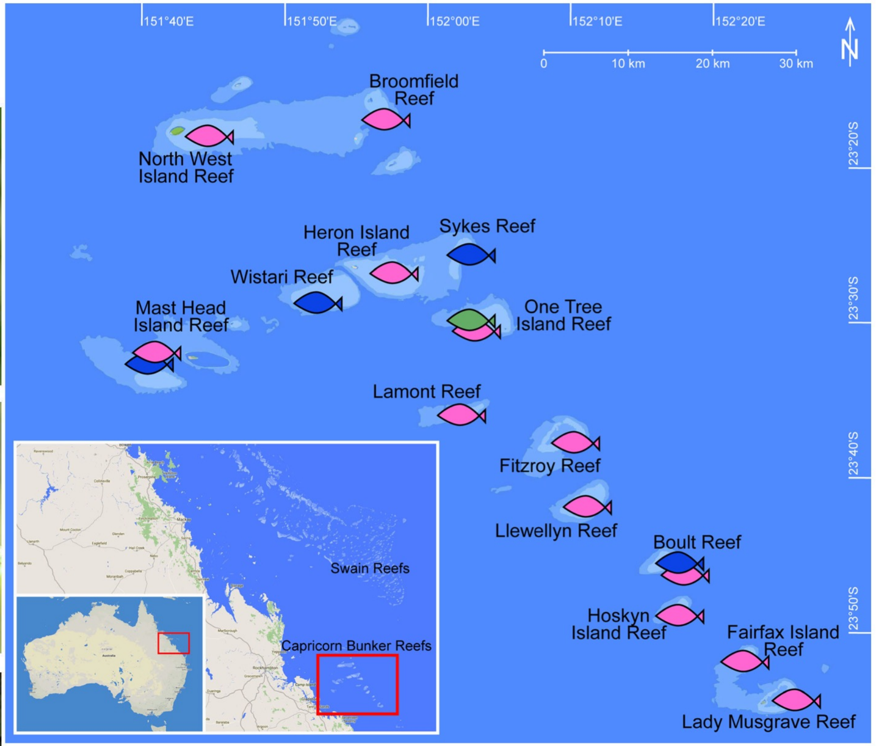


Jain et al. 2021, Frontiers in Ecology & Evolution



Gerlach et al. 2021, Coral Reefs 40: 999-1011

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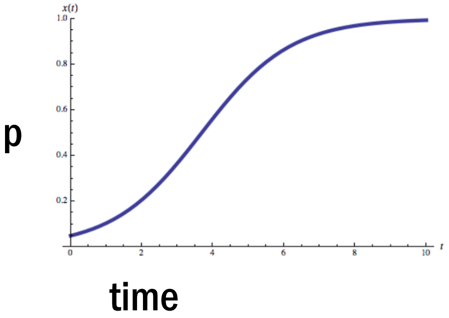
Metapopulation model

$$\frac{dp}{dt} = cp(1-p) - ep$$

Change in the proportion of occupied sites (p) over time (t)



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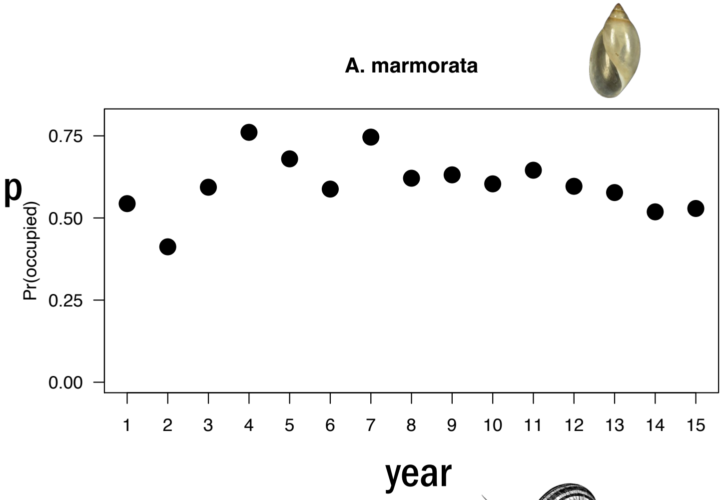
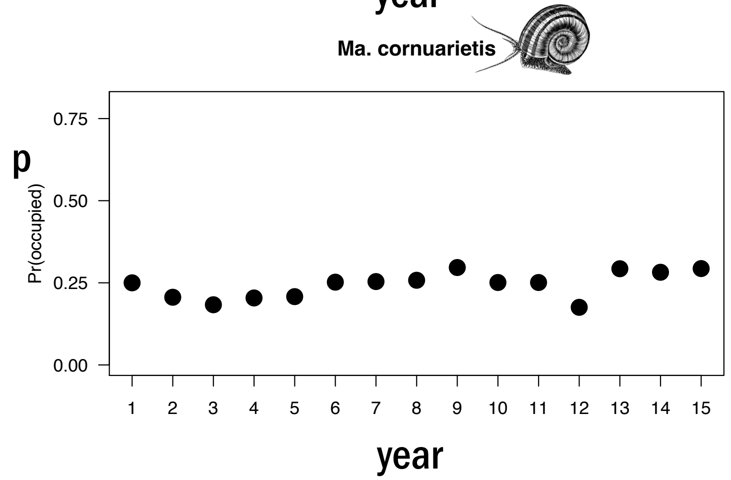




Metapopulation model

$$\frac{dp}{dt} = cp(1-p) - ep$$

Change in the proportion of occupied sites (p) over time (t)

colonization rate (probability a site is colonized)

extinction rate (probability a site becomes extinct)

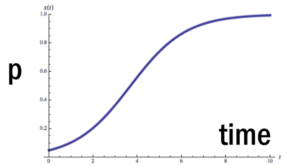
Metapopulation model

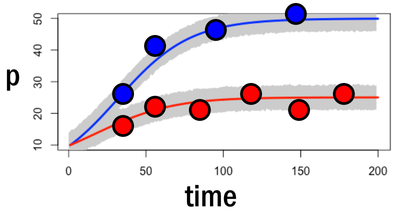
$$\frac{dp}{dt} = cp(1-p) - ep$$

Change in the proportion of occupied sites (p) over time (t)

Probability that an unoccupied site (1-p) becomes colonized by a population from an occupied site (cp)

Equilibrium





$$\frac{dp}{dt} = cp^*(1-p^*) - ep^*$$

$$0 = cp^*(1-p^*) - ep^* \longrightarrow 0 = cp^* - cp^{*2} - ep^* \longrightarrow 0 = p(c - cp^* - e)$$

$$\frac{0}{p} = \frac{p(c - cp^* - e)}{p} \longrightarrow 0 = c - cp^* - e \longrightarrow cp^* = c - e$$

$$\frac{cp^*}{c} = \frac{c-e}{c} \longrightarrow p^* = 1 - \frac{e}{c}$$

When will the metapopulation become extinct ($p^* \leq 0$) ? → e > c

When will the metapopulation persist ($p^* > 0$) ? → c > e

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