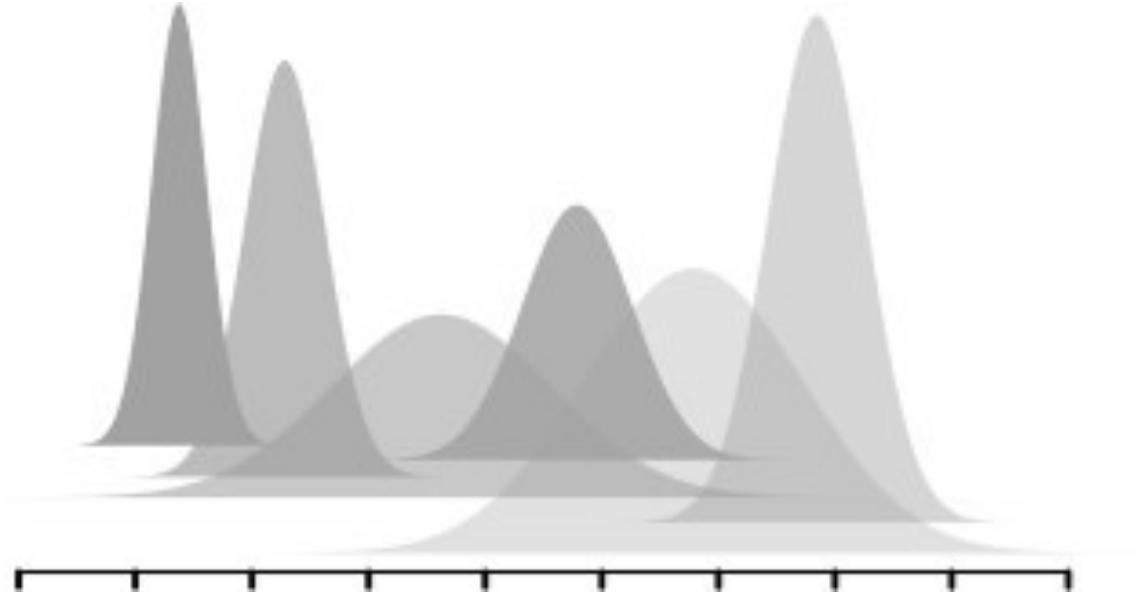
2.1 Population & Community Ecological Models



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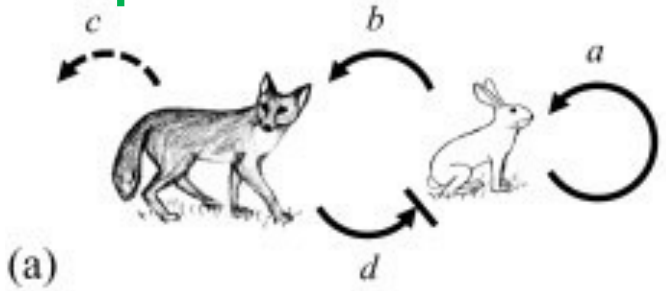
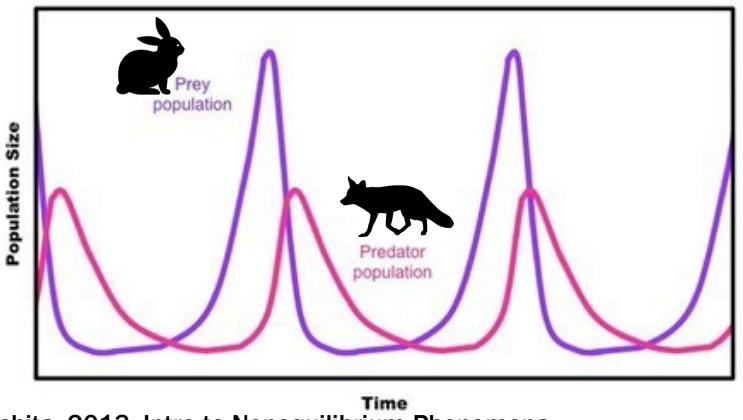
What is a model?

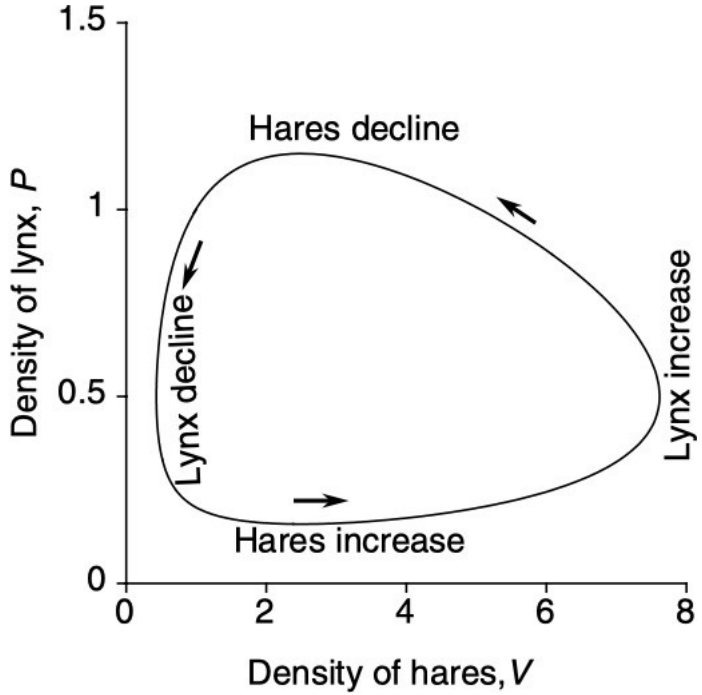
"A *model* is a representation of a particular thing, idea, or condition."

"The *modeling process* is the series of steps taken to convert an idea first into a conceptual model and then into a quantitative model"

Step 1. Formulate a conceptual model







Step 2. Formulate a quantitative model

$$\frac{dV}{dt} = aV - bVP$$

$$\frac{dP}{dt} = -cP + dVP$$

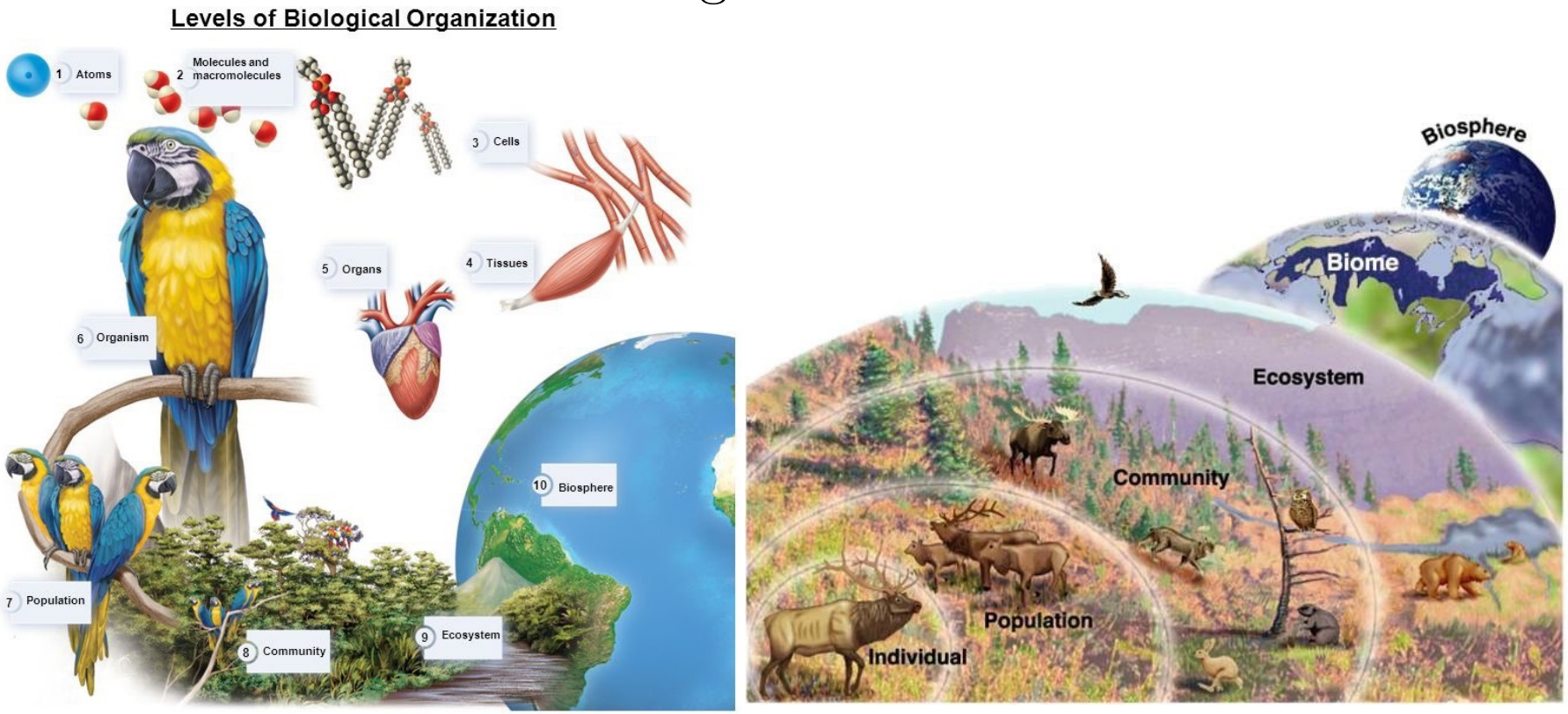
Step 3. Learn about study system through analysis of model behavior

Kinoshita, 2013, Intro to Nonequilibrium Phenomena

What is ecology?

The study of interactions between organisms and their environment, and with one another

The science that investigates the abundance and distribution of organisms



Step 3. Learn about study system through analysis of model behavior

Population ecology

Exponential growth

$$\frac{dn}{dt} = rn$$

Graphical techniques: develop a feeling for your model

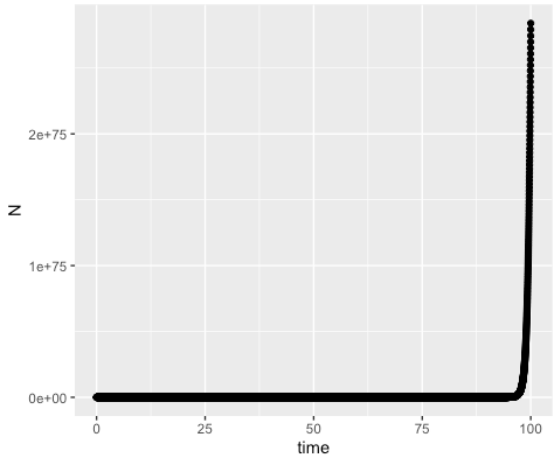
Expected dynamics → depends on r and n_0 (initial population size)

When r > 0?

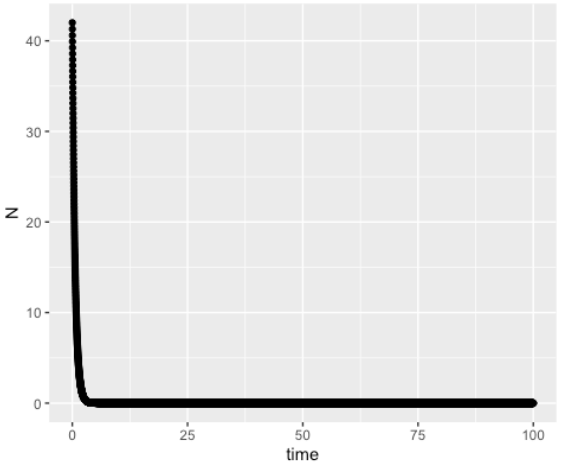
When r < 0?

When r = 0?

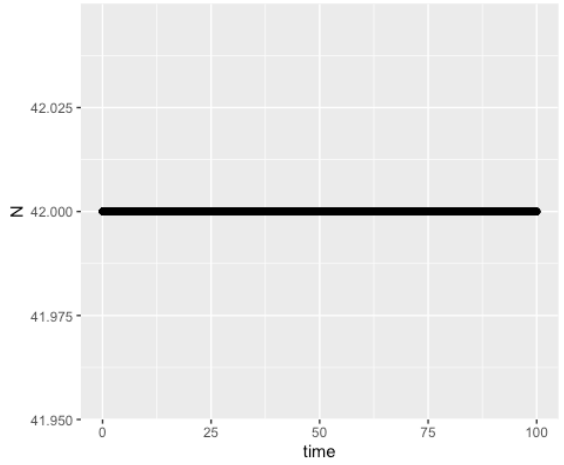
r = 1.7



r = -1.7



r = 0



parameter	
r	population growth rate

Step 3. Learn about study system through analysis of model behavior

Population ecology

Exponential growth

$$\frac{dn}{dt} = rn$$

Graphical techniques: develop a feeling for your model

How did I develop a graph of population size over time for this equation?

I solved for values of n(t) at different points in time.

Let's solve the equation to get a formula for this: we'll need to integrate the formula

$$\frac{dn}{dt} = rn \rightarrow \frac{dn}{n} = rdt \rightarrow \int_{n_0}^{n(t)} \frac{dn}{n} = r\int_0^t dt \rightarrow ln\frac{n(t)}{n_0} = rt \rightarrow n(t) = n_0 e^{rt}$$

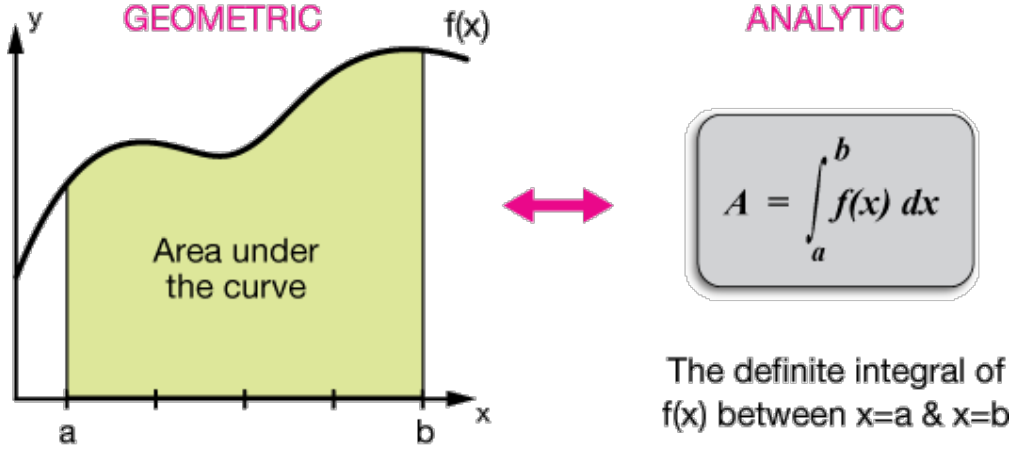


Table of Integrals

BASIC FORMS

(1) $\int x^n dx = \frac{1}{n+1}x^{n+1}$

(2) $\int \frac{1}{x}dx = \ln x$

(3) $\int udv = uv - \int vdu$

(4) $\int u(x)v'(x)dx = u(x)v(x) - \int v(x)u'(x)dx$

RATIONAL FUNCTIONS

(5) $\int \frac{1}{ax+b}dx = \frac{1}{a}\ln(ax+b)$

(6) $\int \frac{1}{(x+a)^2}dx = \frac{-1}{x+a}$

INTEGRALS WITH ROOTS

(18) $\int \sqrt{x-a}dx = \frac{2}{3}(x-a)^{3/2}$

(19) $\int \frac{1}{\sqrt{x\pm a}}dx = 2\sqrt{x\pm a}$

(20) $\int \frac{1}{\sqrt{a-x}}dx = 2\sqrt{a-x}$

(21) $\int x\sqrt{x-a}dx = \frac{2}{3}a(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2}$

(22) $\int \sqrt{ax+b}dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{b+ax}$

(23) $\int (ax+b)^{3/2}dx = \sqrt{b+ax}\left(\frac{2b^2}{5a} + \frac{4bx}{5} + \frac{2ax^2}{5}\right)$

(24) $\int \frac{x}{\sqrt{x+a}}dx = \frac{2}{3}(x+2a)\sqrt{x+a}$

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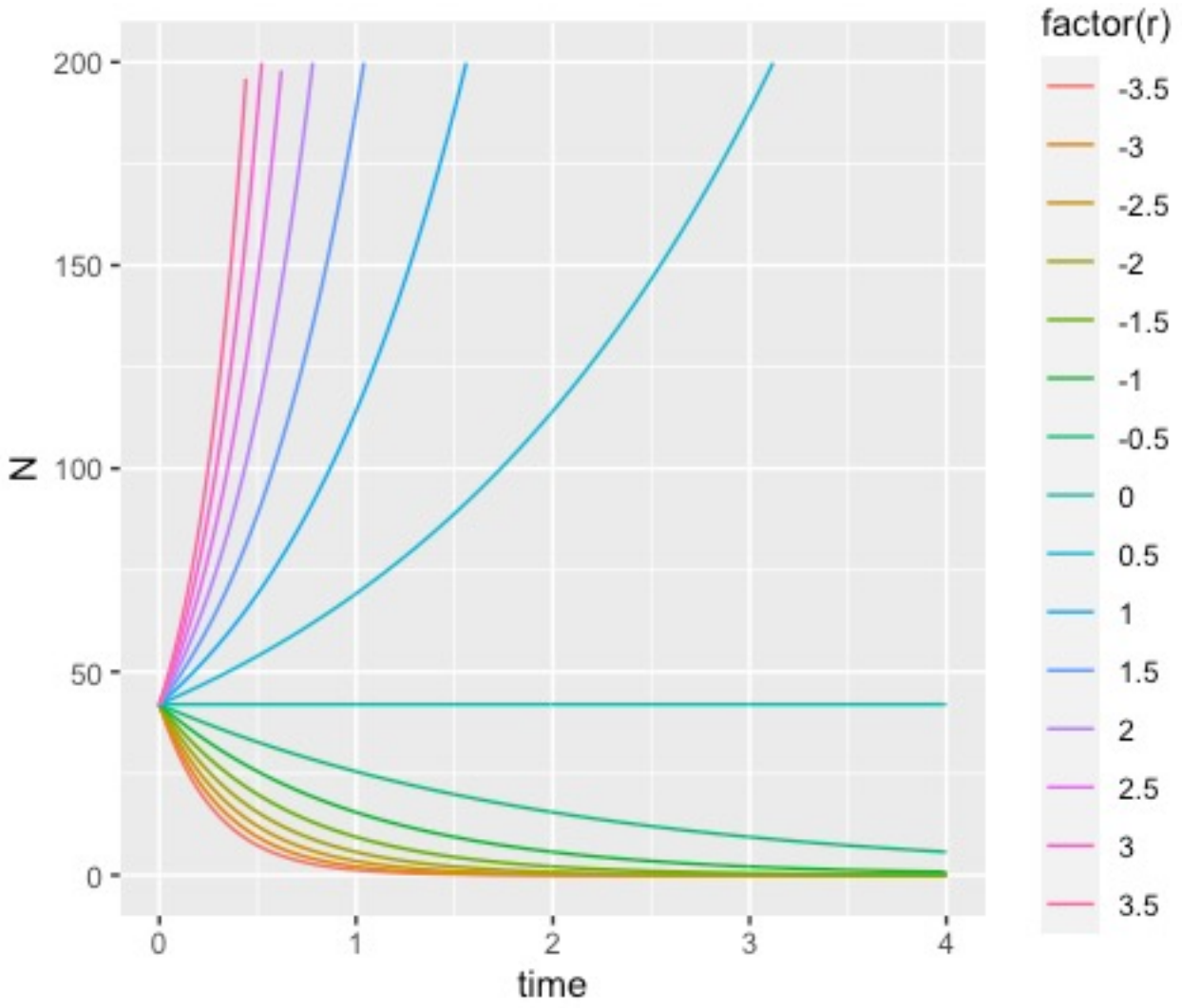
Graphical techniques: develop a feeling for your model

Expected dynamics

When r > 0?

When r < 0?

When r = 0?



parameter	
r	population growth rate

Step 3. Learn about study system through analysis of model behavior

Population ecology

Exponential growth

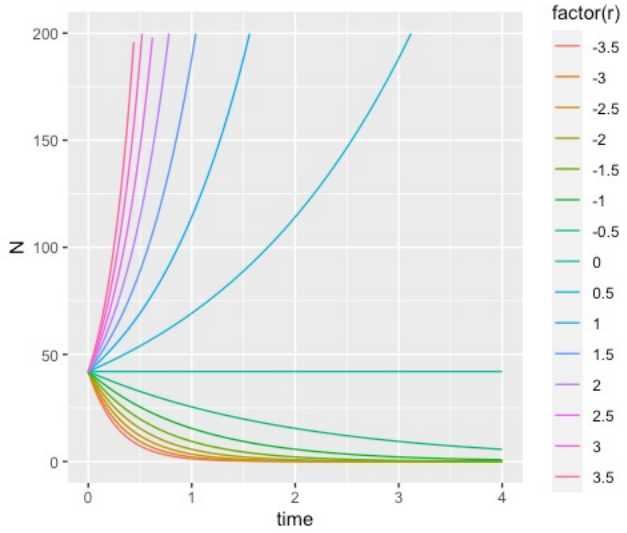
$$\frac{dn}{dt} = rn$$

Logistic growth

$$\frac{dn}{dt} = rn\left(1 - \frac{n}{K}\right)$$

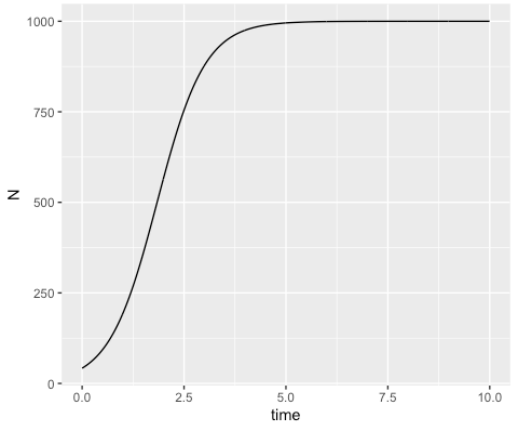
Graphical techniques: develop a feeling for your model

Expected dynamics

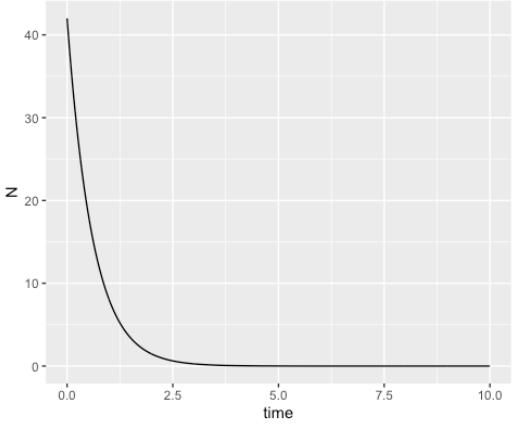


Expected dynamics

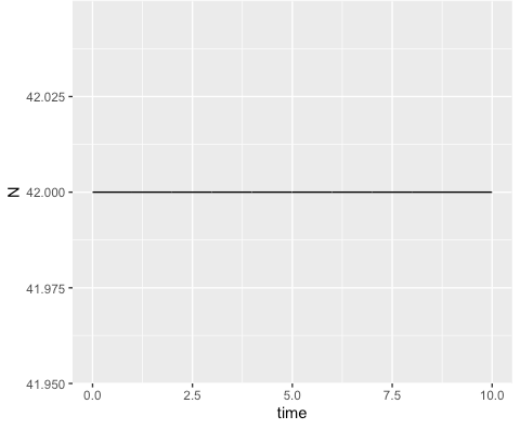
r = 1.7



r = -1.7



r = 0



parameter	
r	population growth rate

parameter	
r	population growth rate
K	Carrying capacity

Step 3. Learn about study system through analysis of model behavior

Population ecology

Exponential growth

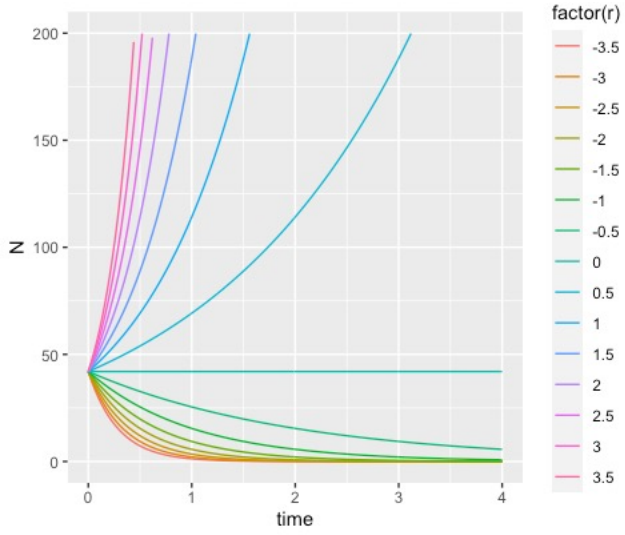
$$\frac{dn}{dt} = rn \rightarrow n(t) = n_0 e^{rt}$$

Logistic growth

$$\frac{dn}{dt} = rn\left(1 - \frac{n}{K}\right) \rightarrow n(t) = \frac{K}{1 + n_0 e^{-rt}}$$

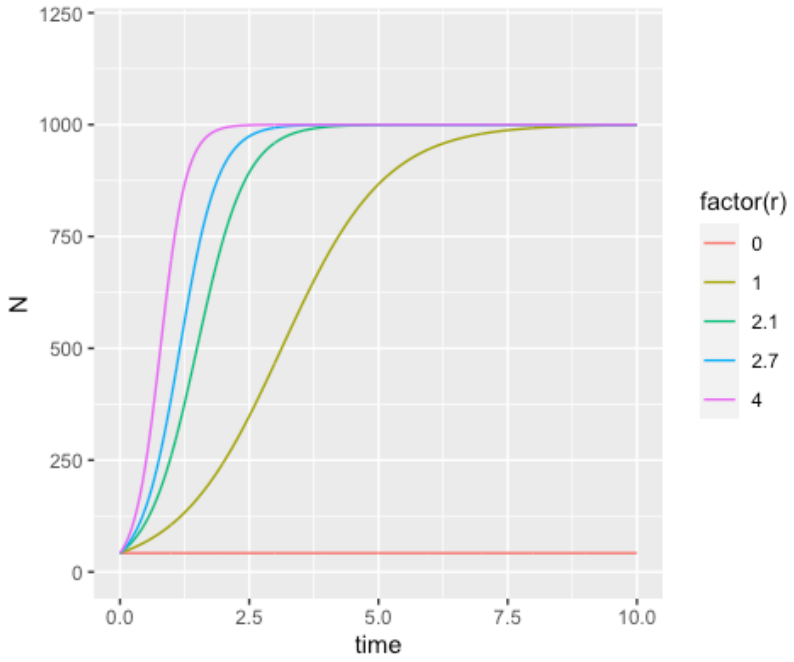
Graphical techniques: develop a feeling for your model

Expected dynamics



parameter	
r	population growth rate

Expected dynamics



parameter	
r	population growth rate
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