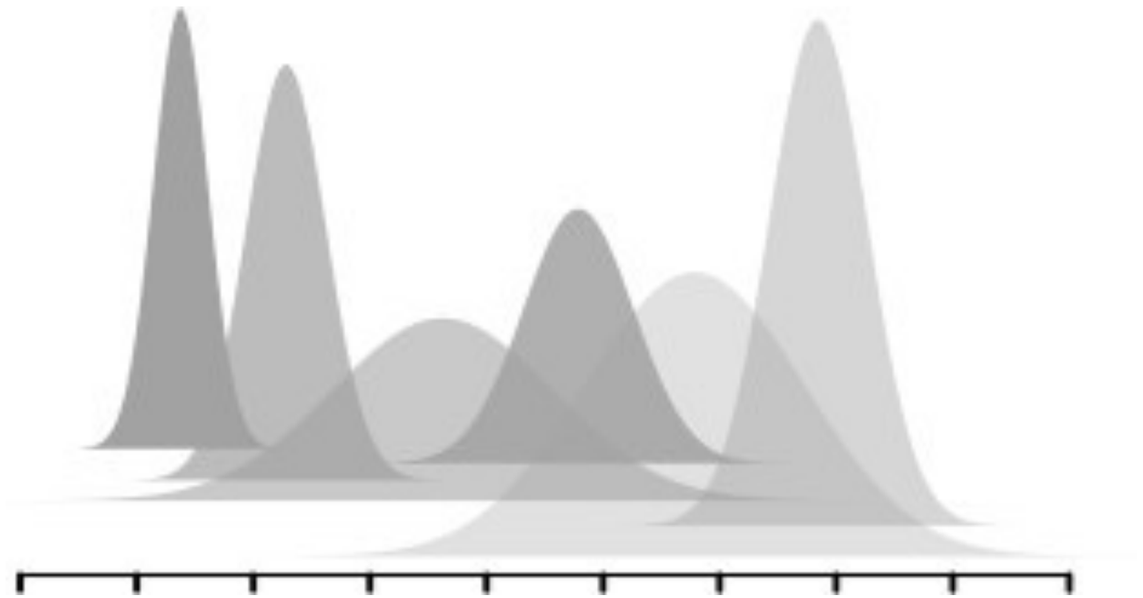
1.2 Introduction to Models in Ecology II



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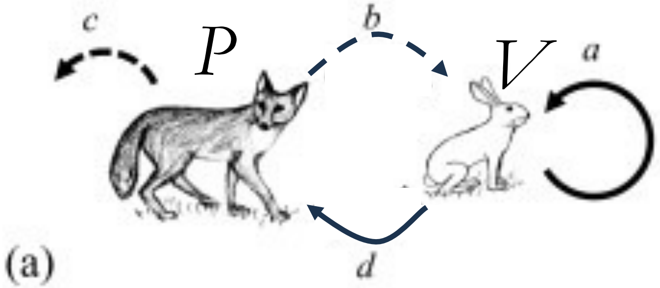
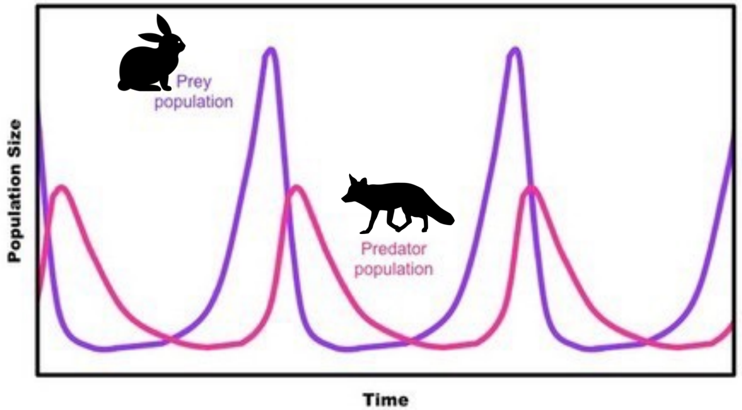
What is a model?

"A *model* is a representation of a particular thing, idea, or condition."

"The *modeling process* is the series of steps taken to convert an idea first into a conceptual model and then into a quantitative model"

Step 1. Formulate a conceptual model

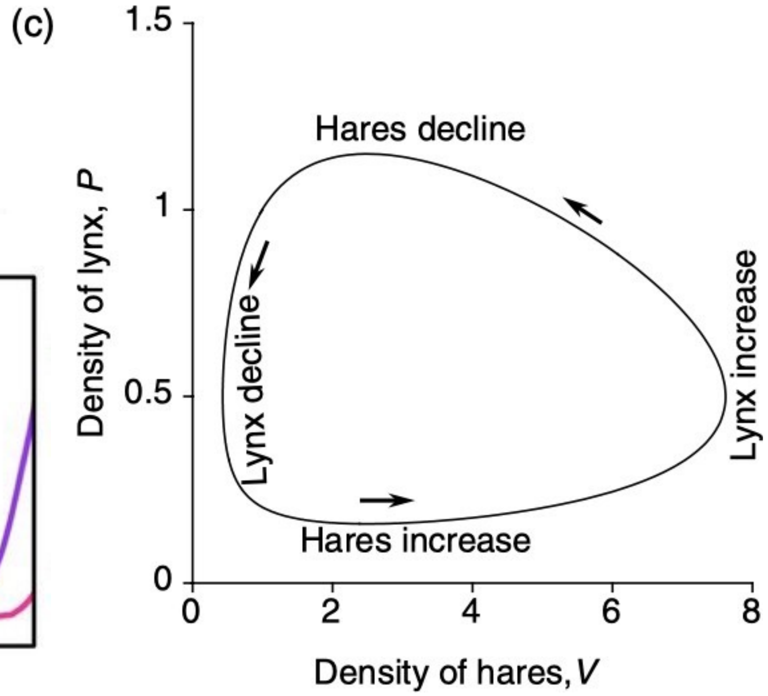




Step 2. Formulate a quantitative model

$$\frac{dV}{dt} = aV - bVP$$

$$\frac{dP}{dt} = -cP + dVP$$



Step 3. Learn about study system through analysis of model behavior

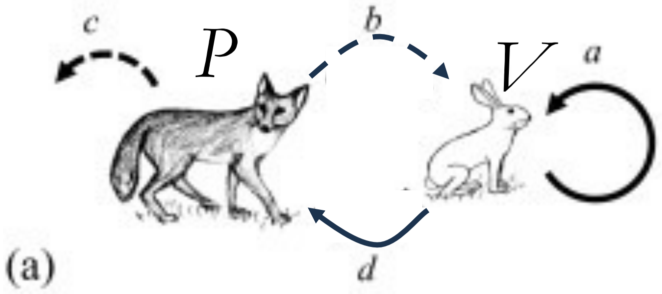
Kinoshita, 2013, Intro to Nonequilibrium Phenomena

What should we consider in ecological models?

"The *modeling process* is the series of steps taken to convert an idea first into a conceptual model and then into a quantitative model"

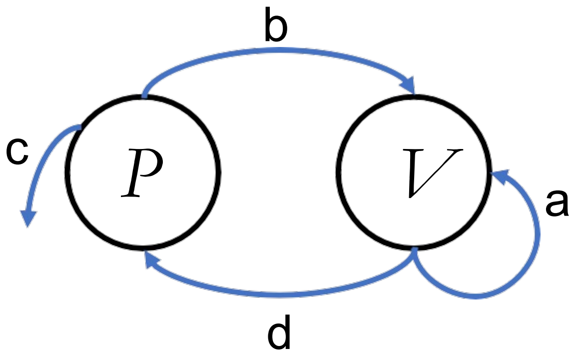
— *Leland Jackson and colleagues (2000)*

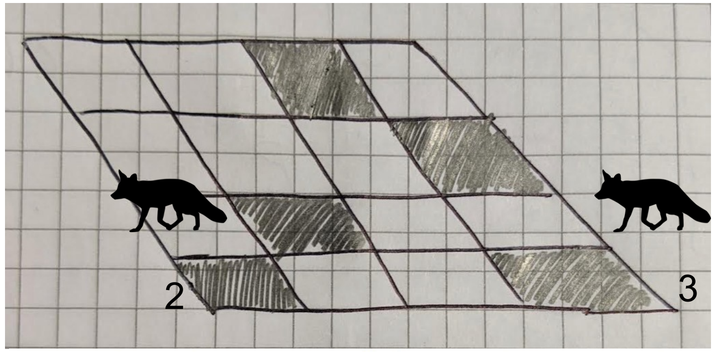
Step 1. Formulate a conceptual model

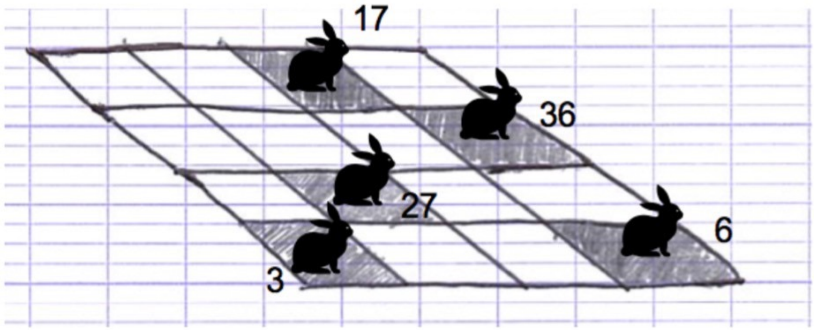


- a ➔ rabbit population growth rate (+ for N_{rabbit})
- b ➔ the rate at which rabbit encounter fox (- for N_{rabbit})
- d ➔ 'conversion' of consumed rabbit into new foxes (+ for N_{fox})
- c ➔ mortality rate of the foxes (- for N_{fox})

- A diagram with boxes/circles and arrows
- Circles are *state variables*, that describe the state of the system
- The arrows illustrate *relationships* among states
- A decision is to be made: what *currency / units* represent the interactions between states? Here we used N, number of individuals (*V* for rabbit, *P* for fox)







Conceptual models, flow diagrams

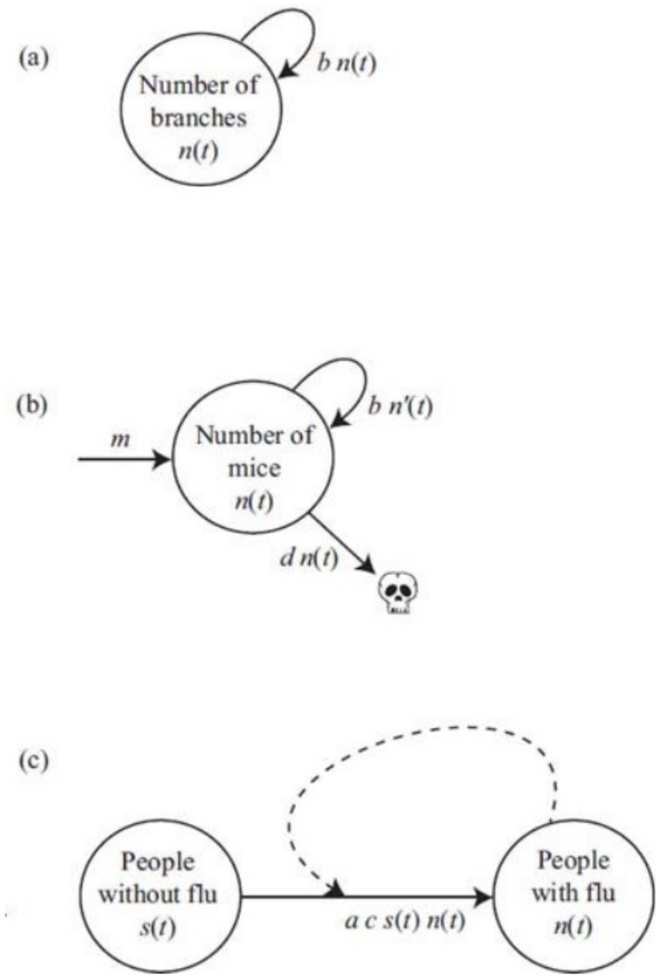
- A flow diagram illustrates the interconnections among the variables and provides a schematic picture of how each variable affects its own dynamics as well as the dynamics of other variables

- In a flow diagram, each circle represents one variable within the model.

- Returning arrows that exit and come back to the same circle represent a variable that can generate more of itself

- Arrows leading into a circle represent the ways a variable can *increase* over time

- Arrows exiting a circle represent the ways a variable can *decrease* over time



Conceptual models, flow diagrams

What is the research question of interest?

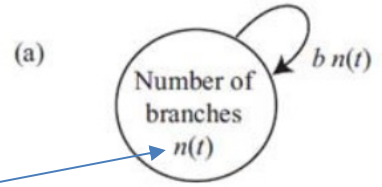
(i) How does the number of branches of a tree change over time?
(ii) How does a cat change the number of mice in a yard?
(iii) How does the number of people with the flu change over the flu season?

What are the *state variables* (the variables of interest that change in the system)?

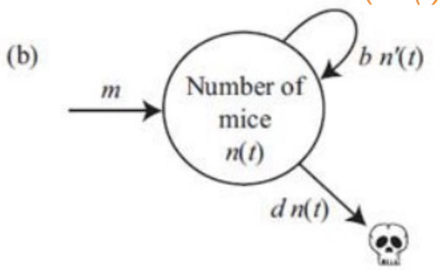
(i) The number of branches on a tree
(ii) The number of mice in a yard
(iii) The number of people without the flu and with the flu

Note a letter is used to represent each variable

Note the variables track changes in number over TIME – n(t) notation is used – "n at time t"



Note – differentiate between constant numbers (i.e. *m*, number mice entering) and those that depend on the variables themselves (*d n(t)*)



What is the "currency" (the units) for the state variables? What are we keeping track of?

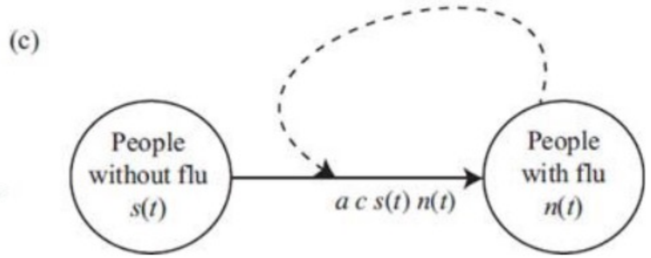
In all instances, the actual *count* (number). Note also we specify the time units (they consider *day* in these cases).

What are the "flows", or changes in state variables? What *parameters* drive these?

-Various quantities that influence the dynamics of the model

-Remain FIXED over time (as the variables change)

-Also given their own symbol (italicized letters – *a, b, c, d, m, r* -, lower-case Greek letters – α, β, θ)



(i) The number of new branches that bud off each old branch per day (*b*)
(ii) The fraction of mice in the yard eaten by cat per day *(d)*, the number of mice born per mouse per day, *(b)*, number of mice arriving / immigrating *(m)*
(iii) The fraction of healthy people that are exposed to a flu carrier per day (*c*), the probability of transmission of the flu between healthy person and carrier upon exposure (*a*)

Conceptual models, flow diagrams

What is the research question of interest?

"the goal of a research project is to determine the relationship between different strategies for stocking exotic salmon in the Great Lakes and the concentrations of potentially toxic contaminants in the salmon and their alewife prey" – page 696

What are the *state variables* (the variables of interest that change in the system)?

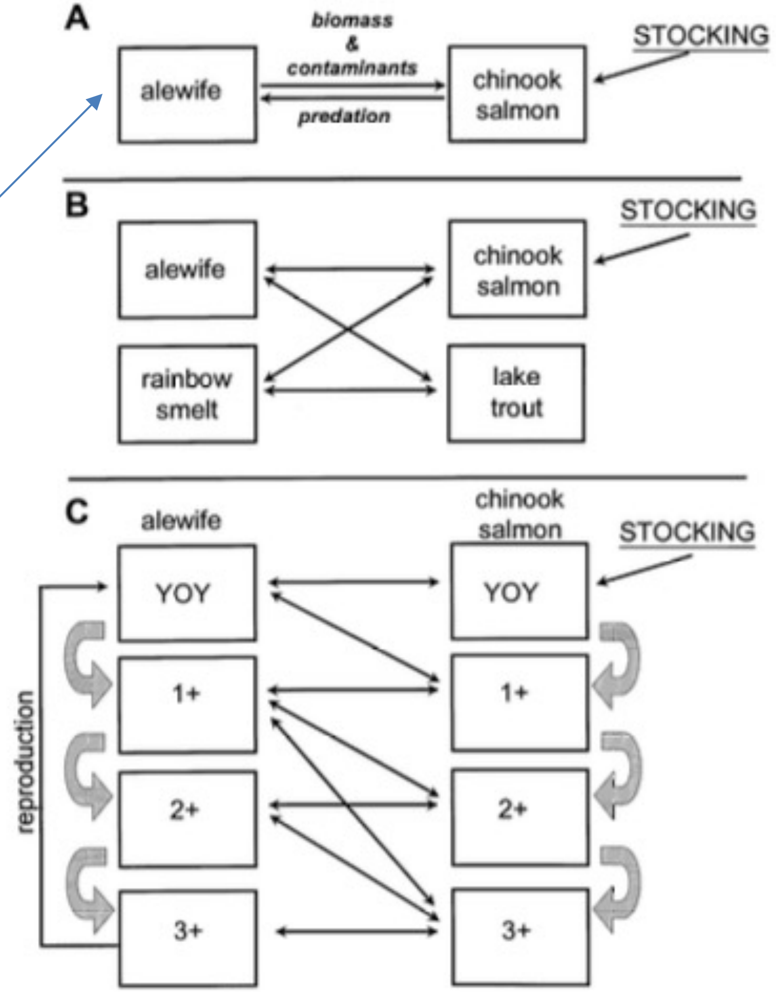
Alewife (prey) and salmon (predator)


What is the "currency" (the units) for the state variables? What are we keeping track of?

Number of individuals (N)? Biomass (grams)? Density (individuals / liter)?

What are the "flows", or changes in state variables? What *parameters* drive these?

Biomass flows from alewife to salmon. Contaminants likewise flow from alewife to salmon. Salmon has a negative impact on the alewife (by eating them!). Salmon flow in at a constant level of stocking.



Jackson et al. 2000

Conceptual models, flow diagrams

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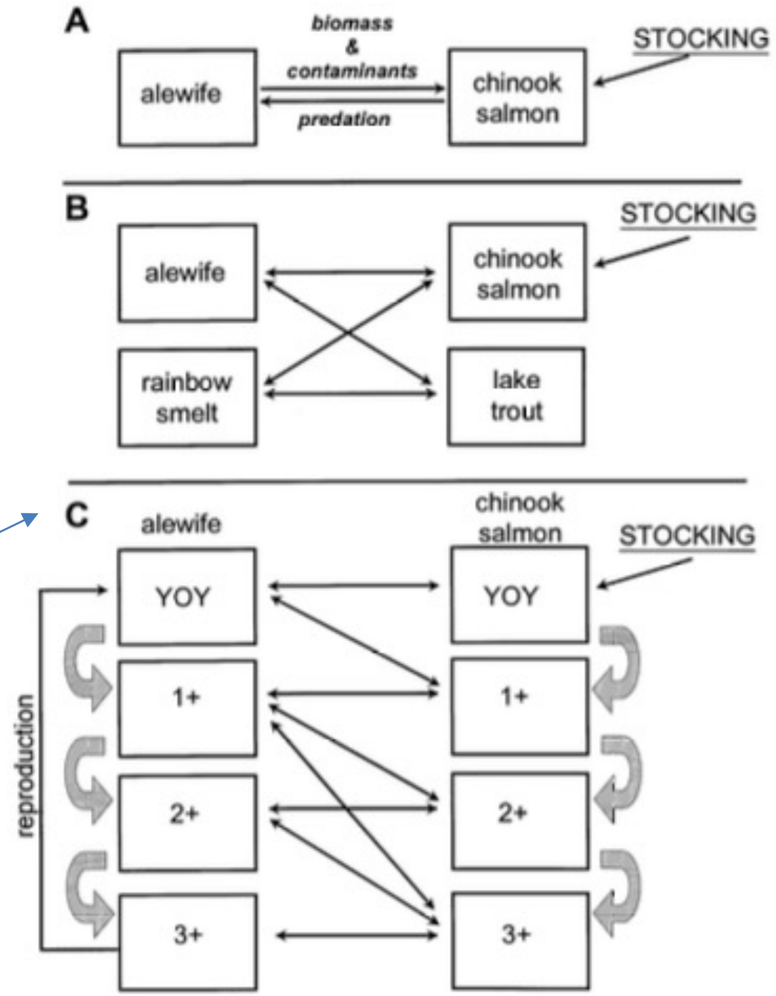

What is an *age-structured model*? A model where the interactions between state variables are dependent on age (there are also size structured models). In other words, for example we can see in Figure 2C that chinook salmon (the predator in the system) don't eat ALL of the alewife (prey), their feeding is size selective. How might this detail change population dynamics? (if there are few YOY alewife and many 3+ salmon, there won't be much food for the salmon).

Jackson et al. 2000

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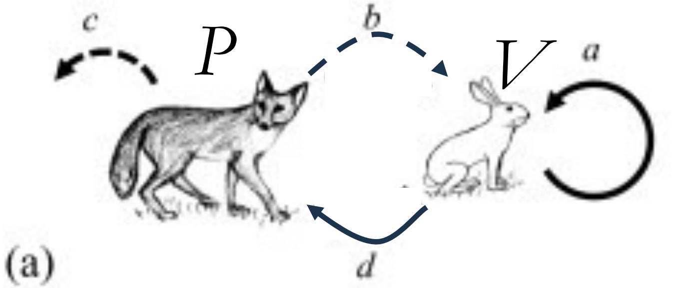

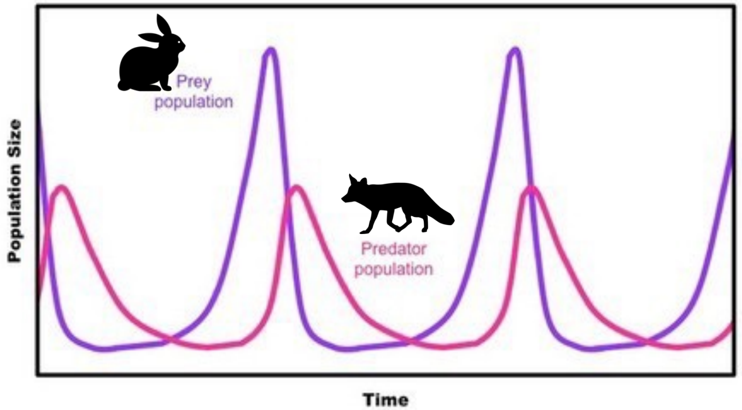

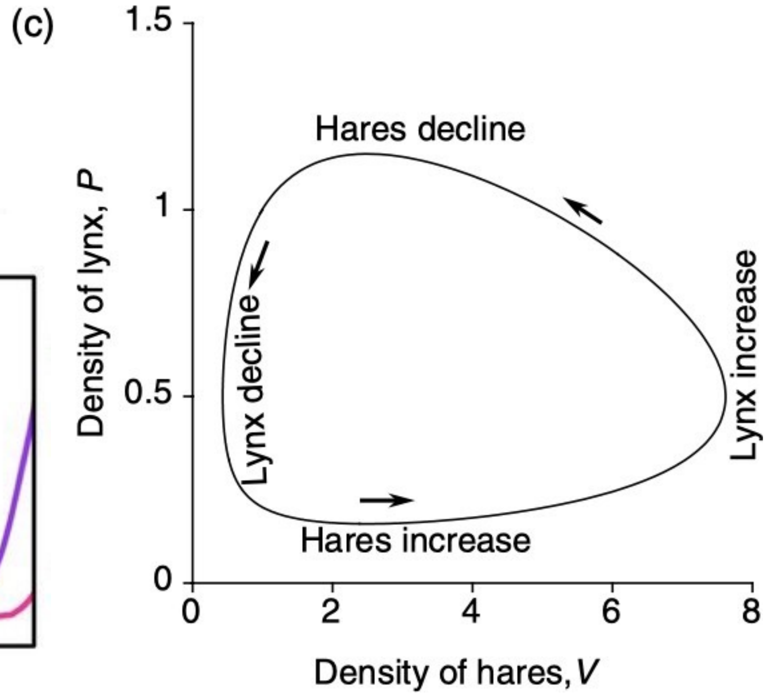

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Kinoshita, 2013, Intro to Nonequilibrium Phenomena

The role of quantitative models in ecological research

What is a *quantitative model*?

A set of mathematical expressions that capture the boxes and arrows of conceptual models

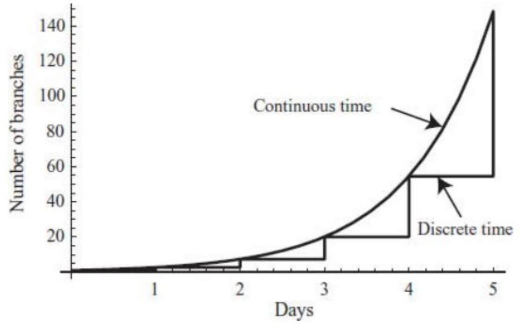
What are *discrete-time* and *continuous* differential equations?

(page 700) – difference equations separate time into discrete intervals, solved by recursion (later predictions depend on the prediction the time step before). Differential equations describe continuous processes

Otto & Day Ch 2: "once you have a list of variables, the next step is to choose a type of dynamical model – two main types – *discrete time* and *continuous time* – depending on whether time is represented in discrete steps or along a continuous axis"

Discrete time – describe how state variables change from one time unit (day, year, generation) to the next

Continuous time – track the variables at all points in time

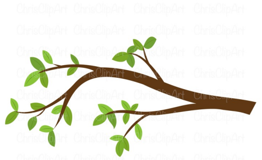


What is the distinction between difference and differential equations?

Why model?

A set of mathematical expressions that capture the boxes and arrows of conceptual models

The role of quantitative models in ecological research



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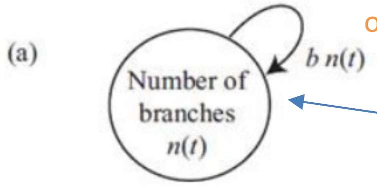

Only growth(+) changes considered

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discrete-time equations

$$n(t+1) = n(t) + b\,n(t)$$

continuous-time equations

$$\frac{dn(t)}{dt} = b\,n(t)$$

CHANGE in number of branches

Per CHANGE in unit time (day)

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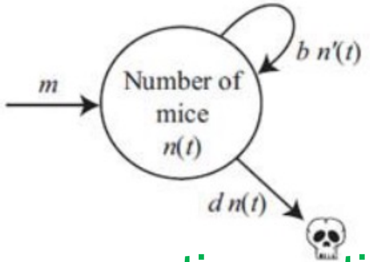
(ii) The fraction of mice in the yard eaten by cat per day *(d)*, the number of mice born per mouse per day, *(b)*, number of mice arriving / immigrating *(m)*



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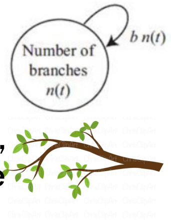

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$$n(t+1) = n(t) + b\,n(t)$$

$$n(t+1) = (1+b)(1-d)\,n(t) + m$$

Growth(+) change
Depends on state variable n(t)

Reduction(-) change
Depends on state variable n(t)

Growth(+) change
Fixed / constant

continuous-time equations

$$\frac{dn(t)}{dt} = b\,n(t)$$

$$\frac{dn(t)}{dt} = b\,n(t) - d\,n(t) + m$$

Why model?

A set of mathematical expressions that capture the boxes and arrows of conceptual models

The role of quantitative models in ecological research

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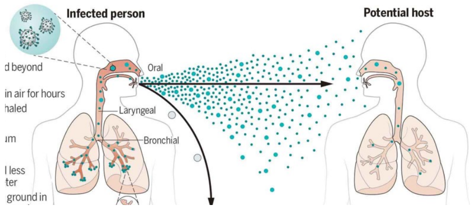

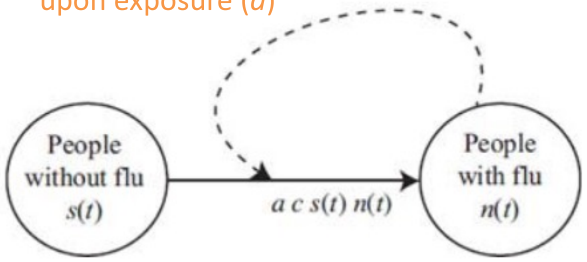

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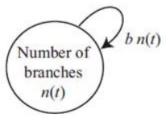

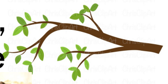

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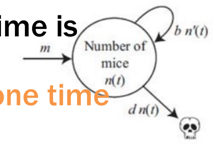

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Continuous time – track the variables at all points in time

discrete-time equations

$$n(t+1) = n(t) + b\,n(t)$$

$$n(t+1) = (1+b)(1-d)\,n(t) + m$$

Growth(+) change

Infected $$n(t+1) = n(t) + a\,c\,n(t)\,s(t)$$

Healthy $$s(t+1) = s(t) - a\,c\,n(t)\,s(t)$$

Exposure and transmission: + for N-Infected, - for N-Healthy

continuous-time equations

$$\frac{dn(t)}{dt} = b\,n(t)$$

$$\frac{dn(t)}{dt} = b\,n(t) - d\,n(t) + m$$

$$\frac{dn(t)}{dt} = a\,c\,n(t)\,s(t)$$

$$\frac{ds(t)}{dt} = -a\,c\,n(t)\,s(t)$$

Why model?

A set of mathematical expressions that capture the boxes and arrows of conceptual models

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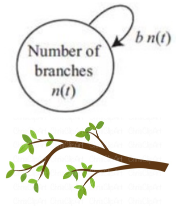

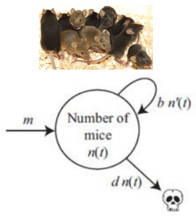

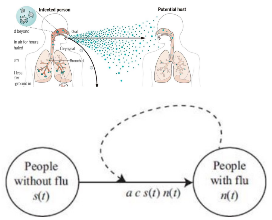

discrete-time equations

$$n(t+1) = n(t) + b\,n(t)$$

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continuous-time equations

$$\frac{dn(t)}{dt} = b\,n(t)$$

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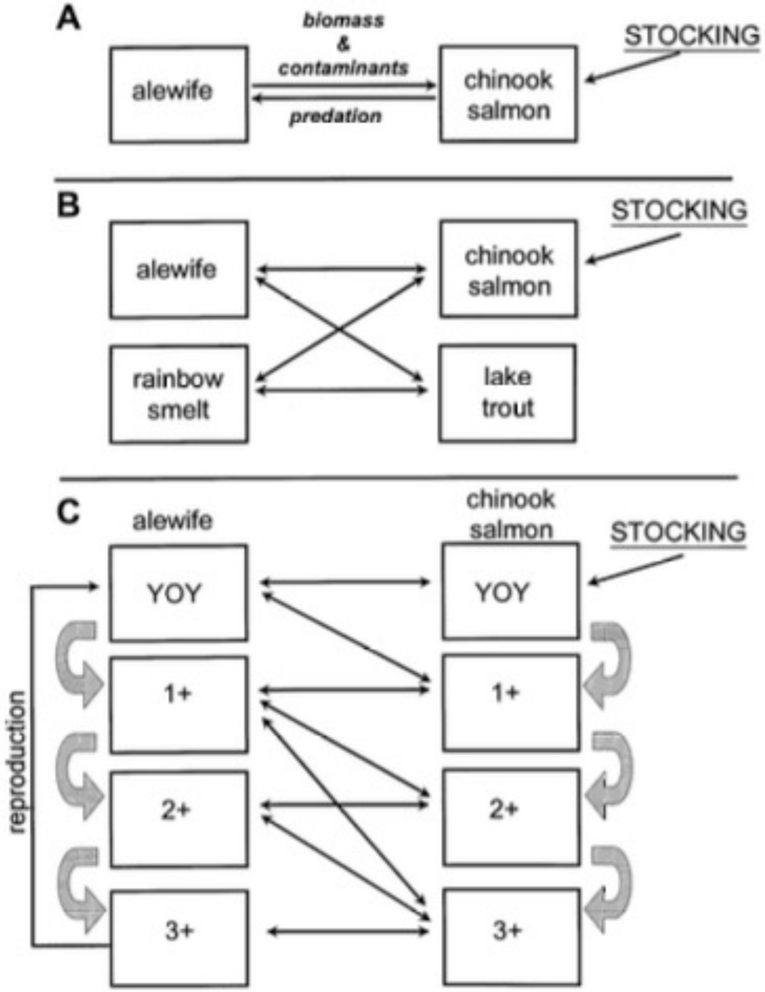
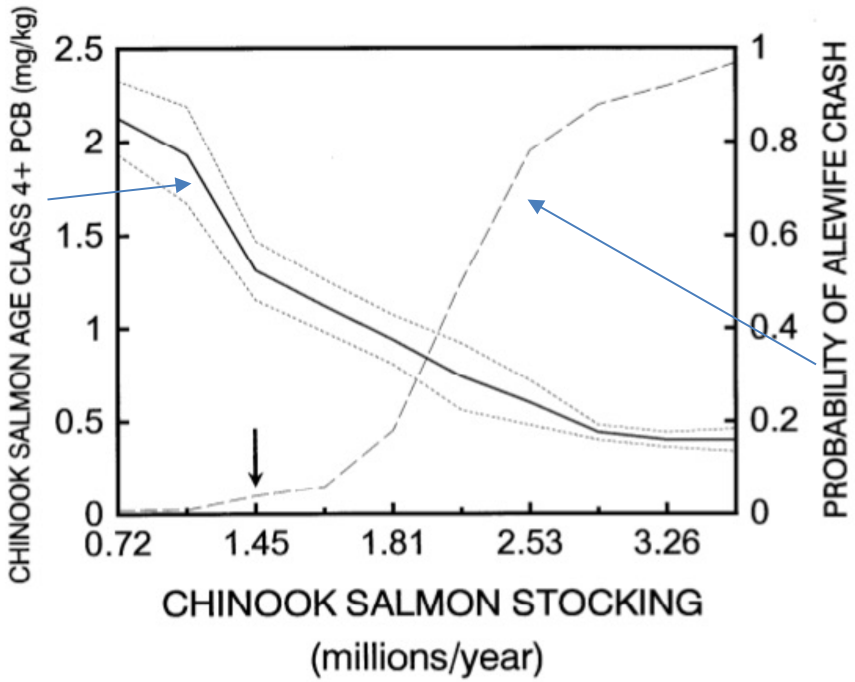
Why model?

- Can use the model to make *predictions* about the system, which can then be *tested*

Alewife-salmon model



Incorporating size-selective predation, increasing contaminant concentrations with increasing prey body size - Prediction: a tradeoff between decreasing PCB concentration in salmon and probability of survival of salmon prey



1. What is on x-axis, left y-axis, right y-axis?

2. What "happens" if you stock very high levels of predatory salmon? Very low levels?

Low PCB concentration but high alewife crash probability!
High PCB concentration but low alewife crash probability!

a tradeoff between contaminant concentration in the oldest salmon (predator) population and the probability that the alewife (prey) population collapses.

Depending on how many salmon you stock, you will either get

- very high salmon contamination + low alewife crash probability (low stocking)
- very low salmon contamination + high alewife crash probability.

Demonstrates why we model, to make decisions and sense from such complexity.

Why model?

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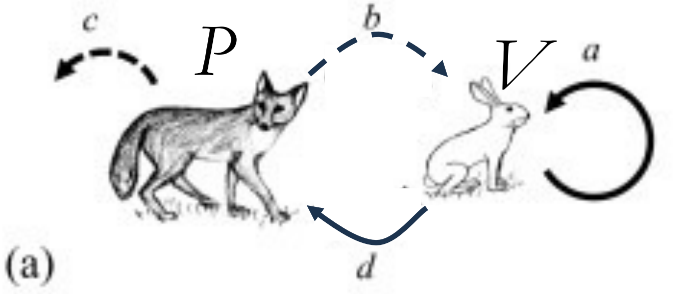

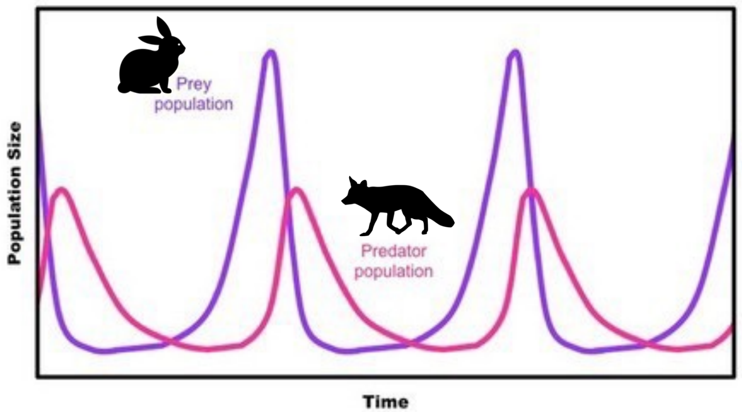

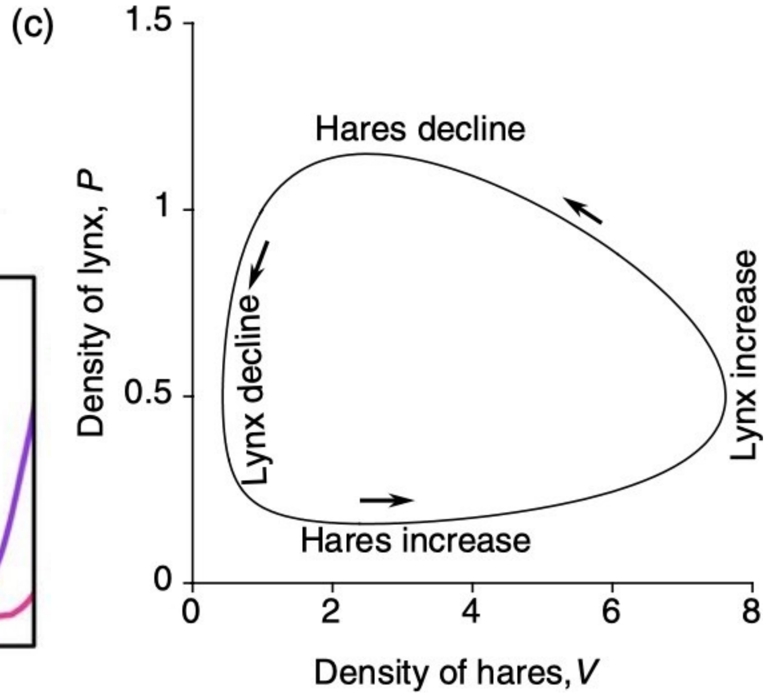

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Step 2. Formulate a quantitative model

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Step 3. Learn about study system through analysis of model behavior

Diatom growth rate model

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— *Leland Jackson and colleagues (2000)*

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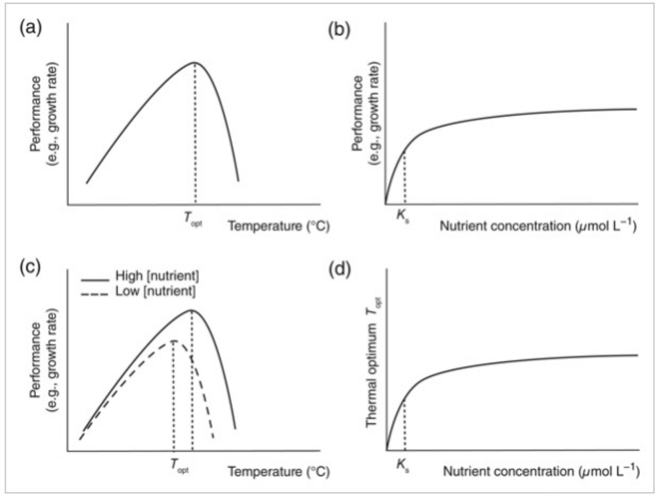

Figure 1 Open in figure viewer | PowerPoint

(a) The thermal performance curve. Performance, which can be measured as growth rate or metabolism, increases with temperature until an optimum T_{opt} and then decreases sharply. **(b)** Monod curve for growth rate as a function of nutrient concentration. The half-saturation constant K_s is the nutrient concentration at which the growth rate is half of the maximum rate at saturating nutrient conditions. **(c)** Impact of both temperature and nutrients on growth rate following Thomas et al. (2017). At low nutrient concentration, the maximum growth rate is lower than at high nutrient conditions, and the thermal optimum shifts to lower temperatures. **(d)** The thermal optimum is a saturating function of nutrient concentration.

Step 2. Formulate a quantitative model