NT 1.2 Operations on vectors

Setting : $\vec{a}, \vec{b} \in \mathbb{R}^2$, $\overset{\text{scalar}}{k} \in \mathbb{R}$.

$\vec{a} = (a_1, a_2)$, $\vec{b} = (b_1, b_2)$

/* Notation :

$\mathbb{R}$: collection of all real numbers

$\mathbb{R}^2$: collection of all 2D vectors

$\mathbb{R}^3$: collection of all 3D vectors

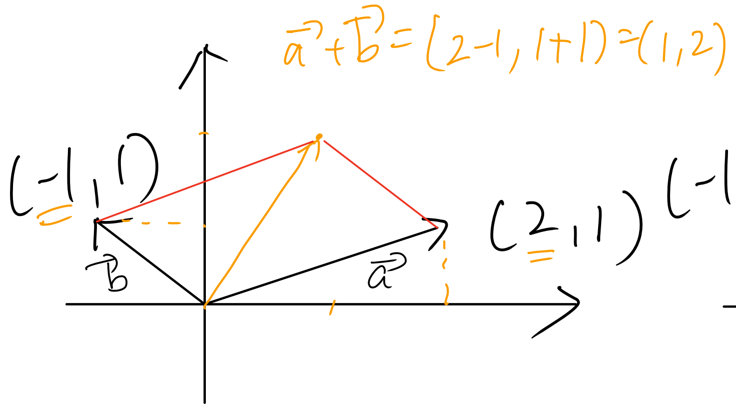
$A \in B$: "A belongs to B"

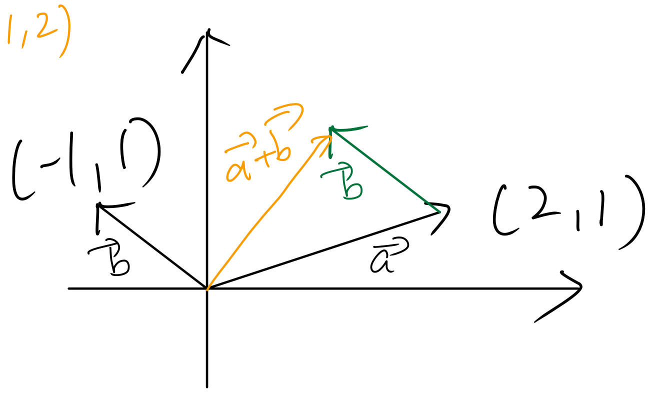
Often used to what kind of object A is.

⊙ sum $\quad \vec{a} + \vec{b} = (a_1 + b_1, a_2 + b_2)$

⊙ difference $\quad \vec{a} - \vec{b} = (a_1 - b_1, a_2 - b_2)$

⊙ scalar multiplication $\quad k\vec{a} = (ka_1, ka_2)$

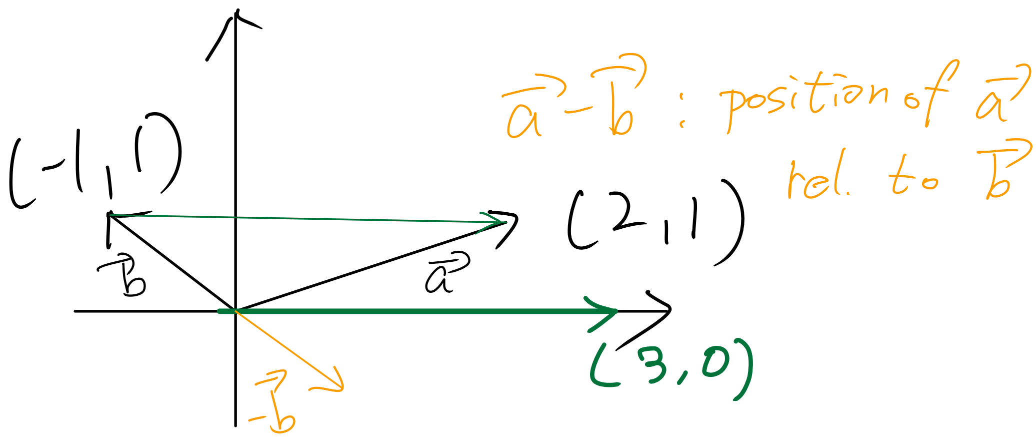




Geometric intuition

① diagonal of parallelogram

② Completing Triangle

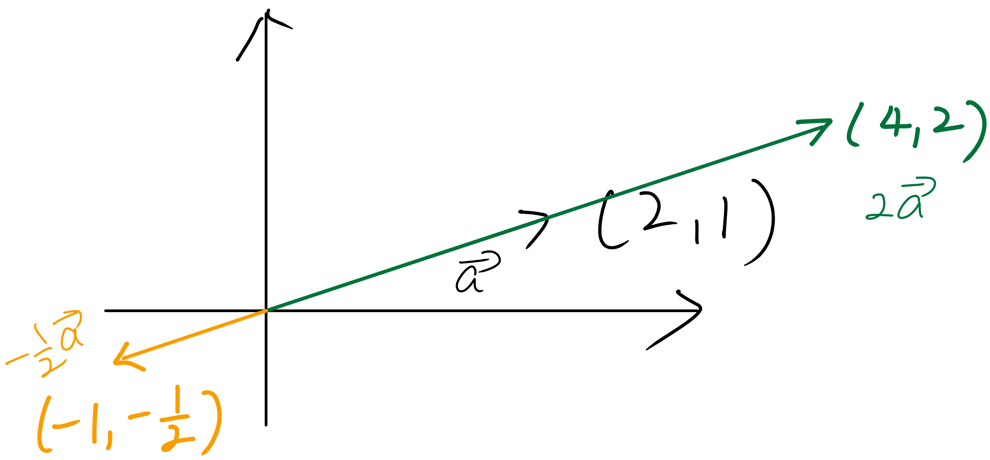


☆ Geometric/Physical intuition

① relative position = object − reference

Change = Final − initial

② $\vec{a} + (-1)\cdot\vec{b}$



Geometric intuition

- stretch, shrink, flip

The same applies to 3D vectors (or higher)

NT 1.3 Basic properties of vector operations.

v, w (vector) α, β (scalar)

$$\mathbf{v}+\mathbf{w}=\mathbf{w}+\mathbf{v} \qquad \text{(commutativity)}$$

$$\mathbf{u}+(\mathbf{v}+\mathbf{w})=(\mathbf{u}+\mathbf{v})+\mathbf{w} \qquad \text{(associativity)}$$

$$\alpha(\mathbf{v}+\mathbf{w})=\alpha\mathbf{v}+\alpha\mathbf{w} \qquad \text{(distributivity)}$$

$$(\alpha+\beta)\mathbf{v}=\alpha\mathbf{v}+\beta\mathbf{v} \qquad \text{(distributivity)}$$

$$(\alpha\beta)\mathbf{v}=\alpha(\beta\mathbf{v}).$$

$\vec{0}=(0,0)$ or $(0,0,0)$

$\vec{v}+\vec{0}=\vec{v}$

$1\cdot\vec{v}=\vec{v}$

$(-1)\cdot\vec{v}=-\vec{v}$

/* Basically, you have all nice properties */.

$\boxed{\text{Clicker}}$ (Connection to Math 4A)

In fact, $\mathbb{R}^2$ is a bona fide vector space in the sense of Math 4A. (Therefore, you can use all appropriate theorem!) What else do you need to check other than the above to show it is a vector space.

① Discuss ② Type in your answer.