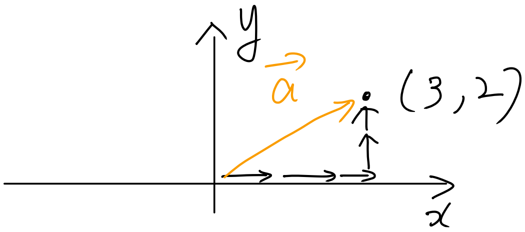
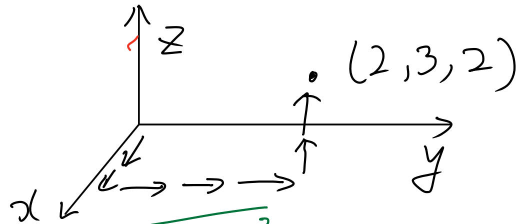
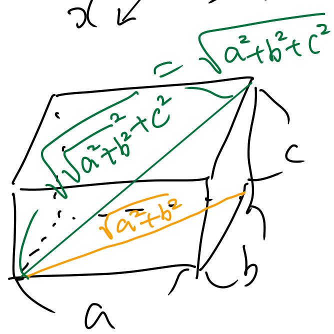
NT 1.4 Length of vectors

Recall Pythagorian theorem.





$$\text{length}(\vec{a}) = \sqrt{3^2+2^2}$$
$$= \sqrt{13}$$



diagonal $=\sqrt{a^2+b^2+c^2}$

$\boxed{\text{Def}}$ (Length of vectors ; norm)

If $\vec{a} \in \mathbb{R}^2$, $\vec{b} \in \mathbb{R}^3$, or even $\vec{c} \in \mathbb{R}^n$,

$$\text{length of } \vec{a} = \|\vec{a}\| = \sqrt{a_1^2 + a_2^2}$$
$$\text{length of } \vec{b} = \|\vec{b}\| = \sqrt{b_1^2 + b_2^2 + b_3^2}$$
$$\text{length of } \vec{c} = \|\vec{c}\| = \sqrt{c_1^2 + c_2^2 + \cdots + c_n^2}$$

This quantity is called the norm of vectors.

/* Notation : If none is mentioned, components has the same letter of the vector and put subscripts for proper coordinates. e.g. $\vec{v} = (v_1, v_2)$ */

$\boxed{\text{Thm}}$ (properties of norm)

1. (Triangle inequality)

If $\vec{a}, \vec{b} \in \mathbb{R}^n$ (in particular, $\mathbb{R}^2$ or $\mathbb{R}^3$)

$$\|\vec{a}+\vec{b}\| \leq \|\vec{a}\| + \|\vec{b}\|$$

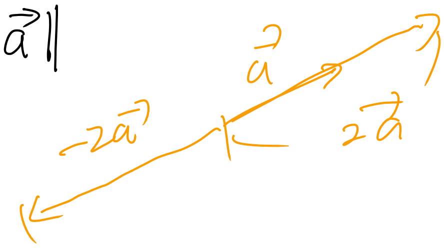
2. (Dilation)

If $\vec{a} \in \mathbb{R}^n$ (in particular, $\mathbb{R}^2$ or $\mathbb{R}^3$) and

$k \in \mathbb{R}$, → notation for scalar

$$\|k\vec{a}\| = |k| \cdot \|\vec{a}\|$$

$2\vec{a}$



Q: Why not $\|k\vec{a}\| = k\|\vec{a}\|$?