Math 6A 1st Day

Communication:

Professor
1. In-person
2. Discord - general
3. Discord - direct message
4. Email

TAs
1. In-person
2. Email

Enrollment: guidelines → attendance + work → sign the sheets.

Activities / Grades (curve only if whole class suffers)

Clicker	HW	Quiz	Midterms	Final
7%	15%	8%	40%	30%
3 drops/15 (20 - 1st week - exam)	1 drop/8 + auto-extension	1 drop/4 (section)	1 drop/3	

Reading → still very important
0%

Grading: inconsistency, skip details
→ expect harsh grading.

Notes: Canvas

Mindmap: Walk around it when you feel something but not quite able to put things together.

Accommodation: drop policy + auto extension (HW). **No more.**

Extra credit: Acknowledgement Quiz (1%)

Course contributors (1-3 people) (1%)

(mind map, Discord, must be responsible) → direct msg by tomorrow

Wisdom hours: Growth/health/life. Open mind expected.

→ Give a heads up msg.

To do today:

- Skim syllabus (bold) and Calendar
- iclicker
- Discord
- Gradescope app
- Reading → Calendar

→ ACTION NEEDED

see syllabus or Discord - #tech-setting

Advertisement: CalPrig → Discord - #general

Building blocks

1. Vectors (Ch 1.1)

NT 1.1 definition of vectors

◦ 2D vector is a tuple of two real numbers, called component, entry, coordinate

similarly, 3D vector (or n-dim vector) is a tuple of 3 (or n) real numbers.

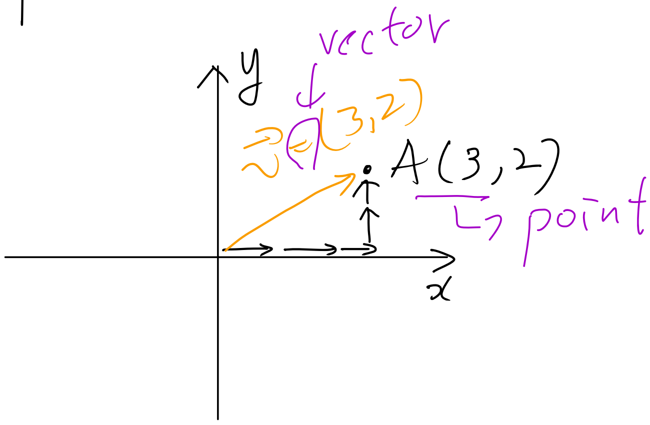
order matters

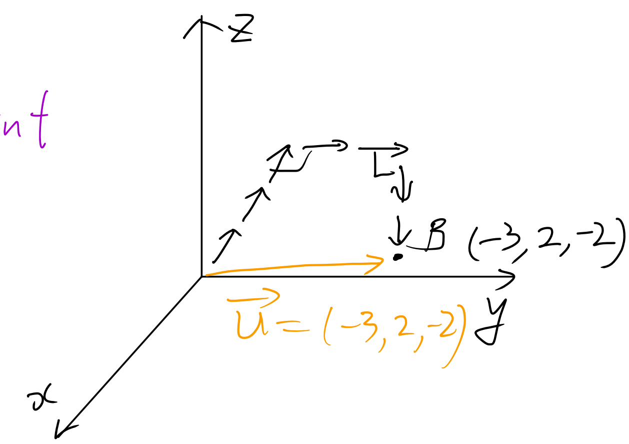
ex) (1, −3), (−1, 5, 0, 3, 14), ...

1st component 2nd

◦ Recall:

We need 2 numbers to locate a position in 2D and 3 numbers in 3D.

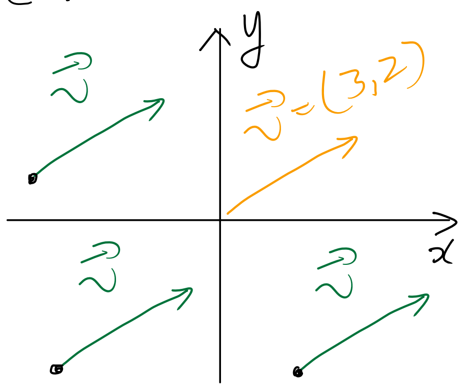




◎ We often visualize a vector as an arrow (directed segment).

◎ Naturally, we can identify each location in space with a vector, called position vector. In this case, the tail of the arrow must be the origin, and the head points to the location that is identified.

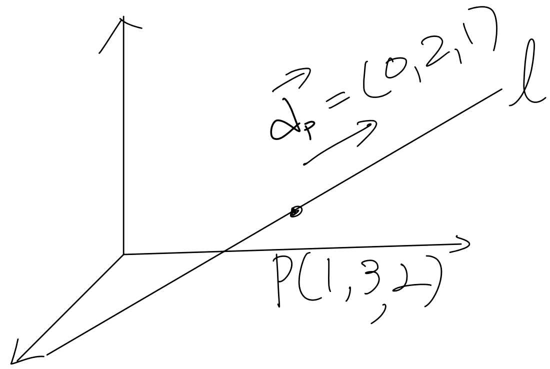
◎ Sometimes, vectors starting at points that are different from the origin are useful. These are called



a representative of the position vector or free vector.

These free vectors are useful for directions. In this case, you need to (at least mentally) move the coordinate axes so that the tail of the arrow and the origin overlap.

Example: ℓ is a line that passes through the point $P(1,3,2)$ and stretches along the direction of $\vec{d}_p = (0,2,1)$.



- $P(1,3,2)$ is usually replaced by a position vector $\vec{a} = (1,3,2)$.
- $\vec{d}_p$ can be viewed as a representative of $\vec{d} = (0,2,1)$

NT 1.2 Operations on vectors

Setting : $\vec{a}, \vec{b} \in \mathbb{R}^2$, $\underline{k \in \mathbb{R}}$ scalar / common way to intro. a scalar
$\vec{a} = (a_1, a_2)$, $\vec{b} = (b_1, b_2)$

/* Notation :

$\mathbb{R}$: collection of all real numbers

$\mathbb{R}^2$: collection of all 2D vectors

$\mathbb{R}^3$: collection of all 3D vectors

$A \in B$: "A belongs to B"

Often used to what kind of object A is.

⊙ sum $\qquad \vec{a} + \vec{b} = (a_1 + b_1, a_2 + b_2)$

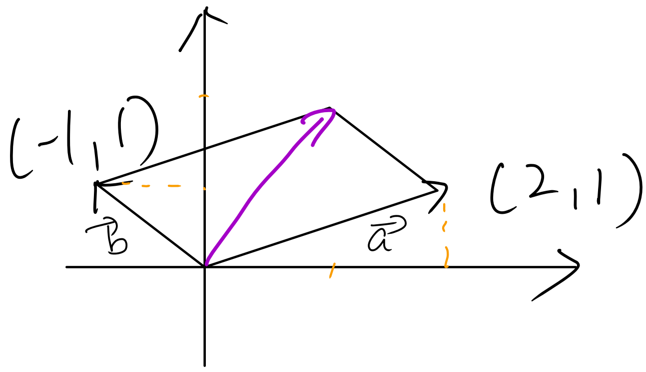
⊙ difference $\qquad \vec{a} - \vec{b} = (a_1 - b_1, a_2 - b_2)$

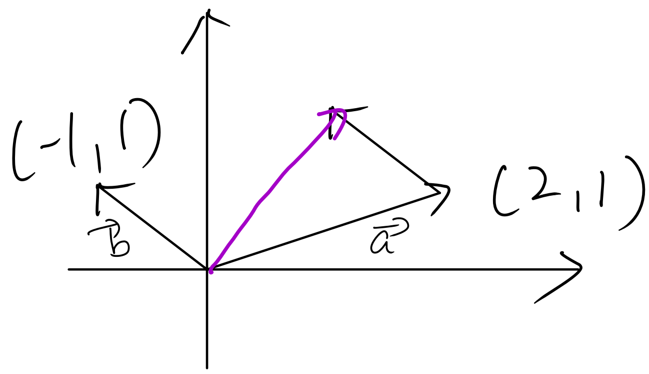
⊙ scalar multiplication $\qquad k\vec{a} = (ka_1, ka_2)$

$\vec{a} = (2, 1)$ $\quad \vec{b} = (-1, 1)$ $\Rightarrow \vec{a} + \vec{b} = (1, 2)$

$\boxed{\text{Clicker}}$ what is $\vec{a} + \vec{b}$ below

(A) (0, 0) $\qquad$ ~~(B)~~ (1, 2) $\qquad$ (C) (4, 0)



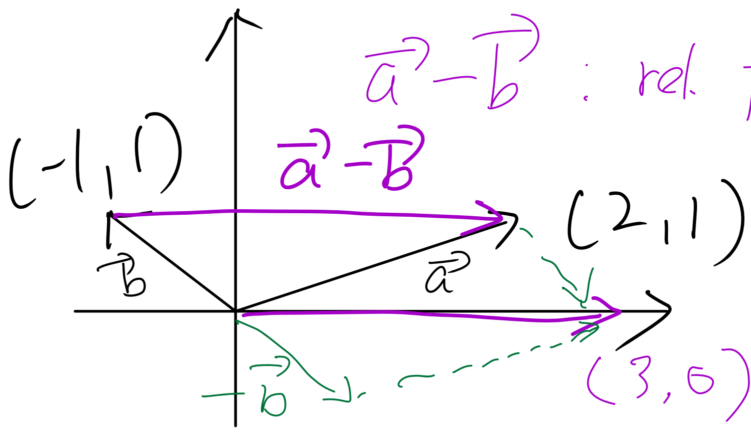


Geometric intuition

① diagonal of parallelogram

② Completing Triangle

$\vec{a}-\vec{b}$: rel. pos. of $\vec{a}$ in ref. to $\vec{b}$.

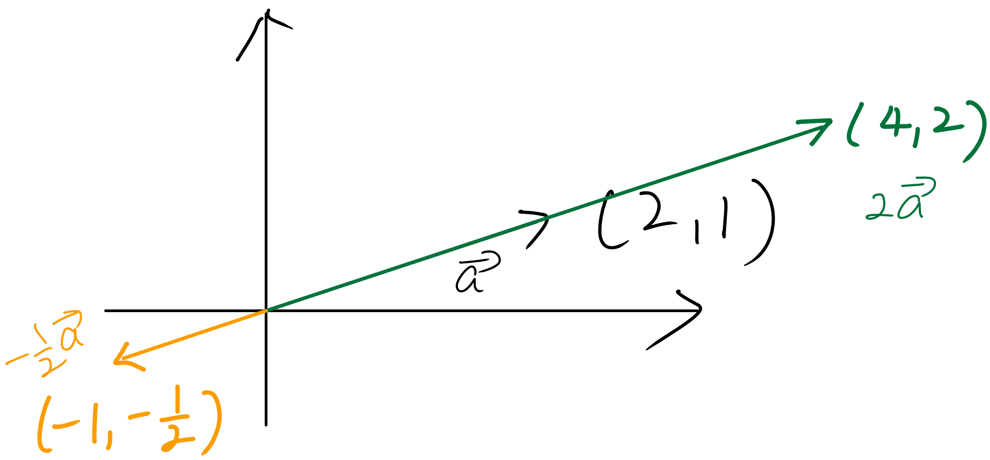


Geometric/Physical intuition

① relative position = object − reference

Change = final − initial

② $\vec{a} + (-1)\cdot\vec{b}$



Geometric intuition

- stretch, shrink, flip

The same applies to 3D vectors (or higher)

NT 1.3 Basic properties of vector operations.

v, w (vector) α, β (scalar)

$$\mathbf{v} + \mathbf{w} = \mathbf{w} + \mathbf{v} \qquad \text{(commutativity)}$$

$$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w} \qquad \text{(associativity)}$$

$$\alpha(\mathbf{v} + \mathbf{w}) = \alpha\mathbf{v} + \alpha\mathbf{w} \qquad \text{(distributivity)}$$

$$(\alpha + \beta)\mathbf{v} = \alpha\mathbf{v} + \beta\mathbf{v} \qquad \text{(distributivity)}$$

$$(\alpha\beta)\mathbf{v} = \alpha(\beta\mathbf{v}).$$

$\vec{0} = (0,0)$ or $(0,0,0)$

$\vec{v} + \vec{0} = \vec{v}$

$1 \cdot \vec{v} = \vec{v}$

$(-1) \cdot \vec{v} = -\vec{v}$

/* Basically, you have all nice properties */.

⊙ In fact, $\mathbb{R}^n$ is a bona fide vector space in the sense of Math 4A. (Therefore, you can use all appropriate theorems!)

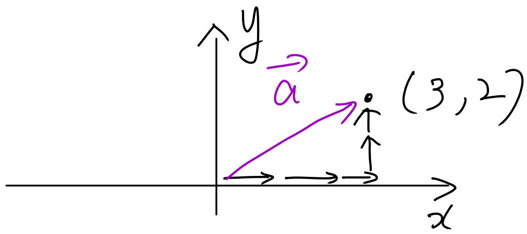
⊙ (Notation)

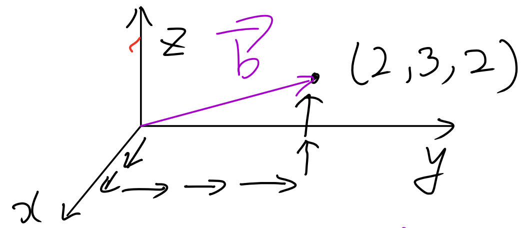
vectors : $\vec{v}$ or $\underline{v}$ (handwritten), $\mathbf{v}$ (printed)

or even just v (need to pay attention to dimension)

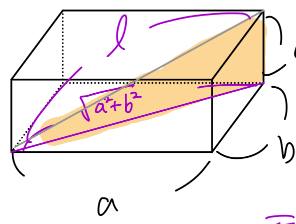
NT 1.4 Length of vectors

Recall Pythagorian theorem.





$$\text{length}(\vec{a}) = \sqrt{3^2+2^2}$$
$$= \sqrt{13}$$



$$l = \sqrt{\left(\sqrt{a^2+b^2}\right)^2 + c^2}$$
$$= \sqrt{a^2+b^2+c^2}$$

$$\text{length}(\vec{b})$$
$$= \sqrt{2^2+3^2+2^2} = \sqrt{17}$$

$\boxed{\text{Def}}$ (Length of vectors ; norm)

If $\vec{a} \in \mathbb{R}^2$, $\vec{b} \in \mathbb{R}^3$, or even $\vec{c} \in \mathbb{R}^n$,

$$\text{length of } \vec{a} = \|\vec{a}\| = \sqrt{a_1^2 + a_2^2}$$
$$\text{length of } \vec{b} = \|\vec{b}\| = \sqrt{b_1^2 + b_2^2 + b_3^2}$$
$$\text{length of } \vec{c} = \|\vec{c}\| = \sqrt{c_1^2 + c_2^2 + \cdots + c_n^2}$$

This quantity is called the norm of vectors.

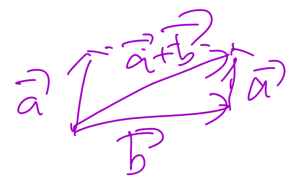
/* Notation : If none is mentioned, components has the same letter of the vector and put subscripts for proper coordinates. e.g. $\vec{v} = (v_1, v_2)$ */

$\boxed{\text{Thm}}$ (properties of norm)

1. (Triangle inequality)

If $\vec{a}, \vec{b} \in \mathbb{R}^n$ (in particular, $\mathbb{R}^2$ or $\mathbb{R}^3$)

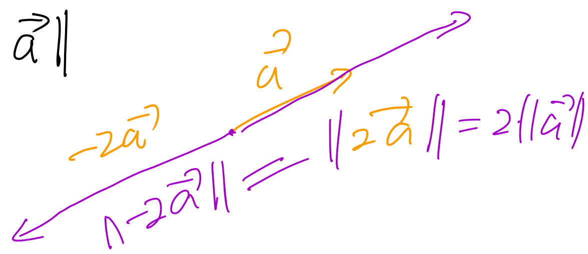
$$\|\vec{a}+\vec{b}\| \leq \|\vec{a}\| + \|\vec{b}\|$$



2. (Dilation)

If $\vec{a} \in \mathbb{R}^n$ (in particular, $\mathbb{R}^2$ or $\mathbb{R}^3$) and $k \in \mathbb{R}$, → notation for scalar

$$\|k\vec{a}\| = |k| \cdot \|\vec{a}\|$$



Q: Why not $\|k\vec{a}\| = k\|\vec{a}\|$?

$\boxed{\text{Clicker}}$ Find the unit vector (of length 1) with the same direction as $\vec{v} = (1, 2, 2)$

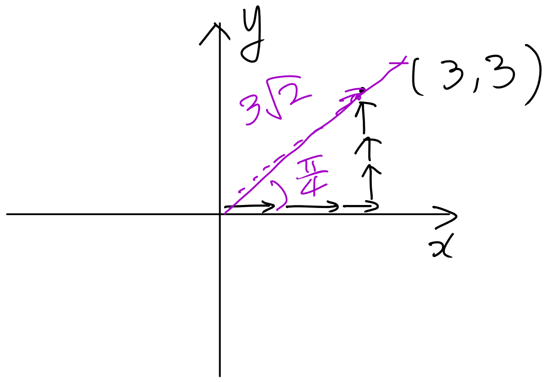
(A) $\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$ (B) $\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$

(C) $(2, 2, 1)$ (D) $(-1, -2, -2)$

we say "normalize $\vec{v}$"

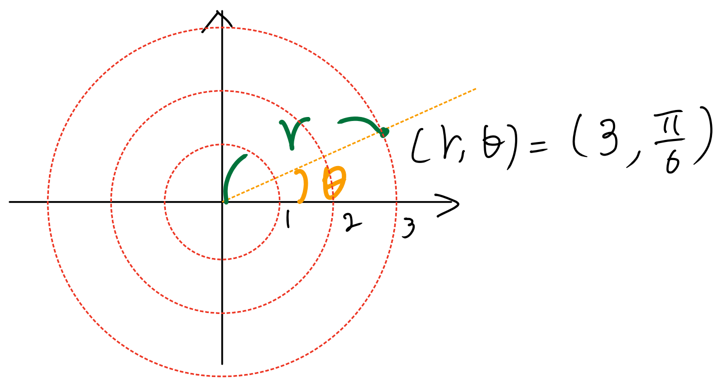
unit vector: $\frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{1^2+2^2+2^2}}(1,2,2) = \frac{1}{3}(1,2,2)$

NT 1.5 Polar/Spherical/cylindrical coordinates



We specified locations (2D) by how many steps we need to take horizontally and vertically. This way of representation is called **Cartesian coordinate** system

We can locate a position using a distance and direction, called **polar coordinate.**



$\boxed{\text{Clicker}}$ Find the polar coodinate of (3,3)

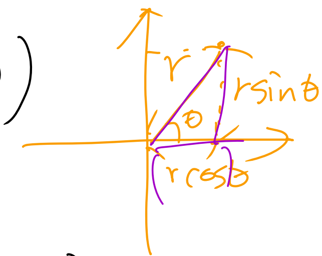
Cartesian

(A) $(0, \frac{\pi}{2})$ (B) $(-3, 0)$ (C) $(3, \frac{\pi}{2})$

(D) $(3\sqrt{2}, \frac{\pi}{4})$ (E) $(3\sqrt{2}, \cancel{45^{\circ}})$ ← must have "o" symbol

In general, they are related:

Cartesian (x, y) Polar (r, θ)

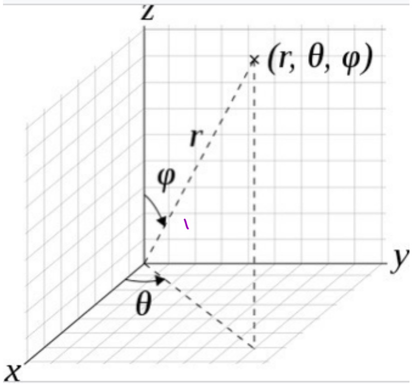


$$x = r\cos\theta$$
$$y = r\sin\theta$$
$\longleftarrow (r, \theta)$

⊙ Observe $r = \sqrt{x^2 + y^2}$.

⊙ spherical coordinate

Similarly, we can specify 3D location by distance (r), angle from x-axis (θ), and angle from z-axis (φ)



$z = r\cos\varphi$

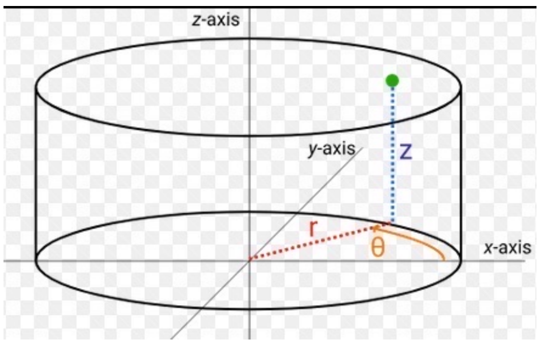
$y = (r\sin\varphi)\sin\theta$

$x = (r\sin\varphi)\cos\theta$

⊙ We see $r = \sqrt{x^2+y^2+z^2}$

◎ Cylindrical coordinates

polar coordinate system for xy-plane plus z-axis gives rise to the cylindrical coordinate system.



$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$

◎ Observe $r = \sqrt{x^2+y^2}$

↳ different from spherical coordinates.

⑩ Graphs in 3D

The following intuition continues to hold:
(if there is no "special issue")

No constraint $\longrightarrow$

An equation $\longrightarrow$

Two equations $\longrightarrow$

Three equations $\longrightarrow$

Four equations $\longrightarrow$

Exercise: Graph the following in 3D

Use the Geogebra links posted on Discord

or fancier ones if you have any.

Cartesian (observe features and sanity-check some pts)

$x=3$

$y=-2$

$z=1$

$x-z=2$

$x+2y+z=0$

$x^2+y^2+z^2=9$

$\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{16}=1$

$x^2+y^2=4$

$z=x^2+3y^2$

$z=xy$

$z=x^2-2y^2$

$z=-x^2-y^2$

$z=\sqrt{x^2+y^2}$

$z=\sin(x)\cdot\sin(y)$

$z=\sin(xy)$

cylindrical

If you have Mac, also try:

$r=3 \longrightarrow r(\theta,z)="3"$ $\quad \theta=\frac{\pi}{4}$

Spherical

$r=2 \longrightarrow \rho(\theta,\varphi)="2"$ $\quad \varphi=\frac{\pi}{4}$

$r=\cos(\varphi)$ $\quad \rho(\theta,\varphi)="\cos(y)"$