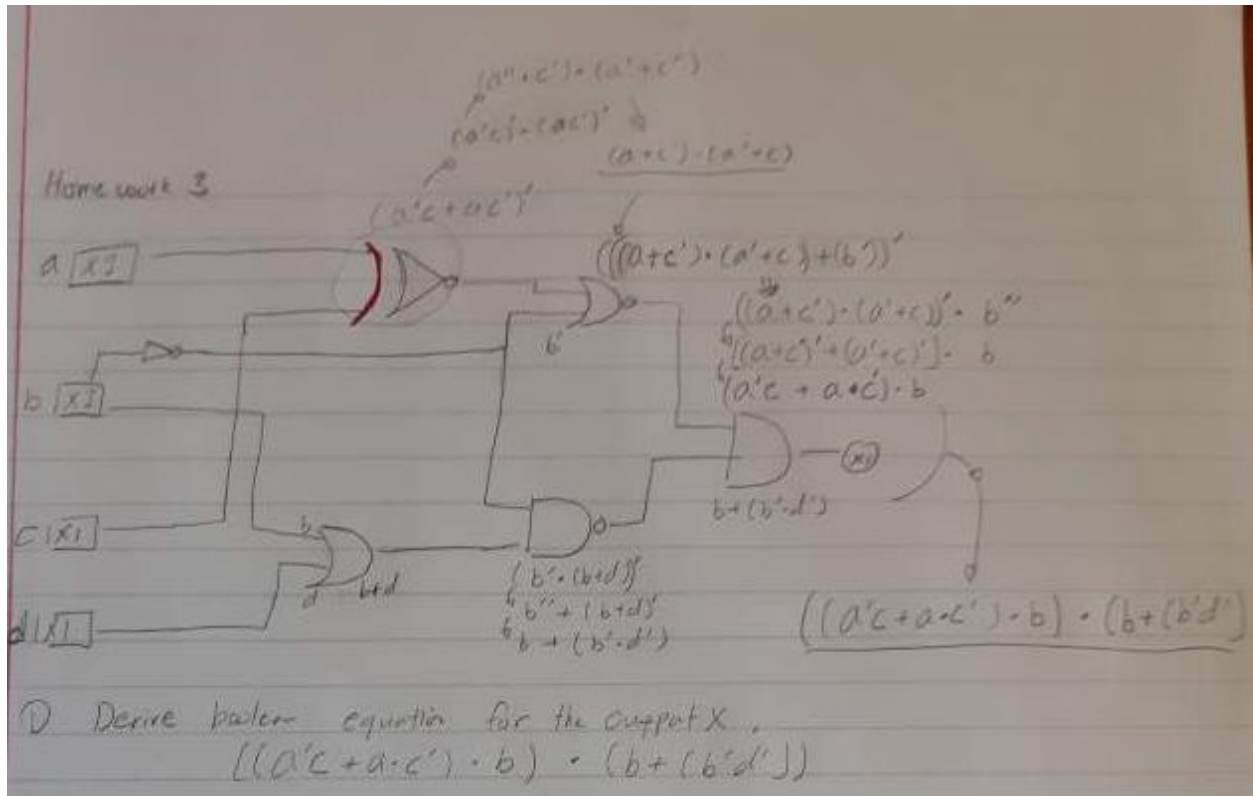


Q1.(5 pts) Consider the logic gate circuit shown below.

(1) Derive a Boolean equation for the output X. You don't need to simplify this equation, but feel free to try!

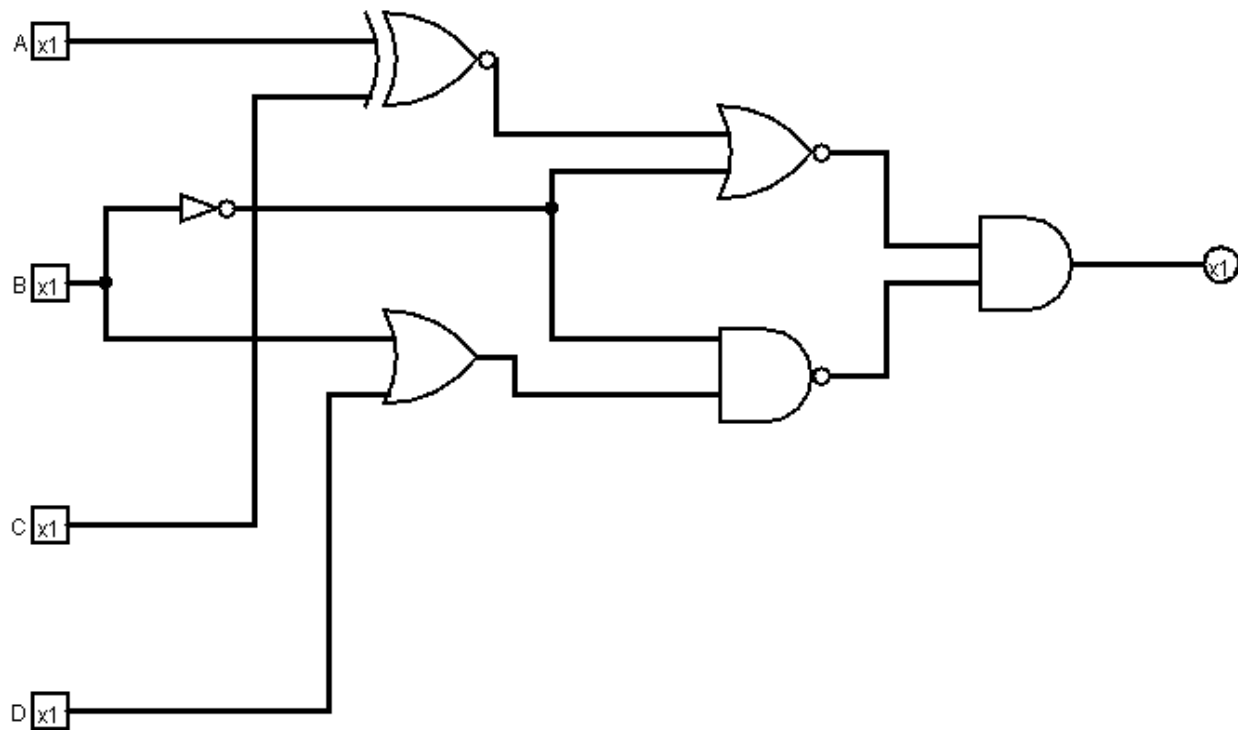


(2) Draw a truth table for the circuit.

2) Draw a truth table for the circuit.

a	b	c	d	Output
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

(3) Make the same circuit with logisim . **Submit the image (export image) and circuit file.**



File had been attached in Zip folder

(4) Generate the truth table with logisim(In logisim, project-->analyze circuit-->table). **Copy & past the table in your submission.**

A	B	C	D	x
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

Q2. (5 pts) Design a combinational circuit system.

(1) x,y,z are inputs and A,B,C are outputs. Draw a truth table for the given function.

Input			Output		
x	y	z	A	B	C
0	0	0	0	1	0
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	1	0	1
1	0	0	0	1	1
1	0	1	1	0	0
1	1	0	1	0	1
1	1	1	1	1	0

(2) Based on the truth table you draw, build Karnaugh maps for the output A,B,C.

Output For A

K Map For A

	$\bar{x}\bar{y}$	$x\bar{y}$	$x\bar{y}$	$x\bar{y}$
\bar{z}	0	1	1	0
z	0	1	1	1

$y + xz$

$$Y + XZ$$

Output For B

K Map For B

	$\bar{x}\bar{y}$	$x\bar{y}$	$x\bar{y}$	$x\bar{y}$
\bar{z}	1	0	0	1
z	1	0	1	0

$$\begin{array}{r} \bar{x}\bar{y}\bar{z} \\ + \bar{x}\bar{y}z \\ \hline \bar{x}\bar{y} \end{array} + \begin{array}{r} \bar{x}\bar{y}\bar{z} \\ + x\bar{y}\bar{z} \\ \hline \bar{y}\bar{z} \end{array} + x\bar{y}z$$

$$\sim X \sim Y + \sim Y \sim Z + XYZ$$

Output For C

KMap For C

	$\bar{X}\bar{Y}$	$\bar{X}Y$	$X\bar{Y}$	XY
\bar{Z}	0	0	1	1
Z	1	1	0	0

$\bar{X}\bar{Y}Z$	$XY\bar{Z}$
$+ \bar{X}YZ$	$+ XYZ$
$\bar{X}Z$	$+ XZ$

$$\sim XZ + X\sim Z$$

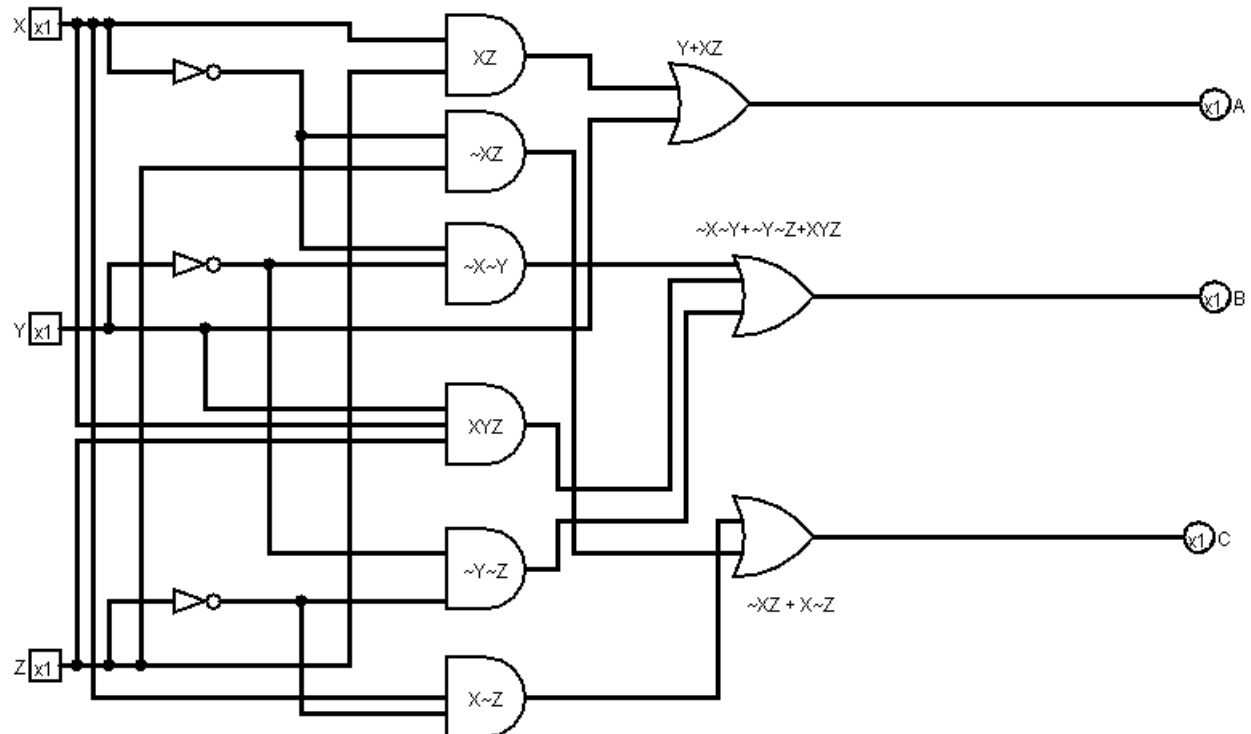
(3) Derive (as simple as possible) Boolean equations for A,B,C using the Karnaugh maps

A) $Y + XZ$

B) $\sim X\sim Y + \sim Y\sim Z + XYZ$

C) $\sim XZ + X\sim Z$

(4) Based on the Boolean equations, draw the logical gate diagram (circuit) for this system in Logisim. **Submit the image (export image) and circuit file.**



Circuit file will be submitted in zip folder.

(5) Test your circuit with the Logisim simulation and generate the truth table (In logisim, project-->analyze circuit-->table). **Copy & past the table, and compare it with your own table from sub-question (2)**

X	Y	Z	A	B	C
0	0	0	0	1	0
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	1	0	1
1	0	0	0	1	1
1	0	1	1	0	0
1	1	0	1	0	1
1	1	1	1	1	0

I did compare, it looks same.

Q3. (5 pts) Combinational circuit and Full Adder.

A full-adder is a combinational circuit that forms the arithmetic sum of three input bits. It consists of three inputs, x, y, z, and two outputs, C and S. Two of the input, that is, x and y, represent the two significant bits to be added. The third input, z, represents the carry from the previous lower significant position. The output S denotes the sum of two bits and C denotes carry. **Answer the following sub-questions.**

(1) Construct a truth table for Full-Adder

z is carry bit

X	Y	Z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

(2) Based on a truth table, construct a K-map for the output S and derive a Boolean equation using K-map. Make the equation as simple as possible

K-map for output S

	$\bar{x}\bar{y}$	$\bar{x}y$	xy	$x\bar{y}$
\bar{z}	0	1	0	1
z	1	0	1	0

eq. $\bar{x}\bar{y}z + \bar{x}y\bar{z} + xy\bar{z} + x\bar{y}z$

(3) Based on a truth table, construct a K-map for the output C and derive a Boolean equation using K-map. Make the equation as simple as possible

K-map for output C

	$\bar{x}\bar{y}$	$\bar{x}y$	xy	$x\bar{y}$
\bar{z}	0	0	1	0
z	0	1	1	1

① $\bar{x}yz$ ② $xy\bar{z}$ ③ $xy\bar{z}$

$+ \frac{\bar{x}yz}{yz}$ $+ \frac{xy\bar{z}}{xy}$ $+ \frac{x\bar{y}z}{xz}$

(4) By algebraic manipulation, show that S can be expressed as the exclusive-OR of the three input variables. That is, show that,

$$S = (x \text{ XOR } y) \text{ XOR } z.$$

You can prove by deriving from $S = (x \text{ XOR } y) \text{ XOR } z$ to the answer you got in the question (2)

$$\begin{aligned}
 & X'Yz + X'Yz' + XYz + XY'z' \\
 & \underbrace{X'(\bar{Y}z + Yz')}_{Y \oplus Z} + \underbrace{X(Yz + \bar{Y}'z')}_{\text{Xnor } Y \oplus Z \text{ Complement}} \\
 & \Rightarrow \bar{X}[Y \oplus Z] + X[\overline{Y \oplus Z}] \\
 & \text{Let } Y \oplus Z = A \\
 & \underbrace{\bar{X}A + X\bar{A}}_{X \text{ XOR } A} \\
 & \text{So, } X \oplus A \\
 & \Rightarrow S = X \oplus Y \oplus Z
 \end{aligned}$$

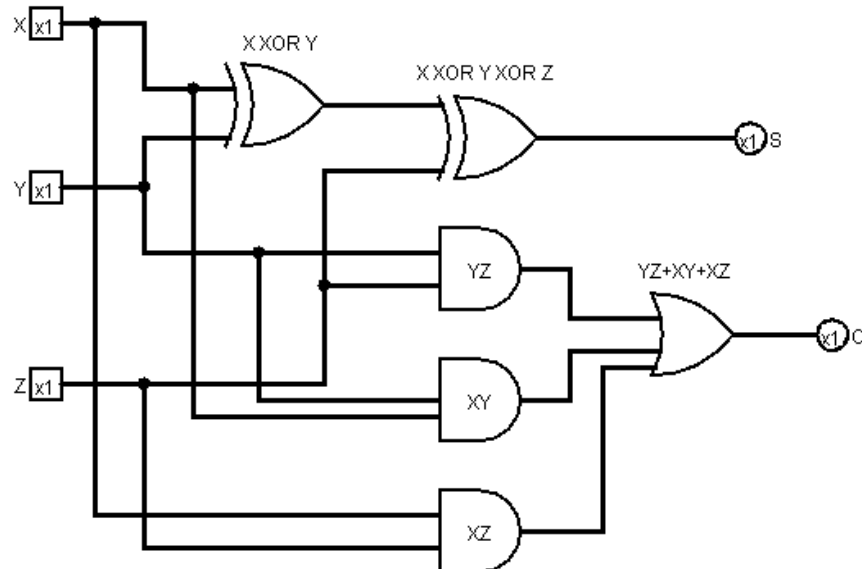
(5) By algebraic manipulation, show that C can be expressed as the following term.

$$C = xy + (x \text{ XOR } y)z.$$

You can prove by deriving from $C = xy + (x \text{ XOR } y)z$ to the answer you got in the question (3)

$$\begin{aligned}
 & yz + xy + xz \\
 & \text{Derive using } C = xy + (x \oplus y)z \\
 & = \Rightarrow xy + (\bar{x}y + x\bar{y})z \\
 & \Rightarrow xy + (\bar{x}yz + x\bar{y}z) \\
 & \Rightarrow y(x + \bar{x}z) + x\bar{y}z \\
 & \quad y(x + \bar{x})(x + z) \\
 & \quad y(x + z) + x\bar{y}z \\
 & \quad xy + yz + x\bar{y}z \\
 & \quad xy + z(y + x\bar{y}) \\
 & \quad xy + z((y + x)(y + \bar{y})) \\
 & \quad xy + z(y + x) \\
 & \quad xy + xz + yz = yz + xy + xz
 \end{aligned}$$

(6) Based on the (4) (5), draw a circuit for the full-adder in Logisim simulator and submit the **image (export the image)** and **circuit file**.



Circuit file will be attached in folder

Q4. (5 pts) Design a sequential circuit.

A sequential circuit has one D flip-flop and one JK-flip-flop, two inputs x and y , and one output z . A and B are the outputs of each D flip-flop, and JK-flip-flop, respectively. The flip-flop **input** equations and the circuit output are as follows. Here D_A is the D input of the D-flip flop of A , and J_B , K_B is the J and K input of the JK-flip flop of B .

$$D_A = \sim xy + yB$$

$$J_B = \sim xB + xy$$

$$K_B = yB + \sim xA$$

$$z = y + x \sim y$$

(1) Draw the logic diagram of the circuit and test it with Logisim. Please attach the **circuit image**, **circuit file**, and attach the **generated table**.

The circuit file will be attached.

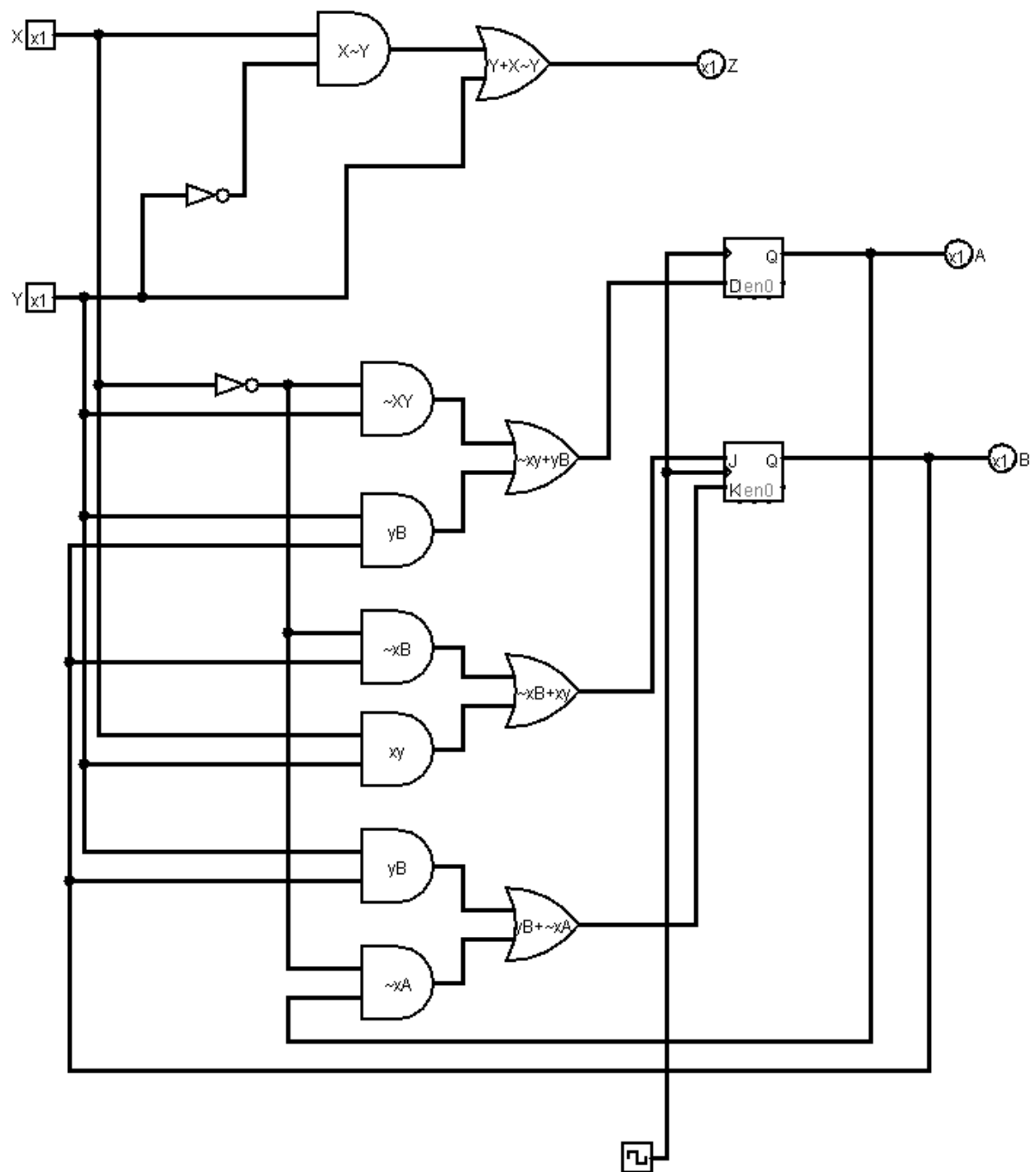


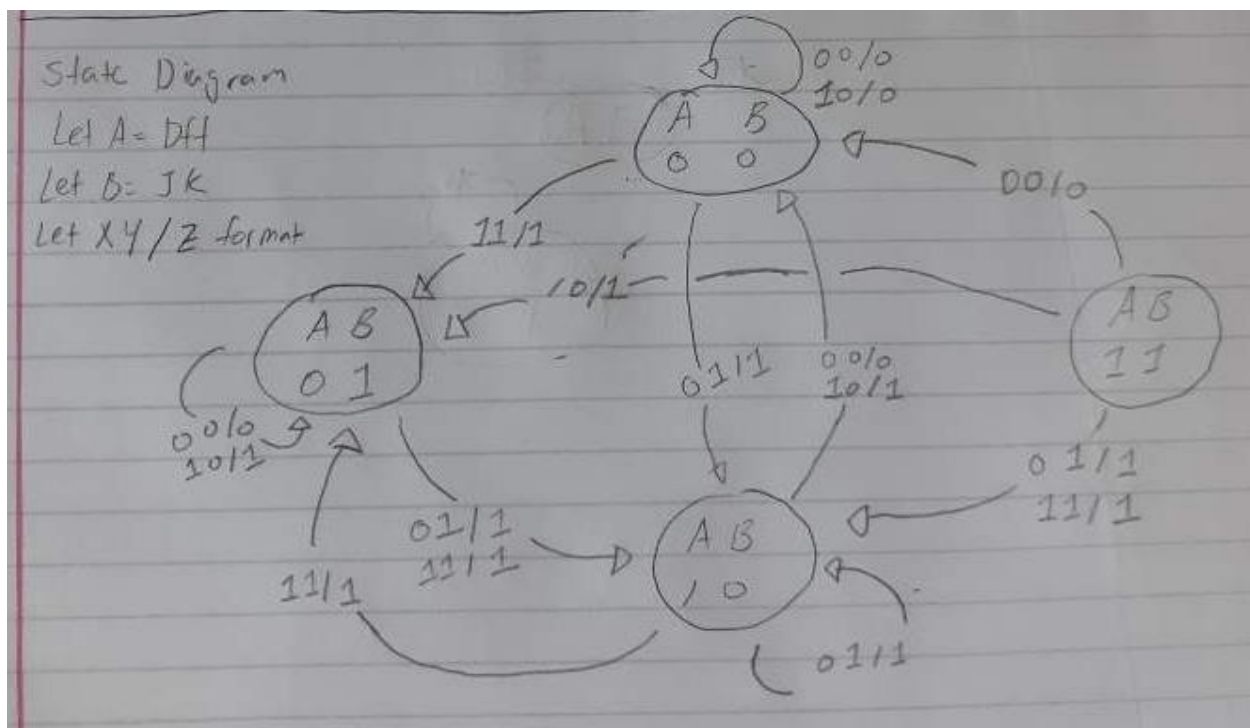
Table it generates in Logisim

X	Y	Z	A	B
0	0	0	0	0
0	1	1	0	0
1	0	1	0	0
1	1	1	0	0

State Table I worked on by Testing

Input X Y	present		next		Output Z
	Dff	JK	Dff	JK	
0 0	0	0	0	0	0
0 0	0	1	0	1	0
0 0	1	0	0	0	0
0 0	1	1	0	0	0
0 1	0	0	1	0	1
0 1	0	1	1	0	1
0 1	1	0	1	0	1
0 1	1	1	1	0	1
1 0	0	0	0	0	1
1 0	0	1	0	1	1
1 0	1	0	0	0	1
1 0	1	1	0	1	1
1 1	0	0	0	1	1
1 1	0	1	1	0	1
1 1	1	0	0	1	1
1 1	1	1	1	0	1

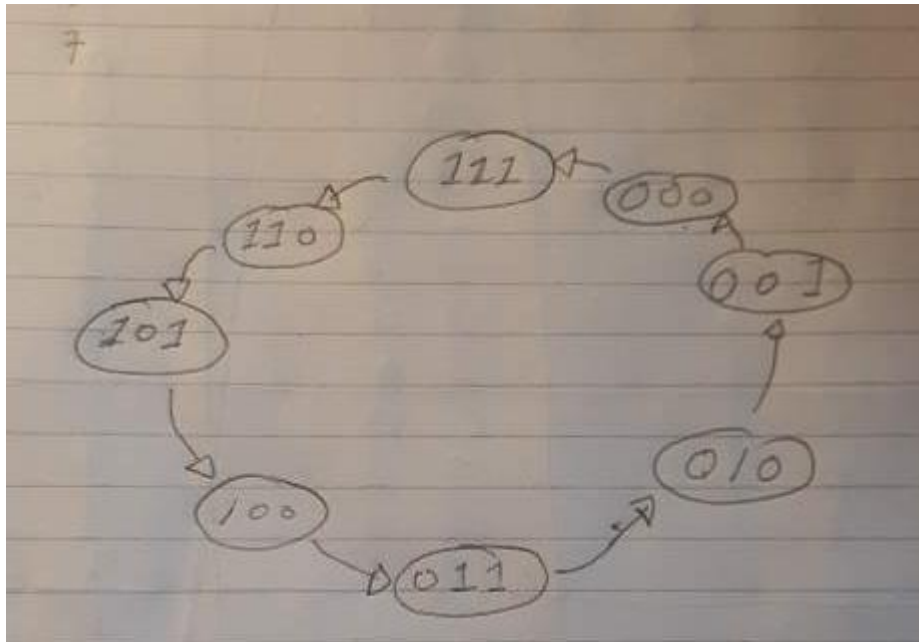
(2) Construct a state diagram of this circuit.



Q5. (5 pts) Design a system

Design a system with the following state changes: This is a sequential circuit with three flip-flops. The state sequence is changed with a clock as in the order of, 111,110,101,100,011,010,001,000,111 and repeat. Use JK flip-flops.

(1) Draw a state diagram



(2) Construct an excitation table

Excitation Table									
Present			Next			JA A		JB B	
A	B	C	A	B	C	JA	KA	JB	KB
1	1	1	1	1	0	X	0	X	0
1	1	0	1	0	1	X	0	X	1
1	0	1	1	0	0	X	0	0	X
1	0	0	0	1	1	X	1	1	X
0	1	1	0	1	0	0	X	X	0
0	1	0	0	0	1	0	X	X	1
0	0	1	0	0	0	0	X	0	X
0	0	0	1	1	1	1	X	1	X

(3) Draw K-maps and derive Boolean equations using K-maps. Make the equations as simple as possible

Kmap for J_A

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
\bar{C}	1	0	x	x
C	0	0	x	x

$\bar{A}\bar{B}\bar{C}$
 $+ A\bar{B}\bar{C}$
 $J_A = \bar{B}\bar{C}$

KMap for K_A

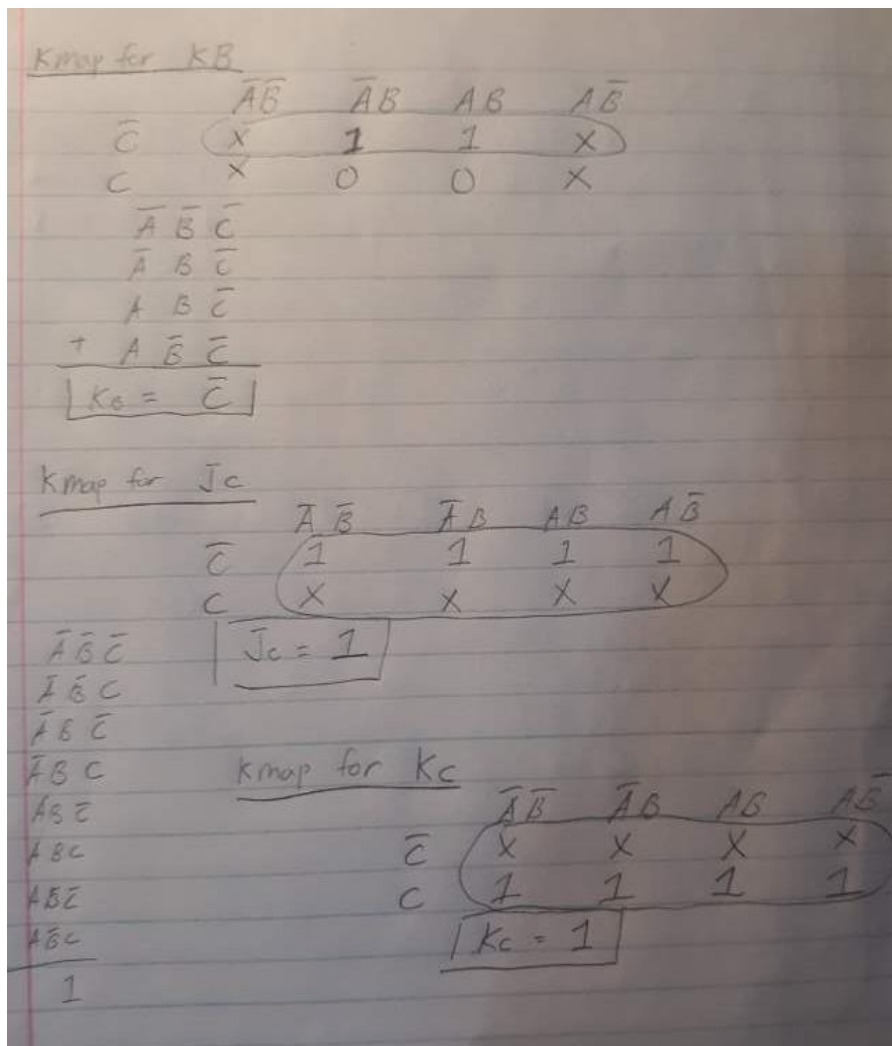
	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
\bar{C}	x	x	0	1
C	x	x	0	0

$A\bar{B}\bar{C}$
 $+ \bar{A}\bar{B}\bar{C}$
 $K_A = \bar{B}\bar{C}$

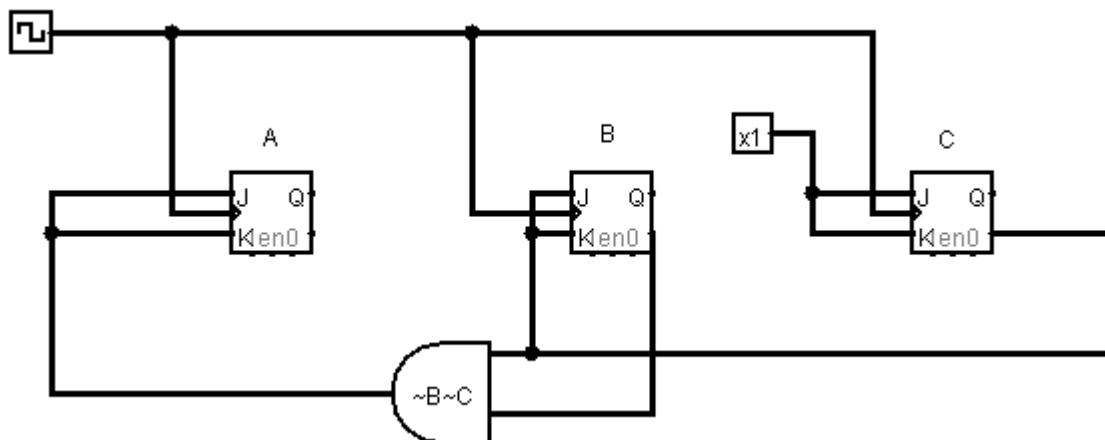
Kmap for J_B

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
\bar{C}	1	x	x	1
C	0	x	x	0

$\bar{A}\bar{B}\bar{C}$
 $+ \bar{A}B\bar{C}$
 $+ AB\bar{C}$
 $+ A\bar{B}\bar{C}$
 $J_B = \bar{C}$



(4) Draw the system in Logisim simulator and include the **circuit image and circuit file** .



Circuit file will be provided.

(5) Test the system and include the **generated table** .

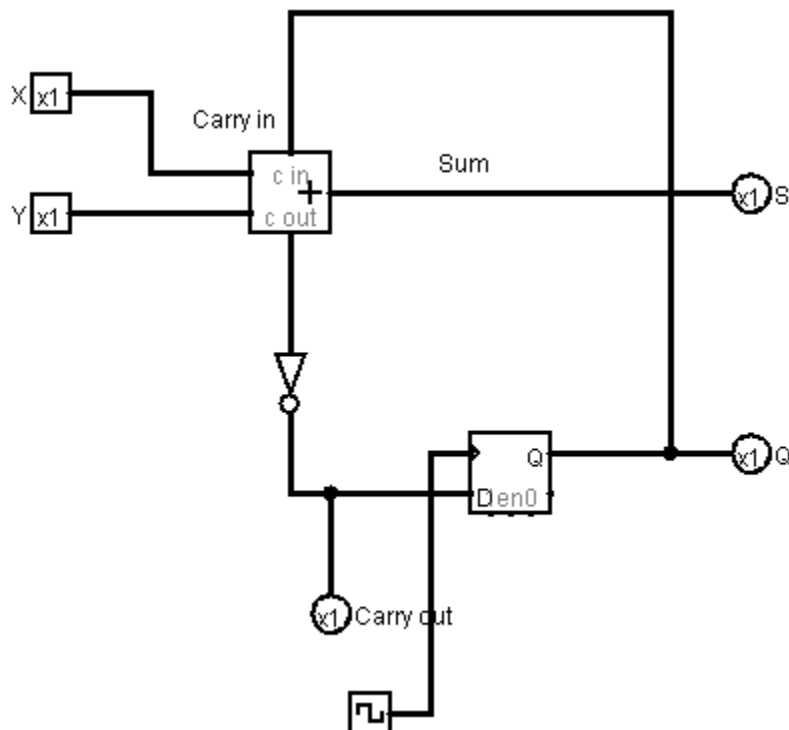
A	B	C
1	1	1
1	1	0
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0
1	1	1

The output of the table matches with FSM, therefore it is correct.

Extra points (5 pts). Sequential circuit and a full-adder

The following sequential circuit includes a full-adder (described in the previous question). Inputs are X, Y and carry-in, and outputs are the next state of S and Q.

(1) Implement the sequential circuit in Logisim simulator and submit the **image** and **circuit** file.



(2) Complete the following truth table for the following sequential circuit: Note that the Carry out signal is output, not the input. The carry in signal is the same as the Q. You can change the Carry-in bit by clicking the D-FF.

X	Y	Carry-in (or Q before clock)	S (before clock)	Carry-out (before clock)	S (after clock)	Carry-out (after clock)
0	0	0	0	1	1	1
0	0	1	1	1	1	1
0	1	0	1	1	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	0
1	0	1	0	0	1	1
1	1	0	0	0	0	0
1	1	1	1	0	0	0