MA1521 Final CheatSheet

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Basic Algebra

$(a\pm b)^3=a^3\pm 3a^2b+3ab^2\pm b^3$

$a^3\pm b^3=(a\pm b)(a^2\mp ab+b^2)$

$\sum_{n=1}^{n}k^2=\frac{n(n+1)(2n+1)}{6}$

Trigonometric Identities

$\csc^2x-1=\cot^2x$	$\sec^2x-1=\tan^2x$
$\sin^2A=\frac{1}{2}(1-\cos 2A)$	$\cos^2A=\frac{1}{2}(1+\cos 2A)$
$\sin 3A=-4\sin^3A+3\sin A$	$\cos 3A=4\cos^3A-3\cos A$
$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}$	$\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$
$\sin A\cos B=\frac{1}{2}(\sin(A+B)+\sin(A-B))$	
$\cos A\sin B=\frac{1}{2}(\sin(A+B)-\sin(A-B))$	
$\cos A\cos B=\frac{1}{2}(\cos(A+B)+\cos(A-B))$	
$\sin A\sin B=-\frac{1}{2}(\cos(A+B)-\cos(A-B))$	
$\sin A+\sin B=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$	
$\sin A-\sin B=2\sin\frac{A-B}{2}\cos\frac{A+B}{2}$	
$\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$	
$\cos A-\cos B=-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$	
$u=\tan\frac{x}{2}(-\pi<x<\pi)\Longrightarrow \sin x=\frac{2u}{1+u^2},\ \cos x=\frac{1-u^2}{1+u^2}$	
$\arcsin x+\arccos x=\frac{\pi}{2}$	

Integral

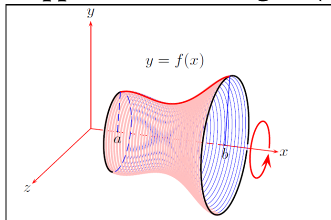
Definite Integral	$\int_a^b f(x)\mathrm{d}x=\lim_{n\to\infty}\left\{\sum_{k=1}^{\infty}\left(\frac{b-a}{n}\right)f\left(a+k\left(\frac{b-a}{n}\right)\right)\right\}$

区间再现公式： $\int_a^b f(x)\mathrm{d}x=\int_a^b f(a+b-x)\mathrm{d}x$

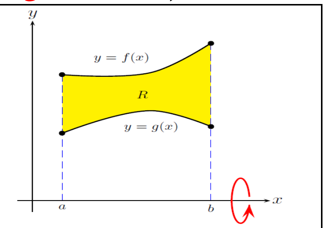
点火公式：

$$\int_0^{\frac{\pi}{2}}\sin^n x\,\mathrm{d}x=\int_0^{\frac{\pi}{2}}\cos^n x\,\mathrm{d}x=\begin{cases}\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\cdots\cdot\frac{2}{3}\cdot 1 & n=2k+1\\ \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\cdots\cdot\frac{1}{2}\cdot\frac{\pi}{2} & n=2k\end{cases}$$

Application of Integral (Singal Variable)

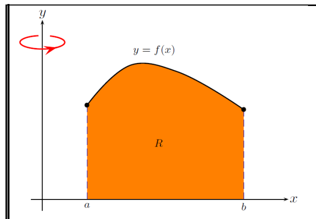


$V=\pi\int_a^b[f(x)]^2\mathrm{d}x$

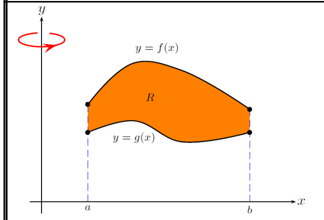


$V=\pi\int_a^b[f(x)]^2-[g(x)]^2\mathrm{d}x$

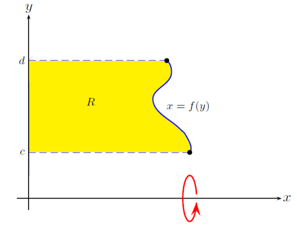
Cylindrical Shell Method



$V=2\pi\int_a^b x|f(x)|\mathrm{d}x$



$V=2\pi\int_a^b x|f(x)-g(x)|\mathrm{d}x$



$V=2\pi\int_c^d y|f(y)|\mathrm{d}y$

The length of the curve $y=f(x);a\leqslant x\leqslant b$ is

$L=\int_a^b\sqrt{1+[f'(x)]^2}\,\mathrm{d}x\quad L=\int_p^q\sqrt{1+[g'(y)]^2}\,\mathrm{d}y$

Series

Test for convergence/divergence

n − th term Test

$\lim_{n\to\infty}a_n\neq 0\Longrightarrow\sum_{n=1}^{\infty}a_n$ diverges

$\lim_{n\to\infty}a_n=0$: Inconclusive

Integral Test $f(x)=a_n>0$

$\int_1^{\infty}f(x)\mathrm{d}x$ converges $\Longrightarrow\sum_{n=1}^{\infty}a_n$ converges

$\int_1^{\infty}f(x)\mathrm{d}x$ diverges $\Longrightarrow\sum_{n=1}^{\infty}a_n$ diverges

Comparison Test: $0\leqslant a_n\leqslant b_n$

$\sum_{n=1}^{\infty}b_n$ converges $\Longrightarrow\sum_{n=1}^{\infty}a_n$ converges

$\sum_{n=1}^{\infty}a_n$ diverges $\Longrightarrow\sum_{n=1}^{\infty}b_n$ diverges

geometric series $\sum_{n=1}^{\infty}ar^{n-1}$ converges $\Longleftrightarrow |r|<1$

$$\sum_{n=1}^{\infty}ar^{n-1}=\lim_{n\to\infty}S_n=\begin{cases}\infty & a>0,\ r\geqslant 1\\ -\infty & a<0,\ r\geqslant 1\\ \frac{a}{1-r} & -1<r<1\\ \text{does not exist} & r\leqslant -1\end{cases}$$

p − series

$\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges $\Longleftrightarrow p>1$

Alternating Series Test

$b_n\geqslant 0,\ b_n\geqslant b_{n+1},\ \lim_{n\to\infty}b_n=0$

$\Longrightarrow\sum_{n=1}^{\infty}(-1)^{n-1}b_n$ converges

Ratio Test $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=L$

$L<1\Longrightarrow\sum_{n=1}^{\infty}|a_n|$ converges

$L>1\Longrightarrow\sum_{n=1}^{\infty}a_n$ diverges

$L=1$: Inconclusive

Root Test $\lim_{n\to\infty}\sqrt[n]{|a_n|}=L$

$L<1\Longrightarrow\sum_{n=1}^{\infty}|a_n|$ converges

$L>1\Longrightarrow\sum_{n=1}^{\infty}a_n$ diverges

$L=1$: Inconclusive

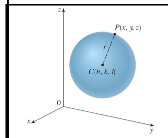
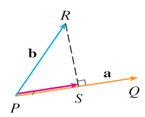
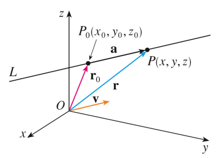
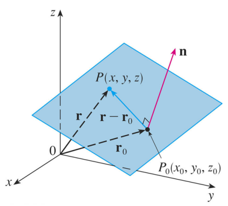
Taylor Series

Taylor series of f at $x=a$, $f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$

$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}$	$\ln(1+x)=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$
$\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n$	$\frac{1}{1+x}=\sum_{n=0}^{\infty}(-1)^n x^n$
$\sin x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$	$\cos x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$
$\arctan x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$	$\arcsin x=\sum_{n=0}^{\infty}\frac{(2n!)x^{2n+1}}{4^n(n!)^2(2n+1)}$

$$(1+x)^{\alpha}=1+\alpha x+\frac{\alpha(\alpha-1)}{2!}x^2+\cdots+\frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^n+\cdots,\begin{cases}x\in(-1,1), & \alpha\leqslant -1\\ x\in(-1,1], & -1<\alpha<0\\ x\in[-1,1], & \alpha>0,\ \alpha\notin\mathbb{N}_+\\ x\in\mathbb{R}, & \alpha\in\mathbb{N}_+\end{cases}$$

空间解析几何(3-Dimensional Vector Geometry)

	**Sphere** $(x-h)^2+(y-k)^2+(z-l)^2=r^2$
Represent	$\overrightarrow{AB}=\langle x_1-x_2,\ y_1-y_2,\ z_1-z_2\rangle$ **[尖括号]**
	vector $\overrightarrow{PS}$ is called the vector projection of **b** onto **a**, denoted by $\mathrm{proj}_{\mathbf{a}}\mathbf{b}$ scalar projection (标量投影, also called component of **b** along **a**) of **b** onto **a** $\mathrm{comp}_{\mathbf{a}}\mathbf{b}=\lVert\mathbf{b}\rVert\cos\theta=\frac{\mathbf{a}\cdot\mathbf{b}}{\lVert\mathbf{a}\rVert}$ $\mathrm{proj}_{\mathbf{a}}\mathbf{b}=\mathrm{comp}_{\mathbf{a}}\mathbf{b}\cdot\frac{\mathbf{a}}{\lVert\mathbf{a}\rVert}$
Distance 点到面	$P(x_0,\ y_0,\ z_0)\rightarrow ax+by+cz=d$ $L=\frac{\lvert ax_0+by_0+cz_0-d\rvert}{\sqrt{a^2+b^2+c^2}}$
Cross Product	$\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\a_1&a_2&a_3\\b_1&b_2&b_3\end{vmatrix}=\lVert\mathbf{a}\rVert\lVert\mathbf{b}\rVert\sin\theta$ Note that $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{a}=0$ $S_{\Delta ABC}=\frac{1}{2}\lVert\overrightarrow{AB}\times\overrightarrow{AC}\rVert$
Line 	$\overrightarrow{P_0P}=t\mathbf{v}$ $\mathbf{r}=\mathbf{r_0}+\overrightarrow{P_0P}\Longleftrightarrow\mathbf{r}=\mathbf{r_0}+t\mathbf{v}$ $\langle x,y,z\rangle=\langle x_0,y_0,z_0\rangle+t\langle a,b,c\rangle$ Hence, **Parametric Equation of Line** $\begin{cases}x=x_0+at\\y=y_0+bt\\z=z_0+ct\end{cases}$
Plane 	$\overrightarrow{P_0P}=\mathbf{r}-\mathbf{r_0}$ normal vector $\mathbf{n}=\mathbf{a}\times\mathbf{b}$ (**a**,**b** 是平面上的两个向量) $\mathbf{n}\cdot(\mathbf{r}-\mathbf{r_0})=0$ $\Leftrightarrow\ \mathbf{n}\cdot\mathbf{r}=\ \mathbf{n}\cdot\mathbf{r_0}$ $\Leftrightarrow\langle a,b,c\rangle\cdot\langle x,y,z\rangle=\langle a,b,c\rangle\cdot\langle x_0,y_0,z_0\rangle$ $\Leftrightarrow ax+by+cz=ax_0+by_0+cz_0$ $\Leftrightarrow ax+by+cz+d=0$

注★★： $\mathbf{v}\cdot\mathbf{n}=0\Longrightarrow$ 线面平行！！不是垂直！！

专有名词

- In 3-D. Nonparallel, nonintersecting lines are called **skew lines**. (异面直线)
- **acute angle** (锐角)
- **Squeeze Theorem** 夹逼定理
- **orthogonal** (正交)
- **Parallelogram** 平行四边形

line of intersection of these two planes.

P_1: $a_1x+b_1y+c_1z=d_1$

P_2: $a_2x+b_2y+c_2z=d_2$

$\because\begin{cases}x_1=\frac{1}{a_1}(d_1-b_1y-c_1z)\\ x_2=\frac{1}{a_2}(d_2-b_2y-c_2z)\end{cases},\ x_1=x_2$

$\therefore\ a_1(d_2-b_2y-c_2z)=a_2(d_1-b_1y-c_1z)$

$\therefore\ z=\frac{(a_1d_2-a_2d_1)-(a_1b_2-a_2b_1)y}{a_1c_2-a_2c_1}$

$\therefore$ 代入x_1，得 $x=\boxed{\text{Result}}$

$\therefore$ Let $y=t$ be the parameter

We obtain the parametric equation

$x=m+a_3t,\ y=t,\ z=n+c_3t.$

方法二：

Step1：找点

Setp2：相交直线方向向量$\mathbf{n_3}=\mathbf{n_1}\times\mathbf{n_2}$

Partial Derivative

Derivation	$\mathbf{r}'(a)=\langle f'(a),\ g'(a),\ h'(a)\rangle$
Arc Length	For $\mathbf{r}(t)=\langle f(t),\ g(t),\ h(t)\rangle,\ a\leqslant t\leqslant b$ $s=\int_a^b\sqrt{f'(t)^2+g'(t)^2+h'(t)^2}\,\mathrm{d}t$ $=\int_a^b\lVert\mathbf{r}'(t)\rVert\mathrm{d}t$
Chain Rule	$\frac{\mathrm{d}z}{\mathrm{d}t}=\frac{\partial f}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t}+\frac{\partial f}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}$ 推广：$\frac{\partial u}{\partial t_i}=\sum_{k=1}^{n}\frac{\partial u}{\partial x_k}\frac{\partial x_k}{\partial t_i}$
Implicit Differentiation (隐函数求导)	$\frac{\partial z}{\partial x}=-\frac{F_x(x,y,z)}{F_z(x,y,z)},\quad\frac{\partial z}{\partial y}=-\frac{F_y(x,y,z)}{F_z(x,y,z)}$

全微分与近似公式：

Increment(增量)：$\Delta z=f(x+\Delta x,y+\Delta y)-f(x,y)$

Total differential(全微分)：$\mathrm{d}z=f_x(x,y)\mathrm{d}x+f_y(x,y)\mathrm{d}y$

Theorem 8.10. Δx and Δy be small increments from (a,b)

$\Delta z\approx\mathrm{d}z=f_x(a,b)\mathrm{d}x+f_y(a,b)\mathrm{d}y=f_x(a,b)\Delta x+f_y(a,b)\Delta y$

方向导数专题（高频考点）

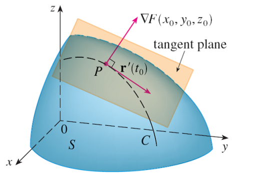
Gradient	$\nabla f(x,y,z)=\left\langle\frac{\partial f}{\partial x},\frac{\partial f}{\partial y},\frac{\partial f}{\partial z}\right\rangle=\frac{\partial f}{\partial x}\mathbf{i}+\frac{\partial f}{\partial y}\mathbf{j}+\frac{\partial f}{\partial z}\mathbf{k}$
Directional Derivative	$\mathrm{D}_{\mathbf{u}}f=\nabla f\cdot\mathbf{u}=\left\langle\frac{\partial f}{\partial x},\ \frac{\partial f}{\partial y},\ \frac{\partial f}{\partial z}\right\rangle\cdot\mathbf{u}$ 注：**u** 需要单位化，即$\lvert\mathbf{u}\rvert=1$

Theorem 8.16: Let P denote a given point， $\nabla f(P)\neq\mathbf{0}$

$D_{\mathbf{u}}f=\nabla f\cdot\mathbf{u}=\lVert\nabla f\rVert\lVert\mathbf{u}\rVert\cos\theta=\lVert\nabla f\rVert\cos\theta\Longrightarrow D_{\mathbf{u}}f(P)=\lVert\nabla f(P)\rVert\cos\theta$

(Since **u** is a unit vector)

$\therefore D_{\mathbf{u}}f(P)\in[-\lVert\nabla f(P)\rVert,\lVert\nabla f(P)\rVert]$

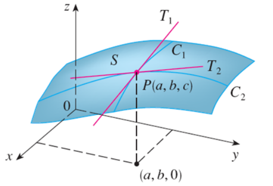
Level Surface vs ∇f	在(x_0,y_0,z_0)处，有
	$\nabla f(x_0,y_0,z_0)\cdot \mathbf{r}'(t_0)=0$ **Theorem 8.15.** Consequently, the tangent plane to the level surface $F(x,y,z)=k$ at (x_0,y_0,z_0) is given by the equation $\nabla F(x_0,y_0,z_0)\cdot\langle x-x_0,y-y_0,z-z_0\rangle=0$

Level Curve:

- A level curve of $f(x,y)$ is the 2-D graph of the equation $f(x,y)=k$ for some constant k.
- A level surface of $f(x,y,z)$ is the 3-D graph of the equation $f(x,y,z)=k$ for some constant k.

Contour Plot: A contour plot of $f(x,y)$ is a graph of numerous level curves $f(x,y)=k$, for representative values of k.

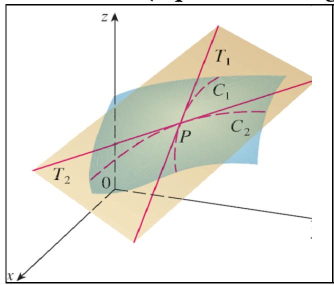
注意： 对于多元函数求切线首先要选定切线所在的坐标系（空间曲面的任意一点有无数条方向不同的切线）。例如，



平面$x=a$交曲线S于C_1: $g(x)=f(x,b)$
$\Longrightarrow$ 切线$T_1=g'(a)$
平面$y=b$交曲面S于C_2: $h(x)=f(a,y)$
$\Longrightarrow$ 切线$T_2=h'(b)$

Theorem 8.4 (Clairaut's Theorem): $f_{xy}(a,b)=f_{yx}(a,b)$

Theorem 8.5 (Equation of Tangent Plane)

	求点$P(a,b,c)$的切平面步骤 1. 分别确定点平面 P 处关于$x=a,\ y=b$的切线方程 2. 根据切线方程确定方向向量 3. 法向量 $\mathbf{n}=\mathbf{r_1}\times\mathbf{r_2}$ 4. 根据点 P 坐标和法向量写出切平面方程

结论： $z=f_x(a,b)(x-a)+f_y(a,b)(y-b)+f(a,b)$

Theorem 8.18 Second Derivative Test

Suppose $f(x,y)$ has continuous second-order partial derivatives on some open disk centered at (a,b).

Suppose $f_x(a,b)=f_y(a,b)=0$. Define the discriminant D for the point (a,b) by

$$D=D(a,b)=f_{xx}(a.b)f_{yy}(a,b)-[f_{xy}(a,b)]^2$$

(a) If D > 0 and $f_{xx}(a,b)>0$, then $f(a,b)$ is a **local minimum**.
(b) If D > 0 and $f_{xx}(a,b)<0$, then $f(a,b)$ is a **local maximum**.
(c) If D < 0, then (a,b) is a **saddle point** (鞍点) of f.
(d) If D = 0, then no conclusion can be drawn.

Critical point (临界点): $f_x(a,b)=f_y(a,b)=0$ **OR** one of the partial derivatives does not exist.

local min.: 极小值 **Absolute min.:** 最小值

Saddle point

- it is a **critical point** of f
- every open disk centered at (a,b) contains points $(x,y)\in D$ for which $f(x,y)<f(a,b)$ and points $(x,y)\in D$ for which $f(x,y)>f(a,b)$

Double Integral

和式极限（其中$D=\{(x,y)|a\le x\le b,c\le y\le d\}$）

$$\iint_R f(x,y)\,\mathrm{d}A=\lim_{n\to\infty}\sum_{i=1}^{n}\sum_{j=1}^{n}f\left(a+\frac{b-a}{n}i,c+\frac{d-c}{n}j\right)\cdot\frac{b-a}{n}\cdot\frac{d-c}{n}$$

Theorem 9.12. Change to Polar Coordinates in Double Integral

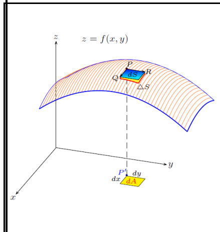
If f is continuous on a **polar rectangle** R given by

$$R=\{(r,\theta):\ 0\le a\le r,\ \alpha\le\theta\le\beta\}$$

Where $0\le\beta-\alpha\le 2\pi$, then

$$\iint_R f(x,y)\,dA=\int_\alpha^\beta\int_a^b f(r\cos\theta,r\sin\theta)\,r\,dr\,d\theta.$$

Surface Area

	Note that $\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\dx&0&f_xdx\\0&dy&f_ydy\end{vmatrix}=\langle -f_x,-f_y,1\rangle dxdy$ $dS=\lvert\langle -f_x,-f_y,1\rangle dxdy\rvert=\sqrt{f_x^2+f_y^2+1}\,dA$ Hence $\boxed{\text{Surface area }=\iint_D dS=\iint_D\sqrt{f_x^2+f_y^2+1}\,dA}$

First Order ODE

Linear: $\boxed{\frac{\mathrm{dy}}{\mathrm{dx}}+P(x)y=Q(x)}$ **Integrating factor:** $I(x)=e^{\int P(x)\mathrm{d}x}$, $(I(x)y)'=I(x)Q(x)$, $y=I(x)^{-1}\left[\int I(x)Q(x)\mathrm{d}x+C\right]$	**Bernoulli equation:** $\boxed{y'+p(x)y=q(x)y^n}$ Put $u=y^{1-n}$ $u'+(1-n)p(x)u=(1-n)q(x)$ 即 $\frac{1}{1-n}\frac{\mathrm{d}u}{\mathrm{d}x}+p(x)u=q(x)$

换元法：
对于 $y'=f(ax+by)$型的微分方程, where f 连续 and $b\ne0$，
则令 $u=ax+by$，故 $u'=a+by'\Rightarrow y'=\frac{1}{b}(u'-a)$

微分方程的物理应用

① 牛顿第二定律，速度

$$v=\lim_{\Delta t\to0}\frac{\Delta s}{\Delta t}=s'(t)\quad a(t)=\lim_{\Delta v\to0}\frac{\Delta v}{\Delta t}=v'(t)=s''(t)$$

$$\text{加速度}a=\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{\mathrm{d}v}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}=v\frac{\mathrm{d}v}{\mathrm{d}x}$$

② 变化率问题

题型 1： 冷却定律 **Newton's law of cooling:** the rate of cooling of an object is proportional to the difference in temperature between the object and its surroundings.

$$\frac{\mathrm{d}T}{\mathrm{d}t}=-k(T-T_0)$$

题型 2： 设总人数为 N，t时刻掌握新技术的人数 x的变化率和已掌握新技术的人数和未掌握新技术人数之积成正比

$$\frac{\mathrm{d}x}{\mathrm{d}t}=+kx(N-x)$$

题型 3:： 半衰期问题：half-life of the substance is T years $y=Ae^{-kt}\Rightarrow\frac{A}{2}=Ae^{-kT}\Rightarrow k=\frac{\ln2}{T}\Rightarrow T=\frac{\ln2}{k}$

题型 4： 盐水问题：一般设 Q 为盐的质量

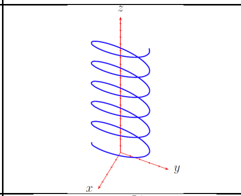
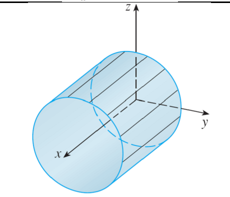
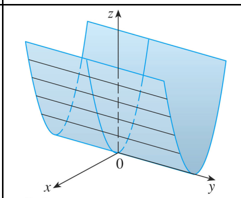
$$\frac{dQ}{dt}=P_{enter}\%v_{enter}-P_{draining}\%v_{draining}$$

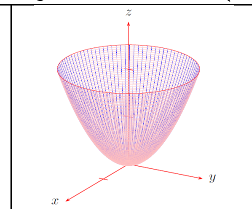
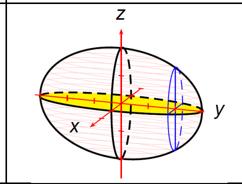
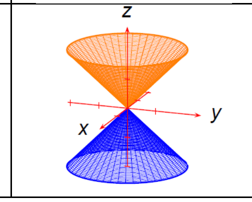
Appendix

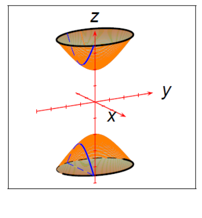
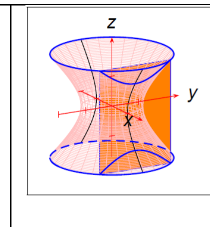
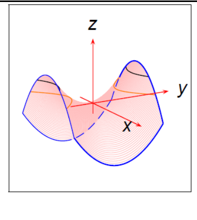
Standard Integrals

$$\int\frac{1}{ax+b}dx=\frac{1}{a}\ln|ax+b|+C$$
$$\int\tan(ax+b)\,dx=\frac{1}{a}\ln|\sec(ax+b)|+C$$
$$\int\sec(ax+b)\,dx=\frac{1}{a}\ln|\sec(ax+b)+\tan(ax+b)|+C$$
$$\int\csc(ax+b)\,dx=-\frac{1}{a}\ln|\csc(ax+b)+\cot(ax+b)|+C$$
$$\int\cot(ax+b)\,dx=-\frac{1}{a}\ln|\csc(ax+b)|+C$$
$$\int\sec^2(ax+b)\,dx=\frac{1}{a}\tan(ax+b)+C$$
$$\int\csc^2(ax+b)\,dx=-\frac{1}{a}\cot(ax+b)+C$$
$$\int\sec(ax+b)\tan(ax+b)\,dx=\frac{1}{a}\sec(ax+b)+C$$
$$\int\csc(ax+b)\cot(ax+b)\,dx=-\frac{1}{a}\csc(ax+b)+C$$
$$\int\frac{1}{a^2+(x+b)^2}dx=\frac{1}{a}\tan^{-1}\left(\frac{x+b}{a}\right)+C$$
$$\int\frac{1}{\sqrt{a^2-(x+b)^2}}dx=\sin^{-1}\left(\frac{x+b}{a}\right)+C$$
$$\int\frac{-1}{\sqrt{a^2-(x+b)^2}}dx=\cos^{-1}\left(\frac{x+b}{a}\right)+C$$
$$\int\frac{1}{a^2-(x+b)^2}dx=\frac{1}{2a}\ln\left|\frac{x+b+a}{x+b-a}\right|+C$$
$$\int\frac{1}{(x+b)^2-a^2}dx=\frac{1}{2a}\ln\left|\frac{x+b-a}{x+b+a}\right|+C$$
$$\int\frac{1}{\sqrt{(x+b)^2+a^2}}dx=\ln\left|(x+b)+\sqrt{(x+b)^2+a^2}\right|+C$$
$$\int\frac{1}{\sqrt{(x+b)^2-a^2}}dx=\ln\left|(x+b)+\sqrt{(x+b)^2-a^2}\right|+C$$
$$\int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$$
$$\int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|+C$$

课标要求空间图形

名称	图像	方程
elliptical helix		$x^2+\left(\frac{y}{3}\right)^2=1$
Cylinder (柱面) [1] 圆柱面		$y^2+z^2=1$
Cylinder (柱面) [2] 抛物柱面		$z=x^2$

Quadric Surface (二次曲面)		
Elliptic paraboloid 椭圆抛物面		$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}$
Ellipsoid 椭球面		$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ 特别的， $a=b=c$时为球面
Double Cone 椭圆锥面		$\frac{z^2}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

其他比较重要的二次曲面		
 Hyperboloid of 2 sheets $-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	 Hyperboloid of 1 sheet $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	 Hyperbolic Paraboloid $\frac{z}{c}=\frac{x^2}{a^2}-\frac{y^2}{b^2}$
双叶双曲面	单叶双曲面	双曲抛物面（马鞍面）

利用二重积分轮换对称性计算 $\int_0^{+\infty}e^{-x^2}\mathrm{d}x$

$$\text{令}I=\int_0^{+\infty}e^{-x^2}\mathrm{d}x\Rightarrow I^2=\int_0^{+\infty}e^{-x^2}\mathrm{d}x\cdot\int_0^{+\infty}e^{-x^2}\mathrm{d}x$$

$$I^2=\int_0^{+\infty}e^{-x^2}\mathrm{d}x\cdot\int_0^{+\infty}e^{-y^2}\mathrm{d}y=\iint_{\substack{0\le x<+\infty\\0\le y<+\infty}}e^{-(x^2+y^2)}\mathrm{d}x\mathrm{d}y$$

$$\underline{\text{极坐标变换}}\int_0^{\frac{\pi}{2}}\mathrm{d}\theta\int_0^{+\infty}e^{-r^2}r\,\mathrm{d}r=\frac{\pi}{4}\Rightarrow I=\frac{\sqrt{\pi}}{2}$$

二重积分的对称性化简（普通对称性）

$$\iint_D f(x,y)\mathrm{d}x\mathrm{d}y=\begin{cases}2\iint_{D_1}f(x,y)\mathrm{d}x\mathrm{d}y & f(x,y)=f(-x,y)\\ 0 & f(x,y)=-f(-x,y)\end{cases}$$

（轮换对称性） $\iint_D f(x,y)\mathrm{d}x\mathrm{d}y=\iint_D f(y,x)\mathrm{d}y\mathrm{d}x$

Fundamental Theorem of Calculus (FTC)

$$\int_a^b F'(x)\mathrm{d}x=F(a)-F(b)$$