Tutorial 1

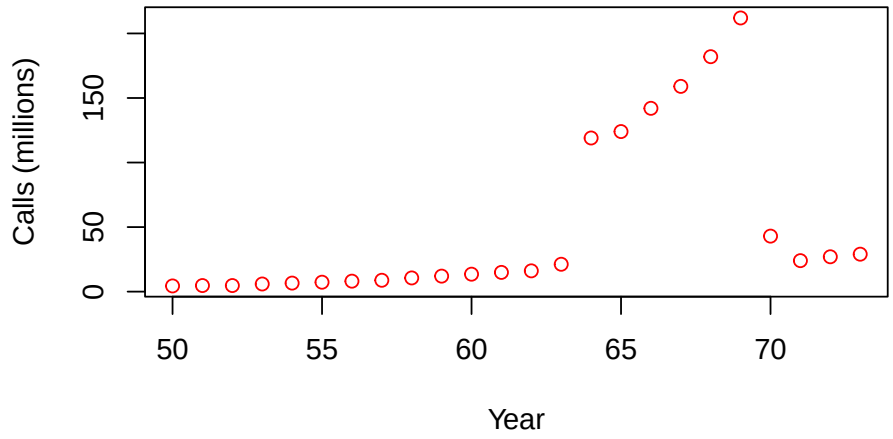
ST2137-2420

Material

This tutorial covers material from chapter 1 of the course textbook. It provides practice on basic R functions. There are many ways to code each question, so try them more than once. The following R functions may be helpful: `combn`, `median`, and `match`.

Dataset: `phones`

The dataset `phones` is available from the `MASS` package (which is installed by default with R). It contains two numeric variables, in a list format. Here is a plot of the two variables:



1. Create a dataframe `df1` with the following columns.

```
   x   y
1 50 4.4
2 51 4.7
3 52 4.7
4 53 5.9
5 54 6.6
6 55 7.3
```

2. Write this dataframe to a csv file in the `data/` folder named `phones-2420.csv`. These are the first few lines of the file:

```
"x","y"
50,4.4
51,4.7
```

```
52,4.7
53,5.9
...
```

3. Answer the following queries about the data:
    1. How many rows are there in the dataset?
    2. How many observations between 100 and 200 million calls?
    3. What are the largest 3 and smallest 3 number of calls?
    4. In which year was the largest number of calls made?

4. In R, matrix multiplication is carried out with the `%*%` operator. For instance, if we have

$$x_{1,2} = \begin{bmatrix} 1 & 1 \end{bmatrix} \quad y_{2,2} = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix}$$

    Then $x \times y$ is computed as

```r
x_mat = matrix(c(1,1), nrow=1)
y_mat = matrix(c(0, 0.5, 0.5, 0), nrow=2)
x_mat %*% y_mat
# solve(y_mat) # computes inverse of a square matrix
# t(y_may)     # returns transpose of a matrix
```

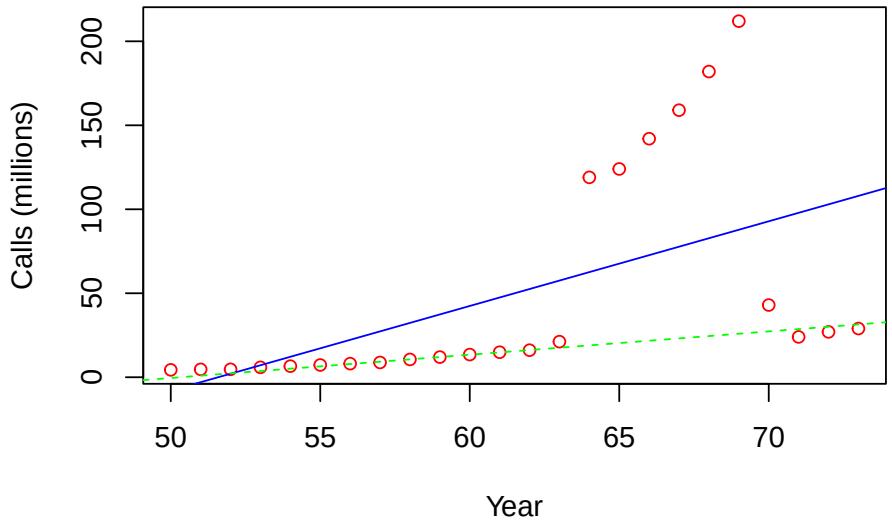
    1. Create a 24×2 matrix $\mathbf{X}$ with the first column all ones, and the second column containing the `year` vector from the `phones` data. Now create a 24×1 matrix $\mathbf{y}$ containing the `calls` column.
    2. Compute the estimate of the slope and intercept for a least-squares best fit to the above data, storing it as `beta_hat`.
$$(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$$
    3. Compute the fitted y-values, storing them as `y_hat`.
$$\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$$

5. The `lm` function performs the above computation for us in R. Inspect the output object and retrieve the parameters and the fitted values.

```r
lm_output <- lm(y ~ x, data=df1)
```

6. The fit of the line has been affected by the anomalous points. Here is an algorithm to compute a fitted line that is not so affected by those points:
    a. Generate all pairwise combinations of observations. (see `combn`)
    b. For each pair of points, compute the gradient.
    c. Compute the median over all these gradients. This returns the fitted slope.

    Write a `for`-loop that will compute this median. Compare it to the earlier slope. The figure below includes both the "usual" line and the one from the above algorith. Any thoughts on how the intercept for this new fitted line should be computed?



7. The file `phones.json` contains corrected readings for particular years. The following commands will read the data into R as a list. Replace the data in `df1` at the appropriate years with the corrected call values. *Hint: read up on* ***`match()`*** *function.*

```
library(jsonlite)
corrected_data <- read_json("data/phones.json", TRUE)
```