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RESEARCH ARTICLE

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Benchmarking commodity investments

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While much is known about the financialization of commodities, less is known about how to profitably invest in commodities. We develop a four-factor asset pricing model of commodity returns. Our four-factor model prices both commodity spot and term risk premia in an intuitive manner related to investable portfolios. The straightforward construction of our factors is an improvement over previous models. Furthermore, our four-factor model prices commodity risk premia using both sorted portfolios and risk adjusted alphas as benchmarks. Thus, we feel it is an appropriate benchmark to evaluate commodity investment vehicles.

KEYWORDS
benchmarking, commodities, spot premia, term premia

> *Like most products in the liquid alternatives space, this is not a simple plug-and-play category, where any above-average fund will suffice . . . The challenge, as always, is finding the right manager. But it doesn't help that the managed futures space is still a very long way from enabling simple and straightforward comparisons. – Investment News, Jan 14, 2015, "Managed futures funds shine anew, but mystery remains"*

> *Commodities "couldn't be hated more" . . .Four years of negative returns for indices tracking futures, with a fifth under way, have undermined the idea that leaving part of one's portfolio in a basket of oil, natural gas, soyabeans, copper and other commodities was prudent. "There's zero interest right now from the institutional space," says Lawrence Loughlin of Drobny Capital. – Financial Times, June 3, 2015, "Investment: Revaluing Commodities"*

1 | INTRODUCTION

The literature on commodities dates back at least to Keynes (1923), but most of it focuses on production and storage decisions or the role of commodities in international trade (Rouwenhorst & Tang, 2012). There is a large and growing literature around the financialization of commodities, the purported cause of which is increasing investment by finance professionals or so-called "speculators" (e.g., Cheng & Xiong, 2014). However, there has been less research about how astute investors should incorporate commodities into a diversified portfolio. Since the global capital (institutional and retail) allocated to commodities is approximately $330B, this is an important question.[1]

[1]$330B comes from investment report from Barclays Capital Commodities Research via a HewittEnnisKnupp Global Invested Capital Report, June 2014.

Indexing of commodity futures, especially equally weighted indexing, is an easily-implemented passive strategy. But this approach has yielded negative or zero returns over much of its history, and practitioners are abandoning it.[2] Figure 1 shows the poor performance of an equally weighted market index since 1987. Gorton and Rouwenhorst (2006) and Erb and Harvey (2006) have a robust discussion of the ambiguous desirability of index strategies.

Some papers have examined Commodity Trading Advisors (CTAs) (e.g., Bhardwaj, Gorton, & Rouwenhorst, 2014; Fung & Hsieh, 1997, 2000). These papers have employed diversified (i.e., including non-commodity factors) factor models because only 19% of CTAs invest exclusively in commodities, despite their name (see Table 1). Fittingly, these studies have typically been interpreted as research on hedge funds, rather than commodities (e.g., Bollen & Whaley, 2009; Kosowski, Naik, & Teo, 2007). Narrowing the portfolio decision to this 19% of CTAs that invest solely in commodities (or "commodity funds") may be the best way to incorporate commodities into a diverse portfolio, but no commodity-specific benchmark exists with which to evaluate these managers. Since Roll (1978) showed that different benchmarks can yield different rankings of "skill," independently identifying the right benchmark is a necessary first step to manager selection.

This paper establishes a parsimonious, tradeable,[3] four-factor model benchmark, with which investors can evaluate commodity fund managers (or other commodity investments, such as Exchange Traded Funds, or ETFs). Our model not only prices commodity spot risk premia, but also commodity term risk premia, identified by Szymanowska, Roon, Nijman, and ven den Goorbergh (2014). Our four-factor model fails to price only two test assets among five different four-way portfolio sorts (two spot premia, three term premia, for a total of 20 portfolios).

The four factors in our model include a market factor, a time series momentum factor, and separate high and low term premia factors, sorted on commodity basis. These factors are drawn from the extant literature and based in commodity fundamentals, and each has been shown separately to capture a risk premium embedded in commodity futures, though never together in the form we propose. While our focus is on benchmarking active managers, our factors can just as easily be thought of as composing a single or multi-factor "smart beta" commodity ETF since they are tradeable and rules-based by construction.

To establish the power of our four-factor model, we run a horserace between our model and two popular models established in the literature. The first is the popular model of Fung and Hsieh (2001), which we call the FH model. This model covers a wide variety of strategies and is intended as a descriptive model to identify the strategies used by hedge funds and CTAs. One drawback of this model is that the factors are not tradeable, making interpretation difficult. This model has also been criticized by Bhardwaj et al. (2014), who argue that the negative performance of the factors means alpha identified based on this model is spurious.[4] The second model comes from Bhardwaj et al. (2014), who include factors for commodities, interest rate derivatives, and currency futures. Since our focus is on commodities, we only test the model's commodity factors, which we call the BGR model.[5]

We find that both our model and the BGR model price spot risk premia adequately. Both estimate an alpha equal to zero for all test assets, have high adjusted R^2, and fail to reject the GRS test that all portfolio alphas are jointly set to zero (Gibbons, Ross, & Shanken, 1989). The FH model fails to price several of the spot premia test portfolios and the GRS test rejects null hypothesis of joint zero alpha for all portfolio sorts. The adjusted R^2 for the FH model is zero for all test portfolios.

Our four-factor model is the only model that can consistently price term premia. The BGR and FH models can only price 3 of 12 well (and BGR is borderline on a fourth). In contrast, our four-factor model, which includes two term premia factors, successfully prices 10 of 12 test asset portfolios. At both the 4 and 6 month horizon, a GRS test of our four-factor model fails to reject the null of zero alpha for all portfolios. At a 2 month horizon, our four-factor model prices 3 of 4 portfolios.

Until now, benchmarking commodity investments has been inhibited by a disagreement in the literature of the drivers of risk premia. Recently, however, the literature has coalesced around a few key drivers, represented by the four factors in our model. Simultaneously, increased interest in commodity investment in the past decade combined with the poor performance of passive market indexes means sophisticated investors are more interested in evaluating the performance of active commodity fund

[2]Investment: Revaluing Commodities, June 3, 2015, Financial Times. http://www.ft.com/cms/s/0/a6ff2818-094c-11e5-8534-00144feabdc0.html, also the source for the second opening quote. Also see Bhardwaj, Gorton, and Rouwenhorst (2015).

[3]We use tradeable to mean that the implementation of our factors as actual strategies is intuitively straight-forward. However, actual implementation on a given set of commodities is subject to open interest considerations, and transaction costs.

[4]In some factor models, negative factor loadings could attribute positive performance to poorly performing factors. But since the Fung–Hseih factors are not tradeable, this interpretation is not applicable.

[5]Bhardwaj et al. (2014) do not test their model, but simply assert it as capturing known trading patterns in commodities. The commodity-only version of their model is also almost identical to a model proposed and tested more thoroughly in a working paper by Bakshi et al. (2014), thus we can refer to it as the "BGR" model and use that to refer to both papers.

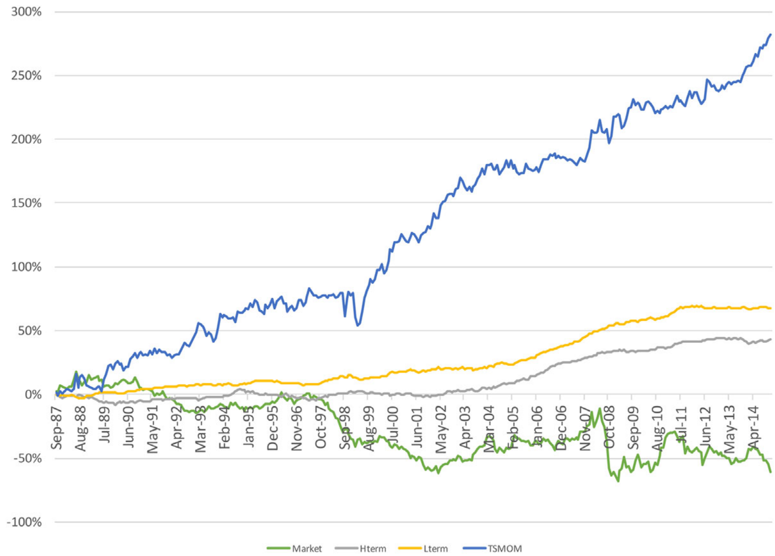

FIGURE 1 Cumulative returns of four factors. Plotted is the cumulative return of the four factors. Market is an equally weighted portfolio of all commodity futures sport returns. H_{term} and L_{term} are portfolios sorted on basis, with high defined as above median and low defined as below median set of high and low commodities, and reflect the equal-weighted average return to calendar spreads that are long 2-, 4-, and 6-month maturity futures and short the spot contract. Cumulative returns are computed by summing returns starting in September 1987, and thus remove the effect of compounding [Color figure can be viewed at wileyonlinelibrary.com]

managers. Financial advisors have even suggested that individuals include commodities in their personal asset allocation.[6] Yet, to our knowledge, there is not a thoroughly tested and established benchmark to evaluate commodity fund managers or commodity ETFs.

While commodity investment often is included as a subset of the hedge fund/CTA literature, there are more similarities between commodity markets and equity markets than between commodities and hedge funds. Both commodities and equities are publicly traded with public closing prices, providing clear, end-of-day portfolio values. Both have a clearly identified regulatory body (the CFTC and SEC). Both represent a defined investment set within which a manager (Commodity Fund or Mutual Fund) must choose either long or short positions. Given these similarities, our paper may be viewed as establishing a factor model benchmark for Commodity Funds in a manner similar to the Fama and French (1993, 2015) work for equity Mutual Funds.

2 | A FACTOR MODEL OF COMMODITY RETURNS

In this first section, we select factors already established in the literature and adjust them for use in benchmarking monthly commodity fund returns. Therefore, while each of our factors has some precursor in the literature, to our knowledge, nobody has combined them together into a single, parsimonious, benchmark, and tested the combined performance at pricing commodity futures spot and term premia.

We follow the general contribution of Szymanowska et al. (2014) and use multiple term premium factors to account for the futures basis. However, we show that two summarized term premium factors are sufficient instead of the six proposed in that paper. Furthermore, we show that a spot basis factor is redundant. We then add a market factor, which is an equal-weighted portfolio of all commodity futures and is common in the commodities literature. We also include a time series momentum factor, which is also present in several commodities papers (e.g., Miffre & Rallis, 2007; Moskowitz, Ooi, & Pedersen, 2012), but is in contrast with cross-sectional momentum common in equities (Jegadeesh & Titman, 2011) and in some commodity models (e.g., the BGR model in both Bakshi, Gao Bakshi, and Rossi (2014) and Bhardwaj et al. (2014). We discuss the differences in these two momentum factors in detail later in this section. Finally, we use a monthly time series of commodity futures returns, in contrast to Szymanowska et al. (2014), who use bimonthly returns and holding periods of up to 8 months. Overall, these adjustments result

[6]“Speculating on commodities can add diversity to your portfolio,” *The Financial Times*, June 16, 2015. http://www.ft.com/intl/cms/s/2/eee82070-ea99-11e4-96ec-00144feab7de.html

TABLE 1 List of commodity trading advisor categories

Categories	Unique funds in dataset	% of funds in dataset
Commodities		
Agriculture	87	2.9
Energy	49	1.7
Financial/metals	435	14.7
Commodities subtotal	571	19.3
Currency	421	14.2
Interest rates	29	1.0
Stock index	184	6.2
Other future subtotal	634	21.4
Arbitrage	50	1.7
Discretionary	53	1.8
Option strategies	153	5.2
Systematic	76	2.6
Diversified	1,196	40.4
No category	228	7.7
General strategies subtotal	1,756	59.3
Total funds in datasets	2,961	100

Listed are the categories for CTAs available in Barclay Hedge, along with counts of unique funds included in each category. They are summarized by strategy. Data from Barclay Hedge obtained monthly from December 2006 through December 2014 and verified to be free of the graveyard bias identified in Bhardwaj et al. (2014). CTA summary includes both active and dead funds.

in fewer factors that are easier to implement, more tradeable, and have comparable explanatory power. We next discuss our data before describing these factors in more detail.

2.1 | Commodity risk premia and motivation for factor selection

what is factor selection

Explaining commodity risk premia dates back at least to Keynes (1923), who proposed a theory of "normal backwardation," in which short hedgers of commodities outnumber long hedgers such that natural hedgers are net short. Thus, the assumed natural state of the market is for futures prices to be lower than expected future spot prices to give speculators a positive expected return for assuming the price risk.[7] In this common "insurance" view of commodity futures risk premia, commodity futures traders accept price risk from hedgers in exchange for a risk premium. Rouwenhorst and Tang (2012) survey the extensive literature and conclude that evidence for this theory is weak.

Keynes' theory predates modern asset pricing theory, embodied in the capital asset pricing model. Early studies find little evidence that this model applies to commodities markets (e.g., Carter, Rausser, & Schmitz, 1983; Dusak, 1973), and recent studies by Gorton and Rouwenhorst (2006), Rouwenhorst and Tang (2012), and Erb and Harvey (2006) confirm these findings. The explanation of commodity risk premia in the context of the capital asset pricing model remains an open question in the commodities literature, and the literature moved to arbitrage pricing models.[8]

The strongest empirical evidence around commodity risk premia associates inventory with commodity risk premia in the context of the theory of storage, which dates back to Kaldor (1939), Working Holbrook (1949), and Brennan (1958). This theory links commodities futures prices to the storage decisions of inventory holders, in terms of financing and warehousing costs net a convenience yield. Gorton, Hayashi, and Rouwenhorst (2012) investigate the fundamentals of commodity investing and find that inventory and storage are the key fundamentals in pricing commodity risk premia and that both correlate with the commodity futures' basis. This finding features prominently in our benchmark four-factor model.

[7] Note that normal backwardation (futures price < expected future spot price) differs from backwardation (futures price < current spot price).

[8] Some literature relates forward and futures premia to the consumption capital asset pricing model (e.g., see Cooper, 1993), which reports that forward and futures contracts respond to time-varying risk premium formulations.

Finally, and most recently, Szymanowska et al. (2014) investigate the term structure of commodity risk premia and show the existence of term premia in commodity futures. They show that factors derived from a sort on futures basis can explain these premia, but use two factors per maturity at 2, 4, and 6 months for a total of six factors. We interpret these factors as capturing expected future changes in commodity inventories given the link between basis and inventory levels established by Gorton et al. (2012).

2.2 | Data and computation of futures premia and returns

We use 21 different commodity futures from Commodity Systems Inc. that represent all major sub-sectors of commodity markets (i.e., energy, agricultural, and metals). The contracts include Soybean Oil, Corn, Cocoa, Light Crude Oil, Cotton, Gold, Copper, NY Harbor ULSD (Heating Oil), Coffee, Lumber, Hogs, Oats, Orange Juice, Soy Beans, Silver, Soy Meal, Wheat (CBT only), Feeder Cattle, Live Cattle, Gasoline RBOB, and Rice Rough for the period between September 1987 and December 2014. Table 2 provides information about Bloomberg codes and exchanges associated with each futures market.

In constructing our factors, we follow convention and consider the spot price to be the price of the contract nearest to expiration and expiring at least 2 months from the current month. This avoids liquidity problems, which can plague the pricing of shorter maturity contracts. It also makes replicating any of our factors with actual trading much cheaper than using the actual spot market would cost.[9] The 2, 4, and 6 month contracts are then defined as the first contract to expire at least 2, 4, and 6 months after the spot contract expires.[10] From the commodity price series, we construct several variables from which all the model's factors are constructed. We define the spot premium of the commodity as the month to month change in the logarithm of the spot price, $s_i(t)$. Therefore, the realized spot premium of commodity i at time t, $\hat{\pi}_{s,i}(t)$, is defined as

$$\hat{\pi}_{s,i}(t) = \ln[S_i(t)] - \ln[S_i(t-1)]. \tag{1}$$

As is standard in the literature, this premium formulation excludes all returns on the required collateral. Intuitively, these returns are comparable to returns in excess of the risk-free rate, because collateral is typically reinvested at that rate.

The n-month basis for commodity i at time t, $y_i^n(t)$ is defined as the logarithm of the ratio of the n-month futures price $f_i^n(t)$ to the spot price. Generally, the n-month maturity term premium $\hat{\pi}_{y,i}^n(t)$ is defined as the change in this value:

$$\hat{\pi}_{y,i}^n(t) = y_i^n(t) - y_i^{n*}(t-1) = \ln\left[f_i^n(t)/f_i^{n*}(t-1)\right] - \ln[S_i(t)/S_i(t-1)] \tag{2}$$

This may be thought of as the return to a calendar spread, which is computed by buying the n-month futures contract and shorting the spot futures contract. The futures returns themselves may be written as[11]

$$r_{f,i}^n(t) = \ln\left[f_i^{n-1}(t)\right] - \ln\left[f_i^n(t-1)\right]. \tag{3}$$

The cost-of-carry relationship for the futures markets allows us to break the n-month expected futures return for commodity i into a spot premium and a term premium.[12] The cost of carry model may be defined as

$$f_i^n(t) = s_i(t)e^{\int_t^{t+n} y_i(\tau)d\tau} \tag{4}$$

where $y(t)$, the time t instantaneous cost of carry includes the risk-free rate, the storage rate for the commodity i, and a generally negative rate known as the convenience yield. The spot price $s_i(t)$ is the true underlying commodity price, and $f_i^n(t)$ is the futures price with maturity n. The total cost of carry over the life of the contract is summarized by the basis, defined as

[9]Using a conservative estimate of nine basis points per side to trade, we estimate any of our model's factors can be implemented for a cost of 8–12 basis points per month, which is a fairly modest cost.

[10]As a simple example, corn has contracts expiring in months 3, 5, 7, 9, and 12. In October (10), the spot contract will be December (12), the 2-month contract will be March (3), the 4-month contract May (5), and the 6-month contract July (7). Some commodities also have monthly expirations, in which case some expiration months would be skipped on any given date. This approach is similar to Szymanowska et al. (2014) and is typical in the commodities literature.

[11]n does not always decrement to $n-1$, because contracts do not necessarily expire every month.

[12]Erb and Harvey (2006), Routledge, Seppi, and Spatt (2000), and Fama and French (1987) establish a link between basis and commodity futures risk premia.

TABLE 2 List of commodities included in study

Name	Exchange	BB symbol	CSI data symbol
Corn	CBOT	C	C2
Rice rough	CBOT	RR	RR2
Lumber	CME	LB	LB2
Wheat	CBOT	W	W2
Oats	CBOT	O	O2
Coffee	ICE−US	KC	KC2
Cocoa	ICE−US	CC	CC2
Cotton	ICE−US	CT	CT2
Hogs lean	CME	LH	LH
Soybean oil	CBOT	BO	BO2
Orange juice	ICE−US	OJ	OJ2
Silver	COMEX	SI	SI2
Gold	COMEX	GC	GC2
Soybean	CBOT	S	S2
Feeder cattle	CME	FC	FC
Cattle live	CME	LC	LC
NY harbor ULSD	NYMEX	HO	HO2
Crude oil light	NYMEX	CL	CL2
Soybean meal	CBOT	SM	SM2
Copper HG	COMEX	HG	HG2
Gasoline RBOB	NYMEX	XB	RB2

Column 1 is the name, column 2 is the exchange on which they are traded, column 3 is the Bloomberg (BB) symbol, and column 4 is the Commodity Systems, Inc. (CSI) symbol.

$$y_i^n = \ln\left[f_i^n(t)\right] - \ln[s_i(t)] = \int_t^{t+n} y_i(\tau)d\tau. \tag{5}$$

Taking derivatives and rearranging yields the equation

$$d\ \ln\left[f_i^n(t)\right] - d\ \ln[s_i(t)] + y_i^n. \tag{6}$$

If we now consider small discrete time changes (so that Equation 6 is still approximately correct), then we can write the expected spot premium as

$$\pi_{s,i}(t) = E_t\left[\ln\left(s_i(t+1) - \ln\left(s_i(t) - y_i^1(t)\right]\right.\right., \tag{7}$$

and the expected term premium as

$$\pi_{y,i}^n(t) = E_t\left[y_i^{n-1}(t) + y_i^1\right] - y_i^n. \tag{8}$$

Equation 7 gives the premium as the difference between the expected change in the spot price and the one-period basis. Equation 8 gives the premium as the deviation from the expectation hypothesis.

We can now expand Equation 6 to expected futures return as

$$E_t\left[r_{f,i}^n(t+1)\right] = E_t\left[f_i^{n-1}(t+1) - f_i^n(t)\right] = E\left[\left(s_i(t+1) + y_i^{n-1}(t+1) - s_i(t) - y_i^n(t) + y_i^1(t) - y_i^1(t)\right]\right.. \tag{9}$$

This reduces into

$$E_t\left[r^n_{f,i}(t+1)\right] = \pi_{s,i}(t+1)\pi^n_{y,i}(t+1), \tag{10}$$

which is in terms of risk premia. Recall that we define the spot commodity as the nearest term futures contract, and this definition includes the true spot price plus the one-period cost of carry. Thus, our spot premium and term premia measures correspond to realizations of the premia in Equation 10.

2.3 | Factor selection and construction—spot premia

We consider factors for each premium in turn, starting with the spot premium. We first include a market factor (MKT), which is an equally weighted average of all commodities' one period spot return. Our market factor is

$$\text{MKT}(t) = \frac{1}{N}\sum_{i=1}^{N}\hat{\pi}_{s,i}(t) \tag{11}$$

where N, the number of total commodities, is 21. This follows the well-known results of Erb and Harvey (2006) and Gorton and Rouwenhorst (2006) who show the value of an equal-weighted market index.

Next, we consider momentum factors, shown necessary by Gorton et al. (2012), among others.[13] First, we consider a time series momentum factor (TSMOM) as in Moskowitz et al. (2012), which is the difference in return between an equally weighted portfolio of commodities with a positive return over the previous 12 months and one with a negative return over the previous 12 months. Specifically, we define momentum as

$$\text{TSMOM}(t) = \left[\frac{1}{N_{\text{pos}}}\sum_{i\in P}\left[\hat{\pi}_{s,i}(t)\right] - \frac{1}{N_{\text{neg}}}\sum_{i\in L}\left[\hat{\pi}_{s,i}(t)\right]\right] \tag{12}$$

where neg and pos refer to the set of commodities with positive and negative trailing 12-month returns. N_{pos} and N_{neg} refer to the number of commodities in each respective group.

Time series momentum differs from typical momentum measures (sometimes called cross-sectional momentum) in the selection of the high and low momentum portfolios. Time series momentum sorts based on sign: only positive momentum stocks are placed in the high momentum portfolio; only negative momentum stocks are placed in the low momentum portfolio. We also consider cross-sectional momentum which uses a ranking process, taking those in the top decile as the high momentum portfolio and the bottom decile as low momentum. This MOM factor is computed as

$$\text{MOM}(t) = \frac{1}{N_g}\left[\sum_{i\in H}\hat{\pi}_{s,i}(t) - \sum_{i\in L}\hat{\pi}_{s,i}(t)\right], \tag{13}$$

where H is the set of High group commodities when sorted on past 12 months return, and L is the set of Low group commodities when sorted on past 12 month return, and N_g is equal to the number of commodities in each group. The two groups in MOM have the same size by definition, in contrast to TSMOM, which allows differently sized groups.

Conceptually, cross-sectional momentum is capturing relative momentum in the cross section, whereas time series momentum captures each security's own trend. In equity markets, the differences are not material, because the large pool of securities ensures that a High cross-sectional momentum portfolio is constructed almost entirely of stocks with positive momentum, and the Low cross-sectional momentum portfolio is constructed almost entirely of stocks with negative momentum. However, in commodity, foreign exchange, and equity index futures markets, where there are significantly fewer securities (e.g., 21 in our sample), time series momentum can differ greatly from cross-sectional momentum. This happens when, for instance, there are only one or two futures contracts with either positive or negative momentum. In those cases, the Low cross-sectional momentum portfolio could contain contracts with zero or positive momentum, and the High cross-sectional momentum portfolio

[13]Others who find evidence for some type of momentum factor include Erb and Harvey (2006), Asness, Moskowitz, and Pedersen (2013), Fuertes, Miffre, and Rallis (2010), and Miffre and Rallis (2007).

could contain contracts with zero or negative momentum. This situation occurs because the sorting does not filter on sign, it simply ranks from high to low.

We confirm that time-series momentum is a much better predictor of commodity risk premia than cross-sectional momentum, in line with the literature (e.g., Baltas & Kosowski, 2012; Miffre & Rallis, 2007; Moskowitz et al., 2012). This choice of the momentum factor is one of the differences between our model and that of Bakshi et al. (2014) and Bhardwaj et al., (2014). While we choose the time series momentum factor, they both use cross-sectional momentum.

Szymanowska et al. (2014) motivate and derive a high-minus-low factor to explain spot premia. Specifically, the HML factor is defined as

$$\mathrm{HML}(t) = \frac{1}{N_g}\left[\sum_{i\in H}\hat{\pi}_{s,i}(t) - \sum_{i\in L}\hat{\pi}_{s,i}(t)\right] \tag{14}$$

where H is the set of commodities with a spot return in the High group when sorted on basis, L is the set of commodities with a spot return in the Low group when sorted on basis, and N_g is the number of commodities in each group. Bhardwaj et al. (2014) split the commodities at the median, and since we have 21 total commodities, N_g is 10. Bakshi et al. (2014) derives a similar factor but set N_g equal to 5. The HML factor we estimate follows the former definition using the median, but our results are robust to this choice. Note that Equations 13 and 14 appear identical. The difference is the sorting variable: past 12 month return in Equation 13 versus basis in Equation 14.

Overall, we consider four possible spot premia factors: MKT, TSMOM, MOM, and HML. We conclude, ultimately, to only include MKT and TSMOM in addition to the two term premia factors discussed in the next section. We present the results justifying this choice later with asset pricing tests. Rouwenhorst and Tang (2012) summarize a long and growing literature of commodities factors that price commodity returns. We do not comprehensively test all possibilities, but rather focus on the factors that most frequently show up as important in pricing commodity risk premia.

find out what above medial basis is

2.4 | Factor selection and construction—term premia

We next consider the term premium. To price the term premium, we choose two factors. First, we construct a high-term premium factor (H_{term}) consisting of the average of the 2-, 4-, and 6-month realized term premia for the 10 commodities with above-median basis (as previously defined in the HML factor). We also construct a low-term premium factor (L_{term}), computed the same way as H_{term}, except using the 10 commodities with below-median basis. These two factors are defined as

$$\begin{aligned} H_{\text{term}}(t) &= \frac{1}{N_g}\sum_{i\in H}\left[\frac{1}{3}\sum_{n=2,4,6}\hat{\pi}^{n}_{y,i}(t)\right] \\ L_{\text{term}}(t) &= \frac{1}{N_g}\sum_{i\in L}\left[\frac{1}{3}\sum_{n=2,4,6}\hat{\pi}^{n}_{y,i}(t)\right], \end{aligned} \tag{15}$$

where H is the set of commodities with above-median basis, and L is the set of commodities with below-median basis. N_{g} is the number of commodities in each group, which is 10.

These two factors follow the intuition of Szymanowska et al. (2014), who also construct their longer term basis factor as separate high and low factors to explain commodity term premia. However, Szymanowska et al. (2014) compute their term structure basis factor using so called "spreading" returns that span the maturity difference of the computed term premia. Thus, to explain 2-, 4-, and 6-month term premia, they require three H factors and three L factors, each with maturities matching those three holding periods. This approach has two implications. First, it is not obvious that a set of factors designed to explain multi-month holding period returns will adequately explain 1-month returns when applied to CTAs or ETFs. Second, including six additional factors in a single benchmark model for commodity funds is unwieldy and likely redundant. Our goal is to preserve the economic intuition and econometric relationships while creating factors with more practical appeal. Our results demonstrate that our factors, though simpler, maintain power in explaining futures returns. Intuitively, our term premium factors capture the equally weighted average of the "expected change in spot basis" across different time horizons. Because of the link between basis and inventories, this naturally maps to trader information about the expected evolution of commodity inventories, which is a key fundamental driver of commodity risk premia.

The various high/low factors may cause confusion. Therefore, to summarize, HML is a factor based on the spot basis meant to capture spot return differences between commodities that have high basis versus those with low basis. H_{term} and L_{term} operate on the forward term premia, and are meant to explain the term premia. We, as have other authors, find that considering each side separately has more power than does considering the difference.

Table 3 summarizes our factors as well as the factors in our competing models (the FH model and BGR model, discussed later). For all asset pricing tests, we apply Newey and West (1987) corrections for heteroskedasticity and autocorrelation with 12 lags, because there is a pronounced seasonal effect in commodities (Gorton et al., 2012). The monthly excess return for almost every factor is modestly positive, but statistically different from zero. Only the market factor and Fung–Hsieh Commodity Factor (FHCOM) are negative, but both are insignificant.

Additionally, Figure 1 shows a time series plot of the performance of our factors. Bhardwaj et al. (2014) critique Fung and Hsieh (2001) because the poor performance of their factors can spuriously indicate a fund delivers alpha. A possible explanation for this resides in the fact that the Fung and Hsieh's factors are not tradeable and therefore cannot be shorted (i.e., cannot have negative coefficients). Our factors do not have this problem. First, as seen in Figure 1, all our factors (excluding the MKT factor) show positive performance since 1987 and for most sub-periods as well. There is a clear, consistent, upward trend. Second, our factors mirror tradeable strategies, meaning an astute investor could trade them long or short.

3 | ASSET PRICING TEST CRITERIA, PORTFOLIOS, AND ALTERNATIVE MODELS

Although the literature provides compelling evidence for the factors (in some form) in our factor model, it has not been tested in the form we propose. Indeed, the only other paper we know of that tests a commodity factor model is current work by Bakshi et al. (2014). To set up the tests of our factor model, we now describe our test criteria, the test asset portfolios employed, and the alternative models against which we evaluate the performance of our model. We then justify the selection of our particular four factors.

TABLE 3 Model factors summary statistics

				Cross−correlations					
Factors	**Monthly excess return (%)**	**Std dev (%)**	***T*-stat for mean = 0**	**MKT**	**HML**	**TSMOM**	H_{term}	L_{term}	**MOM**
Panel A: September 1987–December 2014									
MKT	−0.19	3.46	−0.97	1.00					
HML	0.60	3.46	3.12	0.03	1.00				
TSMOM	0.86	4.39	3.55	0.20	0.39	1.00			
H_{term}	0.13	0.73	3.22	−0.19	−0.35	−0.19	1.00		
L_{term}	0.21	0.63	5.94	−0.36	0.40	0.08	0.11	1.00	
MOM	0.84	5.95	2.56	0.13	0.51	0.80	−0.23	0.20	1.00

				Cross−correlations						
Factors	**Monthly excess return (%)**	**Std dev (%)**	***T*-stat for mean = 0**	**MKT**	**HML**	**TSMOM**	H_{term}	L_{term}	**MOM**	**FHMOM**
Panel B: January 1994–December 2014										
MKT	−0.21	3.72	−0.90	1.00						
HML	0.64	3.33	3.05	0.05	1.00					
TSMOM	0.87	4.54	3.03	0.21	0.43	1.00				
H_{term}	0.18	0.73	3.86	−0.16	−0.34	−0.15	1.00			
L_{term}	0.24	0.67	5.65	−0.38	0.38	0.09	0.17	1.00		
MOM	0.85	5.97	2.26	0.12	0.51	0.79	−0.14	0.20	1.00	
FHCOM	−0.24	14.21	−0.26	0.01	0.02	0.04	−0.08	0.01	0.05	1.00

This table reports summary statistics and the cross-correlations of six candidate factors to explain the cross section of commodity risk premia. The market factor is an equally weighted average of all futures contracts. The high-minus-low (HML) factor is the difference between the above- and below-median portfolios sorted on spot basis. The time series momentum (TSMOM) is an equally weighted return of commodities with positive 12-month trailing return less those with a negative trailing 12-month return. H_{term} and L_{term} are constructed from three equally weighted calendar spread portfolios of 2, 4, and 6 months, split on the median basis for high and low. Cross-sectional momentum (MOM) is an equally weighted return of commodities with above median 12-month trailing return less those with below median 12-month trailing return. FHCOM is the Fung–Hsieh primitive trend following commodity factor from David Hsieh's website. Panel B is subset to start in 1994 because that is when FHCOM is first available.

3.1 | Competing models, tests, and premium statistics

A factor model that prices commodity returns should have an intercept of zero, on average: that is, there should be zero alpha, both economically and statistically. For each portfolio of commodity futures, we report the alpha and t-statistics. For the entire sorted set of portfolios, we also report the GRS p-value from Gibbons et al. (1989), which tests the joint hypothesis that all estimates of alpha are zero for the set of test portfolios. We additionally require that a factor model pricing commodity returns should have a high R^2: that is, it explains a large amount of the variation in the test asset portfolios (Bollen, 2013). We report adjusted R^2 for each test portfolio since the benchmark models have different numbers of explanatory factors.

Our test assets are portfolios sorted on basis and on momentum. Because the literature has converged on basis and momentum as the two key characteristics explaining commodity returns, we focus on those two for brevity.[14] Table 4 provides summary statistics for the test portfolios. Panel A shows spot premia sorts on basis and momentum. Both portfolio sets are monotonically ordered with statistically significant high-minus-low portfolio returns of 0.84%, with t-statistics of 2.7 (Basis) and 2.56 (Momentum). Panel B shows term premia sorts on basis for 2-, 4-, and 6-month term premia. They do not show the same ordering, but almost all portfolios show positive alpha significantly different from zero.

For comparison, we run a horserace against two candidate models in the literature. The FH model is one of the most popular models in the hedge fund/CTA literature. Since we are focused on commodities and not the universe of possible assets available to hedge funds, our FH model only includes their primitive trend following factor for commodities, called FHCOM in Table 7. Other factors related to interest rates, emerging market equities, equity options, currencies, etc. do not apply and only add noise. We obtain this factor from David Hsieh's website.

The second model we called the BGR model, which we now define more rigorously. This is a three-factor model containing the MKT, HML, and cross-sectional MOM factor with two groups, split at the median. This model is asserted (without testing) in Bhardwaj et al. (2014) as an appropriate CTA benchmark, along with additional factors for equity options and currencies. Again, we omit these unrelated, latter factors since our focus is on commodities. Bakshi et al. (2014) uses this same factor but uses the top and bottom five set of commodities to define H and L, respectively, thus setting N_g equal to 5. Since both sets of authors result can be abbreviated as "BGR," we use the abbreviation collectively to refer to both of these models, using the MOM definition from Bhardwaj et al. (2014) with N_g equal to 10.[15]

3.2 | Four factors versus more factors

Finally, we explain why we have a model with four factors and not every factor listed in Table 3. In a nutshell we find some of the factors redundant. This is documented in Tables 5 and 6. In Table 5 we look at the regression of selected spot factors against the other spot premium based factors (for brevity we do not list every possible regression, though we did perform all of them). The first row shows that MOM is redundant compared to TSMOM, HML, and MKT, given a t-statistic of −1.34. Row 2 confirms this finding with only MKT and TSMOM as explanatory variables, giving an intercept of −0.10% and a t-statistic of −0.56. However, row 3 shows that TSMOM is not redundant since it has a positive and significant intercept with t-statistic of 3.07. Removing HML does not change this conclusion regarding TSMOM, as shown in row 4. Finally, row 5 shows that HML is almost redundant with a t-statistic of 1.86, but this is close to the 5% critical value of 1.96, so we reserve judgment for now until we include our two term structure factors based on basis sorts (and thus HML). Recall from Table 3 that the correlation between HML and H_{term} is −0.35 and between HML and L_{term} is 0.40 (in Panel A). This univariate result previews our finding that HML is unnecessary.

In Table 6, we again run regressions of factors on each other to determine if the information collectively contained in subsets fully explains other factors. This time we include the term premium based factors. In Panel A, we test if the two additional term factors drive out any existing factors. Row 1 and 2 of Panel A shows that the MOM factor is still redundant, confirming earlier results. TSMOM still belongs, as shown by the positive and significant intercept in Row 3. Row 4 now indicates the redundancy of HML once H_{term} and L_{term} are included. The intercept of 12 bps and t-statistic of 0.76 clearly show that HML is not necessary to explain alpha. Finally, in Panel B, we see that both H_{term} and L_{term} are not redundant. The intercept is positive and significant for a wide variety of explanatory variables. Thus, we conclude that our four factors are both necessary and sufficient to capture the variation and alpha in commodity futures returns.

[14] Szymanowska et al. (2014) rigorously test a variety of other test assets based on other fundamentals, such as inflation, liquidity, and open interest. These additional tests do not materially change their conclusions.

[15] In the Appendix we also consider the five factor model of Fama and French (2015). Although not a commodity model we agree with an anonymous reviewer who said it might be interesting to compare its ability with that of other models.

TABLE 4 Portfolio performance, sorted on basis and momentum: September 1987 to December 2014

Basis portfolios	Monthly excess return (%)	Standard deviation (%)	*t*-stat for mean = 0	Momentum portfolios	Monthly excess return (%)	Standard deviation (%)	*t*-stat for mean = 0
Panel A							
B1 (bottom)	−0.58	4.54	−0.23	M1 (bottom)	−0.65	4.70	−2.50
B2	−0.42	4.53	−1.68	M2	−0.38	4.09	−1.67
B3	−0.03	4.33	−0.15	M3	0.04	4.09	0.20
B4 (top)	0.26	4.84	0.97	M4 (top)	0.19	5.22	0.67
B4-B1	0.84	5.61	2.70	M4-M1	0.84	5.95	2.56

Basis portfolios	Monthly excess return (%)	Standard deviation (%)	*t*-stat for mean = 0
Panel B			
Panel A. 2 month term premia			
B1 (bottom)	0.18	0.63	5.17
B2	0.10	0.48	3.86
B3	0.07	0.50	2.38
B4 (top)	0.21	0.98	3.78
Panel B. 4 month term premia			
B1 (bottom)	0.26	1.02	4.59
B2	0.16	0.74	4.00
B3	0.05	0.74	1.15
B4 (top)	0.23	1.51	2.76
Panel C. 6 month term premia			
B1 (bottom)	0.37	1.37	4.95
B2	0.17	0.95	3.23
B3	0.01	0.91	0.28
B4 (top)	0.28	1.85	2.70

This table reports summary statistics for portfolios sorted based on basis and momentum. Basis is computed as the log of the ratio of the nearest-dated contract and next-nearest dated contract. Momentum is computed based on excess return over the past year. Each portfolio contains on average of five commodities and is rebalanced monthly. Monthly excess return is computed as the return on the futures contract, without adding in any returns because of collateral reinvestment, and as such is net of the risk-free rate. Panel A shows spot premia, Panel B term premia.

Finally, to recap our model in clear contrast to these two alternatives, our four factors are as follows: a MKT factor, a TSMOM factor, and two basis term premia factors, H_{term} and L_{term}. Thus, we differ from the FH model by including MKT, H_{term}, and L_{term}, and using a TSMOM factor to capture trend-following behavior instead of the lookback option approach in Fung and Hsieh (2001). We differ from the BGR model by using TSMOM instead of MOM, and omitting the single HML in favor of H_{term} and L_{term}, and are similar in that we both use the MKT factor.

4 | PRICING COMMODITY RISK PREMIA

We now present results for our asset pricing tests, starting with spot risk premia. We then move to term premia. Finally we present evidence confirming that separate high and low term factors perform better than a single HML_{term} factor.

4.1 | Pricing tests on spot premia portfolios

Table 7 presents results for each of the three candidate models tested: the four-factor model, the BGR model, and the FH model. Tests are on basis-sorted portfolios B1–B4 in Panel A and momentum-sorted portfolios M1–M4 in Panel B. Our four-factor model performs well, pricing all test assets in both Panels A and B. Alphas range from about 0 to 26 bps in absolute value and

TABLE 5 Tests for redundancy among factors in factor model—spot premia based factors

Dependent variable	Intercept (%)	*t*-statistic	Adj-R2	Independent variables
MOM	−0.23	−1.34	0.68	MKT, HML, and TSMOM
MOM	−0.10	−0.56	0.63	MKT and TSMOM
TSMOM	0.40	3.07	0.64	MKT, HML, and MOM
TSMOM	0.61	3.31	0.19	MKT and HML
HML	0.31	1.86	0.15	MKT and TSMOM

Data include 21 commodities' monthly spot returns from September 1987 to December 2014. This table reports results from regressions of the factors on each other to test for redundancy. If a factor has an intercept no different from zero, then it is redundant. Momentum (MOM) is a cross-sectional momentum factor not used in our final model. It is defined as the top-quartile portfolio less the bottom-quartile portfolio of commodities sorted on the previous 12 months of spot returns. The market (MKT) factor is an equally weighted average of all futures contracts. The high-minus-low (HML) factor is the difference between the above- and below-median portfolios sorted on spot basis. Basis is computed as the log of the ratio of the nearest-dated contract and next-nearest dated contract. Time series momentum (TSMOM) is an equally weighted return of commodities with positive 12-month trailing return less those with negative trailing 12-month return. Spot returns are based on the nearest-dated contract. *T*-statistics are computed based on standard errors with a Newey–West correction of 12 lags.

t-statistics range from 0.03 to 1.80 in absolute value. R^2 ranges from 0.64 to 0.81, and the GRS *p*-values are 72.89% and 60.54% for Panels A and B, respectively.

Next, we turn to the BGR model. This model successfully prices all the test asset portfolios. In Panel A, monthly alpha ranges from 8 to 12 bps in absolute value, and *t*-statistics range from 0.63 to 0.73 in absolute value. Adjusted R^2 is between 0.73 and 0.79. The GRS *p*-value is 95.65%, supporting the hypothesis that joint alpha is statistically zero. In Panel B, the results are not as strong in magnitude, but statistically give the same conclusion. Monthly alphas range from 6 to 25 bps in absolute value, and *t*-statistics from 0.54 to 1.72 in absolute value, thus indicating that alphas are statistically zero. The adjusted R^2 varies from 0.62 up to 0.90, and the GRS *p*-value is 65.57%, again supporting the hypothesis that joint alpha is zero.

The FH model can price most of the individual portfolios (6 of 8), but finds significant and economically large negative monthly alpha in portfolios B1 (−68 bps monthly) and M1 (−61 bps monthly). It also rejects the null hypothesis of each portfolio's alpha jointly set to zero with the GRS test, since the *p*-value is less than 5%. Most importantly, the adjusted R^2 for all portfolios is 0–2 significant figures. This means that this factor model has zero explanatory power of these portfolio returns.

Overall, our Four-factor model performs very well. The magnitudes of alpha are larger than BGR, but all are insignificant statistically. The adjusted R^2 and the GRS *p*-values give somewhat mixed results in terms of ordering. However, it is clear both

TABLE 6 Tests for redundancy among factors in four-factor model

Dependent variable	Intercept (%)	*t*-statistic	Adj-R2	Independent variables
Panel A				
MOM	−0.28	−1.56	0.68	MKT, HML, TSMOM, H_{term}, L_{term}
MOM	−0.23	−1.31	0.66	MKT, TSMOM, H_{term}, L_{term}
TSMOM	0.63	3.06	0.18	MKT, HML, H_{term}, L_{term}
HML	0.12	0.76	0.40	MKT, TSMOM, H_{term}, L_{term}
Panel B				
H_{term}	0.17	3.30	0.15	MKT, HML, and TSMOM
L_{term}	0.15	3.11	0.29	MKT, HML, and TSMOM
H_{term}	0.12	3.08	0.19	MKT, HML, and TSMOM L_{term}
L_{term}	0.12	3.04	0.33	MKT, HML, and TSMOM H_{term}
H_{term}	0.17	3.33	0.15	MKT, HML, MOM
L_{term}	0.15	3.07	0.29	MKT, HML, MOM

This table reports results from regressions of the factors on each other to test for redundancy. If a factor has an intercept no different from zero, then it is redundant. Momentum (MOM) is a cross-sectional momentum factor not used in our final model. It is defined as the top-quartile portfolio less the bottom-quartile portfolio of commodities sorted on the previous 12 months of spot returns. The market (MKT) factor is an equally weighted average of all futures contracts. The high-minus-low (HML) factor is the difference between the above- and below-median portfolios sorted on spot basis. Basis is computed as the log of the ratio of the nearest-dated contract and next-nearest dated contract. Time series momentum (TSMOM) is an equally weighted return of commodities with positive 12-month trailing return less those with negative trailing 12-month return. H_{term} is the equally weighted average of the above median portfolio of 2-, 4-, and 6-month calendar spread returns. L_{term} is the same for those below the median. Spot returns are based on the nearest-dated contract. *T*-statistics are computed based on standard errors with a Newey–West correction of 12 lags.

TABLE 7 Asset pricing tests of spot returns: comparing factor models

FH model				BGR model				Four-factor model			
	Alpha	*T* stat	Adj-R2		Alpha	*T* stat	Adj-R2		Alpha	*T* stat	Adj-R2
Panel A. basis portfolios											
B1	−0.68%	−2.19	0.00	B1	−0.08%	−0.63	0.74	B1	−0.00%	−0.03	0.65
B2	−0.41%	−1.51	0.00	B2	0.08%	0.63	0.77	B2	−0.22%	−1.53	0.72
B3	0.07%	0.23	0.00	B3	0.10%	0.73	0.76	B3	0.09%	0.57	0.71
B4	0.13%	0.43	0.00	B4	−0.12%	−0.73	−0.73	B4	0.11%	0.69	0.69
GRS (*p*-value) 2.98%				GRS (*p*-value) 95.65%				GRS (*p*-value) 72.89%			
Panel B. momentum portfolios											
M1	−0.61%	−2.39	0.00	M1	0.06%	0.54	0.89	M1	0.22%	1.46	0.74
M2	−0.48%	−1.67	0.00	M2	−0.25%	−1.72	0.62	M2	−0.26%	−1.80	0.64
M3	−0.04%	−0.11	0.00	M3	0.11%	0.85	0.75	M3	0.06%	0.42	0.76
M4	0.25%	0.82	0.00	M4	0.06%	0.54	0.90	M4	−0.03%	−0.17	0.81
GRS (*p*-value) 4.71%				GRS (*p*-value) 65.57%				GRS (*p*-value) 60.54%			

Data include 21 commodities' monthly spot returns from September 1987 to December 2014. This table reports asset pricing tests for spot returns when futures are sorted on basis and momentum. Basis is computed as the log of the ratio of the nearest-dated contract and next-nearest dated contract. Momentum is computed based on excess return over the past year. Each portfolio contains, on average, five commodities and is rebalanced monthly. Monthly excess return is net of the risk-free rate (no collateral reinvestment). The four-factor model includes two spot factors (MKT and TSMOM) and two basis term premia factors (H_{term} and L_{term}) to form our our four-factor model. The BGR model contains the three commodity factors (MKT, HML, and MOM) in Bhardwaj et al. (2014), similar to Bakshi et al. (2014). The FH model includes the commodity primitive trend-following factor from Fung and Hsieh (2001). *T*-statistics are computed based on standard errors with a Newey–West correction of 12 lags. GRS *p*-values are computed as in Gibbons et al. (1989).

the BGR model and the four-factor model successfully price commodity spot premia very well. The FH model shows clear inadequacies. In unreported results, we find that adding the HML factor to our model can improve its performance in these tests. This improvement is trivial, however, since all tests already show the portfolios to be fully priced. These results did not affect our decision to go with the results of our previous tables and leave HML from the model.

4.2 | Pricing tests on term premia portfolios

We next turn to pricing term premia in Table 8. This table is a horserace of our four-factor model versus the BGR model versus the FH model in pricing basis portfolios term premia. Portfolios are composed in the same manner as in Table 7, but portfolio performance is based on term premia rather than spot premia. This is where our four-factor model is strongest, performing significantly better than the other two. First, the magnitudes of alpha are very low, ranging from 0 bps (Panel A, portfolio B3) to −12 bps (Panel C, portfolio B3). It prices 10 of 12 portfolios across Panels A, B, and C. The GRS test shows in both Panel B and Panel C that all four portfolios jointly have alpha of zero. In Panel A, the 2-month term premia, does not fare well on the GRS test due to portfolio B4. But even here, the measured alpha is only 9 bps monthly.

In contrast, the BGR model and FH model both perform worse at pricing term premia. The BGR model can price 3 of 12 portfolios well and is borderline for a fourth portfolio. The FH model can price 3 of 12 (portfolio B3 in all three panels). The magnitudes of alpha are much higher, as high as 51 bps monthly for the BGR model (Panel C, B4) and 45 bps monthly for the FH model (Panel C, B1). The GRS test gives a *p*-value of zero to two significant figures for all groups of portfolios. The BGR model gives R^2 substantially lower than our four-factor model and the FH model has R^2 that are zero for all portfolios.

4.3 | Single versus separate term factors

Overall, these results provide robust evidence that our four-factor model performs the best of these three models given our test assets and test criteria. Now, we consider whether a single HML_{term} factor, computed as H_{term}–L_{term}, suffices to price term premia? We answer this question in Table 9 by comparing the two methods and how they price the term premia test assets. As can be seen, including a HML_{term} factor as a single fourth factor does not adequately price term premia. The second column replicates the results from Table 8 for the Four-factor model including both term factors separately for reference. The combined HML_{term} factor performs about the same as the BGR model, pricing only 2 of 12 portfolios and obtaining a *p*-value of zero for all GRS

TABLE 8 Asset pricing tests of term premia: basis portfolios

FH model				BGR model				Four-factor model			
	Alpha	***T* stat**	**Adj-R2**		**Alpha**	***T* stat**	**Adj-R2**		**Alpha**	***T* stat**	**Adj-R2**
Panel A. 2 month term premia											
B1	0.20%	3.31	0.00	B1	0.15%	2.48	0.15	B1	0.01%	0.50	0.61
B2	0.13%	2.82	0.00	B2	0.11%	2.25	0.02	B2	0.03%	0.97	0.32
B3	0.07%	1.83	0.00	B3	0.07%	1.95	−0.01	B3	−0.00%	−0.05	0.16
B4	0.26%	3.38	0.00	B4	0.31%	4.24	0.06	B4	0.09%	2.68	0.68
GRS (*p*-value) 0.00%				GRS (*p*-value) 0.00%				GRS (*p*-value) 2.45%			
Panel B. 4 month term premia											
B1	0.29%	3.07	0.00	B1	0.19%	2.11	0.21	B1	-0.04%	−1.17	0.70
B2	0.18%	3.64	0.00	B2	0.14%	2.52	0.09	B2	0.02%	0.38	0.40
B3	0.06%	1.05	0.00	B3	0.06%	1.14	0.01	B3	−0.07%	−1.69	0.20
B4	0.33%	3.02	0.00	B4	0.41%	4.22	0.10	B4	0.06%	1.16	0.71
GRS (*p*-value) 0.00%				GRS (*p*-value) 0.00%				GRS (*p*-value) 21.25%			
Panel C. 6 month term premia											
B1	0.45%	3.79	0.00	B1	0.30%	2.79	0.25	B1	0.01%	0.20	0.67
B2	0.17%	2.92	0.01	B2	0.12%	1.74	0.15	B2	−0.02%	−0.49	0.39
B3	0.02%	0.35	0.00	B3	0.03%	0.45	0.03	B3	−0.12%	−2.62	0.21
B4	0.40%	3.06	0.00	B4	0.51%	4.33	0.13	B4	0.09%	1.38	0.73
GRS (*p*-value) 0.00%				GRS (*p*-value) 0.00%				GRS (*p*-value) 15.83%			

Data include 21 commodities' 2-, 4-, and 6-months term premia from September 1987 to December 2014. This table reports asset pricing tests for term premia with holding period returns of 2, 4, and 6 months when futures are sorted on basis and computed at sequentially longer dated maturities. Basis is computed as the log of the ratio of the nearest-dated contract and longer dated contract (2, 4, or 6 months later). Each portfolio contains, on average, five commodities and is rebalanced monthly. The four-factor model includes two spot factors (MKT and TSMOM) and two basis term premia factors (H_{term} and L_{term}) to form our four-factor model. The BGR model contains the three commodity factors (MKT, HML, and MOM) in Bhardwaj et al. (2014), similar to Bakshi et al. (2014). The FH model includes the commodity primitive trend-following factor from Fung and Hsieh (2001). The FH model is tested on data starting in 1994 because that is when those factors are first available. *T*-statistics are computed based on standard errors with a Newey–West correction of 12 lags.

tests. The adjusted R^2 are substantially lower. Thus, we reject the single HML_{term} factor in favor of the proposed separate H_{term} and L_{term} factors. This result aligns with a similar test in Szymanowska et al. (2014).

5 | THE FOUR FACTOR MODEL AND SUB-PERIODS

Having established our model over our entire data set we wish to consider relevant sub-periods on either side of an important shift in financial markets.[16] To accomplish this we break the data at January of 2003, the approximate time of rising high frequency trading and financialization in the marketplace.[17] Our results show that while there are interesting changes in the factors themselves, pre- and post-January 2003, the four-factor model performs well in both sub-periods.

Table 10 gives statistics of the various factor premia considered in Table 3, overall and broken down by sub-period. We include HML and MOM since they are intuitive factors which appear on Table 3 and, even though we removed them later as redundant, we would like to examine their behavior. The sub-periods show some interesting variations.

The factors MKT, HML, and TSMOM are consistent in their behavior. Whatever level of significance they have in one sub-period tends to be qualitatively like what they have in the other sub-period. For example MKT is insignificant in both sub-periods. Both TSMOM and HML are significant in both periods, though both have higher *t*-stats in the first sub-period than the second.

However, the term factors and the MOM factor have a different story. Looking at the sub-periods one can see that MOM went from significant to insignificant as the return collapsed. On the other hand, H_{term} and L_{term} have both gotten far more significantly

[16]We would like to thank an anonymous referee for suggesting the tests in this section.

[17]For a discussion of the linkage between high frequency trading and financialization, readers are referred to Cooper, Seddon, and Vanvliet (2017).

TABLE 9 Tests for HML_{term} factor versus separate H_{term} and L_{term}

HML_{term}				H_{term} and L_{term}			
	Alpha	T stat	Adj-R2		Alpha	T stat	Adj-R2
Panel A. 2 month term premia							
B1	0.16%	3.44	0.20	B1	0.01%	0.50	0.61
B2	0.09%	2.48	0.10	B2	0.03%	0.97	0.32
B3	0.08%	2.31	0.07	B3	−0.00%	−0.05	0.16
B4	0.26%	4.53	0.39	B4	0.09%	2.68	0.68
GRS (*p*-value) 0.00%				GRS (*p*-value) 2.45%			
Panel B. 4 month term premia							
B1	0.22%	3.11	0.24	B1	−0.04%	−1.17	0.70
B2	0.14%	3.41	0.15	B2	0.02%	0.38	0.40
B3	0.06%	1.41	0.08	B3	−0.07%	−1.69	0.20
B4	0.31%	3.84	0.41	B4	0.06%	1.16	0.71
GRS (*p*-value) 0.00%				GRS (*p*-value) 21.25%			
Panel C. 6 month term premia							
B1	0.32%	3.57	0.26	B1	0.01%	0.20	0.67
B2	0.14%	2.87	0.14	B2	−0.02%	−0.49	0.39
B3	0.03%	0.62	0.06	B3	−0.12%	−2.62	0.21
B4	0.38%	3.93	0.43	B4	0.09%	1.38	0.73
GRS (*p*-value) 0.00%				GRS (*p*-value) 15.83%			

Data include 21 commodities' monthly spot returns from September 1987 to December 2014. This table reports asset pricing tests for term premia with holding period returns of 2, 4 and 6 months when futures are sorted on basis and computed at sequentially longer dated maturities. It compares a combined HML_{term} factor versus individual H_{term} and L_{term} factors. Basis is computed as the log of the ratio of the nearest-dated contract and longer dated contract (2, 4, or 6 months later). Each portfolio contains, on average, five commodities and is rebalanced monthly. H_{term} is the equally weighted average of the above median portfolio of 2-, 4-, and 6-months calendar spread returns. L_{term} is the same for those below the median. HML_{term} is the difference between the two. *T*-statistics are computed based on standard errors with a Newey–West correction of 12 lags.

positive. These results indicate possible interesting future research in the effects of financialization on risk premiums. However, the question for the current paper is whether, like the factor returns, our four-factor model has performance that varies by sub-period.

Table 11 gives the answer to our question with respect to spot premia. In both sub-periods the joint hypothesis that all alphas are zero is not rejected, though the result is stronger for the momentum sorted portfolio in sub-period 2. Only 1 of 16 possible sub-period sorted portfolios has a significant unexplained performance (alpha t-stat = −2.16). This correlates well with the GRS results.

Table 12 answers the question with respect to term premia. There are no significant sub-period t-stats in the alpha regressions, though the 2 month premium in the later sub-period rejects the joint hypothesis that all alphas are zero (GRS = 3.17%). This second sub-period result drives the overall result reported previously. So, our model performs well overall in the sub-periods but

TABLE 10 Model factors summary statistics by sub-periods

Entire sample				Sept 1987–Dec 2002			Jan 2003–Dec 2014		
Factors	Monthly excess return (%)	Std dev (%)	*T*-stat for mean = 0	Monthly excess return (%)	Std dev (%)	*T*-stat for mean = 0	Monthly excess return (%)	Std dev (%)	*T*-stat for mean = 0
Market	−0.19	3.46	−0.97	−0.28	2.41	−1.59	−0.06	4.46	−0.17
HML	0.60	3.46	3.12	0.61	3.66	2.27	0.58	3.21	2.17
TSMOM	0.86	4.39	3.55	0.88	4.49	2.64	0.84	4.27	2.36
H_{term}	0.13	0.73	3.22	0.01	0.75	0.26	0.28	0.69	4.86
L_{term}	0.21	0.63	5.94	0.11	0.60	2.48	0.33	0.65	6.12
MOM	0.84	5.95	2.56	1.00	6.07	2.24	0.64	5.81	1.31

This table repeats the summary statistics of Table 3. The sub-periods are broken down to approximate the period before and after the widespread advent of high frequency trading (and corresponding financialization).

TABLE 11 Four-factor model tests of spot premia by sub-period

Entire sample				Pre 2003				2003 onward			
	Alpha	*T* stat	Adj-R2		Alpha	*T* stat	Adj-R2		Alpha	*T* stat	Adj-R2
B1	−0.01%	−0.03	0.65	B1	−0.24%	−0.88	0.63	B1	0.27%	1.20	0.66
B2	−0.22%	−1.53	0.72	B2	−0.08%	−0.53	0.55	B2	−0.38%	−1.87	0.78
B3	0.09%	0.57	0.71	B3	0.26%	1.23	0.49	B3	−0.10%	−0.43	0.77
B4	0.11%	0.69	0.69	B4	0.01%	0.05	0.59	B4	0.23%	0.99	0.73
GRS (*p*-value) 72.89%				GRS (*p*-value) 68.15%				GRS (*p*-value) 63.46%			
M1	0.22%	1.46	0.74	M1	0.30%	1.67	0.68	M1	0.10%	0.44	0.79
M2	−0.26%	−1.80	0.64	M2	−0.46%	−2.16	0.32	M2	−0.06%	−0.30	0.73
M3	0.06%	0.42	0.76	M3	−0.16%	−0.77	0.50	M3	0.28%	1.33	0.82
M4	−0.03%	−0.17	0.81	M4	0.35%	1.89	0.76	M4	−0.37%	−1.60	0.83
GRS (*p*-value) 60.54%				GRS (*p*-value) 29.11%				GRS (*p*-value) 58.99%			

This table repeats the tests of Table 7 for the four-factor model with two sub-periods. The data set is broken at Dec 2002 which is the approximate date of increased high frequency trading and financialization in the market place.

there is a shift in the performance on the nearest term premium which provides a basis for further interesting research, to link this result and the term factor performance possibly to a common cause.

6 | CONCLUSION

We establish and test a monthly, implementable, four-factor model of commodity returns. Our factors include a market factor, a time series momentum factor, and H_{term} and L_{term} factors, sorted on basis and containing returns to calendar spreads, to price the term premia inherent in commodity futures. This model outperforms the existing models of commodity returns existing in the literature Bhardwaj et al. (2014), Bakshi et al. (2014), Fung and Hsieh (2001) when tested against test portfolios of commodity futures spot and term premia. We have shown it to be the most parsimonious model possible.

TABLE 12 Four-factor model tests of term premia by sub-period

Entire sample				Pre 2003				2003 onward			
	Alpha	*T* stat	Adj-R2		Alpha	*T* stat	Adj-R2		Alpha	*T* stat	Adj-R2
Panel A. 2 month term premia											
B1	0.01%	0.50	0.61	B1	−0.01%	−0.25	0.64	B1	0.04%	1.00	0.58
B2	0.03%	0.97	0.32	B2	0.05%	1.63	0.25	B2	0.01%	0.16	0.36
B3	−0.00%	−0.05	0.16	B3	−0.05%	−1.14	0.15	B3	0.06%	1.79	0.20
B4	0.09%	2.68	0.68	B4	0.11%	1.81	0.67	B4	0.07%	1.75	0.70
GRS (*p*-value) 2.45%				GRS (*p*-value) 36.88%				GRS (*p*-value) 3.17%			
Panel B. 4 month term premia											
B1	−0.04%	−1.17	0.70	B1	−0.09%	−2.18	0.71	B1	0.02%	0.49	0.68
B2	0.02%	0.38	0.40	B2	0.09%	1.94	0.40	B2	−0.07%	−1.35	0.42
B3	−0.07%	−1.69	0.20	B3	−0.11%	−1.67	0.14	B3	−0.02%	−0.46	0.27
B4	0.06%	1.16	0.71	B4	0.11%	1.42	0.71	B4	8.88E-05	0.14	0.73
GRS (*p*-value) 21.25%				GRS (*p*-value) 26.41%				GRS (*p*-value) 54.46%			
Panel C. 6 month term premia											
B1	0.01%	0.20	0.67	B1	−0.05%	−0.66	0.70	B1	0.08%	1.09	0.64
B2	−0.02%	−0.49	0.39	B2	0.02%	0.32	0.38	B2	−0.08%	−1.38	0.40
B3	−0.12%	−2.62	0.21	B3	−0.16%	−2.12	0.11	B3	−0.09%	−1.54	0.31
B4	0.09%	1.38	0.73	B4	0.18%	1.78	0.73	B4	0.00%	-0.06	0.75
GRS (*p*-value) 15.83%				GRS (*p*-value) 43.03%				GRS (*p*-value) 16.65%			

This table repeats the tests of Table 7 for the four-factor model with two sub-periods. The data set is broken at Dec 2002 which is the approximate date of increased high frequency trading and financialization in the market place.

Considering the recent poor performance of passive investments in commodities, our benchmark for active commodity management is a timely addition to the literature. In spite of recent performance, large investors like CalSTRS (California State Teachers Retirement System) are still willing to invest in commodities.[18] This benchmark can be used to draw comparisons between passive management, active management via Commodity Trading Advisors, and commodity-focused Exchange Traded Funds as investors large or small consider commodities exposure.

Finally, we show that our commodity factor model is stable in sub-period analysis and that more traditional factor models are not a good way to model commodities. These late results make our model look even more attractive,

An additional application of our factor model could also be its conversion into an Exchange Traded Fund itself. Since our factors are tradeable, they could easily be incorporated into a "smart beta" style commodity ETF that uses algorithms to rebalance the portfolio based on market data and factor construction. Such a product would likely be well received in the current marketplace given the prevalence of smart beta products and the desire for commodity exposure.

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REFERENCES

Asness, C. S., Moskowitz, T. J., & Pedersen, L. H. (2013). Value and momentum everywhere. *Journal of Finance*, *68*, 929–985.

Bakshi, G., Gao Bakshi, X., & Rossi, A. G. (2014). Understanding the Sources of Risk Underlying the Cross-Section of Commodity Returns. SSRN Working Paper.

Baltas, A- N., & Kosowski, R. (2012). Momentum Strategies in Futures Markets and Trend-following Funds. SSRN Working Paper, 1–60.

Bhardwaj, G., Gorton, G. B., & Rouwenhorst, K. G. (2014). Fooling some of the people all of the time: The inefficient performance and persistence of commodity trading advisors. *Review of Financial Studies*, *27*, 3099–3132.

Bhardwaj, G., Gorton, G. B., & Rouwenhorst, K. G. (2015). Facts and Fantasies About Commodity Futures Ten Years Later. SSRN Working Paper.

Bollen, N. P. B. (2013). Zero-R2 hedge funds and market neutrality. *Journal of Financial and Quantitative Analysis*, *48*, 519–547.

Bollen, N. P. B., & Whaley, R. E. (2009). Hedge fund risk dynamics: Implications for performance appraisal. *Journal of Finance*, *64*, 985–1035.

Brennan, M. J. (1958). The supply of storage. *The American Economic Review*, *48*, 50–72.

Carter, C. A, Rausser, G. C., & Schmitz, A. (1983). Efficient asset portfolios and the theory of normal backwardation. *Journal of Political Economy*, *91*, 319–331.

Cheng, I- H., & Xiong, W. (2014). Financialization of commodity markets. *Annual Review of Financial Economics*, *6*, 419–441.

Cooper, R. (1993). Risk premia in the futures and forward markets. *Journal of Futures Markets*, *13*, 357–371.

Cooper, R., Seddon, J., & Van Vliet, B. (2017). High-frequency trading and conflict in the financial markets. *Journal of Information Technology*, *32*, 270–282.

Dusak, K. (1973). Futures trading and investor returns: An investigation of commodity market risk premiums. *Journal of Political Economy*, *81*, 1387–1406.

Erb, C. B., & Harvey, C. R. (2006). The strategic and tactical value of commodity futures. *Financial Analysts Journal*, *62*, 69–97.

Fama, E. F., & French, K. R. (1987). Commodity futures prices: Some evidence on forecast power, premiums, and the theory of storage. *The Journal of Business*, *60*, 55–73.

Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, *33*, 3–56.

Fama, E. F., & French, K. R. (2015). A five-factor asset pricing model. *Journal of Financial Economics*, *116*, 1–22.

Fuertes, A.-M., Miffre, J., & Rallis, G. (2010). Tactical allocation in commodity futures markets: Combining momentum and term structure signals. *Journal of Banking and Finance*, *34*, 2530–2548.

Fung, W., & Hsieh, D. A. (1997). Survivorship bias and investment style in the returns of CTAs. *The Journal of Portfolio Management*, *24*, 30–41.

Fung, W., & Hsieh, D. A. (2000). Performance characteristics of hedge funds and commodity funds: Natural vs. spurious biases. *Journal of Financial and Quantitative Analysis*, *35*, 291–307.

Fung, W., & Hsieh, D. A. (2001). The risk in hedge fund strategies: Theory and evidence from trend followers. *Review of Financial Studies*, *14*, 313–341.

Gibbons, M. R., Ross, S. A., & Shanken, J. (1989). A test of the efficiency of a given portfolio. *Econometrica*, *57*, 1121–1152.

[18]*As other funds bail on commodities, CalSTRS pursues test drive*, Reuters, Sept 4, 2015. http://www.reuters.com/article/us-california-investment-commodities-idUSKCN0R416J20150904

Gorton, G. B., & Rouwenhorst, G. (2006). Facts and fantasies about commodity futures. *Financial Analysts Journal*, *62*, 47–68.

Gorton, G. B., Hayashi, F., & Rouwenhorst, K. G. (2012). The fundamentals of commodity futures returns. *Review of Finance*, *17*, 35–105.

Jegadeesh, N., & Titman, S. (2011). Momentum. *Annual Review of Financial Economics*, *3*, 493–509.

Kaldor, N. (1939). Speculation and economic stability. *The Review of Economic Studies*, *7*, 1.

Keynes, J. (1923). Some aspects of commodity markets. *Manchester Guardian Commercial*, *13*, 784–786.

Kosowski, R., Naik, N. Y., & Teo, M. (2007). Do hedge funds deliver alpha? A Bayesian and bootstrap analysis. *Journal of Financial Economics*, *84*, 229–264.

Miffre, J., & Rallis, G. (2007). Momentum strategies in commodity futures markets. *Journal of Banking and Finance*, *31*, 1863–1886.

Moskowitz, T. J., Ooi, Y. H., & Pedersen, L. H. (2012). Time series momentum. *Journal of Financial Economics*, *104*, 228–250.

Newey, W. K., & West, K. D. (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica*, *55*, 703–708.

Roll, R. (1978). Ambiguity when performance is measured by the securities market line. *Journal of Finance*, *33*, 1051–1069.

Routledge, B. R, Seppi, D. J., & Spatt, C. S. (2000). Equilibrium forward curves for commodities. *Journal of Finance*, *55*, 1297–1338.

Rouwenhorst, K. G., & Tang, K. (2012). Commodity investing. *Annual Review of Financial Economics*, *4*, 447–467.

Szymanowska, M., Roon, F. A. D., Nijman, T. E., & van den Goorbergh, R. (2014). An anatomy of commodity futures risk premia. *Journal of Finance*, *69*, 453–482.

Working Holbrook (1949). The theory of price of storage. *The American Economic Review*, *39*, 1254–1262.

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APPENDIX

In this appendix we consider the ability of a more traditional equity factor model, namely the Fama–French (2015) five-factor model (henceforth FF5) to explain term and spot premia. To the extent that investment markets have cross correlations one might expect to find some power in this model. Also, the test is useful so that we may feel more certain we have made a thorough study of the alternatives to our four-factor commodity model. To conduct these tests we take the factor returns directly from French's website.

Table A1 looks at the performance of the FF5 model at explaining spot premia. The FF5 model prices the better faring portfolios as sorted by basis, but cannot explain the worse performing portfolios, either for the overall sample, or for the sub-periods. Moreover, the GRS statistic also indicates rejection of the null hypothesis that all coefficients are zero in the overall sample period. The results for the momentum sorted portfolios are slightly worse than this.

In Table A2, we see the results for the term premia are much worse. The model misses pricing almost every premium in the overall sample and the second sub-period. Also, the GRS tests reject all alphas equal to zero. The model performs a little better in the first sub-period. However, the adjusted R^2's are uniformly extremely low across all cells in the table. These results lead us to conclude that a traditional model like FF5 is likely not a good way to model commodity premia.

TABLE A1 Fama–French five factor model test of the spot premia

Entire sample				Pre 2003				2003 onward			
	Alpha	*T* stat	Adj-R2		Alpha	*T* stat	Adj-R2		Alpha	*T* stat	Adj-R2
B1	−0.86%	−2.69	0.11	B1	−0.81%	−2.05	0.07	B1	−0.81%	−2.43	0.18
B2	−0.72%	−2.40	0.12	B2	−0.70%	−2.06	0.03	B2	−0.66%	−1.59	0.25
B3	−0.25%	−0.81	0.15	B3	−0.04%	−0.10	0.02	B3	−0.27%	−0.64	0. 28
B4	−0.05%	−0.15	0.04	B4	−0.04%	−0.07	0.04	B4	−0.03%	−0.08	0.18
GRS (*p*-value) 1.21%				GRS (*p*-value) 20.37%				GRS (*p*-value) 17.63%			
M1	−0.69%	−2.60	0.10	M1	−0.38%	−0.81	0.10	M1	−0.76%	−2.59	0.23
M2	−0.83%	−2.77	0.12	M2	−0.89%	−2.20	−0.03	M2	−0.70%	−1.90	0.26
M3	−0.32%	−0.95	0.12	M3	−0.53%	−1.50	0.02	M3	−0.12%	−0.28	0.24
M4	−0.04%	−0.10	0.06	M4	0.31%	0.53	0.01	M4	−0.22%	−0.61	0.20
GRS (*p*-value) 1.13%				GRS (*p*-value) 2.18%				GRS (*p*-value) 10.52%			

This table shows the performance of a more traditional equity factor type model for explaining the commodity spot premia. This table follows the same format as Table 11.

TABLE A2 Fama-French five factor model test of the term premia

Entire sample				Pre 2003				2003 onward			
	Alpha	***T* stat**	**Adj-R2**		**Alpha**	***T* stat**	**Adj-R2**		**Alpha**	***T* stat**	**Adj-R2**
Panel A. 2 month term premia											
B1	0.21%	3.15	0.01	B1	0.05%	0.80	0.00	B1	0.30%	3.57	0.02
B2	0.14%	3.07	0.00	B2	0.09%	2.10	−0.03	B2	0.18%	2.49	−0.01
B3	0.08%	2.03	0.00	B3	−0.07%	−1.30	−0.03	B3	0.18%	6.21	0. 03
B4	0.27%	3.13	0.01	B4	0.16%	1.30	−0.01	B4	0.35%	3.14	−0.02
GRS (*p*-value) 0.00%				GRS (*p*-value) 19.58%				GRS (*p*-value) 0.00%			
Panel B. 4 month term premia											
B1	0.32%	3.02	0.06	B1	0.04%	0.39	0.02	B1	0.49%	4.01	0.09
B2	0.20%	3.78	0.03	B2	0.18%	2.87	0.01	B2	0.21%	2.64	0.03
B3	0.07%	1.35	0.01	B3	−0.10%	−1.28	−0.04	B3	0.18%	4.06	0.06
B4	0.34%	2.86	0.01	B4	0.22%	1.14	0.00	B4	0.44%	3.06	0.00
GRS (*p*-value) 0.00%				GRS (*p*-value) 12.10%				GRS (*p*-value) 0.00%			
Panel C. 6 month term premia											
B1	0.46%	3.35	0.05	B1	0.12%	0.72	0.02	B1	0.66%	4.56	0.09
B2	0.21%	3.42	0.04	B2	0.16%	2.26	0.01	B2	0.24%	2.63	0.04
B3	0.05%	0.81	0.01	B3	−0.13%	−1.44	−0.04	B3	0.16%	2.28	0.07
B4	0.39%	2.78	0.00	B4	0.29%	1.23	0.00	B4	0.49%	2.66	−0.01
GRS (*p*-value) 0.00%				GRS (*p*-value) 21.10%				GRS (*p*-value) 0.00%			

This table shows the performance of a more traditional equity factor type model for explaining the commodity term premia. This table follows the same format as Table 12.