Inferring Perception from Continuous Control

Joshua J.H. Ryu
Department of Psychology, ICME
Stanford University

Background

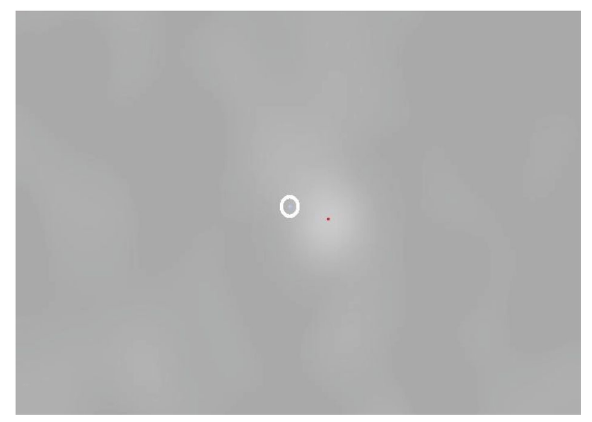
Try quickly blinking your eyes. The visual information available to us is very limited during the brief duration for which your eyes are open. The goal of the visual system is to infer a likely state of the external world by integrating information from the past with the transient sensory information from the present.

I work with the following set of assumptions about human perception and control to infer perception from a continuous task paradigm:

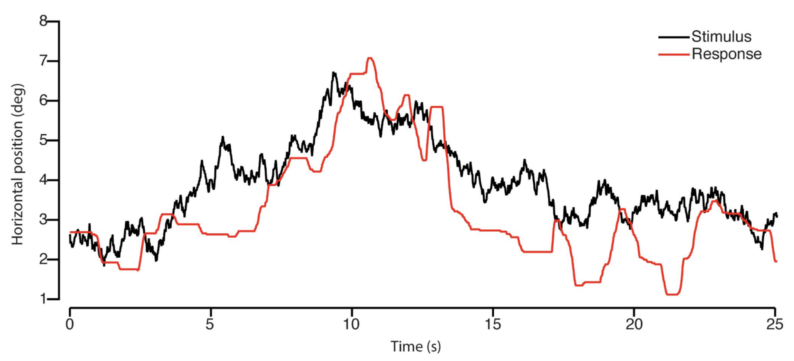
- Humans receive very noisy visual information about the world in a small time frame.
- Humans continuously integrate noisy visual information in an optimal fashion, given the world dynamics, weighing its uncertainty via the Kalman filter.
- Perception and control are modular (e.g. Firestone and Scholl, 2016)
- Human control is limited by our biology.
- Human control movements are approximately linear (e.g. Todorov, 2002)

Task

Track a (hard-to-see) blob as it moves across the screen. The blob follows a brownian motion (a fixed dynamic). The subjects fixate in the center and use a cursor to follow the center of the blob.



Sample frame from the task



Example trial, target (black) and cursor (red)

EM Algorithm

ELBO

$$\begin{aligned}\log P(\{x_t\}_{t=1}^{T}, \theta, \phi) &= \log P(\{x_t\} \mid \theta, \phi) + \log P(\theta, \phi)\\ &\geq \mathbb{E}_{q(\{z_{t-1}\})}\left[\log P(\{x_t, z_{t-1}\} \mid \theta, \phi) + \log q(\{z_{t-1}\}))\right] + \log P(\theta, \phi)\\ &\triangleq L(q, \theta, \phi)\end{aligned}$$

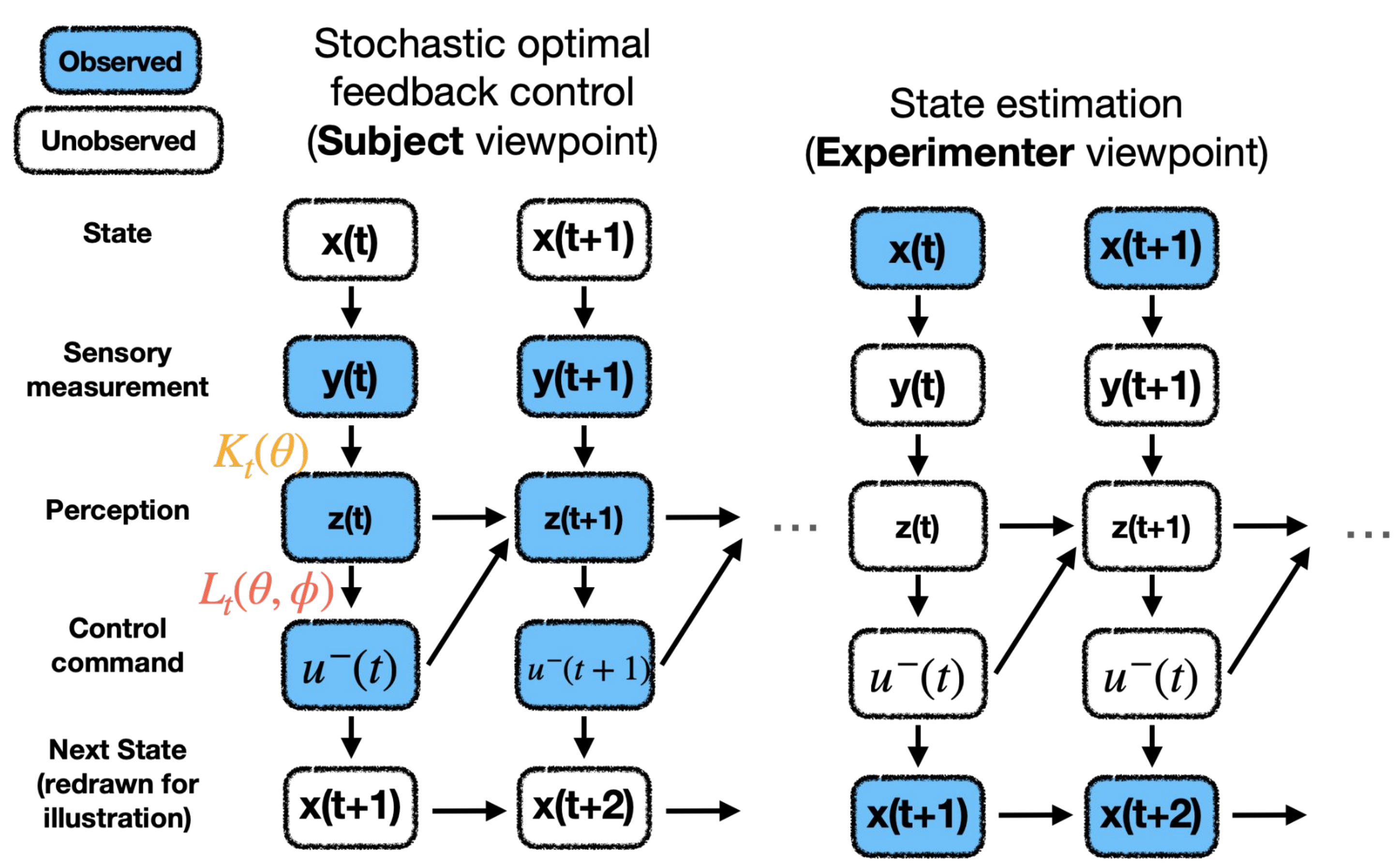
E-step (Kalman smoothing)

- We make the Elbo tight in the E-step by calculating the smoothed marginals (Jordan, 2007; Murphy, 2012)

M-step (gradient descent)

- Calculate expected elbow with the smoothed marginals. (Shumway and Stoeffer, 1982)
- Use gradient descent on the elbo (for differentiable filters)

The Generative Model



world $x_{t+1} = Ax_t + Bu_t + Cu_t^{external} + \xi_t$

measurement $y_t = Hx_t + \omega_t$

perception $z_{t+1} = Az_t - BL_t z_t + K_{t+1}(y_t - Hz_t) + \epsilon_t$

control $u_t = -L_t z_t + \varepsilon_t$

Optimal perceptual filters maximizes the probability of seeing the actual stimulus
Optimal control filters maximizes the task reward, subject to constraints

$$p(u_t \mid z_t; \theta, \phi) = \mathcal{N}(u_t \mid -L(t, \theta, \phi)z_t, \Omega_u)$$

$$p(x_{t+1} \mid z_t, x_t; \theta, \phi) \sim \mathcal{N}(x_{t+1} \mid Ax_t - BL(t, \theta, \phi)z_t, \Omega_x + B\Omega_u B^T)$$

$$p(z_t \mid z_{t-1}; x_t, \theta, \phi) \triangleq \mathcal{N}\left(a_{z_{t|t-1}} z_{t-1} + b_{z_{t|t-1}}, \Omega_{z_{t|t-1}}\right)$$

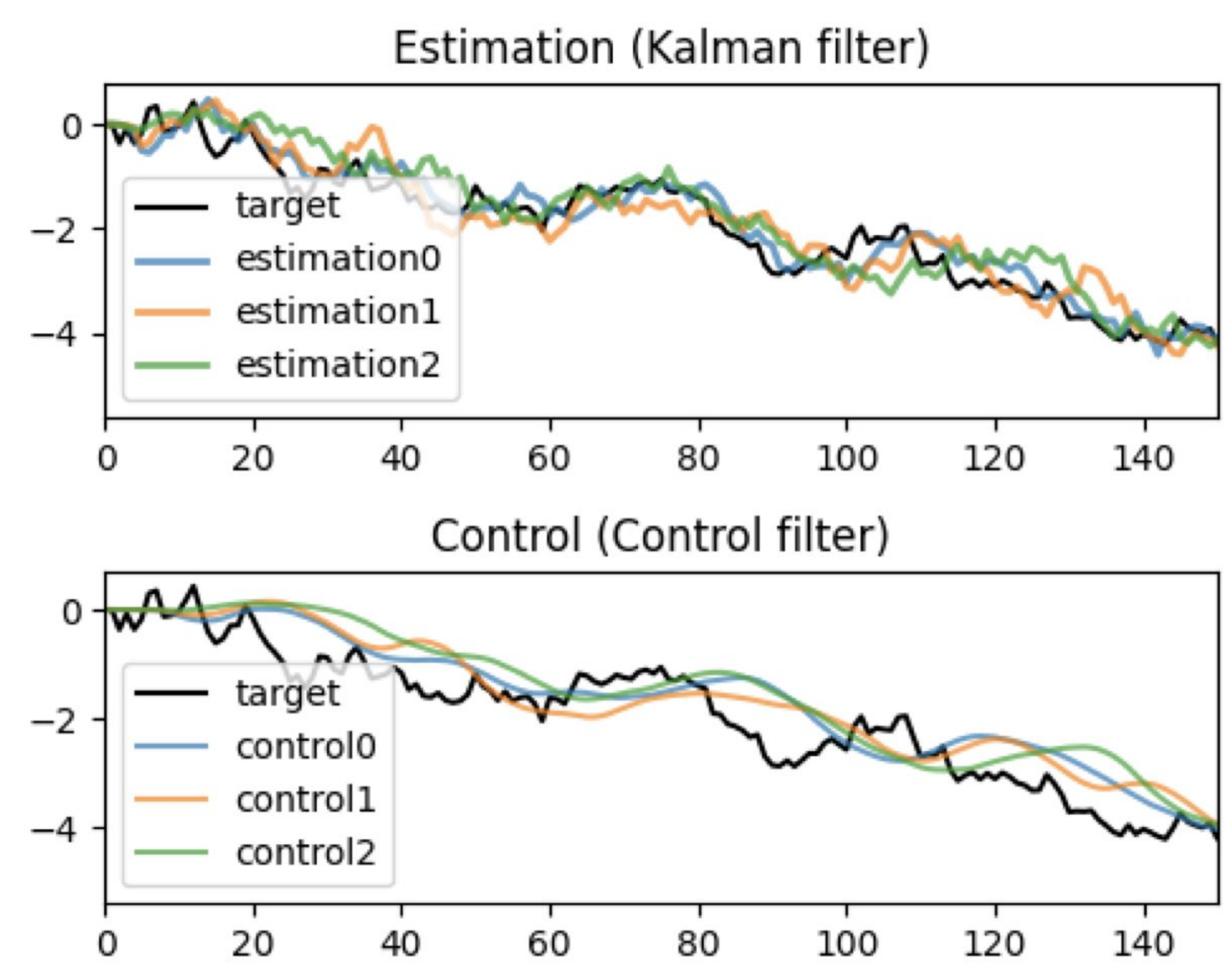
where:

$$a_{z_{t|t-1}} := A - BL(t-1, \theta, \phi) - K(t, \theta)H$$

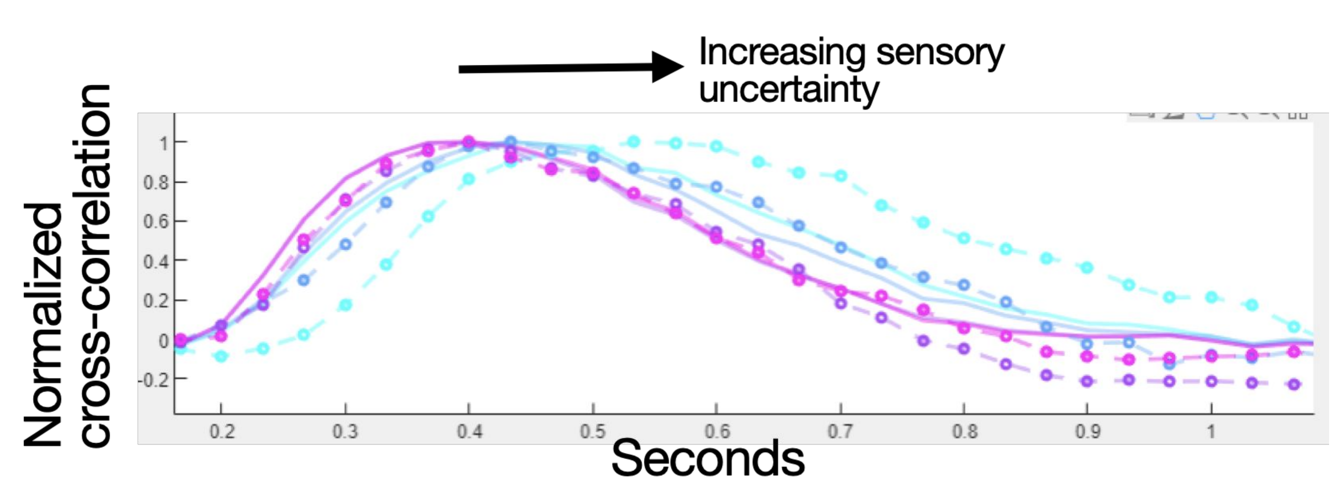
$$b_{z_{t|t-1}} := K(t, \theta)Hx_t$$

$$\Omega_{z_{t|t-1}} := \Omega_z + K(t, \theta)\Omega_y K(t, \theta)^T$$

Optimal Feedback Control



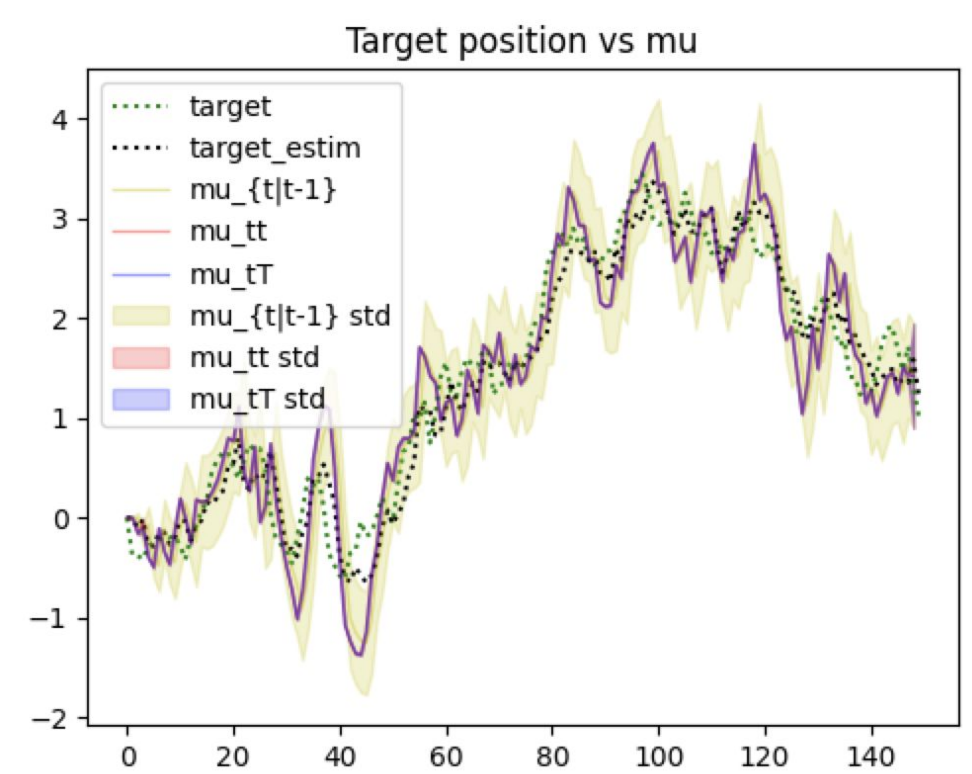
Simulated trajectory from the generative model. (Top) State estimation. The state estimation is more delayed with increasing uncertainty (blue → orange → green), indicating a stronger use of prior information. (Bottom) Control. The estimated state is then used to make a control movement. We see similar patterns of delay. Control costs also remove high frequency components.



Human data shows increasing peak delay with increasing sensory uncertainty. Plotted are the cross correlation between target and cursor velocities for human (circles) and model fit to the cross correlation (lines). Colors indicate different stimulus uncertainty levels (red being most certain and light blue being more uncertain). Model parameters were manually fit to match the cross correlation kernels.

Inference and Estimation

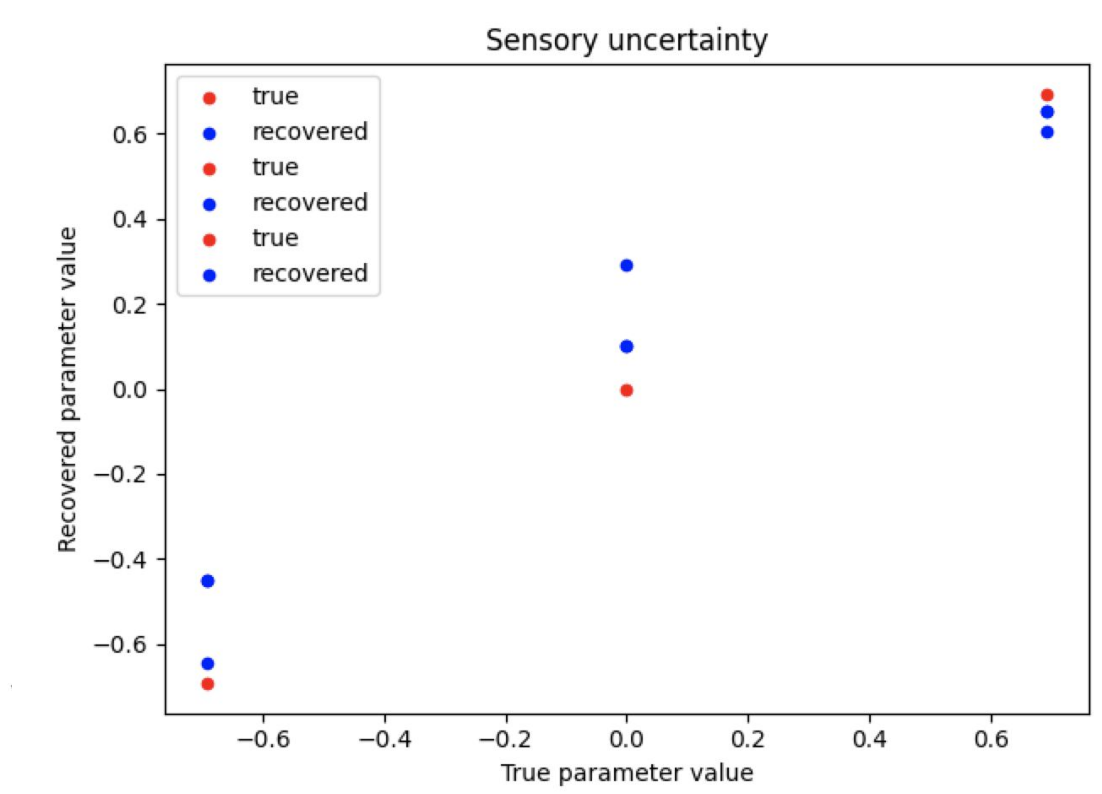
Inferred Marginal Latent States



Probability and 3 std of inferred target position perception p(z_t)
(Green dotted): Simulated target trajectory
(Black dotted): Simulated target perception
(Yellow) prior for filtered marginals: p(z_t | x_{1:t}), given true parameters
(Red) filtered marginals: p(z_t | x_{1:t+1}), given true parameters. This is the prior with likelihood of seeing the
(Blue) smoothed marginals: p(z_t | x_{1:T}).

The latent state estimates are highly dependent on the control parameters and noise. Observing the control that arises from the latent state significantly decreases the uncertainty of the latent state

Estimated Parameters



Sensory uncertainty parameter estimated using the EM algorithm
(Red): true parameter value
(Blue): estimated parameter value. Each dot corresponds to different random initializations.

For the given simulation, control parameters were fixed to true values, and only the sensory parameters were modified. Ran 10 iterations of the EM-algorithm, with each iteration consisting of 3 continuous M-steps.

References

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[2] K.P. Murphy. Machine Learning: A Probabilistic Perspective. The MIT Press, 2012
[3] R.H. Shumway and D.S. Stoeffer. AN APPROACH TO TIME SERIES SMOOTHING AND FORECASTING USING THE EM ALGORITHM. Journal of Time Series Analysis, 3 (4): 253-264, 1982.
[4] E. Todorov. Optimal Feedback Control as a theory of motor coordination. Nature Neuroscience, 5 (11). 2002
[5] C. Firestone and B.J. Scholl. Cognition does not affect perception: Evaluating the evidence for "top-down" effects. Brain and Behavioral Sciences. 2016.