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Kernel Estimation Methods for Seismic Hazard Area Source Modeling

by Gordon Woo

Abstract In probabilistic seismic hazard analysis, the representation of seismic sources by area zones is a standard means of data reduction. However, where the association between seismicity and geology is complex, as it is in many tectonic regimes, the construction of zone geometry may become contentiously subjective, and ambiguities may end up being resolved through appeal to the nonscientific rule of conservatism or pragmatism. Although consideration of alternative zonations within a logic-tree framework provides a channel for some of the uncertainty, it does not address the fundamental validity of the zonation procedure itself. In particular, neither the minimal assumption of uniform seismicity within a zone nor the Euclidean geometry of a zone accord with the fractal spatial distribution of seismicity, and the magnitude insensitivity of zonation ignores the spatial extent and correlations of different-sized earthquakes. An alternative procedure for area source modeling avoids Euclidean zones and is based statistically on kernel estimation of the activity rate density inferred from the regional earthquake catalog. The form of kernel is governed by the concepts of fractal geometry and self-organized criticality, with the bandwidth scaling according to magnitude. In contrast with zonal models for intraplate regions, the kernel estimation methodology makes provision for moderate earthquakes to cluster spatially, while larger events may migrate over sizeable distances.

Introduction

Apart from integrating diverse branches of the earth sciences and establishing a physical explanation for continental drift, the theory of plate tectonics provided the first scientific framework for the assessment of seismic hazard. Prior to the validation of plate tectonic concepts in the 1960s, it was customary for earthquake catalogs to be taken at face value as representative empirical guides to the sources of hazard, future as well as past. However, given the lack of direct tectonic input, sole reliance on historical earthquake data was deprecated by Cornell (1968) on the grounds that insufficient weight was given to known correlations between geological structure and most seismic activity. A quarter of a century later, it still remains common practice in seismic hazard analysis to construct a source model of seismicity, which includes a geographical partition of the region around the site of engineering interest, into disjoint Euclidean seismic area zones, sometimes called tectonic provinces.

The basic principle underlying a zonal partition is that, whereas significant differences may exist between zones, the characteristics of seismicity within each zone are supposed to be sufficiently homogeneous for seismological parameters to be assigned adequately on a zonal basis. As nonseismological criteria for delineating zones, geological data have been used with varying degrees of scientific conviction. There are tectonic provinces defined either (1) from causal relationships established between geological structures and earthquakes or (2) from faults that have been historically aseismic but show recent geological displacement.

Other than these two classes of zone, Thenhaus (1986) defines a third and fourth type, which are far more common but also more intractable and which demand the exercise of considerably more judgment in their construction. The third is based on an association of seismicity with geology, which falls short of direct evidence of active faulting, and lacks development of a clear history relating contemporary seismic activity with geological structure. The fourth type of zone is one constructed solely using the spatial distribution of historical seismicity. If there is an association with known geological structure, only a small portion may be currently active, and it may not be clear which of several possible structures could be active.

Thenhaus himself describes the delineation of the latter zones as arbitrary and judgmental. No wonder then that seismologists not involved with engineering seismic hazard assessment are apt to express misgivings over zonation and, consequently, with the edifice of hazard computation built on zonation. The idiosyncracy of judgment in delineating zones is a challenge for seismic hazard analysts striving to improve the scientific standing of the process of earthquake hazard assessment and trying to minimize the recourse to subjective judgment, however expert this might be. To some extent, the manifold ambiguities in zone delineation might be represented probabilistically within a zonal branch of the hazard model logic tree. This has become quite a widespread practice (Reiter, 1990), which at least recognizes the uncertainties of zonation and attempts to quantify them, albeit by the elicitation of expert opinion. But, recalling that the original rationale for introducing zones was to respect the observed correlations between geological structure and seismicity (Cornell, 1968), the practice of zonation should not be merely routine when applied to areas where such correlations are tenuous.

Fractals and Area Source Modeling

Where there is a lack of correlation between geological structure and seismicity, it has been assumed from the early days of probabilistic hazard computation (Cornell, 1968) that earthquakes are equally likely to occur anywhere over the area. However, the assumption of spatial uniformity within an area zone is now known from extensive studies of inter-event correlation functions (Kagan and Knopoff, 1980) to conflict with the actual spatial distribution of seismicity, which is capable of arrangement in a far more intricate and complex pattern. The gaps, clusters, and chains of seismicity (Korvin, 1992) are increasingly revealed by high-quality seismological data and remain to be fully explored.

Thus, sets of earthquake epicenters, rather than uniformly spanning a Euclidean area zone, can exhibit spatial clustering and display a fractal geometry so that the number of events per unit area at a distance R from any earthquake epicenter is proportional to R^{-a}, where the exponent a corresponds to the fractal dimension $D = 2 - a$ (Ito and Matsuzaki, 1990). Values of D obtained from regional and global catalogs vary from 1.0 to 1.7, which contrasts with the value of 2, which would pertain if the epicenters were uniformly distributed. Thus, rather than the density of epicenters being independent of distance, as it would be if seismicity were uniformly distributed within an area, it falls off in power-law fashion. To the extent that the fractal dimension is controlled by the spatial distribution of active faults, the clustering of earthquake epicenters provides indirect insight into the geometry of faulting (Hirata, 1989). The universality of the geometrical structure of seismic activity is highlighted by constant stress microfracturing experiments on rock (Hirata *et al.,* 1987), which demonstrate the fractal spatial characteristics of acoustic emissions.

Even if the nonuniformity of seismicity is accepted on purely seismological grounds, it might be argued that uniformity best characterizes the uncertain association of seismicity with structure and the uncertainty in the predictability of earthquake locations. In a state of relative ignorance about future seismicity, there may be no reason to believe one part of a zone to be more active than another. From this perspective, statistical tests for the uniformity of seismicity within a zone are redundant.

The degree to which a hazard analyst is persuaded by the nonuniform distribution of historical and instrumental epicenters inevitably depends on the quality of the seismological catalogs, their completeness, their accuracy, etc. Errors in catalogs tend to randomize earthquake geometry and obscure the actual spatial patterns of seismicity. Historical research into Norwegian seismicity, for example (Muir Wood *et al.,* 1988), has served to reduce the noise in the underlying macroseismic data and so establish clear spatial correlations with well-constrained instrumental epicenters. A feature of seismological research over the past several decades has been a resurgence of interest in historical earthquakes, with extensive research programs being undertaken to acquire new documentary information. This research has led to great progress in refining historical catalogs worldwide, which makes it now possible to contemplate using them directly for hazard assessment.

Self-Organized Criticality and Source Magnitude Modeling

Power laws are a manifestation of scale invariance, indicative of the capability of earthquake-generating systems to self-organize to a critical state with no characteristic length scale other than the size of the system (Bak *et al.,* 1988). Such systems are far from equilibrium, being driven by the input of energy that is stored and then dissipated in earthquake activity. Over a long period of geological time, the lithosphere has evolved to a steady state where the buildup of stress is balanced by the release of stress during earthquakes. The network of faults is self-organized in such a way that earthquake generation takes place as a critical chain reaction, with the process able to continue—but only marginally. If the process were supercritical, it would run away, but if the process were subcritical, it would terminate rapidly, as happens in some other physical avalanche processes (e.g., Bak and Flyvberg, 1992).

Power laws provide insight into the fundamental physical characteristics of the systems in which they are observed. The theory of self-organized criticality can explain the Gutenberg-Richter relation, where it holds [i.e., for moderate to large earthquakes (Scholz, 1990)]. However, over large areas, the vital issue to be addressed is what are the full dynamical implications for a system in a state of self-organized criticality. Earthquakes form a cascade process, with the cascade transferring the energy injected into the system on large lithospheric scales to a wide range of smaller scales, where it is dissipated. Self-organization results in the formation of self-similar clusters, which can lead to fractal spatial correlations of earthquakes.

Given the fundamental physical significance of the magnitude-frequency power law, its status within probabilistic seismic hazard assessment deserves to be re-evaluated. In particular, the implications for area source modeling need to be addressed. For logistical reasons, it is usually the case that only after seismic zones have been delineated on geological and seismological grounds is an attempt made to estimate zonal magnitude-frequency relations. Exploratory studies of magnitude-frequency statistics prior to zonation are not routinely undertaken as an aid to dynamical understanding. If a substantial weight has been accorded to geological knowledge, relative to seismological information, zonal magnitude-frequency relations may indeed be poorly constrained.

Within contemporary methods for seismic area source modeling, little would change if the magnitude-frequency relation were not a power law, i.e., if earthquake occurrence were not a self-organized critical phenomenon. The mathematical formula for deriving zonal recurrence relations would be different, but the actual zonation would not be altered, still less the underlying principles of zonation.

With the scale-invariant power law dictating earthquake occurrence, long-range interactions typical of many critical phenomena can take place, which means that small perturbations can trigger earthquakes at a distance. This arises dynamically through the complex interaction of faults, rather than the direct transmission of stress (Turcotte, 1992). The possibility of long-range correlations of seismic activity may inhibit the partitioning of a region into zones of independent seismicity. Across the boundaries of such zones, there is no mechanism by which earthquakes can migrate or be triggered.

Cellular automaton stick-slip models (Nakanishi, 1991) show that two types of seismic region can exist: one type is where large events occur periodically, and the other is where only smaller events occur. Separation of these two types of regions into zones of apparently contrasting and unrelated seismicity would purport to identify anomalous local variations in *b* value, but it would obscure the reality that the two types of regions inherently belong to the same interacting dynamical state.

The identification of dynamically interacting areas remains an important scientific task for geophysicists to address. Impetus for this study can come from actual earthquake observations as well as from theoretical studies. The Landers earthquake (Anderson *et al.*, 1994) provides a recent example of an event that triggered an increase in activity over a much larger area than would have been previously regarded as dynamically coupled. Triggered earthquakes occurred more than 500 km from the epicenter.

The concept of self-organized criticality offers a spatial interpretation of the Gutenberg-Richter magnitude-frequency relation, which otherwise is apt to be regarded merely as a routine arithmetic accounting device for keeping track of event numbers. In particular, the new spatial perspective encourages a review of procedures used by seismic hazard analysts to microzone *b* value, a practice that is not customarily considered as being subject to geometrical constraints.

Zoned b *Values*

The physics behind the power-law relation argues that zonal distinctions should be made for events of different magnitudes. Spatial distinctions between sets of events of different magnitudes are hard to assimilate within zonal models. Area zonation is not a magnitude-dependent concept: the geometry of a zone is invariant with respect to the magnitudes of earthquakes that occur within. This makes for awkward zonation decisions. As an illustration, Figure 1 shows a schematic plot of epicenters of major regional earthquakes, categorized as magnitudes 4, 5, and 6. If the magnitude threshold of engineering significance were 6, then the entire square might reasonably be taken as constituting a single-area zone for hazard modeling. However, if the magnitude threshold were 5, then a north–south partition of the square into equal upper and lower halves might be deemed to constitute a valid two-zone source model. And, if the magnitude threshold were lowered to 4, then the clustering of earthquakes in the four quadrants of the square would suggest a fourfold zonation of the square.

Partitioning the square into four equal quadrants recognizes the clustering of the magnitude 4 events and renders the seismicity within each zone reasonably homogeneous. However, such a fourfold zonation segregates the larger events, with the consequence that maximum magnitudes might be defined on a local rather than a more appropriate regional seismotectonic scale, and differing *b* values might be assigned to each quadrant.

In partitioning a zone with a designated *b* value into subzones, there are mathematical constraints on *b*-value assignment imposed by adherence to the Gutenberg-Richter magnitude-frequency relation. For an illustrative case, consider a general zone partitioned into a finite number S of subzones. Let the magnitude-frequency relation for the whole zone be

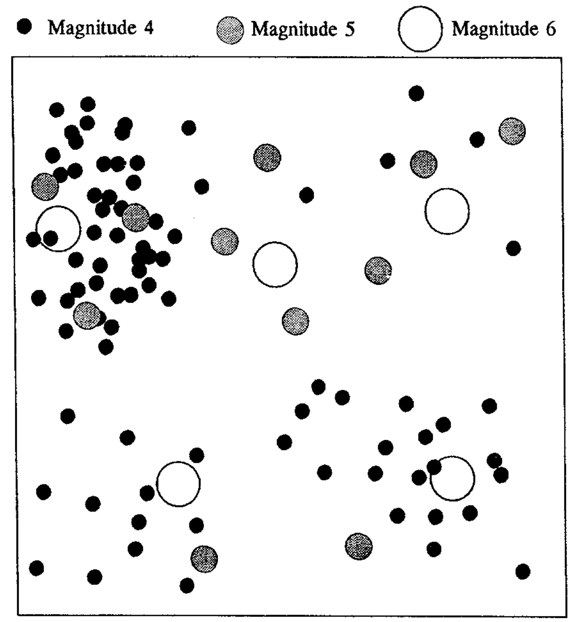

Figure 1. Illustrative plot of regional epicenters of earthquakes of magnitude 4, 5, and 6, showing the magnitude dependence of the spatial distribution.

$$\log N_{\text{whole}} = a_{\text{whole}} - b_{\text{whole}}M \quad (1)$$

and the magnitude-frequency relation for the ith subzone be

$$\log N_i = a_i - b_iM. \quad (2)$$

In these equations, N is the number of events greater or equal to magnitude M, normalized to some time period.

Since $N_{\text{whole}} = N_1 + N_2 + N_3 + \ldots N_s$, the above two equations are only algebraically compatible for general values of M, if $b_i = b_{\text{whole}}$ for all values of i. The same b-value algebraic constraint applies if a truncated form of Gutenberg-Richter relation holds, with a maximum magnitude for each subzone. Thus the partition of a principal zone into subzones of nonequal Gutenberg-Richter b value is algebraically inconsistent. This mathematical result reflects the geometrical principle of self-similarity that underlies the power law: if the fault geometry within a main zone is self-similar with a certain fractal dimension, then, in any subzone, it cannot be self-similar with another fractal dimension. A practical consequence is that within a zone with a designated b value, earthquakes cannot be clustered in a particular subregion in such a way as to cause an apparent internal b-value variation. This constraint may be difficult to maintain in the course of a zonation where seismicity parameters are determined separately for each zone.

Avoidance of this b-value pitfall, while maintaining the uniformity of seismicity within each zone, is one of the tribulations of constructing a mathematically consistent area zonation. The problems are exacerbated by the migration of epicenters, and other temporal fluctuations in seismicity, which destabilize the computation of zonal b values.

Estimation of Activity Rate Density

The language of probability is well suited to describing seismogenic processes, as with many other complex dynamical systems, because of the erosion of deterministic predictability (Turcotte, 1992). For quite different reasons connected with the practice of engineering risk evaluation, probability happens also to be the primary language of contemporary seismic hazard assessment. The fact that probability is well suited both for describing seismic processes and communicating hazard suggests that it also be used to model sources derived from epicenters, inasmuch as future earthquakes are likely to occur near past earthquakes in a vicinity whose scale is likely to be magnitude dependent.

Allowing for dynamical variations, each earthquake can be considered to be an "automodel" of a sequence of other earthquakes (Kagan and Knopoff, 1980), which constitutes a multiple realization of the basic stochastic crustal process. This underlies the approach to seismic area modeling that is described here.

The Poisson process is a standard model to describe the time dependence of the recurrence of all but very large earthquakes. Apart from affording a reasonable phenomenological description of arrival times of moderate to large earthquakes, the Poisson process is consistent with analog studies of self-organized criticality (Huang *et al.*, 1992). To implement the Poisson model, the mean rate of activity has to be defined as a function of location. Within the context of zonal area source models, each zone is associated with a single activity rate, expressed in terms of the expected annual number of zonal events exceeding the engineering threshold magnitude.

Venturing beyond the restrictions of zonation, the general functional representation of mean activity rate is $\lambda(M, \mathbf{x})$, which is the expected annual number of events of magnitude M occurring at location $\mathbf{x}$. The assignment of $\lambda(M, \mathbf{x})$ may be based partly on geological grounds and partly on seismological data. The simplest assumption is to take $\lambda(M, \mathbf{x})$ to be nonzero only at the epicenters of historical earthquakes of magnitude M. However, the principal weakness of methods such as this, which are based directly on the historical catalog (e.g., Milne and Davenport, 1969; Wiechert and Milne, 1979), is that they fail to take account of the fundamental stochastic variability in the location of an event. Such methods are dynamically inert: earthquakes only occur where they have occurred before, except if allowance is made for the possible occurrence elsewhere of surprise events within large background zones.

Here, an alternative procedure is followed. From a catalog of N historical epicenters $\mathbf{x}_i$, each event being of magnitude M and associated with an effective observation time period $T(\mathbf{x}_i)$, the activity rate for events of magnitude M at a general point $\mathbf{x}$ within the region, $\lambda(M, \mathbf{x})$, can be estimated via a statistical smoothing operation, which recognizes the fundamental probabilistic nature of the discrete sample of historical observations. The smoothing operation involves the introduction of a kernel $K(M, \mathbf{x})$, which is a magnitude-dependent multi-variate probability density function.

The mean activity rate at a point $\mathbf{x}$ is written as a kernel sum over the historical dataset, in which the contribution of each event is inversely weighted by its effective return period:

$$\lambda(M, \mathbf{x}) = \sum_{i=1}^{N} K(M, \mathbf{x} - \mathbf{x}_i)/T(\mathbf{x}_i). \quad (3)$$

Various forms of kernel $K(M, \mathbf{x})$ might be chosen. An exponential form yields a rapid decay. Another example suggested by Vere-Jones (1992) is the following infinite-range power-law decay, dependent on the radial separation distance $r(M)$, a fall-off parameter a and a magnitude-dependent bandwidth parameter $h(M)$:

$$K(M, \mathbf{x}) = [(a - 1)/\pi]\, h(M)^{-2}\, (1 + r^2/h(M)^2)^{-a}. \quad (4)$$

According to Vere-Jones (1992), a typical value of a is between 1.5 and 2, which yields a cubic or quartic fall-off of probability density with epicentral distance. This contrasts with the more rapid Gaussian fall-off sometimes adopted in smoothing schemes. In actual implementation, the parametric form of kernel tends to be less important than the bandwidth. For five values of h varying from 10 to 50 km, Figure 2 shows a cumulative probability plot of the infinite-range kernel against epicentral distance.

An alternative finite-range form for the kernel, based on the fractal dimension of epicenters $D = 2 - a$, is

$$K(M, \mathbf{x}) = [D/2\pi h(M)]\,\{h(M)/r\}^{2-D} \quad \text{for } r \leqq h(M); \qquad (5)$$
$$= 0 \quad \text{for } r > h(M).$$

With D set at 1.5 and for five values of $h(M)$, varying from 20 to 100 km, Figure 3 shows a cumulative probability plot of this kernel against epicentral distance. Both of the above kernel expressions are isotropic, which is appropriate where there is no preferred perturbation direction. In some seismotectonic circumstances, there may be information to allow for explicit orientation dependence.

There are statistical methods for estimating the bandwidth $h(M)$ via a least-squares cross-validation procedure, in which kernel estimates are made with successive individual events removed. This technique was suggested by Hall (1983) and further developed by Stone (1984). However, where earthquake data are relatively sparse, this approach may not be viable. A more practical and more physical alternative is to make use of arguments from the theory of dynamical systems.

For a dissipative system displaying power-law behavior, the probability of an avalanche-type event occurring within a given spatial extent should scale with the event size (Kadanoff, 1990). The kernel bandwidth $h(M)$ should therefore

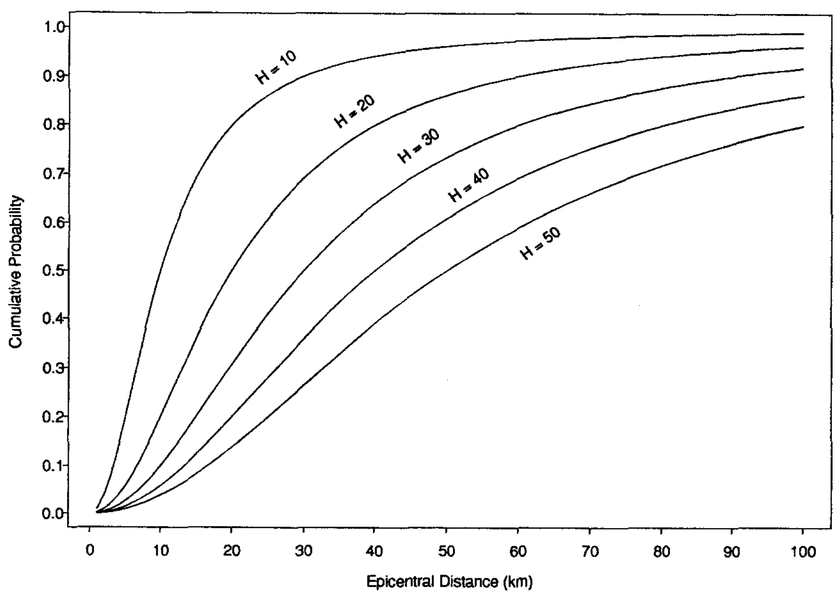

Figure 2. Cumulative probability distribution graph of the infinite range power-law kernel for various bandwidths, ranging from 10 to 50 km.

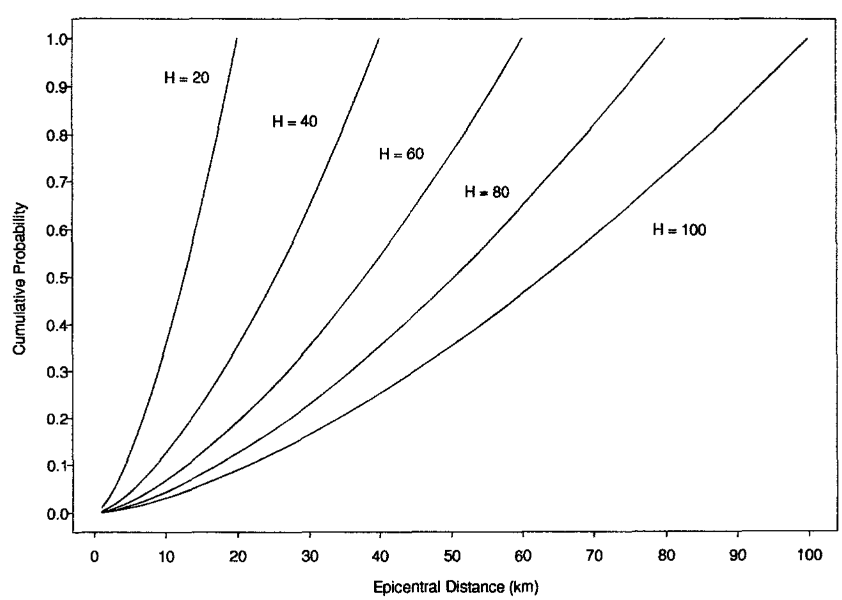

Figure 3. Cumulative probability distribution graph of the finite range kernel, with fractal dimension 1.5, for various bandwidths, ranging from 20 to 100 km.

scale according to earthquake size, which might be taken in the present context to be fault length L. Given the standard form of logarithmic correlation of fault length L with magnitude M, $h(M)$ can be parameterized as $h(M) = H \cdot \exp(kM)$, where H and k are constants that are region specific and should be estimated from regional seismological and geological considerations. The size of k should reflect the size of region over which power-law magnitude-frequency statistics are observed. If seismic activity tends to cluster in magnitude terms, then k may be quite large, independently of the absolute frequency of earthquake occurrence. The large bandwidth in this situation is required by the migration of seismicity within the power-law region.

Although problematic within the context of area zonation, magnitude dependence of the spatial distribution of earthquakes is automatically represented within the kernel estimation methodology, through bandwidth scaling with fault size. For the data shown in Figure 1, the bandwidth would be narrow for events of magnitude 4 to reflect the manifest clustering of epicenters and be broad for events of magnitude 6 to reflect a larger distance between epicenters. Through this kind of magnitude scaling of the bandwidth, it is possible to replicate many spatial patterns of intraplate seismicity, which do not conform to the plate boundary archetype of events of all magnitudes packed into a confined fault zone. In particular, it is possible to simulate some intraplate scenarios where damaging magnitude 5 or 6 events occur unexpectedly in regions of low ambient seismicity, where moderate magnitude events are historically rare.

Once the kernel $K(M, \mathbf{x})$ is parameterized, a set of activity rate grid plots $\lambda(M, \mathbf{x})$ may be constructed covering the spatial region making a nontrivial contribution to the hazard at a designated site of engineering concern. The magnitude range M would be divided into incremental intervals ranging from the threshold of engineering interest to the historical maximum magnitude. For magnitudes exceeding the historical maximum value, activity rate grid plots may be synthesized from neotectonic information, where available, or they may just be assigned, using expert judgment, up to the notional regional maximum magnitude. Because maximum magnitude is a regional rather than local concept, this procedure is an improvement over the practice of assigning maximum magnitude values separately for individual zones of moderate size. To the extent that earthquake epicenters are the basic currency in which seismicity is expressed, neotectonic information could be converted into this form and thus be assimilated into the kernel procedure.

Observational uncertainty in the estimated magnitude of a historically documented or instrumentally recorded event and the intrinsic stochastic fluctuation of magnitude with fault rupture can both be assimilated via the introduction of a probability distribution, which assigns finite chances of the event contributing to a suite of different activity rate grid plots, each corresponding to a different magnitude value. The construction of these magnitude-dependent intensity grid plots collectively obviates the need to introduce an explicit b value to represent magnitude dependence in a subregional scale seismic source model. Thus, any characteristic feature of local seismicity is respected, with the Gutenberg-Richter magnitude-frequency relation being maintained on the critical regional scale. The recurrence rate of large earthquakes might then be estimated from regional magnitude-frequency statistics.

Apart from accounting for the magnitude variability of events in this direct fashion, observational uncertainty in epicenter location can also be explicitly recognized by allowing each epicenter to vary spatially according to its error distribution. This will vary from event to event according to the state of recording technology at the time, whether the epicenter was estimated from macroseismic information, instrumentally estimated from teleseismic data, or determined from a local network. Such refinement of data uncertainty is uncommon within the context of zonal modeling, because issues of completeness and detection thresholds are treated more coarsely on a zonal basis. It is a merit of the kernel density approach that magnitude and epicentral uncertainty form an integral part of the source modeling procedure, since they contribute an important observational element of event smoothing.

Application to Seismic Hazard in Britain

To illustrate the usefulness of the kernel estimation method, consideration is given here to area source modeling in Great Britain, an intraplate environment of modest seismicity, with the advantage of a long earthquake record. Following the major reassessment of British earthquakes in the early 1980s (e.g., Woo and Muir Wood, 1984), various site-specific hazard studies have been carried out during the past decade. These have all incorporated zonal seismic source models with zone geometries defined as polygons, a procedure convenient for computerized data entry, as exemplified by the program EQRISK (McGuire, 1976).

If the rationale for the introduction of seismic zones was to give adequate weight to known correlations between geological structure and seismic activity, Britain would not have provided the original inspiration. Correlations between geological structure and seismicity are rather obscure, as they are in many intraplate regions. With the small spatial scale of the sources of recorded British earthquakes, and the mid to lower crustal depth of some of the most accurately located hypocenters, the causative faults are not readily identifiable from surface mapping, geophysical surveys, or remote sensing investigations.

Even if the causative geological structures are unresolved, in principle they might be manifested indirectly via the alignment of earthquake epicenters. However, such alignments deserve to be treated with statistical caution, because apparent colinearities may arise merely through chance: stochastic geometry theory (Kendall and Kendall, 1980) shows that approximate colinearities may happen surprisingly often accidentally. Given the limitations of available geological constraints, zone boundaries in Britain have been delineated largely through reference to the spatial pattern of seismicity and the practical engineering requirements of robustness and conservatism in seismic source modeling.

Inevitably, the more intractable decisions on zone construction are left to expert judgment; a recourse that may be circumvented to a large extent if the earthquake data are not forced to fit a Euclidean zonation. Much of the difficulty in zonation may be traced to its rigid magnitude invariance: zone geometry is independent of the size of earthquakes. In the entire millenial historical catalog of Britain, there are no documented events of 6 M_S or higher, but there have been a number of lesser events of 5 M_S (see Fig. 4). The number of magnitude 4 M_S events is more substantial; epicenters for an observationally complete set of events since 1800 are mapped in Figure 5. The spatial characteristics of the two datasets are markedly different. As quantified by comparing average nearest-neighbor distances with average event distances, the magnitude 4 events are significantly clustered, whereas the set of magnitude 5 events are more regularly spaced.

This disparity in spatial statistics is difficult to uphold within a zonation procedure, but it can be represented via a kernel bandwidth that scales with magnitude. The usefulness of this facility is illustrated in Figure 6, which shows the epicenter of the 1931 North Sea earthquake, which, at 5.5 M_S, is the largest historical British earthquake. In deciding on the kernel bandwidth, there are seismological arguments to guide expert judgment. A broad bandwidth of several hundred kilometers would be consistent with the high magnitude

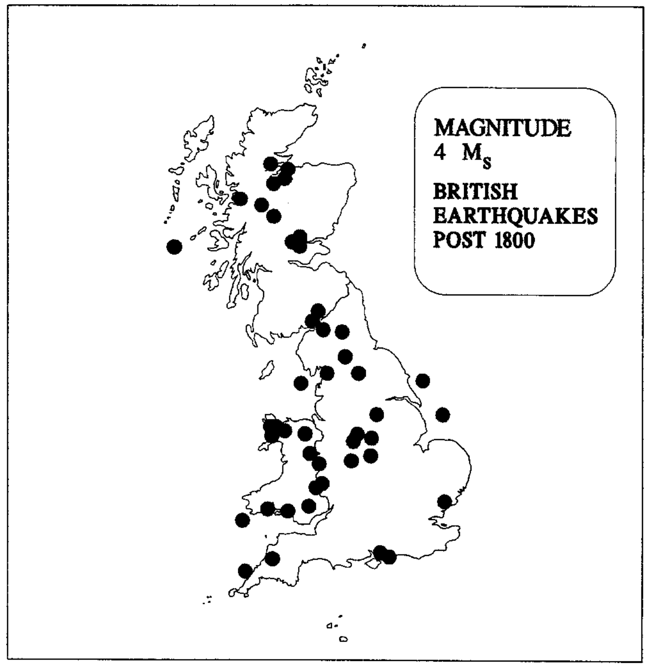

Figure 5. Map of Britain showing the epicenters of all earthquakes of magnitude 4 M_S, since 1800.

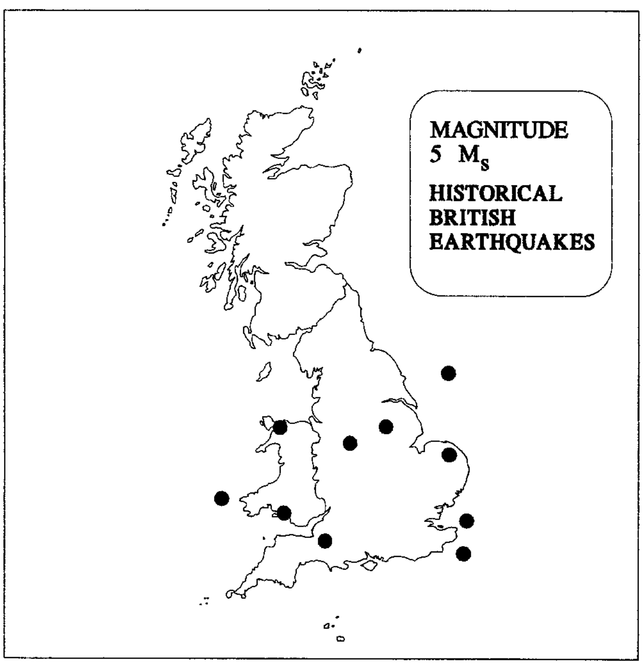

Figure 4. Map of Britain showing the epicenters of all documented historical earthquakes of magnitude 5 M_S.

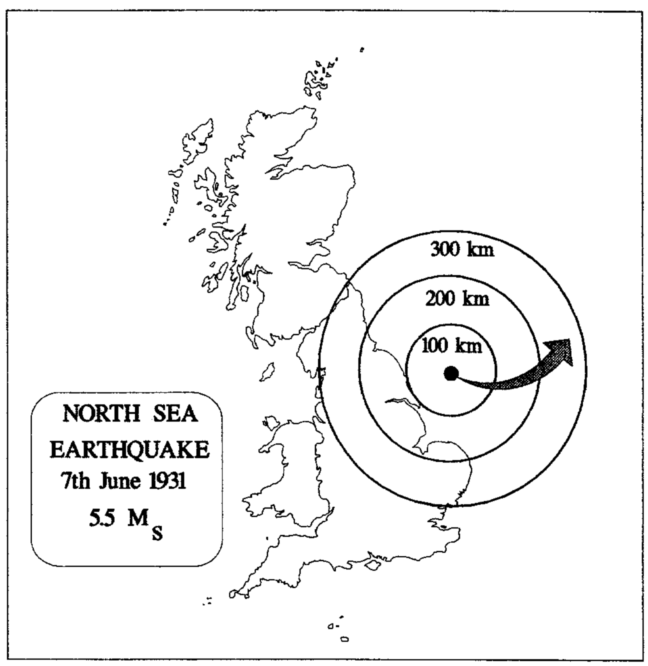

Figure 6. Map of Britain showing concentric bandwidth circles of radii 100, 200, and 300 km, around the epicenter of the 5.5 M_S North Sea earthquake of 7 June 1931, the largest historical British event.

and also with the fact that this event sets a crucial uppermost reference point in the Gutenberg-Richter magnitude-frequency relation for the British Isles as a whole: the magnitude-frequency statistics are incomplete in subregions of the British Isles. Thus, from a dynamical viewpoint, this event is one that should not be confined to a narrow zone but should be free to migrate over a large distance. Within the context of Euclidean zonation, this could only be realized if much of Britain were subsumed within a single homogeneous zone, which would be discordant with clear geographical variations in the distribution of lesser-magnitude events.

Because of the presence of significant magnitude-dependent variations in British seismicity and regional anomalies in b-value estimation, there is no accepted scientifically based zonation of Britain, nor even consensus on the principles for zoning this seismotectonically inhomogeneous territory. With conservatism being paramount, different national zonations have been constructed for different sites. It is against this background of zonation controversy that the concept of kernel estimation is advocated.

Application to Seismic Hazard in Other Regions

The North Sea earthquake cited above is but one example of the potential migration of a major isolated earthquake, epicentered in a region of seismotectonic complexity. The same problem is posed in many countries and most continents. If the singular major event (which may be accompanied by some lesser activity) lies in a region of otherwise modest seismicity, then there may be a tendency for a hazard analyst to fence the event within its own special zone. The implication being that the level of hazard is especially high within the narrow confines of the epicentral region of the major event but comparatively low beyond the zone boundary fence.

Unfortunately, the spatial attributes of seismicity are treated so simplistically within zonation schemes that the task of constructing a zone around the singular event is beset with uncertainty. The neglect of the spatial extent of an event within the point source approximation leads inevitably to the neglect of essential geometrical and dynamical differences in the characteristics of earthquakes of differing magnitudes. Because the dynamical perturbation distance of the epicenter of an earthquake scales with event magnitude, the zone must in fact be adequately spacious in order for the large event not to migrate outside the zone boundary. The b values estimated for small zones are liable to significant temporal fluctuations and may be discordant with regional b values. However, if the zone is of sizeable dimension, then the lesser-magnitude events may be observed to cluster within it, in a manner incompatible with the inherent zonal assumption of uniformity. Within the zonation methodology, magnitude dependence is only recognized as a multiplicative factor on a spatially uniform activity rate; there is no allowance for magnitude dependence of the spatial distribution of events within a zone.

Although primarily intended for application in regions of poor correlation between seismicity and geological structure, the kernel estimation method can be extended to cover those regions of active tectonics, where seismicity is distributed geographically over a connected network of faults, some of which may be mapped or identified by geophysical means, but others may be hidden or otherwise unknown.

A standard simplification is to conceive of the network of active faults as a homogeneous Euclidean zone of activity, in which seismicity is spread uniformly throughout. This is a viable approach numerically, but it fails to do justice to the spatial complexity of fault geometry, which, although immaterial for far-field seismic hazard, may be significant in the near field. The alternative is to make more direct use of seismicity data. Provided the local earthquake catalog is sufficiently extensive, the record of seismicity itself constitutes a realization of the underlying complex fault geometry, which may be generalized stochastically by the kernel smoothing of epicenters. The effect of the smoothing is to allow probabilistically for the dynamical perturbation of events from one fault to another. For fault zones where the Gutenberg-Richter relation is violated at high magnitudes, i.e., where the number of magnitude 7 or 8 earthquakes is anomalously large, scaling breaks down, and criticality may cease to be self-organized (Rundle and Klein, 1993). As a consequence, the kernel bandwidth for such magnitudes will be narrow and anisotropic, reflecting the spatial clustering of these events.

For fault zones where the Gutenberg-Richter relation does not hold because of an excess of large earthquakes, the maximum magnitude can be inferred from the statistics of these events or from the physical dimensions of faulting. But for zones where power-law behavior is observed, care has to be taken to define the extent of the region on which the maximum magnitude is based. This is because the maximum magnitude will tend to scale with the size of the region. Choice of too small a region may exclude a larger extra-regional event that might migrate into it.

For reasons of brevity or incompleteness of the historical record, where it is reckoned that the maximum magnitude earthquake is missing from the regional earthquake catalog, the activity rate density field may be adjusted to accommodate such an extreme occurrence. In a similar way, other events with a return period longer than the historical record could be accommodated, if there were adequate geological evidence upon which to make the required adjustment to the activity rate density field. Through these measures, geological data on recent faulting might be converted to a form in which they could be included within the overall framework of the kernel estimation method.

Comparison of Seismic Area Source Models

The simplest and most direct approach to seismic source modeling is to take the raw historical catalog as a literal guide to the short-term future as well as a record of the historical past. Each epicenter is then taken to be the source of a Poisson process (Lomnitz, 1974). In some highly active regions of the world, where the earthquake catalog spans several seismic cycles and most active seismic sources have already contributed some observable seismicity, this method has found favor as a computationally intensive but methodologically transparent means of hazard computation. Within GSHAP, the Global Seismic Hazard Assessment Program, this is one category of approach that has been adopted (e.g., in parts of Latin America) as an alternative to more formal deductive methods, based on the characterization of faults and area sources.

Where active faults are clearly identified geologically or geophysically as displaying signs of recent tectonic movement, their inclusion as explicit sources within a seismic hazard source model might be justified on the grounds of proximity to the site of engineering concern. The residual seismicity not associated with the modeled active faults has to be assimilated within area zones, which, among other ambiguities, may lack a clear scientific guide as to boundary definition. For single-site hazard studies, such uncertainty can be expressed probabilistically within a logic tree or enveloped by conservatism. For multiple site studies, where the artificial discontinuity of seismological parameters across zone boundaries could lead to anomalous contouring results, a boundary smoothing procedure can be implemented. Whereas within a standard zonal model, each area segment of a zone has the potential for being the epicenter of an earthquake, Bender and Perkins (1982) devised a procedure whereby each epicenter is taken to be normally distributed around the area segment, thus smoothing out activity rates near zone boundaries sufficiently for continuous hazard contour maps to be drawn.

Within the kernel estimation methodology, the absence of zone boundaries circumvents the technical problem of boundary anomalies. It also avoids much of the subjectivity in the representation of seismic area sources, although other subjective judgments are introduced, such as the bandwidth. The kernel methodology is empirically rooted in the regional earthquake catalog and indeed is able to make fuller use of seismological data, since rigid zonal completeness thresholds no longer apply. But, in contrast with methods based simply on the raw catalogue, it is recognized that the actual historical earthquake dataset is only one realization of a nonlinear stochastic process that can readily lead to dynamical perturbations of event epicenters and magnitudes. Accounting for these dynamical perturbations as well as observational errors generates a natural smoothing of the earthquake data. Through the magnitude scaling of kernel bandwidths, allowance is made for the possible migration of epicenters of large historical events toward gaps in seismicity, which may presage future activity. Where broad kernel bandwidths are deemed relevant to a specific region and event magnitude, the migration distances can be large. In this way, kernel estimation provides a mechanism for nontrivial probabilities to be assigned to the occurrence of earthquakes in regions of historically sparse seismicity, hitherto regarded as of extremely low seismic hazard.

Figure 7 provides a graphical comparative summary of the four area source methods described, which are based respectively on the raw catalog, fixed zone boundaries, smoothed zone boundaries, and kernel smoothed epicenters. Ultimately, a seismic hazard analyst's methodology should be data adaptive: the choice of method (or the weight for a logic-tree methodology branch) should be decided on the merits of the regional data and not on circumstance, such as the availability of specialist software or even the hazard analyst's own philosophical inclination (Giardini *et al.,* 1993).

The kernel estimation procedure meets the principle of data adaptivity more than zonal models in that all events are treated on an individual basis, with due recognition for the locational and magnitude uncertainty of each event as well as its effective historical period of observation. Furthermore, in contrast with the rigid formalism of zonation, it is acceptable for spatial patterns to display complex forms of spatial nonuniformity, just as magnitude values may locally clump together in a narrow characteristic range.

Given that the kernel estimation procedure is intrinsically empirical, it will tend to be less applicable to regions of sparse historical and instrumental seismological data, unless the shortcomings of the earthquake catalog can be significantly compensated by supplementary geological information. Shortage of regional data cannot but be a detriment to the quality of seismic source modeling. However, any

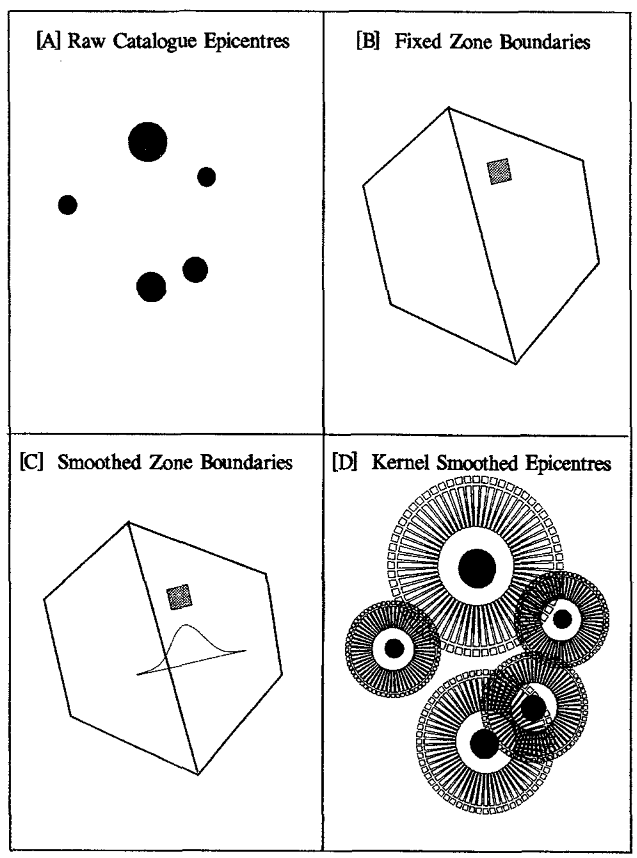

Figure 7. Chart displaying four alternative methods for area source modeling in seismic hazard assessment.

methodological deficiency due to lack of regional data is likely to be shared by zonation schemes, which might purport to represent sparseness of data as indicative of a superficial uniformity of seismicity. In practice, as the illustrative example of Britain shows, this caveat on data sparseness may not be unduly restrictive of the method's applicability.

Conclusions

There are many parts of the world where the association between seismicity and individual geological structures is complex. The rationale may exist for the explicit fault modeling of some individual well-mapped structures, but most of the seismicity in these regions has to be accommodated, for purposes of seismic hazard computation, within area sources. With the many ambiguities in zone definition, the geometrical representation of such area sources as Euclidean zones of uniform seismological character typically lacks support or justification. The kernel estimation method proposed here constructs an area source model by applying a magnitude-dependent probabilistic smoothing procedure to catalog epicenters. Through allowance for observational error as well as the intrinsic nonlinear dynamical perturbation of event locations and magnitudes, the method is capable of representing both the fractal clustering of moderate magnitude epicenters and the haphazard migration of major isolated earthquakes over large continental regions. Moreover, the method is motivated by universal scaling principles of modern statistical physics, which may ultimately prove to be a more fruitful basis for understanding the spatial characteristics of seismicity than traditional doctrines of seismology.

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