

R Project 3:

Multiple Regression Analysis

Data Analytics 2 – ADS523

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Introduction

The object of this project is to perform multiple regression analysis (using R) to find the least-squares regression line that would enable home sale price predictions based on related quantitative real estate (independent) variables such as square footage, lot size, number of bedrooms, year built, etc. The components of the least-squares multiple regression line (partial slopes and the y-intercept) will be interpreted to describe the relationship between several independent variables and a dependent variable (home sale price).

Data Description & Initial Variable Selection

The provided data set contains 100 data points related to real estate in the greater Seattle, WA area. It includes home prices and other variables associated with each home, displayed in Table 1. Inspecting the variables reveals a number that should be removed prior to further analysis. These include nominal

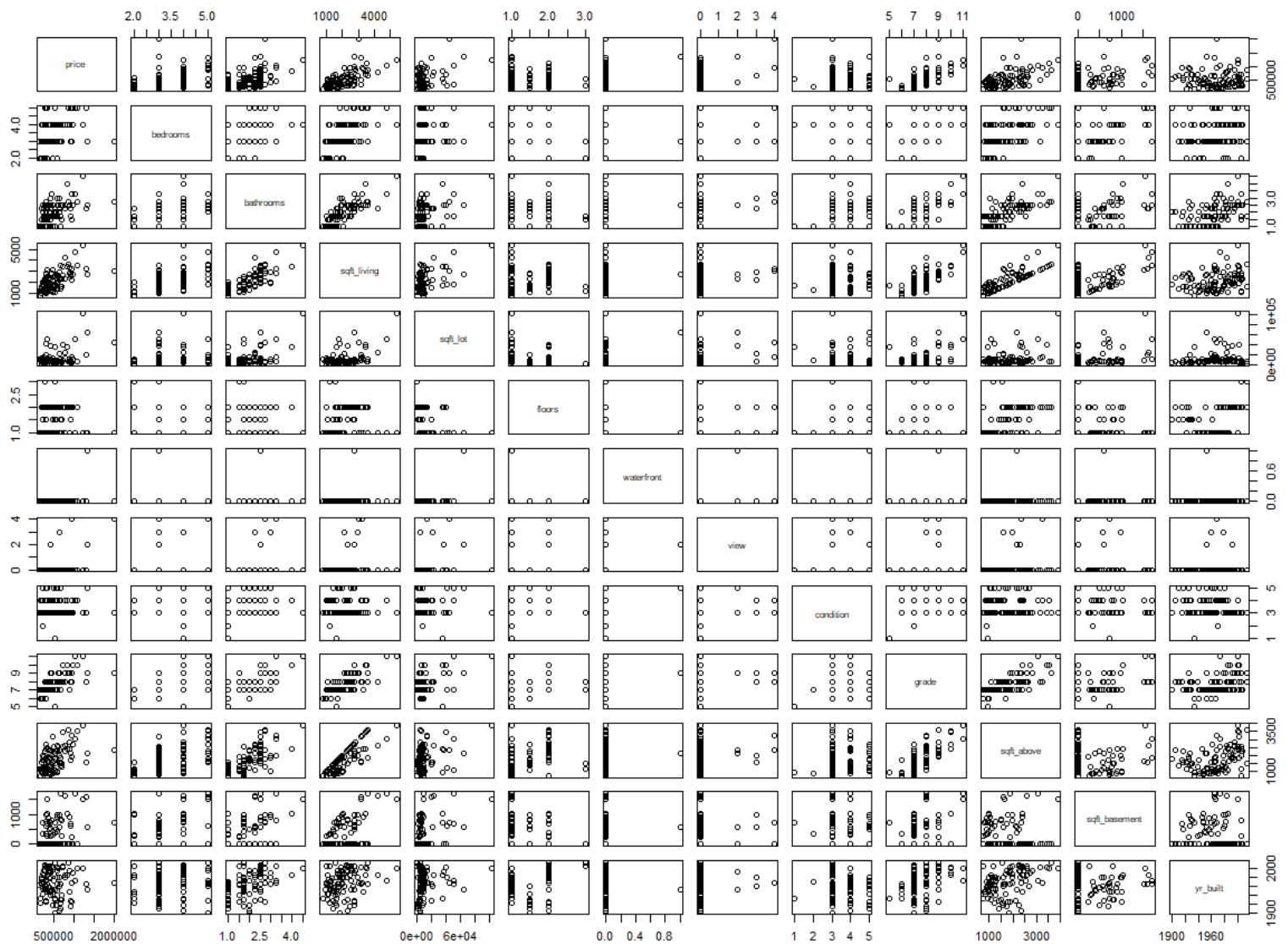
Table 1. Independent variable description and initial variable selection.

Variable	Description	In/Exclude	Reason for Exclusion
id	Transaction ID	Exclude	Nominal data
date	Date of Sale or Listing	Exclude	Only of interest if able to transform to days on market
bedrooms	Bedrooms, Each	Include	
bathrooms	Bathrooms, Each	Include	
sqft_living	Size of House, Square Feet	Include	
sqft_lot	Size of Lot, Square Feet	Include	
floors	Floors, Each	Include	
waterfront	On Waterfront, 0 (No) or 1 (Yes)	Include	
view	View Rating, Weighted Scale of 0, 2, 3, 4	Include	
condition	Condition Rating, Scale of 1-5	Include	
grade	Grade, Scale of 5-11	Include	
sqft_above	Size of House Above Ground, Square Feet	Include	
sqft_basement	Size of Basement, Square Feet	Include	
yr_built	Year Built	Include	
yr_renovated	Year Renovated, 0 (Not Renovated) or Year	Exclude	Improper scale, 0 for Not Renovated would mean renovated in the Year 0
zipcode	Zip Code	Exclude	Nominal data
lat	Latitude, Degrees	Exclude	Unable to transform to meaningful continuous variable
long	Longitude, Degrees	Exclude	Unable to transform to meaningful continuous variable

variables such as the `id` (Transaction ID) and `zipcode`. There are also GPS coordinates, latitude (`lat`) and longitude (`long`) that are continuous variables, but not of any useful scale if they are not being compared to a single point (if distance from city center, for example, was of interest). The same can be said about the `date` variable; the date alone is not useful. Without knowing if it is a sale date or date of listing, nor the current date to compare them, calculating a valuable attribute, like number of days on the market, is not possible. Improper scaling is of concern for the variable `yr_renovated`, associated

with the year in which the home was renovated. In cases where the home was not renovated, the value of zero was assigned, which is interpreted as the year zero; it is assumed the renovation did not occur over two thousand years ago and prior to the home's initial construction. The discrete variables `waterfront`, `view`, `condition`, and `grade` raised concerns as well, but they will be retained for their significance to the multiple regression model, explained below.

Figure 1. Scatterplot matrix after initial variable selection.

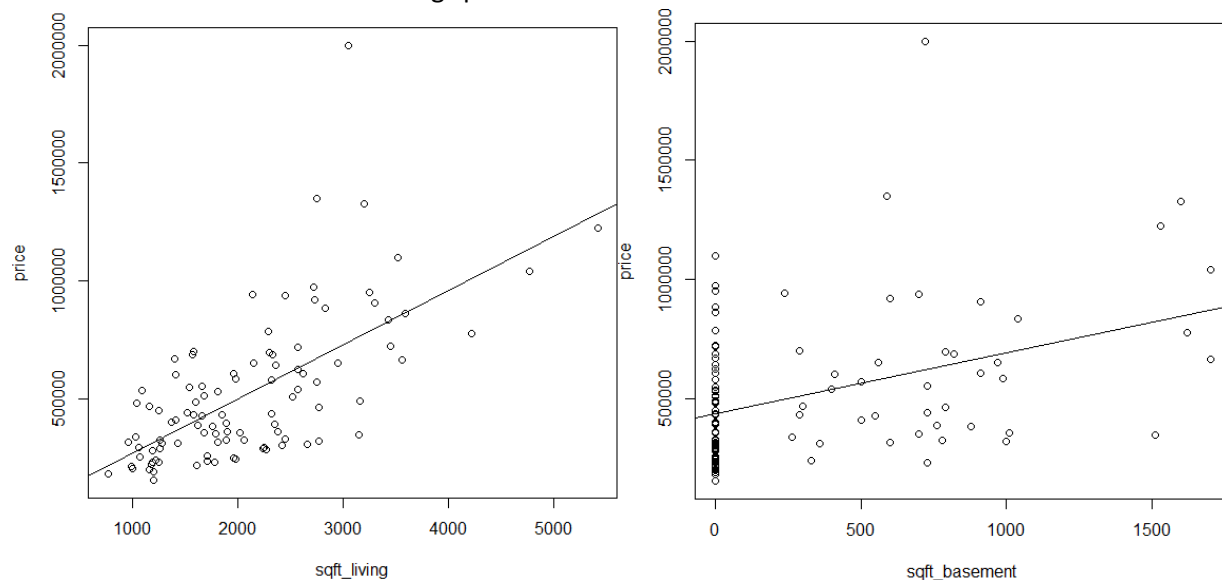


Bivariate Analysis: Linear Correlation and Regression

With initial variable selection complete, scatterplot (Figure 1) and correlation (Table 2) matrices of the remaining variables were constructed to aid in bivariate analysis. The scatter plot matrix plots the dependent variable (`price`) against each independent variable - seen in the first column or first row of Figure 1 - as well as each possible independent variable pair.

A scatterplot with data points clustered together in a linear fashion indicates a linear correlation between the two variables; a plot of more dispersed data points indicates a lack of linear correlation. For example, the scatterplot (also see Figure 2) that compares the dependent (response) variable `price` and the independent (explanatory) variable for living space (`sqft_living`) somewhat resembles a diagonal line, suggesting that the two variables are potentially linearly correlated. Further, the two variables are positively correlated as an increase in the size of the house is associated with an increase in price. Conversely, the plots that compare `price` and the number of floors (also see Figure 2) does not appear to resemble a line. This indicates that the number of floors does not have a linear association with the price of a home.

Figure 2. Scatterplots (with least-squares regression lines) of price associated with the size of living space and basement.



A more precise measure of the strength and direction of the linear relation between two quantitative variables may be achieved by calculating their linear correlation coefficient, r . The formula for a sample's linear correlation coefficient (r) is the sum of the products of the z-scores for the explanatory and response variables (divided by the degrees of freedom (sample size minus one)). As the numerator is a standardized measurement of each variables' relationship to its respective mean, their product is a measurement of the relationship between the two variables once it is normalized by dividing it by the degrees of freedom (which may strengthen or weaken the relationship depending on how large or small, respectively, the sample size is) (Sullivan, page 188).

The linear correlation coefficient (r) is on a scale from -1 to +1, with values at those extremes representing perfect negative and positive linear relationships, respectively. A value of zero signifies that a linear relationship between the two variables does not exist (however correlation may still exist). Values closer to zero ($r -0.3 \leq r \leq +0.3$) represent weak relationships, and those close to the extremes ($r \leq -0.7$ or $r \geq +0.7$) represent strong ones. The sign of the coefficient represents the nature of the relationship. Positive values result when the response variable increases with the explanatory variable, or they both decrease. A negative value results when the response variable decreases with an increase in the explanatory variable, or when the response variable increases with a decrease in the explanatory variable.

Table 2. Correlation matrix after initial variable selection.

	price	bedrooms	bathrooms	sqft_living	sqft_lot	floors	waterfront	view	condition	grade	sqft_above	sqft_basement	yr_built
price	1.000												
bedrooms	0.354	1.000											
bathrooms	0.465	0.486	1.000										
sqft_living	0.651	0.560	0.790	1.000									
sqft_lot	0.452	0.119	0.372	0.498	1.000								
floors	0.072	0.142	0.373	0.224	-0.231	1.000							
waterfront	0.276	-0.051	0.070	0.078	0.364	-0.093	1.000						
view	0.443	0.065	0.258	0.206	0.283	0.014	0.247	1.000					
condition	-0.007	-0.004	0.018	0.011	0.092	-0.114	0.218	0.061	1.000				
grade	0.654	0.362	0.650	0.732	0.438	0.318	0.135	0.235	0.010	1.000			
sqft_above	0.516	0.541	0.729	0.838	0.333	0.511	0.056	0.192	-0.003	0.726	1.000		
sqft_basement	0.396	0.192	0.323	0.540	0.398	-0.377	0.057	0.081	0.025	0.221	-0.008	1.000	
yr_built	-0.034	0.094	0.505	0.282	0.124	0.424	-0.049	0.021	-0.264	0.335	0.417	-0.127	1.000

The line that is “seen” when viewing a scatter plot can be calculated formally as the least-squares regression line, noted in Figure 2. It represents a line that minimizes the sum of the squared errors (or residuals), which is the distance between the observed values of the response variable and those predicted by the least-squares regression line. The equation for this line is given by Formula 1:

$$\hat{y} = b_1x + b_0$$

where $b_1 = r \cdot \frac{s_y}{s_x}$ is the regression, or slope coefficient of the least-squares regression line,

and $b_0 = \bar{y} - b_1\bar{x}$ is the y-intercept of the least-squares regression line.

It is noted in the equations above that \bar{x} is the sample mean and s_x is the sample standard deviation of the explanatory variable x ; \bar{y} is the sample mean and s_y is the sample standard deviation of the response variable y (Sullivan, page 205). The linear correlation coefficient (r) can be thought of as a representation of how closely the data points (of the scatter plot) adhere to the least-squares regression line. More significant r values (closer to -1 or +1) means there is a more significant linear relationship that results from a tighter scatter plot.

The correlation matrix Table 2 supplies the correlation coefficients that correspond to the scatter plot matrix of Figure 1. Applying the concepts related to correlation from above enables confirmation of the previously noted relationships derived from visual inspection of the scatterplot matrix Figure 1. The noted positive linear correlation between the dependent response variable `price` and the independent explanatory variable for living space (`sqft_living`) is confirmed with a relatively strong r value of 0.651. This is reasonable, as when all other variables are held constant, larger houses are typically more expensive than smaller ones.

The r value for `price` and the number of `floors` is weak at 0.072, confirming that the number of floors does not likely have a linear association with the price of a home and is therefore removed from the multiple regression model (noted on Table 4). The variables `condition` and `yr_built` also have r values near zero, but they will be retained for their significance to the multiple regression model, explained below.

The scatterplot (Figure 1) and correlation (Table 2) matrices also compare each possible independent variable pair. When building a multiple regression model, care is taken to avoid independent variables that exhibit multicollinearity. Multicollinearity exists when two independent variables are highly correlated ($r \leq -0.7$ or $r \geq +0.7$), suggesting that they represent the same thing, or perhaps are the result of, or associated with, the same thing.

For example, the linear correlation coefficient (r) that represents the relationship between living space (`sqft_living`) and `bathrooms` is 0.790. This strong positive relationship is reasonable as the number of bathrooms in a house is expected to be greater for a larger house. Similarly, the number of square feet above ground (`sqft_above`) is also highly positively, and reasonably, correlated with living space (`sqft_living`) with an r value of 0.838. Therefore, due to concerns for multicollinearity, the variables `bathrooms` and `sqft_above` will be removed from the multiple regression model (noted on Table 4) as their association with the response variable `price` ($r = 0.465$ and $r = 0.516$, respectively) is weaker than that of `sqft_living` ($r = 0.651$).

Removing independent variables due to multicollinearity is not always practiced. For example, there is a strong relationship ($r = 0.732$) between the independent variables `sqft_living` and `grade` that would suggest multicollinearity and the subsequent removal of `grade` due to its weaker relationship with the response variable `price`. However, the variables are not necessarily collinear as size and rating are not obviously related. Further, `grade` will be retained for its significance to the multiple regression model, explained below.

Multiple Regression Analysis, Part One

As linear regression enables bivariate analysis of the dependent response variable and a single independent explanatory variable, multiple regression expands on that concept to describe the relationship when more than one explanatory variable is present. While linear regression describes a sloped line on a plane, with additional variables, multiple regression describes a plane with multiple partial slopes in a multi-dimensional space. Its equation is like that of Formula 1, but with the additional variables and their partial slope coefficients, given by Formula 2:

$$\hat{y} = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + \cdots + b_kx_k$$

where b_0 is the intercept when all x 's are zero; b_k refers to the sample regression coefficients - or partial slopes - for k independent variables; and x_k refers to the value of the various independent variables. In the home sale price scenario, multiple regression analysis will be applied to predict the dependent response variable `price` (\hat{y}) based on a selection of (k) independent variables and their derived coefficients (b_k).

The process begins by determining whether it is possible that all the independent variables have zero as regression coefficients. In other words, if all the regression coefficients equal zero, then the multiple regression equation is reduced to only the y -intercept (b_0); none of the selected independent variables are associated with the response variable. The null and alternative hypotheses are therefore:

$$\begin{aligned} H_0: \beta_1 = \beta_2 = \beta_3 = \cdots = \beta_k = 0 \\ H_1: \text{Not all } \beta_i \text{'s are 0} \end{aligned}$$

The F -distribution is used to find the P -value and determine the significance of regression coefficients of the selected variables. Technology (R or Excel) was used to perform Analysis of Variance (ANOVA) to calculate the F -test statistic and subsequent P -value, based on a 0.05 level of significance (α), the results are shown in the middle section of Table 3. As the P -value of 6×10^{-18} is very low, the null hypothesis is rejected; at least one of the selected variable's regression coefficient is not equal to zero.

As the null hypothesis is rejected, indicating that at least one of the variable's regression coefficient is not equal to zero, the next step is calculating (also with technology, R or Excel) the regression coefficient of each independent variable. A test statistic (based on the Student's t -distribution) and subsequent P -value are also calculated for each variable to determine which coefficients differ significantly from zero (P -value $< \alpha$). The variables that have regression coefficients that are not significant (differ from zero) are usually excluded from the analysis as they signify a lack of association with the response variable.

The bottom section of Table 3 displays the calculated regression coefficient and subsequent P -value of each independent variable in the home sale price scenario. The variables whose calculated regression coefficient are determined as insignificant ($\alpha = 0.05$) in terms of their P -value include: bedrooms (0.5458), bathrooms (0.5784), floors (0.7014), sqft_lot (0.7748), sqft_above (error) and sqft_basement (error). Therefore, these variables will also be excluded from the multiple regression model, noted on Table 4.

It was noted in the linear regression section above that certain variables that likely should be removed from the multiple regression model for various reasons, but were exempted due to their significance (P -value $< \alpha$ of 0.05) to the initial model. These include the discrete variables `waterfront`, `view`,

Table 3. Multiple regression analysis after initial variable selection.

Regression Statistics								
Multiple R	0.824							
R Square	0.679							
Adjusted R Square	0.6275							
Standard Error	182997							
Observations	100							
ANOVA								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Signifi- cance F</i>			
Regression	12	6E+12	5E+11	16.919	6E-18			
Residual	88	3E+12	3E+10					
Total	100	9E+12						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	5,253,699.004	2E+06	2.9928	0.0036	2E+06	9E+06	2E+06	9E+06
grade	122,991.845	28271	4.3505	4E-05	66809	179175	66809	179175
view	105,439.198	27056	3.897	0.0002	51670	159208	51670	159208
yr_built	-2,947.521	887.98	-3.319	0.0013	-4712	-1183	-4712	-1183
waterfront	433,964.582	206823	2.0982	0.0388	22947	844982	22947	844982
sqft_living	111.974	54.234	2.0646	0.0419	4.1948	219.75	4.1948	219.75
condition	-55,836.396	27526	-2.029	0.0455	-1E+05	-1134	-1E+05	-1134
bedrooms	18,419.305	30377	0.6064	0.5458	-41949	78788	-41949	78788
bathrooms	-28,402.248	50921	-0.558	0.5784	-1E+05	72793	-1E+05	72793
floors	20,995.862	54574	0.3847	0.7014	-87458	129450	-87458	129450
sqft_lot	0.515	1.7937	0.2869	0.7748	-3.05	4.0792	-3.05	4.0792
sqft_above	0.000	0	65535	#NUM!	0	0	0	0
sqft_basement	56.553	63.201	0.8948	#NUM!	-69.04	182.15	-69.04	182.15

condition, and grade which have the potential that their scales are arbitrary and not proportional or linear. And the variables condition and yr_built have a weak relationship with the response variable price. Also, the variable grade had the potential for multicollinearity with another independent explanatory variable sqft_living, yet both variables were retained. In each case, their *P*-values, see Table 3 indicated that their inclusion was significant to the model.

In addition to these variables, sqft_living is considered significant with a *P*-value of 0.0419 less than the 0.05 level of significance (α).

Table 4. Further variable selection from linearity, multicollinearity, and significance checks.

Variable	Description	In/Exclude	Reason for Exclusion
bedrooms	Bedrooms, Each	Exclude	p-Value > α
bathrooms	Bathrooms, Each	Exclude	Multicollinearity with sqft_living ($r = 0.790$), p-Value > α
sqft_living	Size of House, Square Feet	Include	
sqft_lot	Size of Lot, Square Feet	Exclude	p-Value > α
floors	Floors, Each	Exclude	No correlation with price ($r = 0.072$), p-Value > α
waterfront	On Waterfront, 0 (No) or 1 (Yes)	Include	
view	View Rating, Weighted Scale of 0, 2, 3, 4	Include	
condition	Condition Rating, Scale of 1-5	Include	No correlation with price ($r = -0.007$), p-Value < α
grade	Grade (?), Scale of 5-11	Include	
sqft_above	Size of House Above Ground, Square Feet	Exclude	Multicollinearity with sqft_living ($r = 0.838$), error calculating p-Value
sqft_basement	Size of Basement, Square Feet	Exclude	error calculating p-Value
yr_built	Year Built	Include	No correlation with price ($r = -0.034$), p-Value < α

Multiple Regression Analysis, Part Two

As the initial multiple regression model included insignificant ($P\text{-value} > \alpha$ of 0.05) variable coefficients, they are removed, and multiple regression analysis is run again on the remaining significant ($P\text{-value} < \alpha$ of 0.05) variables. The results are displayed in Table 5.

The process repeats by first determining whether it is possible that all the independent variables have regression coefficients of zero. The null and alternative hypotheses are therefore:

$$H_0: b_1 = b_2 = b_3 = \dots = b_k = 0$$

H_1 : Not all b_i 's are 0

The F -test statistic is calculated, as is the subsequently low (based on a $\alpha = 0.05$ level of significance) P -value of 1×10^{-20} . The null hypothesis is once again rejected, meaning that at least one of the selected variable's regression coefficient is not equal to zero.

The regression coefficient of each independent variable is then calculated. A test statistic (based on the Student's t -distribution) and subsequent P -value are also calculated for each variable to determine which coefficients differ significantly from zero ($P\text{-value} < \alpha$ of 0.05 level of significance). The results are displayed in the bottom section of Table 5. It is noted that every variable's regression coefficient is deemed significant with P -values less than the 0.05 level of significance. Therefore, the multiple regression model is created by substituting the y -intercept, variables, and their coefficients into Formula 2:

$$\text{price} = 6,121,222.19 + 136.12 \times \text{sqft_living} + 445,690.40 \cdot \text{waterfront} + 102,040.90 \cdot \text{view} - 60,784.97 \cdot \text{condition} + 116,865.80 \cdot \text{grade} - 3,350.53 \cdot \text{yr_built}$$

This model can be interpreted in terms of the y -intercept and the effect of each independent variable on the dependent response variable `price`. A y -intercept of \$6,121,222.19 indicates that when all independent variables are equal to zero ($x = 0$), the value of the home is predicted to be \$6,121,222.19. As these values are outside of the scope of the data, it is unsupported by the model, and not practical as a home with zero square feet of living space is not a home.

Each independent variable's coefficient indicates the change in `price` that results from an incremental change (increase or decrease) of that variable, provided that all other variables are held constant in each case. For example, if the living space (`sqft_living`) increases by one square foot, the prediction of the price of the home increases by \$136.12. Property categorized as being `waterfront` (1 = yes, 0 = no) will gain \$445,690.40 in value, and those that are, will not change in value. The coefficients for these variables are positive terms, indicating that `price` will increase as they do. The same is the case for `view` and `grade` which increase the `price` of the home by \$102,040.90 and \$116,865.80 with each increase of those variables, respectively. These are reasonable as houses are more valuable when they are larger, are on water, have better views, and/or are graded/rated higher.

Regression coefficients with negative signs indicate a negative linear relationship with `price`, meaning that, provided that all other variables are held constant, as the independent variable increases, the `price` of the home decreases. For example, the negative regression coefficient for the variable `condition` indicates a price decrease of -\$60,784.97 for each increase on the variables discrete scale

of 1 through 5. This likely means that the scale is inverted where a 1 indicates the best condition and a 5 indicates the worst; the interpretation of the coefficient is reasonable with this evaluation.

However, the variable `yr_built` has a negative regression coefficient of -3,350.53 that appears counterintuitive. The variable is reported in the actual year (e.g., 1950) the home was built and not the age of the house, meaning the model calculates that there is a decrease of -\$3,350.53 for every year *newer* the house is. In other words, more value is placed on older houses, and less value is placed on newer ones.

Table 5. Multiple regression analysis after initial variable selection.

<i>Regression Statistics</i>								
Multiple R	0.8213237							
R Square	0.67457262							
Adjusted R Square	0.653577305							
Standard Error	179222.6432							
Observations	100							
ANOVA								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	6	6E+12	1E+12	32.12967397	1E-20			
Residual	93	3E+12	3E+10					
Total	99	9E+12						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	6,121,222.187	1E+06	4.4605	2.28655E-05	3E+06	9E+06	3E+06	9E+06
sqft_living	136.117	30.839	4.4138	2.73423E-05	74.877	197.36	74.877	197.36
waterfront	445,690.403	191064	2.3327	0.021822239	66274	825106	66274	825106
view	102,040.904	25635	3.9805	0.000136239	51135	152947	51135	152947
condition	-60,784.970	26333	-2.308	0.02319588	-1E+05	-8494	-1E+05	-8494
grade	116,865.797	26235	4.4546	2.33825E-05	64769	168963	64769	168963
yr_built	-3,350.533	702.78	-4.768	6.87397E-06	-4746	-1955	-4746	-1955

Conclusions

Upon review of the final multiple regression model, it is noted that some components are dubious:

- The negative regression coefficient for `yr_built` is counterintuitive.
- The `waterfront` variable is zero for all data points except one; its inclusion is questionable.
- Explanations for the variables `grade` and `condition` are not provided, and though their r value is close to zero, multicollinearity is suspected as it is reasonable to assume they both describe the same concept: the state of the home.

Potential improper initial variable evaluation may be responsible for the model's mediocre Adjusted R^2 value of 0.6536. The Adjusted R^2 value is a coefficient of determination that measures the percentage of total variation in the response variable that is explained by the least-squares regression lines of the multiple regression model (Sullivan, page 717) and adjusted based on the number of degrees of freedom. Adjusted R^2 is a good measure of the model's predictive accuracy.

A model that is 65.36% accurate in predicting the price of a home is more reasonable than not but should be considered mediocre at best. If transformation of the existing variables - or additional ones - were available, it is possible that a more accurate model could be derived. The geography of the homes suggests that they are suburban, so the variables of interest would be tied to what is of interest to that demographic and housing market. These could include distance to the city (Seattle) center as an evaluation of commuting distance, rating of school systems, and rating of public safety.

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References

Sullivan III, Michael. Statistics: Informed Decisions Using Data; 6th Edition, ISBN-13: 9780135780121.