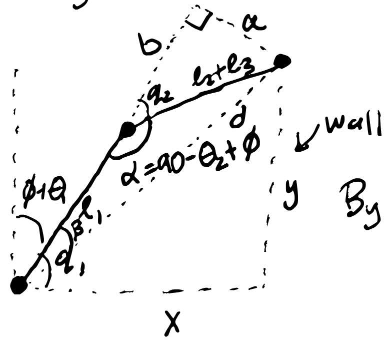
Placing the arm at an vertical point on the wall



WANT: θ_1, θ_2

KNOW: $l_1, l_2, l_3, \phi, \underbrace{X, y}_{\text{target}}$ ← offset from arm

By Pythagorean, $d^2 = X^2 + y^2$ and $(l_2+l_3)^2 = a^2 + b^2$

By Law of Cosines: $d^2 = l_1^2 + (l_2+l_3)^2 - 2l_1(l_2+l_3)\cos(\alpha)$

Thus, $\cos(\alpha) = \dfrac{l_1^2 + (l_2+l_3)^2 - d^2}{2l_1(l_2+l_3)}$, $q_2 = \pi - \alpha$, and $\frac{\pi}{2} - \theta_2 + \phi = \pi - q_2$

$\theta_2 = q_2 + \phi - \frac{\pi}{2}$. Thus, $\cos(q_2) = \cos(\pi - \alpha) = \cos(\pi)\cos(\alpha) + \sin(\pi)\sin(\alpha)$
$= -\cos(\alpha)$

We then have $\cos(q_2) = -\dfrac{l_1^2 + (l_2+l_3)^2 - d^2}{2l_1(l_2+l_3)}$. Therefore,

$$q_2 = \begin{cases} \pi - \cos^{-1}\left(\dfrac{l_1^2 + (l_2+l_3)^2 - X^2 - y^2}{2l_1(l_2+l_3)}\right) \\ \cos^{-1}\left(-\dfrac{l_1^2 + (l_2+l_3)^2 - X^2 - y^2}{2l_1(l_2+l_3)}\right) \end{cases}$$

which means

$$\theta_2 = \begin{cases} \dfrac{\pi}{2} - \cos^{-1}\left(\dfrac{l_1^2 + (l_2^2 + l_3)^2 - X^2 - y^2}{2l_1(l_2+l_3)}\right) + \phi \\ \cos^{-1}\left(-\dfrac{l_1^2 + (l_2+l_3)^2 - X^2 - y^2}{2l_1(l_2+l_3)}\right) - \dfrac{\pi}{2} + \phi \end{cases}$$

Depending on q_2 choice

To solve for θ_1, we first find q_1. We know that:

$\tan(q_1 - \beta) = \dfrac{y}{X}$ and $\tan(\beta) = \dfrac{a}{b + l_1} = \dfrac{(l_2+l_3)\sin(q_2)}{l_1 + (l_2+l_3)\cos(q_2)}$.

Thus, $q_1 = \tan^{-1}\left(\dfrac{y}{X}\right) + \beta = \tan^{-1}\left(\dfrac{y}{X}\right) + \tan^{-1}\left(\dfrac{(l_2+l_3)\sin(q_2)}{l_1 + (l_2+l_3)\cos(q_2)}\right)$

Then, $\phi + \theta_1 = \frac{\pi}{2} - q_1$, so $\theta_1 = \frac{\pi}{2} - \phi - q_1$.

For Drawing Vertically, we have:

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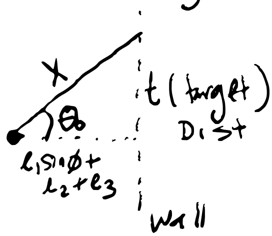
$$X = \text{Distance from Robot to Board} = \overbrace{l_1 \sin\phi}^{\text{"box"}} + l_2 + l_3$$

$$y = (\text{Height of arm at } (0,0)) - (\text{target distance in meters})$$

from datasheet

For Drawing Horizontally, we have a top-down view of:



$$\theta_0 = \tan^{-1}\left(\frac{t}{l_1 \sin\phi + l_2 + l_3}\right) \text{ and}$$

$$X = \sqrt{t^2 + (l_1 \sin\phi + l_2 + l_3)^2}$$

For "vertical" movement (to ensure we stay on wall), set

$$X = \sqrt{t^2 + (l_1 \sin\phi + l_2 + l_3)^2} \text{ and}$$

$$y = \text{Height of arm at } (0,0)$$

technically, this can be any height, but keep it constant from starting point

We can use the equations on last slide to move to any point in the (X, y) plane (dist from wall, vertical dist). To move arm off the wall, set $X < l_1 \sin\phi + l_2 + l_3$.