Case Study # 3 – Structural Analysis: Perforated Plate in Tension

Background:

The quantification of stress distribution in a perforated plate in uniaxial tension and its departure from the stress that would exist in the same, but intact, plate is a classical elasticity problem. An analytical solution exists for the case of a thin plate of infinite extent with a hole of radius R subject to uniaxial tension σ [1]:

$$\sigma_{xx} = \sigma \left[1 - \frac{R^2}{r^2} \left(\frac{3}{2} \cos 2\theta + \cos 4\theta \right) + \frac{3}{2} \frac{R^4}{r^4} \cos 4\theta \right] \tag{1}$$

$$\sigma_{yy} = \sigma \left[-\frac{R^2}{r^2} \left(\frac{1}{2} \cos 2\theta - \cos 4\theta \right) - \frac{3}{2} \frac{R^4}{r^4} \cos 4\theta \right] \tag{2}$$

$$\sigma_{xy} = \sigma \left[-\frac{R^2}{r^2} \left(\frac{1}{2} \sin 2\theta + \sin 4\theta \right) + \frac{3}{2} \frac{R^4}{r^4} \sin 4\theta \right] \tag{3}$$

where the cartesian components, $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$, of the stress tensor are described using polar coordinates (r, θ). This case study requires that you perform numerical simulations of this problem for a square $(L =)$ 4 m steel plate with a central $(R =)$ 0.5 m hole with $\sigma = 10$ kPa using OpenFOAM and that you compare your predictions to the analytical solution presented above. This problem is identical to that that described in OpenFOAM's "stress analysis of a plate with a hole" tutorial.

Tasks:

- Use (OpenFOAM) to solve the problem described above.
- Quantitatively compare your results to the analytical solution (Eq. 1).
- Investigate the effect of plate length (keeping the width unchanged) on the numerical solution and its departure from the analytical solution (Eq. 1).

Report:

Prepare a report (2-4 pages max in ASME's two-colum article format, templates are available on-line) describing your work and including:

1. Short description of the problem
2. Numerical Solution Approach
3. Results and Discussion
   - Mesh sensitivity study
   - Selected key results for the base case
   - Effect of the plate length study
   - Summary of the difficulties that you have encountered in running the various cases and how you have addressed them.
4. Conclusion.

The Report (single file, PDF format only) is due on **November 16, 2014 at 6pm** and must be submitted electronically using the class SmartSite.

References

[1] S. P. Timoshenko and J. N. Goodier. *Theory of Elasticity*. McGraw-Hill, New York, N. Y., 3rd edition, 1970.